Quantum Electronics
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Module No. # 04
Third Order Effects
Lecture No. # 22
Third Order Non - Linear Effects (Contd.)

Ok so we continue with our discussions on third order effects.

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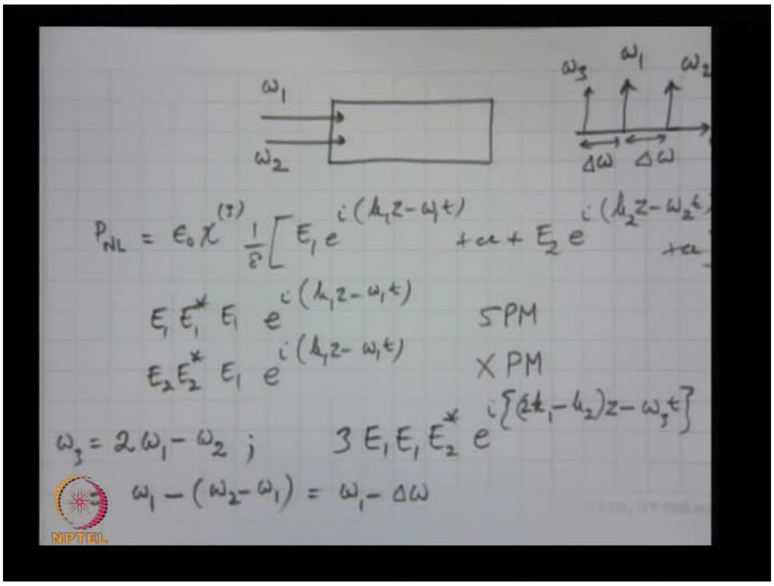

So, towards the end of last class, I said that let us look at a situation, where I have two waves coming in to the medium at frequencies omega 1 and omega 2. So, the nonlinear polarization total nonlinear polarization is epsilon zero chi 3, 1 by 8 of E 1 E to the power i, K 1 Z minus omega 1 T plus complex conjugate plus E 2 E to the power i, K 2 Z minus omega 2 T plus complex conjugate whole cube. So, the two incident waves will now generate a nonlinear polarization, which will which is given by this equation. Now, the nonlinear polarization we will calculate the nonlinear polarization and any new frequency that appear in the nonlinear polarization will try to generate those new frequencies and so when I calculate finally the nonlinear polarization, I need to include the electric fields of those waves also. For example; in second harmonic generation, I initially took omega calculated the nonlinear polarization and I found that there is nonlinear polarization at 2 omega and I also found out under what condition the efficient efficiency for generation of 2 omega will be high. So, if I am close to that condition the total electric field will now consist of omega and 2 omega.

So, similarly here I start with an electric field at input at omega M 1 and omega 2. This will lead to certain nonlinear polarization terms. If those nonlinear polarization terms generate new waves at new frequencies and if that efficient if that efficiency is significant then to calculate the nonlinear polarization, I must include those electric fields also finally, so this is not complete in that sense exactly like I did for the second order nonlinearity. So, when you expand this term as I told last yesterday, there is a cubic term, so you will have terms like mode E 1 square in to E 1, which is the same frequency as omega 1 and that leads to self-phase modulation of omega 1 wave. There will be terms like mode E 2 square E 3 this in to this in to this it is a cubic term, so it is a product of three terms, so mode E 2 square E 2 will give me self-phase modulation at omega 2. I will also have E 2 in to E 2 star in to E 1, that will also be at omega 1 and that leads to cross phase modulation. So you will have for example; you have E 1 E 1 star E 1 and this will have a space dependence of the form K 1 Z minus omega 1 T, this is self-phase modulation then you will have E 2 E 2 star E 1 and that will also have K 1 Z minus omega 1 T. This is cross-phase modulation. E 2 E 2 star has no time and space dependence even E 1 star has no time and space dependence, so the only space and time dependence comes from the E 1 term. So, the propagation of omega 1 wave not only is determined by the electric field at omega 1, because of self-phase modulation, it also depends on the electric field at omega 2, because of cross-phase modulation. So, the velocity of propagation of the wave at omega 1 can be controlled by another wave at a different frequency.

So, I could have for example; a very low energy wave or low intensity wave at omega 1 and so a self-phase modulation is very weak. I could have a very strong wave at omega 2 frequency. So, although this term will be negligible, because E 2 is large there will be a significant contribution to the propagation of E 1 wave, because of E 2 E 2 star. So, the velocity of the omega 1 wave can be changed by the intensity at the omega 2 frequency. So, I can use this process for example; to change the phase of the omega 1 wave by switching on and off another wave and another frequency. So, if I can change the phase of a wave I can use that change of phase to to go from constructive to destructive interference when I use it as a part of an interferometer. So, this process of cross-phase modulation can be used in applications like all optical switching, so I can switch the omega 1 wave from constructive to destructive interference by changing the intensity of light at omega 2 frequency. So, I can build an interferometer say for example; a Mach–Zehnder interferometer and I can ensure that the output goes from one to the other depending on the intensity of the omega 2 wave. This is a very interesting application in all optical switching, but I am interested in another term here, which is in a different frequency.

