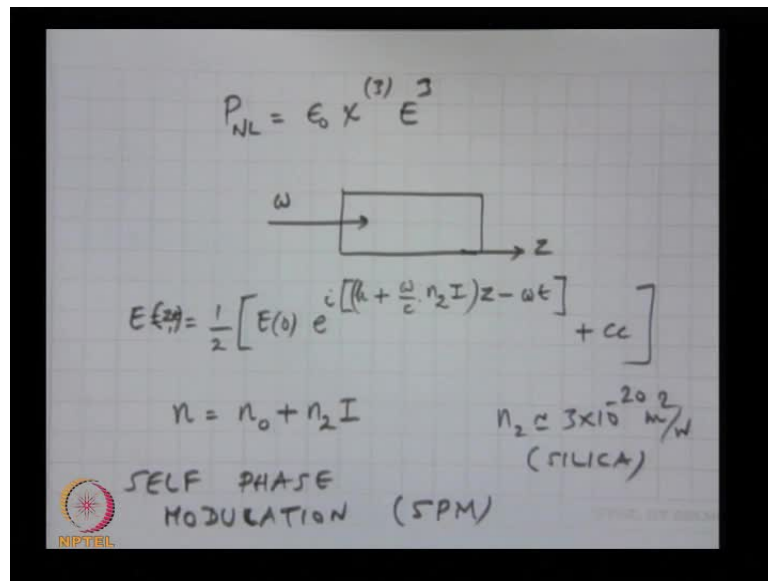


**Quantum Electronics**  
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**Module No. # 04.**  
**Third Order Effects**  
**Lecture No. # 21**  
**Third Order Non - Linear Effects (Contd.)**

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We are actually discussing third order nonlinear effects, that is brought about by this term  $\epsilon_0 \chi^{(3)} E^3$ , this is the first nonlinear term that is present in media in which there is a center of inversion symmetry, which means all the second order tensor elements are 0 and media like glass or liquids they have this first nonlinear term in the expansion.

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$$\phi = \left(k + \frac{\omega}{c} n_2 I\right) z - \omega t$$
$$= \left[\frac{\omega}{c} n_0 + \frac{\omega}{c} n_2 I(t)\right] z - \omega t$$
$$e^{i\phi(t)}$$
$$\omega(t) = \frac{d\phi(t)}{dt}$$

CHIRPED PULSE

We started looking at the effect of this term and just recall the first thing we looked at is, if I have a medium possessing a  $\chi^3$  value. I launch light at single frequency  $\omega$  then, I found that the electric field as a function of  $z$  goes as  $e^{i(kz - \omega t)}$ ,  $E(z, t) = E_0 e^{i(kz - \omega t)}$ . I found that the electric field as a function of  $z$  goes as  $e^{i(kz - \omega t)}$ ,  $E(z, t) = E_0 e^{i(kz - \omega t)}$ . I found that the electric field as a function of  $z$  goes as  $e^{i(kz - \omega t)}$ ,  $E(z, t) = E_0 e^{i(kz - \omega t)}$ .

The electric field as a function of  $z$  and time for an incident electric field of the form  $E(z, t) = E_0 e^{i(kz - \omega t)}$  as it propagates through the medium the electric field gets modified. What effectively is happening is the refractive index changes from  $n_0$  to  $n_0 + n_2 I$ . The propagation constant has changed from  $k$  to  $k + \frac{\omega}{c} n_2 I$ , effectively if  $n_0$  the refractive index of the medium for low intensities, the intensity of the medium at high intensity becomes  $n_0 + n_2 I$ ,  $n_2$  is the nonlinear coefficient and as I mentioned to you for glass it is about  $3 \times 10^{-20}$  meter square per what this is for silica, glass fibers are made of silica and that is a typical value of  $n_2$  and what we also found is that the change in index is very small.

If you propagate over long distances the accumulated effect of the change in phase can become significant in comparable to  $\pi$ , which means you can go from constructive to destructive interference. This term leads to what is called as self phase modulation or SPM. This is very important in experiments in optics in which the intensity becomes quite high and you are propagating through medium over long distances or if you have a very

small cross sectional beam with a reasonably high power then the coefficient this term can significantly affect light propagation. Although this analysis is valid for a monochromatic input at a single frequency, I can use some heuristic arguments to tell you what will happen. If I had a pulse of light incident in such a medium pulse means the intensity is not remaining constant but, changes with time.

Although this analysis has assumed monochromatic waves, I will try to argue it heuristically. What is the effect of having an intensity dependent refractive index on a pulse of light that is launched into this medium? in this case  $n$  will not remain constant with time,  $n$  itself will change with time, if  $n$  repels for example, I have a Gaussian pulse of light the intensity increases and decreases with time, which means the intensity at the input increases and decreases with time which means that the output phase also changes with time within the pulse.