For example; if I calculate if I calculate so, let me look at this frequencies, let me assume omega 1 is here and omega 2 is some frequency here. Now, let me look at a term, can you tell me the nonlinear polarization at a frequency, which I call omega 3 is equal to 2 omega 1 minus omega 2 omega 3 is equal to 2 omega 1 minus omega 2. So, what will be the nonlinear polarization term? So, 2 omega 1 minus omega 2, which it must have E 1, E 1, E 2 star and it have exponential i because of E 1, I will have a K 1, because of E 1 another K 1, so 2 k 1, E 2 star will give me minus K 2 z and what will be the frequency this will be omega 3, because 2 omega 1 minus omega 2, I have defined as omega 3. Actually if you expand this A plus B plus C plus D whole cube you will get a lot of terms and I am just picking up some of this term. So, this term is of course there will be another corresponding complex conjugate and what will be the factor multiplying A square B, A B square, A square C all this will have a factor of 3, when you have product of three different terms, then you will have a factor of 6, A B C will have a factor of 6, B C D will have a factor 6, A square B or A square C or C square D all of them will have a factor of 3 ok. So, this is the term, which I will get of course with epsilon zero chi 3 by 8, but what is interesting here is what is the frequency now. You see this is actually omega 1 minus omega 2 minus omega 1. So, if I call this delta omega this is omega 1 minus delta omega. I am assuming here omega is omega 2 is bigger than omega 1, so this is omega axis. So, this omega 3 will be here equally spaced on the other side.

So, when I launch a wave at omega 1 and a wave at omega 2, there is nonlinear polarization in the medium at the frequency 2 omega 1 minus omega 2 and this frequency is exactly equally spaced on the other side of omega 1 and omega 2. If omega 2 was here omega 3 will be here, if omega 2 is here omega 3 is here ok. So, it is just a mirror mirror image omega 3 is a mirror image of omega 2 around omega 1. Now, suppose I neglect self-phase modulation and cross-phase modulation then you can see that means let me assume that E 1 is almost a constant that is a pump here. This the propagation constant of this wave now, so what is the phase matching condition I will have, this is trying to generate an omega 3 wave, the velocity of this wave this polarization wave must be equal to the velocity of the wave at omega 3 frequency. So, if I neglect please remember normally it may happen that as you can see here the omega 3 wave the velocity of omega 3 wave will depend on intensity at omega 1, because of cross-phase modulation note down. Here, I showed you that the velocity of omega 1 depends on E 2 square mode E 2 square, so the velocity of omega 1 depends on the intensity at omega 2. Similarly, the velocity of omega 3 will also depend on the intensity at omega 1 and intensity at omega 2 by cross-phase modulation and the velocity of omega 1 itself depends on its intensity, because of self-phase modulation. In principle, I need to worry about all this to find out under what condition I will have what is called as phase matching; that means what will be the condition under which I will have significant generation, but let me assume that the intensities of these lights are very low. So, this typically happens in a fiber optic communication system, where I have a many number of channels going through. Now, these frequencies are defined by the international telecommunication union ITU grid, it is called ITU grid. These frequencies are well defined frequencies spaced by 200 gigahertz or 100 gigahertz or something and so these are well defined frequencies. So, what it implies is in an optical fiber I can use these various frequencies for communication at different channels, these are different channels like you have in radio transmission you have different channels del E A del E B del E C etcetera etcetera, so these are different frequencies. So, here also in fiber optic communication I can use omega 1 for 1 channel, I can use omega 2 another channel omega 3 another channel, so at the receiver if I pick up omega 1 in my receiver I will hear the signal at omega 1, my receiver can be tuned to receive omega 3 frequency, so I will receive omega 3 frequency.

Now, what it this implies is, if I am using these different channels the presence of light at omega 1 and omega 2 can create problems at omega 3 frequency, because the propagation of omega 3 now is altered, because of E 1and E 2, that means the presence of omega 1 and omega 2 frequencies in the communication link can alter the propagation of omega 3 through this effect and this is called four way mixing.

Mohit: Sir you said that the velocity depends on the intensity also.

Because self-phase modulation, what is self-phase modulation? The refractive index changes, which means the velocity changes.

So, when suppose these were of high powers all of them, the velocity at omega 3 the velocity as seen by the omega 3 wave depends on the intensity at omega 3, because of its self-phase modulation, it depends on the intensity at omega 1, because of cross-phase modulation, it depends on the velocity at omega 2, because of cross phase modulation. Now, the velocity of omega 1 itself depends on the intensity at omega 1, because of self-phase modulation. So, when I look at this process completely, if I assume that these are not low power low power signals then I need to worry about all the self-phase modulation terms all the cross-phase modulation terms, in calculating this phase matching condition. So, I am not going to discuss that, but I am going to assume that these signals are low, which means I can forget about self-phase and cross-phase modulation terms. We are not very significantly high. So, what will be the phase matching condition, if I if I neglect XPM and SPM, the phase matching condition implies that velocity of this wave must be equal to the velocity of the wave at omega 3 frequency.