The total phase change will become like  $\phi = k z + \omega \int_0^z n^2 dz - \omega t$ . This is the phase at the output  $z = z(t)$ .

which is at  $z$  as a function of time, if intensity is the function of time, this actually this is  $\omega \int_0^z n^2 dz - \omega t$ . This is a function of time into  $z$  minus  $\omega t$ , now this is a term that means you have function of time now at the output which is not exponential  $e^{i \omega t}$  but, it has another time dependence, suppose I have a pulse which varies as  $e^{i \phi(t)}$  I define, what is called as the instantaneous frequency, instantaneous frequency as  $\omega(t) = d\phi/dt$ , the rate of changes of the phase with time is defined as instantaneous frequency. If the phase changes linearly with time, if  $\phi(t) = \omega t$ , then  $d\phi/dt = \omega$  the frequency which remains constant with time. If  $\phi(t)$  is not a linear function of time then  $d\phi/dt$  is not a constant and the frequency instantaneous frequency of the pulse will change with time, that means within the pulse the frequency does not remain constant have you heard of a pulse like this, what is such a pulse called? CHIRPED PULSE, the frequency within the pulse change with time this term comes from bird calls which have which chirp the birds chirp

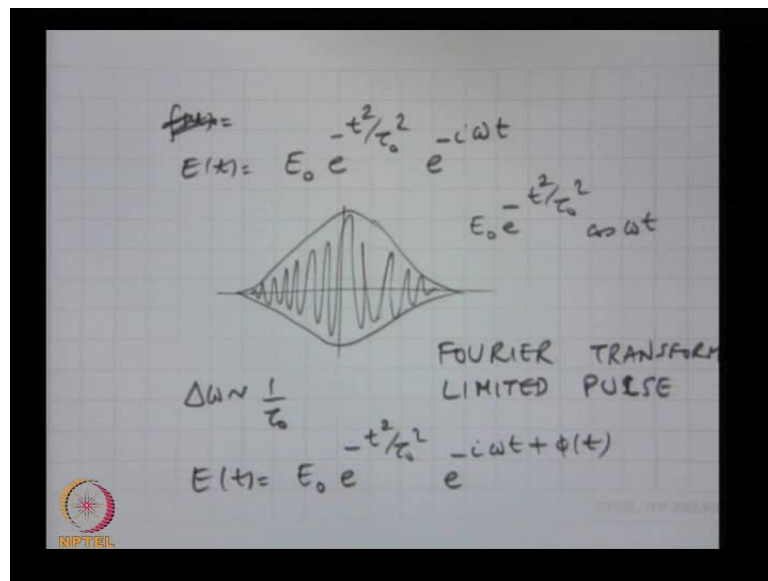
So, there is a frequency which varies the time and that is essentially a situation here where, the frequency of the signal is not constant within the pulse but, changes

with time in general in any pulse we say there are multiple frequencies vary in any finite duration pulse

Yes

So, how is it different from that case?

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Ok now let me let me show you 2 kind of pulses one for example, i can have a function of time which is equal to electric field, let me write electric field as a function of time is equal to  $E_0$  exponential minus  $t$  square by  $\tau_0$  square, exponential minus  $i$  omega  $t$ , now this is the pulse if i were to draw the real part of this equation the electric field will like this is the envelop and inside there is the oscillations because i am plotting what i am plotting is  $e$  naught minus  $t$  square by  $\tau_0$  square  $\cos$  omega  $t$ . Now this is an amplitude term and that is a phase term the instantaneous frequency within the pulse is constant because the phase term is omega  $t$  exponential minus  $i$  omega  $t$ , frequency remains constant with time but, this pulse contains many frequencies, because if you take a Fourier spectrum of this pulse if you take a Fourier transform you will find that this pulse contains many frequencies what will be by the spectrum of the frequencies of the order of,  $\Delta\omega \sim 1/\tau_0$ , narrower the pulse the broader the spectrum the broader the pulse narrower the spectrum, this is the Fourier transform pair the function of time

and the function of frequency which are, Fourier transform pairs they have an inverse relationship invert width

So, this is kind of an uncertainty relationship you cannot talk of very narrow pulse with a very narrow frequency spectrum, it is not possible the moment you define  $\tau$  naught. The width of a pulse the duration you have no choice as far as  $\Delta\omega$  is concerned, there is a minimum  $\Delta\omega$ , this called an unchirped, if you had a function of time which as which was  $E_0 \exp(-t^2/\tau_0^2) \exp(-i\omega_0 t + \phi(t))$  the envelope is still Gaussian the intensity varies till as intensity means proportional to square of  $E$  mod  $E$  square. It is still proportional to exponential minus  $2t^2/\tau_0^2$  the intensities varies in a Gaussian fashion but, this one has now a phase term which is also a function of time apart from  $\omega_0 t$  dependence, the instantaneous frequency of this pulse is not constant with time this, this has also a spectrum and this spectrum is not related is not given by this equation is valid for what are called as Fourier transform limited pulses.

$\Delta\omega \tau \geq 1$  is greater than equal to one it is not always equal to 1, it cannot have less than equal to less than 1. Solid means this pulse has the same width but, as a spectrum it is much broader, if you take a Fourier transform of this for example, if  $\phi(t)$  happens to be say  $\alpha t^2$  or something, Some function which is not a function of linear function of time  $\alpha t^2$  or something else. Then you can show that the spectrum of this pulse is broader than the spectrum of this pulse, both of them have the same width if you were to detect both the optical pulses they will give you a same intensity dependence pulse versus time. They will give you the same electrical signal because the detector responds to intensity of this is proportional to mod  $E$  square, intensity of this and intensity of this vary in a same fashion with time, a photo detector will give you the same current variation with this and with this but, if you were to pass this through what is called as an optical spectrum analyzer which means it gives you the optical spectra. What are the frequencies optical frequencies present in the medium for example, grating if  $i$  were to push on the grating spreads out the various frequencies you will find that the frequency spread of this is smaller than the frequency spread of this