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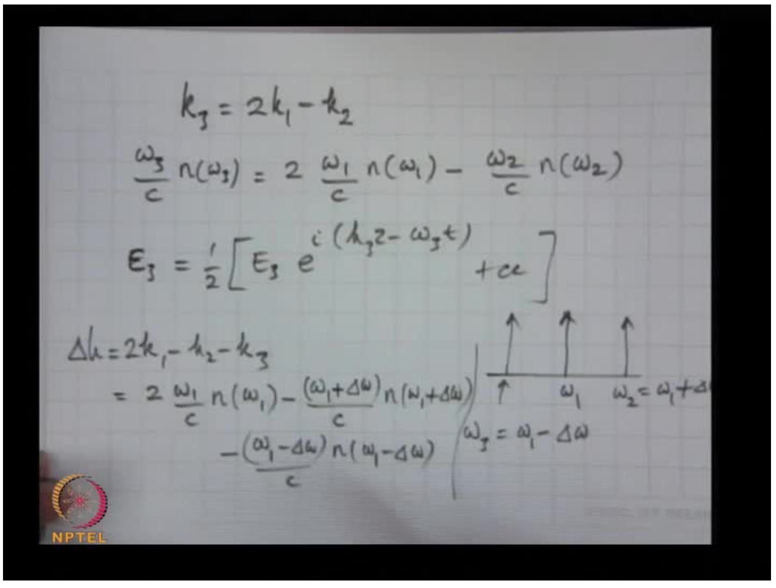

So, the omega 3 frequency will have a propagation constant K 3, so the phase matching condition will be K 3 is equal to 2 K 1 minus K 2. This is neglecting XPM and SPM, (Refer Slide Time: 00:36) because I am neglecting any contribution to the Z dependence from E 1and E 2 star. Please note that, E 1, I have written as exponential i omega 1 T, i K K 1 z minus omega 1 T, but actually K 1 will itself get gets modified, because of self-phase modulation, K 1 will get modified because of cross-phase modulation. So, all those terms I need to worry about in a complete analysis, but here for low signal values, which is what we will discuss here the phase matching condition implies that K 3 must be equal to T K 1 minus K 2. That means it implies omega 3 by C refractive index at omega 3 must be equal to 2 times omega 1 by C refractive index at omega 1 minus omega 2 by C refractive index at omega 2. Propagation constant is the frequency omega by C into refractive index at that frequency. Yes.

Sir omega 3 is generated by the nonlinear polarization, so this K 3 that you have written in this measure what does it stands for, I mean (( )).

So, no the electric field at the omega 3 frequency will be written as half E 3 exponential i K 3 Z minus omega 3 let me call it by some other symbol here.

This is the velocity the velocity of the wave at omega 3 frequency is omega 3 by K 3. If I neglect If I neglect the effect of the presence of omega 1 and omega 2 and omega 3, because of self-phase and cross-phase modulation, because otherwise please note, as I showed you in self-phase modulation or cross-phase modulation term E 3 does not propagate like this. It also has a phase change, because of the presence of the other signals, which I am neglecting here. (Refer Slide Time: 00:36) The nonlinear polarization has this term; the electric field has this term, so omega 3 by K 3 must be equal to omega 3 by 2 K 1 minus K 2, which is what is this condition.

This is very similarly, to K P is equal to K S plus K i, which I got in the 3 wave mixing term. The idler the idler propagation constant must be equal to omega K P minus K S, which is what I got in the 3 wave mixing, in the 4 wave mixing at this example it tells me K 3 must be equal to 2 K 1 minus K 2. So, if I assume for example; so let me let me do a simplified analysis let me assume this is omega 1 this is omega 2, which is equal to omega 1 plus some delta omega, so omega 3 is equal to omega 1 minus delta omega. So, this condition so let me called it delta K, delta K is equal to 2 K 1 minus K 2 minus K 3 ok. So, this is 2 omega 1 by C N at omega 1 minus omega 1 plus delta omega by C N at omega 1 plus delta omega minus omega 1 minus delta omega by C N at omega 1 minus delta omega.

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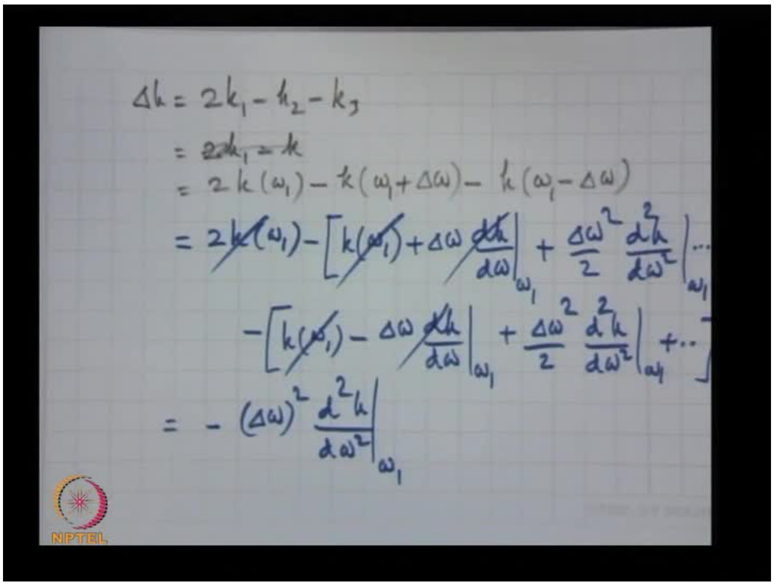

Now, let me let me do a simplified analysis from here, so let me write like this, so K delta K is equal to 2 K 1 minus K 2 minus K 3, so this is equal to 2 K 1 minus so ok let me write this 2 times K at omega 1 minus K at omega 1 plus delta omega minus K at omega 1 minus delta omega ok. So, this is 2 K at omega 1 minus, let me make a Taylor Taylor series expansion K at omega 1 plus delta omega D K by D omega at omega 1 plus delta omega square by 2 D square K by D omega square at omega 1 minus K at omega 1 minus delta omega D K by D omega at omega 1 plus delta omega square by 2 D square K by D omega square at omega 1. If delta omega is small compared to omega, omega is of the order of 10 to the power of 15 hertz second inverse radians per second. So, delta omega is 10 to the power 10 or 10 to power 9 then delta omega is small compared to omega and I can make a Taylor series expansion and as you can see here this plus this cancels off with this and this cancels off. So, I will essentially land up with minus delta omega square. I am trying to find out is how does delta K depend on the frequency difference delta omega under the condition that delta omega is small compared to omega, so delta K is proportional to the second differential of K with respect to omega. This quantity is termed group velocity dispersion, second differential of K with respect to omega is actually depends on its effectively proportional to second differential of the refractive index with frequency of wavelength. This is related to dispersion. So, if I want this process to be efficient this must be zero and a medium which possess this property media may will possess this property at some frequency or a wavelength and that frequency of wavelength is called the zero dispersion wavelength or zero dispersion frequency.

For example; silica if you take silica glass it has this equal to zero around 1.27 micrometers (( )) nanometers this is zero, because of the way refractive index varies, actually if you plot refractive index versus wavelength it will go like this;

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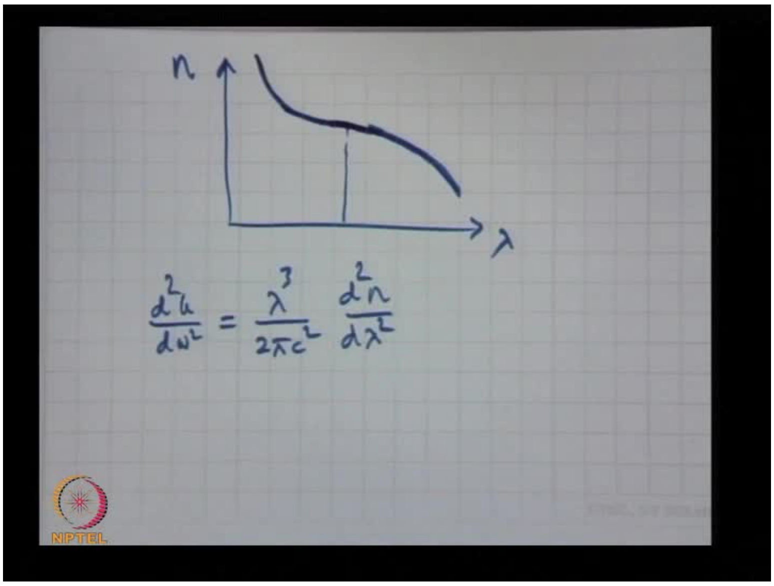

And in fact this term E square K by 2 omega square is proportion so let me I will leave this you to to calculate, so D square K by D omega square is actually lambda cube by 2 pi C square. Please use expression for K in terms of omega or wavelength and refractive index and convert this differential from omega to lambda, you know the relationship between omega and lambda, you know the relationship between K and refractive index, so you just need to change differential from D square K by D omega square to D square N by D lambda square. So, you see here D N by D lambda is negative and as you come like this it decreases and then it starts to increase again the first differential. This second differential will be zero here, but the curvature changes from positive to negative. So, this is the wavelength around this wavelength you will have D square N by D lambda square is zero or D square K by D omega as zero and that as I said for silica silica material it is about 1.27 microns. For different materials it may it will be different and so if you want to use silica and achieve good 4 wave mixing, you must be close to this wavelength of 1.27 microns.

Now, in optical fibers it is not K, but the propagation constant of the mode (Refer Slide Time: 18:43) that is propagating through the fiber and this condition actually gets little more modified and (Refer Slide Time: 22:06) by choosing fibers of different diameters and different refractive index profiles you can move this point elsewhere. So, if you want good 4 wave mixing at a different wavelength, say 50 50 nanometers you can shift this position of this inflection point to a 50 50 nanometer wavelength by designing an optical fiber, because when you take an optical fiber and light propagation of the medium it not only depends on the medium refractive index, but also in the geometry. If you have studied metallic waveguides the velocity depends on the dimensions of waveguide. Similarly, here the velocity of the waves depend on the dimensions of the waveguide of the fiber, (Refer Slide Time: 18:43) which means K depends on the dimension of the fiber K 1 K 2 K 3 are all dependent on the materials as well as the geometry of the fiber. So, by appropriately designing (Refer Slide Time: 22:06) it is possible to shift this wavelength of zero dispersion to other wavelengths and so I can actually achieve a good efficiency for this process of 4 wave mixing. If I do not want 4 wave mixing, I can ensure that this is not zero. In communication systems, I do not want cross talk. (Refer Slide Time: 00:36) I do not want that signals taking carrying information at these two frequencies signals carrying information at these two frequencies should alter anything to do with at omega 3 frequency. (Refer Slide Time: 22:06) So, I would operate at a wavelength at which this is not zero close to zero then this 4 wave mixing is very much reduce. So, depending on this parameter, I can have high large 4 wave mixing high efficiency 4 wave mixing or I can decrease the 4 wave mixing efficiency by an appropriate choice of the wavelength of operation or in optical fibers by an appropriate choice of the geometry of the optical fiber. So, we will not discuss this here, but this is a very important aspect in the field of optical fiber communication, because this particular 4 wave mixing crosses problems in distorting the signals at other wavelengths, it leads to cross talk ok.