So, a CHIRPED PULSE always has the frequency spectrum which is broader than the corresponding unchirped pulse and that comes about simply by a Fourier transform relationship. If you take a Fourier transform of this you will get a spectrum of the order

of  $1$  by  $\tau$  naught, width of the order of  $1$  by  $\tau$  naught, if you take a Fourier transform of this it will always be bigger than  $1$  by  $\tau$  naught, actually for a given spectrum the narrowest pulse you can make is given by  $1$  by  $\Delta\omega$  of the order of  $1$  by  $\Delta\omega$ , suppose I give you a spectrum over a certain frequency range I have light at the frequency range  $\Delta\omega$  the shortest pulse you can make with this spectrum is of the order of  $1$  by  $\Delta\omega$ , you cannot with a spectrum you cannot make a pulse which is  $1$  by  $10\Delta\omega$ , I cannot make because they are inconsistent with the Fourier transform relationship. I can have a broader pulse with the same spectrum I cannot have a narrower pulse So, this is the  $\Delta\omega$  into  $\tau$  naught is greater than of the order of  $1$  exactly like an uncertainty it is all uncertainty relationship actually, that is the difference between a the moment you have any time dependence other than exponential minus  $i\omega t$  it has a spectrum this has a spectrum this has a spectrum in this case what is actually happening is the within the pulse the frequency remains constant instantaneous frequency does not change with time here the instantaneous frequency is changing with time, this is different this is called the CHIRPED PULSE this is called an unchirped pulse and this is a Fourier transform limited pulse because the spectra and the temporal duration are limited are related to  $\Delta\omega$  into  $\tau$  naught of the order of  $1$ .

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$$\phi = \left(k + \frac{\omega}{c} n_2 I\right) z - \omega t$$

$$= \left[\frac{\omega}{c} n_0 + \frac{\omega}{c} n_2 I(t)\right] z - \omega t$$

$$e^{i\phi(t)}$$

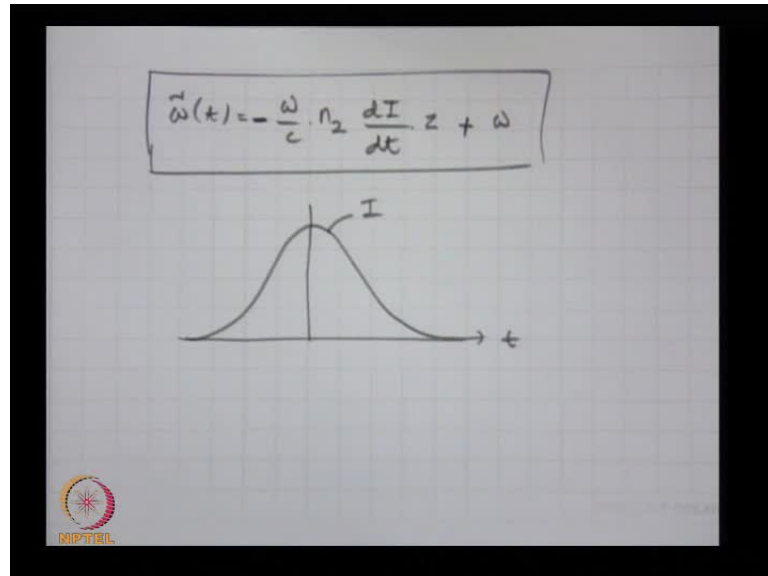
$$\omega(t) = \frac{d\phi(t)}{dt}$$

CHIRPED PULSE

This one will have a  $\Delta\omega$  into  $\tau$  naught greater than equal to one greater than one So,, now what is going to happen is in our case with self phase modulation what I

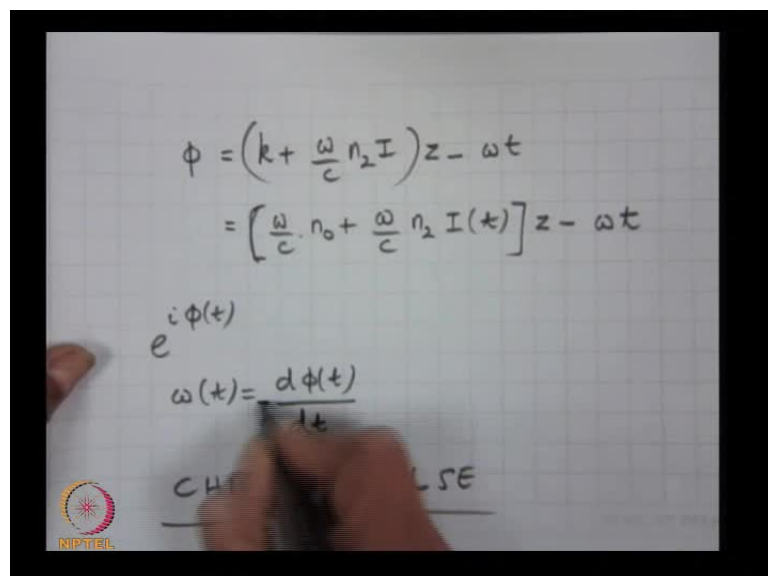
see is at the output of this of this medium the phase is varying with time not linearly but, with Some arbitrary function of time i of into i of t here,