There are other interesting things that can happen here (Refer Slide Time: 00:36). For example; this is also used for converting signals at 1 wavelength to the same signals at another wavelength. So for example; if I have signals at omega 2 frequency I can mix this with another wave at omega 1 and I will get the same signal at now omega 3 frequency, these are called wavelength converters. I have some signals, some signals means I have a sequence of pulses coming at omega 2 frequency and I want these pulses to shift to another frequency for another wavelength. So, what I can do is, I can mix this signals with another wave such that this omega 2 waves get converted to omega 3 wave. These are called wavelength converters and in applications like routing of optical signals in optical networks this is a very important component, these are called wavelength converters. So, this is another application and for this obviously I need to choose the fiber to have high 4 wave mixing efficiency ok.

As you can see this process this process what is this process, if I write it slightly differently this process is satisfying this condition.

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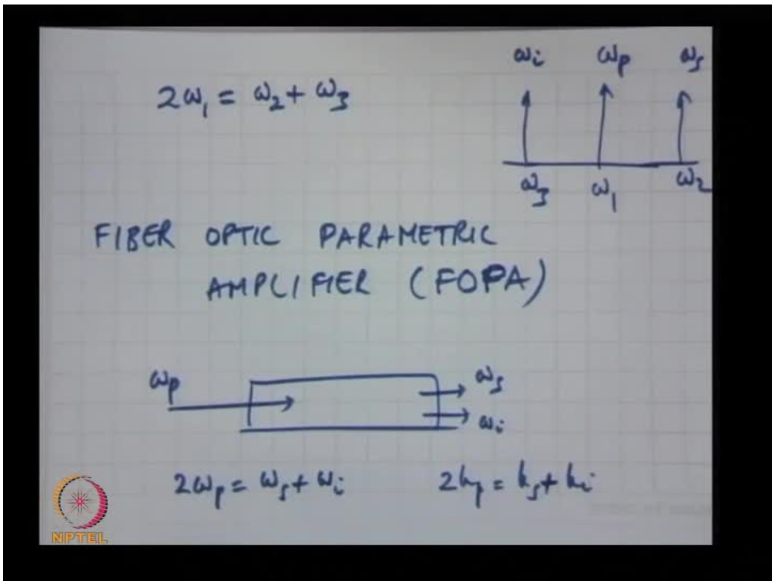

So, 2 omega 1 is equal to omega 2 plus omega 3. This is the same equation I am writing in a slightly different fashion this is omega 1 this is omega 2.

Now, if I want to solve the problem of how does the electric field at omega 1, omega 2 and omega 3 vary with propagation how would I solve. I will find out the nonlinear polarization at the 3 frequencies. I will substitute at the wave equation. I will use the same procedure that we had used to obtained equations for omega S omega P and omega i for the 3 wave mixing process. Here, I will have three equations one for omega 1, one for omega 2 and one for omega 3. The conditions now is 2 omega 1 is equal to omega 2 plus omega 3, there it was omega P is equal to omega S plus omega i. So, I will not go into details, but you can show that this process can lead to amplification of the signal at omega 2 exactly like the amplification of omega S by omega P. In the 3 wave mixing process, I can amplify the omega 2 signal, when I generate omega 3, I will also amplify omega 2, because what is this process, it is 2 photons at omega 1 frequency splitting together in to 1 photon at omega 2 and 1 photon at omega 3 frequency. So, every time I generate omega 3 I will have to I will have to generate omega 2 also, so omega 2 will automatically get amplified as it propagates through and as it generates omega 3. So, this is called again an optical parametric amplifier and this uses chi 3 nonlinearity and this amplifier based on optical fibers, because you have optical fibers in which you can have long distance propagation and this is called the fiber optic parametric amplifier or (( )) fiber optic parametric amplifier, this is a very important component again and there is a lot of research going on in achieving high efficiency amplifiers with broadband amplifiers and so on using the process. This is a chi 3 process compared to chi 2 process for crystals and this will happen in all media. For this of course, I need to satisfy the conditions for phase matching.

Ok now to obtain the gain coefficient and all those things, I need to solve all the 3 equations. I do the similar kind of approximation may be a pump depletion neglect pump depletion, which means usually this is the pump so this is pump, which is called omega P, I can call this omega S and omega i. So, this equation is 2 omega P is equal to omega S plus omega i, there it was omega P is equal to omega S plus omega i. So, it is a 4 wave mixing because there are 4 frequencies, which are interacting in this equation omega 1 plus omega 1 is equal to 2 plus omega 3. There it was 3 wave mixing here it is 4 wave mixing. So, again we will not solve this equation, but you got enough basics here to sort of use this analysis and parallel analysis to what we did for chi 2 process to analyze the problem of fiber optic parametric amplifier. Also, if I take a medium and launch only omega P classical equations do not predict the generation of the signal and idler pairs, but I will find generation of omega S and omega i simultaneously coming out, satisfying this equation satisfying this equation 2 omega P is equal to omega S plus omega i, of course provided I satisfy 2 K P is equal to K S plus K i.

So, again this is a process in which I can spontaneously generate pairs of photons at symmetrically located around the pump frequency one higher one lower, in that case both where lower frequencies. In the 3 wave mixing case omega P is equal to omega S plus omega i, so omega P will generate 2 photons at 2 lower frequencies. Here waves at omega 1 omega P will generate 1 at a slightly higher frequency and equal 1 at a slightly lower frequency exactly situated on the other side and so just like the photon generated in that process this is also an interesting way to generate photon pairs and true experiment in quantum information or quantum communication and so on. So, again this is a lot of interesting physics in this area, what we will do in the class later on as we go through quantum mechanics is solve the problem or look at the problem of chi 2 nonlinearity and the photons generated in that process and we will not consider this in the class.