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So, i can calculate from here an instantaneous frequency by just differentiating phi with respect to time and what is instantaneous frequency i will get, let me call this omega tilde of t is equal to this is differential of this, this is omega by c, n 2 d I by d t into, z minus omega. What is omega? omega is the frequency of the input pulse this be that center of frequency the input pulse varied has exponential minus i omega t

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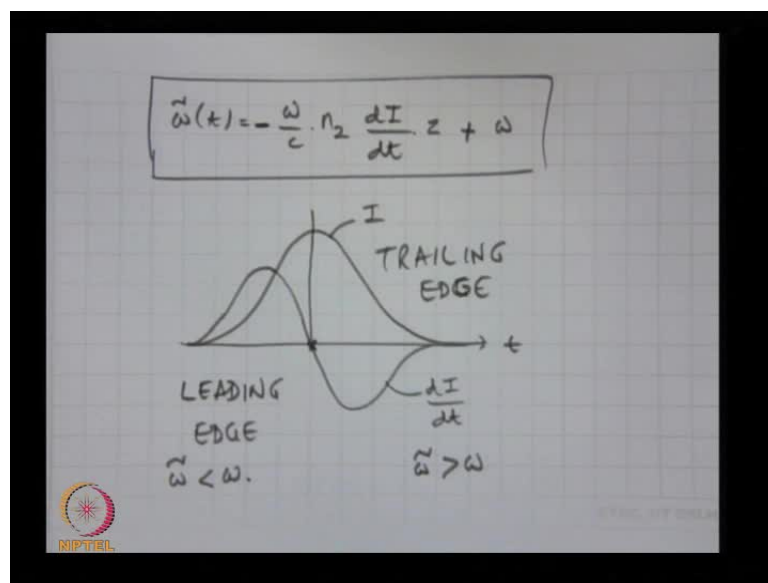
$$\phi = \left(k + \frac{\omega}{c} n_2 I\right) z - \omega t$$

$$= \left[\frac{\omega}{c} n_0 + \frac{\omega}{c} n_2 I(t)\right] z - \omega t$$

$$e^{i\phi(t)}$$

$$\omega(t) = -\frac{d\phi(t)}{dt}$$

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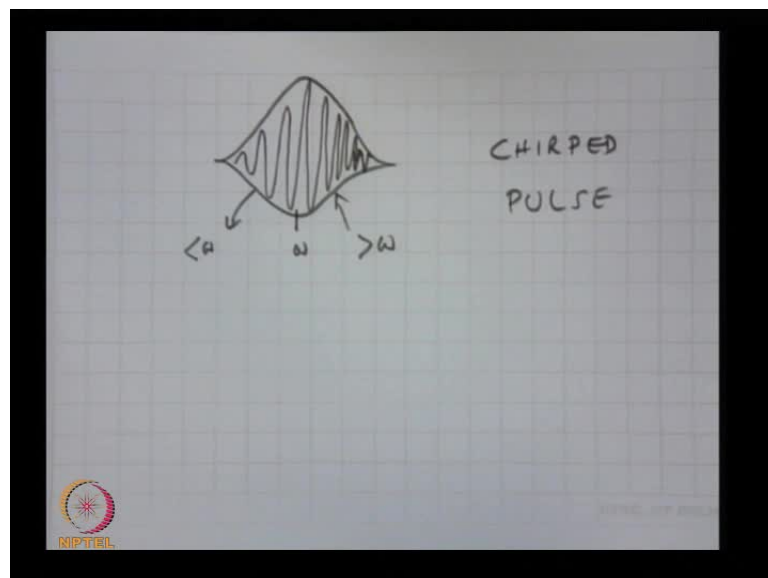


So, let me plot for example, if I had pulse like this is I as a function of time I must define a with a negative sign because I am defining exponential minus i omega t, please define with a negative sign here, the frequencies are always positive, this is actually this because the exponential minus i omega t here, the frequency of this is omega, I must define as minus d phi by d t. If it was exponential i omega t minus k z, I would have defined d by d t of omega t. This is to get the correct expression, if I had minus i omega t then I would have call as minus d phi by d t, now you see here that if I plot d I by d t, d I

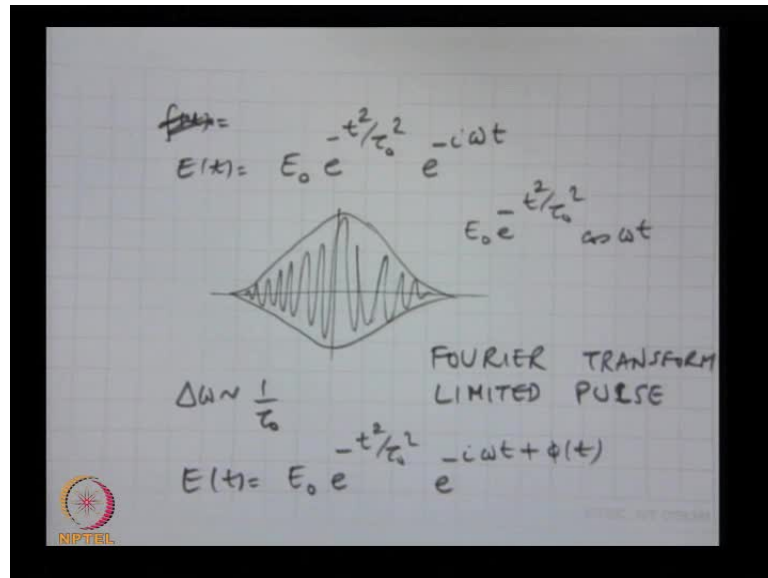


by  $d t$  is positive. Here,  $d i$  by  $d t$  become negative, on this side, this is  $d i$  by  $d t$  differential of this  $i$  as a function of time, what does it imply ?, this is the center of the pulse look here, this is center of pulse, at the center of the pulse  $d i$  by  $d t$  is 0 and the frequency instantaneous frequency is  $\omega$  the same at the frequency of the input. This side corresponds to the leading edge of the pulse, please remember this is earlier time than the center, this is not plotted as a function of space coordinate it is being plotted as a function of time coordinate, this is earlier and this is later, this is the front of the pulse and this is the back of the pulse because if you are standing at some point you will start to see from here, the first one to arrive will be here then this and then this

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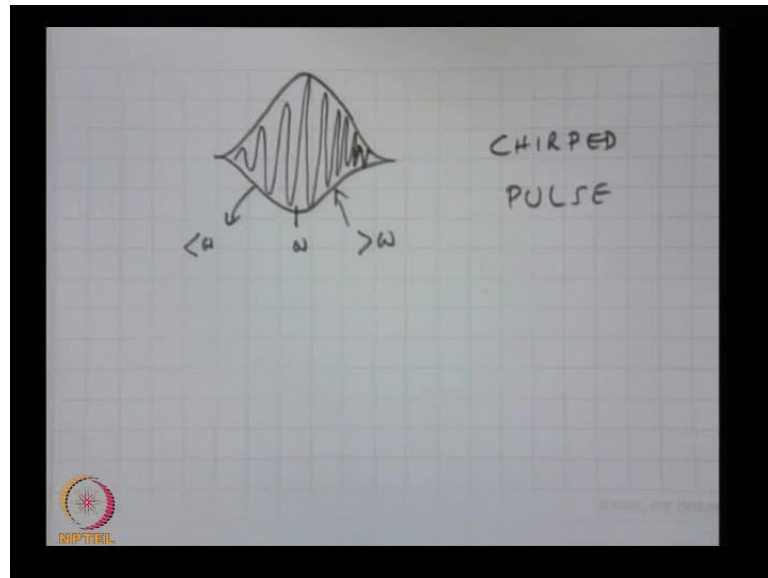


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So, the leading edge has  $di$  by  $dt$  as positive, which means the instantaneous frequency is lesser than  $\omega$ , the leading edge this is the leading edge and this is the trailing edge, the leading edge has an  $\omega$  tilde less than  $\omega$  naught  $\omega$ , this has  $\omega$  tilde greater than  $\omega$ , what will happen is if  $i$  were to draw this pulse it will look like this, now I have drawn a lower frequency here,  $\omega$  here, and a higher frequency here, the oscillations are faster here slower here, this is  $\omega$  naught  $\omega$  this is less than  $\omega$  and here, it is greater than  $\omega$  and this is CHIRPED PULSE. Now the way we have analyzed this problem is, what is happening is the pulse enters the enters the medium and comes out with a same width but, now chirped, as i mentioned to you if you have 2 pulses of the same duration but, one is chirped and one is not chirped the CHIRPED PULSE has a higher frequency larger frequency spectrum than the unchirped pulse

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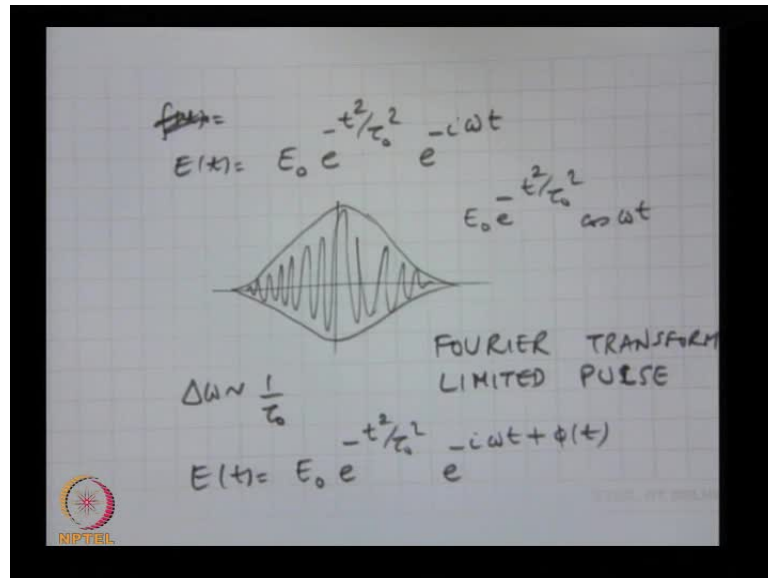
So, what is actually happening is, the self phase modulation chirps the pulse that is propagating through the medium, the self phase modulation actually leads to the generation of new frequencies and that is coming because of nonlinear effect.

Yes mohith.