So, this is also an interesting aspect to this 4 wave mixing process. In fact you can further generalize from 2 input waves to 3 input waves and you will find all kinds of frequencies and so on ok, so we will not continue with that ok. So, with that I want to finish the 4 wave mixing problem and before we start the quantum aspects I want to just briefly discuss one more very interesting aspect of this chi 3 process ok. Before that do you have any questions?

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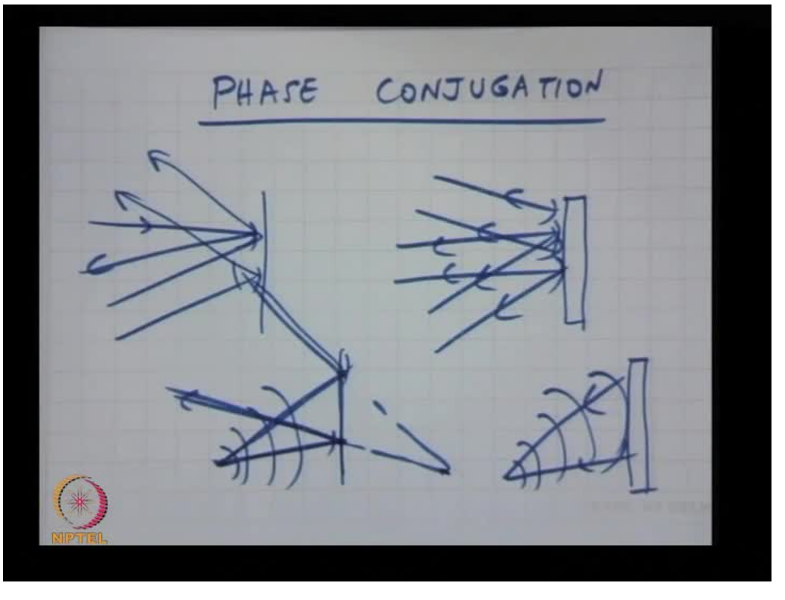

Ok what I am going to discuss is what is called as optical phase conjugation. I will briefly discuss this as an interesting example of chi 3 nonlinearity. Now, all the time we have been discussing, we have assumed that all waves are travelling in the same direction collinear, but it is not required, because I can have waves going in different directions total electric field I calculate and I calculate the nonlinear polarization and so on. So, in phase conjugation, I am going to discuss an example in which the waves are not propagating in the same direction, but may be in different directions and the phase matching conditions now become vector equations rather than scalar equation. So, what you had as K P is equal to K S plus K i, will become K P vector is equal to K S vector plus K i vector. Similarly, this will be 2 K P vector is equal to K S vector plus K i vector, they all vector equations. We could write the scalar equations earlier, because we assume that all the waves were propagating in the same direction, so the vectors there are all parallel, so it just becomes the scalar equation.

Ok now, before I discuss this problem, let me look at the following problem, suppose I take a mirror ok if I take a normal mirror and I send a wave like this wave goes here. Suppose I had different mirror in which if I send a wave like this it comes back, no matter which direction I input. So, I have another wave coming from here it goes back here is the wave this goes back. Here, if I send this wave it will go here, so I mean obviously in the normal mirror the reflection the the reflected wave is not going back towards the incident wave. This is a normal mirror. This is a phase conjugate mirror. It is phase conjugate because a wave going like this is reflected in to a wave going backwards. It retraces its path in the reverse direction. Now, imagine a mirror like this and if you look at this mirror what do you see, can I see myself, what will I see, wave from here not reach my eye, they will go back here waves from here will go back here, my right eye will see the right eye the left eye will see the left eye, because light from here goes and comes back, light from here does not does not it goes back here I cannot see that, it is a very strange mirror. Just imagine the phase conjugate mirror is very strange, because it reflects the wave exactly in the reverse direction. So, if you had for example; again a normal mirror if you had a point source here this wave would have got reflected and you will see a virtual image here, in this phase conjugate mirror goes like this and comes back. This wave will go reflect here this wave will reflect here, so you will form a virtual image. This diverging wave further diverges after reflection. This diverging wave which becomes a converging wave on reflection and it retraces retraces its path exactly. So, if you play the reflected wave here on a movie it will be exactly a time reversed forward wave. The wave started like this went like this and so if I if I if I play the reflected wave will go back to the starting point, just like time reversal. I am not reversing time, but what is actually happening is the wave is propagating as if the time has got reversed, a diverging wave become a converging wave after reflection.

Now, this is a very interesting mirror because this can be used for primarily the one of the major applications is in overcoming distortions of light waves as they propagate through distorting media. For example; the atmosphere has a lot of distortions, because of temperature fluctuations and refractive index variation of air, so the image of the stars and the images of various astronomical objects I see is not very clear. The twinkling of the star is primarily, because of fluctuations. I could use a phase conjugate mirror to cancels this fluctuation and I could use this to image an object lying on the other side of a distorting objecting distorting surface for example or something like that. So, it is a very very interesting area of application and this uses the chi 3 nonlinearity.

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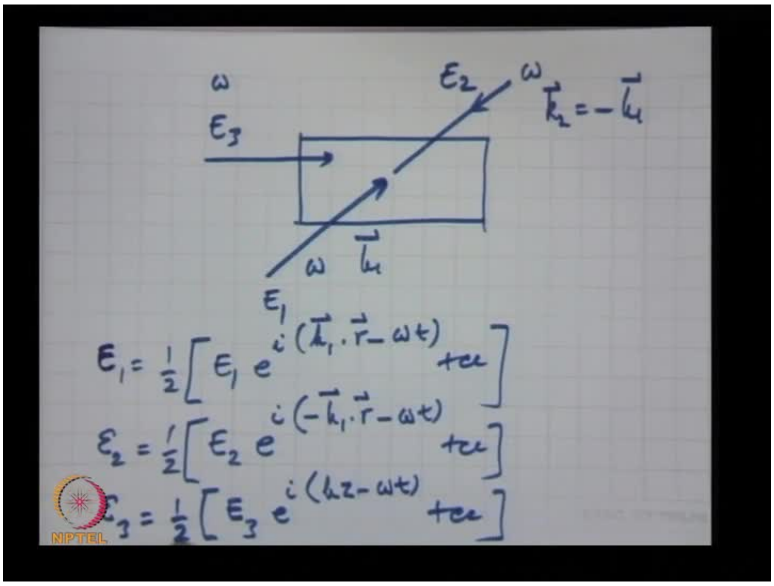

Now, one of them there are many other different techniques. Now, let me tell you what the process is, suppose I take a medium ok I will launch a wave like this and I will launch another wave like this, so both of them have same frequency omega, so let me call this E 1 and let me call this E 2 and let me now launch a wave E 3 from here and also let me assume all of them have the same frequency ok. So, let me write E 1 is equal to half of let me write this curly so that I just differentiate this is E 1 E to the power i K 1 dot R minus omega T plus complex conjugate. K 1 vector is the propagation direction of the E 1 wave, so this K 1 vector K 1 vector here. What is E 2 the second wave is half E 2 E 2 the power i, so K 2 so this is K 2 minus K 1 and this is K 1, so this is minus K 1 dot R minus omega T and I have a third wave which is E 3 is equal to half E 3 E to the power i A Z minus omega T ok.

Now, the nonlinear polarization will be epsilon zero chi 3 in to some of these three terms whole cube right. The total electric field is now consisting of all the three. So, I am writing scalar equations forgetting about polarization states. So, the total electrical field is E 1 plus E 2 plus E 3, so the nonlinear polarization is epsilon zero chi 3 in to E 1 plus E 2 plus E 3 whole cube. Now, among the many terms, there will be one term looking like this, so you will have for example; E 1, E 2 E 1, E 2, E 3 star this in to this in to star of this, you will have terms like this cube this cube this cube this square in to this all kinds of terms will be there, so one of the terms let me pick it up, so this is this in to this in to this star.

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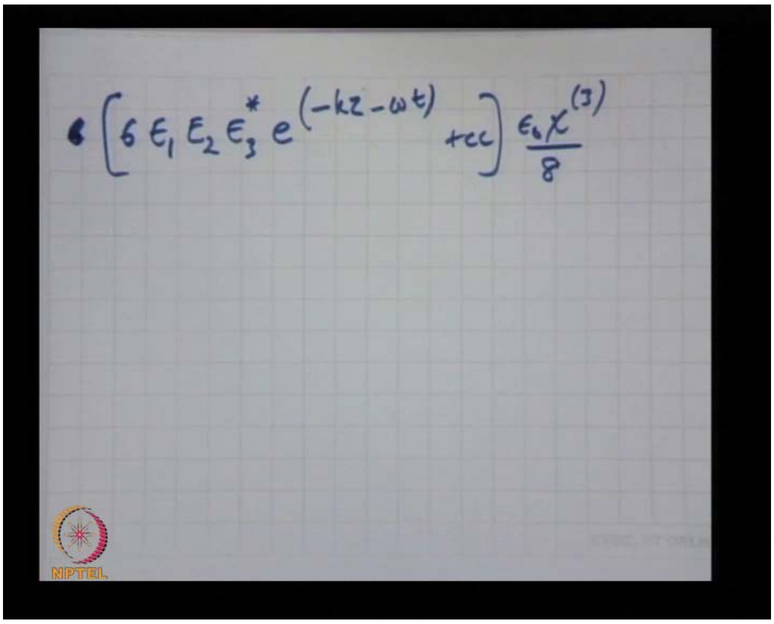

What will be the term? So, there will be nonlinear polarization at the following following dependence, which will be E 2 E 3 sorry E 1, E 2, E 3 star. Now what about the exponentials (Refer Slide Time: 40:13) this into this will cancel and this star will give me minus K Z and then what about time dependence. So, I will have actually there will be a factor of 6 6 and of course there will be epsilon zero chi 3 by 8, which which is appearing because of the half (Refer Slide Time: 40:13) this half there will be a cube of this it will be 1 by 8 and there is an epsilon zero chi 3. So, this is the term there will be a nonlinear polarization term, which will have this as one of the terms plus of course its complex conjugate, so this multiplied by epsilon zero chi 3 by 8.