Sir in this chirp pulse we have i mean earlier we say that there are frequency of this pulse is less than  $\omega$  then it becomes constant at  $\omega$  which the incidentally

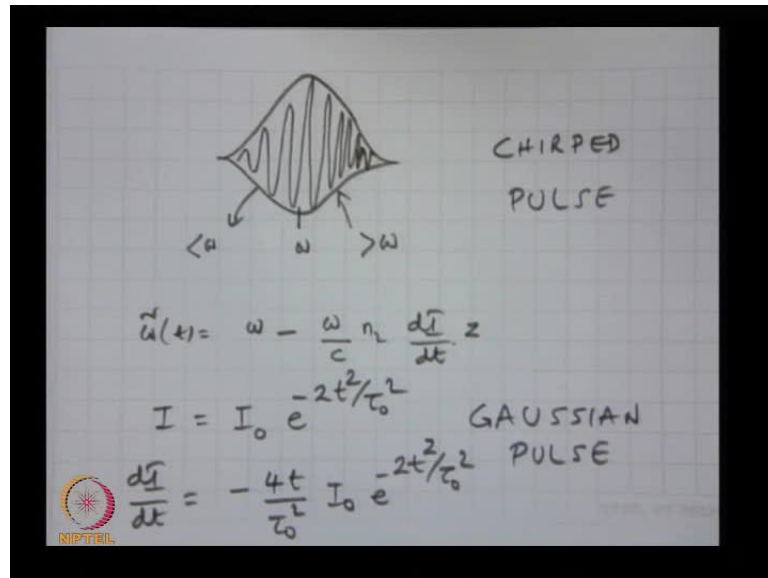
No, instantaneous frequency is constant with time but, there that input pulse has a spectrum there are frequencies present in the input pulse over a spectrum width of the order of  $1/\tau$ , suppose I take a one picosecond pulse it has a spectral width of the order of inverse of our picosecond, which is  $10^{12}$  radians per second.

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Now what I am saying is the output has the same one picosecond duration but, now it is chirped, its spectrum is not inverse of 1 picosecond but, more than that. If I start with the 1 picosecond pulse, if I start with tau naught out now input has a spectrum of the order of 1 by tau naught, the output has the same width tau naught but, as a spectrum which is more than 1 by tau naught, that means surface modulation has led to the generation of new frequencies within the medium and those new frequencies have been generated primarily because of a nonlinear effect, we have seen nonlinear effect generate new frequencies, this new frequencies that come out is, actually because of the nonlinear effects within the fiber and the spectrum of the output is now more than the spectrum of the input

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So, we can actually estimate the increase in spectral width by a simple argument, let me calculate what is the spectral width increase.

Sir,

Yes.

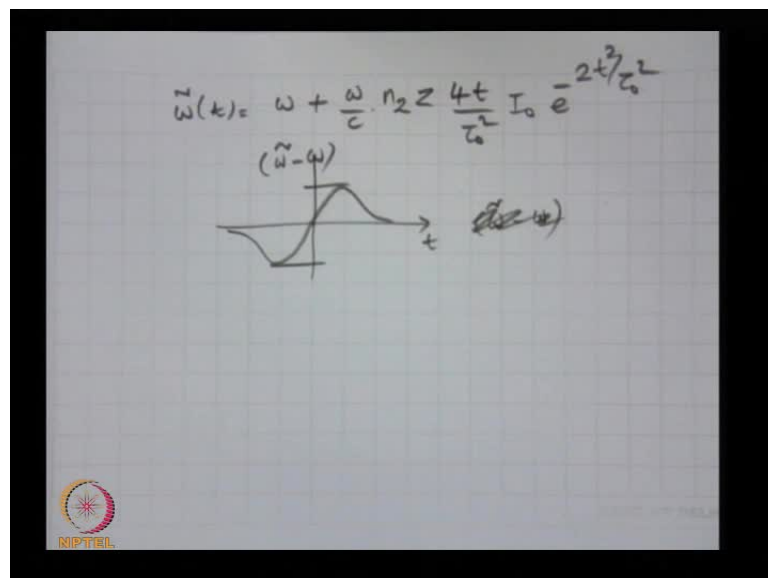
In fact the photons does it mean that 3 photons combine to make?

Yes, they have they have all mixed together to create new frequencies some photons have disappeared from some the total energy will remain the same, if I assume there is no loss in the system whatever energy I have put in will be coming out from the medium the only difference that is happening is the frequency spectrum is different. it is exactly like in second harmonic whatever energy is you are putting in is coming out either in  $\omega$  or in  $2\omega$ . Similarly, here you have put in a certain spectrum of frequencies at input what will come out is the broader spectrum of frequencies by mixing of energy into various new frequencies, new frequencies have got generated by taking up energy from the other frequencies, it just leading to some and all kinds of mixing of this frequencies when we come to putting more input into this nonlinear medium we will see that how the new frequencies can get generated, let me calculate what is the typical let us put Some numbers and calculate

Let me rewrite this equation  $\omega(t)$  is equal to  $\omega$  minus, is there something wrong in the equation dimensionally? that is fine, this is length by time then cancels off this is time goes off, this fine, Yes ok.

Now let me take Some typical example,  $i$  is equal to, let me take a Gaussian pulse minus intensity varies this is called a Gaussian pulse, this function of dependence is a Gaussian and, this means the intensity increases and decreases like this in a Gaussian fashion,  $d i$  by  $d t$  is equal to minus  $4 t$  by  $\tau_0^2$  square,  $i$  naught exponential minus  $2 t$  square by  $\tau_0^2$  square.