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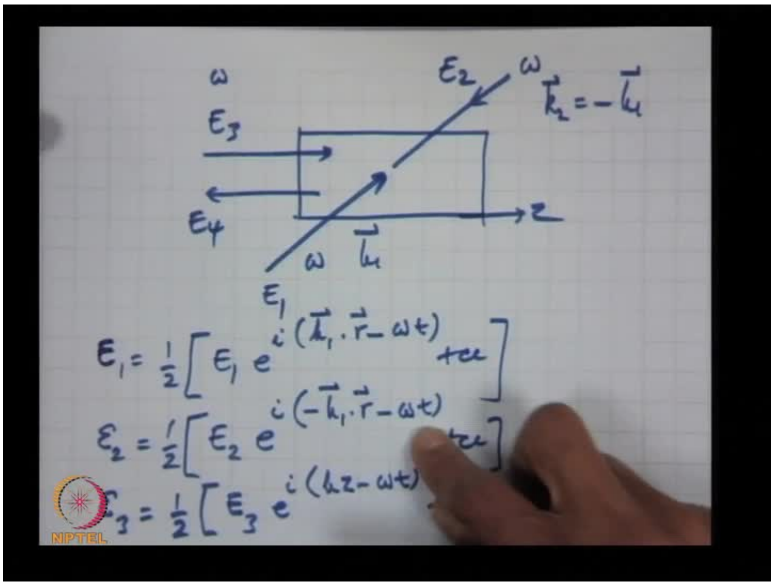

What is this term, this is a wave at the same frequency, but propagating in the negative Z direction and this was this was Z this was positive Z. This is called this wave this will try to generate a wave going in minus Z direction and because the frequency and the propagation constant of this it is phase matched. The frequency omega has a propagation constant K in this medium and so the polarization that it generates at the frequency omega, but reverse direction is also K propagation constant. So, this wave is automatically phase matched in the interaction process and so this polarization term will lead to a wave going in the reverse direction. This is called the phase conjugate wave and as you can see here the only difference is this space variation space dependence has been changed from plus K Z to minus K Z, the time variation is still minus omega T, so there is i factor sorry. The time variation is still minus omega T what has happened is the exponential plus i K Z terms has been changed to minus i K Z term. So, if you shine two laser beams like this into a medium and send a wave from here this nonlinear polarization will generate a wave in the reverse direction exactly a replica of E 3, but in the reverse direction.

Sir E 1 and E 2 are strong waves or weak.

Strong, E 1 and E 2 are supposed to be the strong waves, (Refer Slide Time: 43:22) so that this polarization is having enough strength otherwise; the polarization term will be very weak and there will not be sufficient generation. So, I should make E 1 and E 2 to be strong waves if I want this to be a mirror. So, this is this is this is a mirror which is active mirror, so I have this wave coming in and if I send a weak wave from here it gets reflected and goes back towards the source again exactly back.

Now please note that any wave can be written as a super position of plane waves propagating in different directions; have you done this somewhere in the course. Any time dependent function can be written as a super position of various frequencies by Fourier transform. Any X Y Z depend wave can be written as a super position of plane waves going in different directions. So, if each plane wave has its phase conjugate generated by the mirror the entire wave when it goes will generate its own phase conjugate and that is why I drew (Refer Slide Time: 34:22) that if I had a spherical diverging wave hitting this mirror after this reflection it becomes a converging spherical wave. In a normal mirror a diverging spherical wave remains a diverging spherical wave after reflection. That is very interesting mirror, which is a phase conjugate mirror and what I will do in the next class I will just give you a very short description of couple of equations on of this process and I will show you some interesting features of this phase conjugation problem.

Sir, here we have two lasers K 1 K 2 does a direction of as to where (( )).

No, as long as they are exactly opposite to each other, (Refer Slide Time: 44:33) because then I need to cancel these, K 1 does not come in to picture at all finally. So, what I can do is send a beam from here put a mirror here ok, so the beam goes and reflects and exactly comes back that is a normal mirror, so I will have, so what is happening is these 4 waves are exchanging energy in this process.

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No, it's automatically phase matched. I need to make sure, I need to consider polarization states, I need to calculate the chi I J K L, which I have not done here, so I need to do all the tensor analysis there, but this process will be reasonably efficient and it is an efficient efficient process.

But to optimize mean to use the chi (( )).

Also chi matrix and also overlap, the beam should overlap significantly. See if I come like this for example; only the volume of interaction is only so much, but if I come at larger angle may be the volume of interaction increases, so I need to worry about those things to make the process of generation of the conjugate wave more efficient and plus the medium chi chi tensor that is that is available to me in the medium. So, so next class what I will do is very briefly write down a couple of equations and show you this process and it has it is a very interesting mirror people have tried to use this mirror as a mirror in a laser. I can I can make a laser out of a normal mirror and a phase conjugate mirror. So, any distortions inside the laser cavity gets cancel, because of the phase conjugate mirror.

Sir in this in this kind of mirror the the intensity of E (( )) are coming out it's would be what will be.

We will we will go with this equation then I will tell you. I have to write the coupled equations now, what I must do is write the coupled equations (Refer Slide Time: 44:33) for E 3 and E 4 forgetting about the depletion of E 1 and E 2, so E 3 and E 4 get coupled because of E 1 and E 2 ok. See E 3 gets E 4 gets generated by E 3, but E E 4 will generate E 3, because E 4 is a wave going like this, it will generate it says conjugate which is this direction, so that is the coupling mechanism that is taking place.

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Yes yes so that is coupling. (Refer Slide Time: 44:33) So, E 3 interacts with E 1 E 2 generates E 4, E 4 interacts with E 1 E 2 to generate E 3 and so that is gets mixed and I will have a couple set of equations it is a very simple equations, which you will solve and I will show you some interesting features. Ok any questions ok thank you.