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So,  $i$  can substitute  $d i$  by  $d t$  into this equation and find out the instantaneous frequency as  $\omega(t)$  is equal to  $\omega$  plus  $\omega$  by  $c$ ,  $n_2 z$  into  $4 t$  by  $\tau_0^2$  square  $i$  naught minus  $2 t$  square,  $\tau_0^2$  square, this minus sign with a minus sign here becomes plus,  $\omega$  by  $c$ ,  $n_2$  into  $z$  into  $4 t$  by,  $\tau_0^2$  square  $I$  naught exponential minus  $2 t$  square by,  $\tau_0^2$  square, as you can see here the center of the pulse is  $t$  is equal to 0 and this because of this factor here  $\omega(t)$  becomes  $\omega$  and the leading edge of the pulse is less than  $t$ , less than 0, this is negative, the frequency is less than  $\omega$  the trailing edge of the pulse is  $t$  greater than 0, which means this is positive and the frequency is higher than  $\omega$ .

So, instantaneous frequency is the frequency is increasing with time you could also have a chirping which the frequency decreases with time, one is called up chirp one is called down chirp you can have either way, this leads to a frequency increasing with time and, our estimation. If I plot the frequency versus time it will look like this, let me plot this as  $\omega_{\text{tilde}} - \omega$  versus time, leading edge  $\omega_{\text{tilde}}$  is less than  $\omega$  trailing edge  $\omega_{\text{tilde}}$  is more than  $\omega$ , i want to calculate this increase in spectral width because of chirping which is given by this approximately, let me calculate this quantity approximately what is this number how much is this,

Yes,

If there is you know instantaneous frequency rate of the pulse the spectral width of the pulse

No, spectral width of the pulse depends on  $\tau$ , the actual width of the pulse in time domain. That is called Fourier transform limited pulse always. But in general for CHIRPED PULSES and we have calculated  $\omega_{\text{tilde}}$  that is, I mean this let us say first, we are saying that it is less than  $\omega$  and afterwards in the tailing as the later part of the pulse.

Yes,

Approaches.

Yes.

This it becomes greater than  $\omega$

Yes

So,

Instantaneous frequency

Yes

There is an instantaneous frequency which was constant with time within the pulse at the input now is changing with time within the pulse itself.

So, for this chirp pulse.

It is also has a spectrum.

If you wish to calculate the spectral width.

Yes

Then we have to subtract these 2 width

No, we have to actually take a Fourier transform of the electric field distribution and calculate but, I just want to know how much is the kind of chirping that is this creating whatever frequency variation within the pulse.

No but, ideally what will happen is that the highest frequency minus the lowest frequency should ideally give us just a spectral width, I am not asking about the constituent frequencies different frequencies i am just saying that if you wish to calculate the spectral width we can just subtract the value of upper beam and the lower beam

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Handwritten notes on a grid background showing the mathematical representation of a Fourier transform limited pulse. The notes include the electric field equation  $E(t) = E_0 e^{-t^2/\tau_0^2} e^{-i\omega t}$  and its complex conjugate  $E_0 e^{-t^2/\tau_0^2} e^{i\omega t}$ . A diagram shows a Gaussian pulse with an oscillating carrier wave. The text "FOURIER TRANSFORM LIMITED PULSE" and  $\Delta\omega \sim 1/\tau_0$  are also present. The equation  $E(t) = E_0 e^{-t^2/\tau_0^2} e^{-i\omega t + \phi(t)}$  is written at the bottom.



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$$\tilde{\omega}(t) = \omega + \frac{\omega}{c} \cdot n_2 z \frac{4t}{\tau_0} I_0 e^{-2t^2/\tau_0^2}$$

$$\omega_{max} \approx \omega + \frac{\omega}{c} \cdot n_2 z \frac{4}{\tau_0} \cdot \frac{\tau_0}{2} I_0 e^{-1/2}$$

$$\delta\omega = \omega_{max} - \omega$$

$$\approx \frac{\omega}{c} \cdot n_2 z \frac{2}{\tau_0} I_0 e^{-1/2}$$

See, the problem is in this pulse the frequency remains constant with time, I cannot say that the spectral width of pulse is 0 it is  $\omega \delta\omega$  of the order 1 by  $\tau_0$  naught, this is only an indication of what kind of chirping I have generated within the pulse, how much of chirping the instantaneous frequency, if it remains constant the pulse at the certain spectral width, if I keep the same temporal width of the pulse and now chirp the pulse the spectral width will increase, I am sort of estimating from this quantity because this quantity must be finally, related to the increasing spectral width if this quantity is 0, if  $n_2$  was 0 for example, then this is not there and it has a certain spectral width instantaneous frequency does not change with time. The presence of this term now which means there is chirping in the pulse tells me that the frequency spectrum has increased and the increase in frequency spectrum must be related with this quantity here by how much i am not telling, this is the exactly the increase in spectral width but, i am saying that this is an indication of how much is the increase in spectral width that i am going to see because of this, if this quantity is much bigger than  $\delta\omega$  which i have calculated then obviously this is the spectral width otherwise it is related it is related to the total spectrum.

Ok So, what i am trying to do is only from this quantity guessing or try to find out how much is the increasing spectral width, because of the nonlinear effect now how do I calculate at what time will the frequency be maximum and what time will the frequency be minimum can you calculate and tell me what is the value of  $t$  at which the frequency

is maximum and what is the value of  $t$  at the which frequency is minimum what is this time and what is this time.  $\tau$  naught plus  $\tau$  naught by 2, just differentiate this equation and calculate put is equal to 0 and you will get  $t$  is equal plus  $\tau$  naught by 2 and minus  $\tau$  naught by 2, the frequencies are maxima and minima, let me try to calculate what is the maximum frequency within the chirp,  $\omega$  max within the pulse is  $\omega$  plus,  $\omega$  by  $c$  and  $2 z 4$  by  $\tau_0$  square,  $t$  is  $\tau$  naught by 2 I naught exponential minus half.

Ok So, if i call this  $\delta\omega$  which is equal to  $\omega$  max minus  $\omega$  is of the order of  $\omega$  by  $c$   $n 2 z 4$  by  $\tau_0$  square, this goes out 2 by  $\tau_0$ , I naught phi. This just an estimate of the chirp that is taking place within the pulse, the frequency will go from  $\omega$  at the center,  $\omega$  plus  $\delta\omega$  and then, go back to  $\omega$  because the incidence frequency goes back to  $\omega$  and similarly, on the other side on the leading edge it goes from  $\omega$  to  $\omega$  minus  $\delta\omega$  and then goes back to  $\omega$  again. Because these are the peak values within the pulse, there are 2 positions where the frequencies are maximum and minimum with respect of chirping, let me put some numbers and calculate how much is this quantity.

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$$\delta\omega = \frac{\delta\omega}{2\pi} = \frac{v}{c} \cdot n_2 z \frac{2}{\tau_0} \frac{I_0}{\sqrt{\epsilon}}$$

$$= \frac{1}{\lambda_0} \frac{2 n_2 z}{\tau_0} \frac{I_0}{\sqrt{\epsilon}}$$

$n_2 = 3 \times 10^{-20} \text{ m}^2/\text{W}$ ,  $\lambda_0 = 1 \mu\text{m} = 10^{-6} \text{ m}$   
 $\tau_0 = 1 \text{ ps} = 10^{-12} \text{ s}$ ,  $P = 5 \text{ W}$   
 $z = 1 \text{ km} = 10^3 \text{ m}$ ,  $A = 100 \mu\text{m}^2$

$$\delta\omega = \frac{2 \times 3 \times 10^{-20} \times 10^3}{10^{-6} \times 10^{-12}} \cdot \frac{10 \times 5}{10^{10}} \frac{1}{\sqrt{2.7}} \approx 2 \times 10^{-20+4+18} \approx 2 \times 10^{12} \text{ s}^{-1}$$

Let me calculate also  $\delta\nu$ ,  $\delta\nu$  is equal to  $\delta\omega$  by  $2\pi$ , the actual frequency, this is  $\nu$  by  $c$  and  $2 z$  into,  $2$  by  $\tau_0$  naught,  $I_0$  naught by  $\nu$  by  $c$ , is actually  $1$  by  $\lambda_0$  naught, the free space wave length and  $2 z$  by,  $2$  into  $z$  by,  $\tau_0$  naught  $2$ . Let

me take silica glass, 3, 10 to the minus 20 meter square per watt. Let me take a pulse which is one picoseconds 10 to the minus 12 seconds, let me take a length of 1 kilometer which is 10 to the power 3 meters and let me assume a wave length of one micro meter which is into 6 meter. And let me assume a power of say 5 watts, area of cross section which is 100 micro meter square, these are some numbers typical of optical fibers if you launch light into an optical fiber the cross sectional area of the of the beam is about 100 microns square and I can launch 5 watts of power into the fiber, that gives me an intensity, I need an intensity here peak intensity value, this is the peak power of the pulse that is a area of the pulse area of the cross sectional area, let me substitute here, delta nu is equal to 2 Into 3, 10 to the minus 20 by 10 to the minus 6, 10 to the minus, 12 into ten the power 3 into five by ten to the minus ten one by root 2.7 approximately.

Can you estimate how much is this?, root 2 .7 is about 1.6, 1.6 5 Something, I can strike this off with a factor of 2 here, you will have 2 Into, 5 into 10, into 10 to the power minus 20 plus 4 minus plus 18 plus 10, I will write here. We put it as 8 into 10 to the power of 12 hertz ,yes sir that is fine, Yes 2, 10 to the power of 12 seconds inverse hertz actually, that is 2 terahertz, what is the spectral width of 1picosecond pulse operate terahertz. 10 to the power minus 12 seconds, delta nu is about 1 by 2 pi of 10 to the power 12, about 1.6, 10 to the power 11, this is broaden much more spectral width as broaden much more, what is the change in wave length what is the spectral width in wave length space now how do I calculate.

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Handwritten mathematical derivation on a grid background:

$$\delta\lambda = \frac{\lambda}{c} \delta\nu = \frac{\lambda^2}{c} \delta\nu$$

$$= \frac{10^{-12}}{3 \times 10^8} \times 2 \times 10^{12}$$

$$\approx 0.6 \times 10^{-8} \text{ m}$$

$$\approx 6 \text{ nm}$$

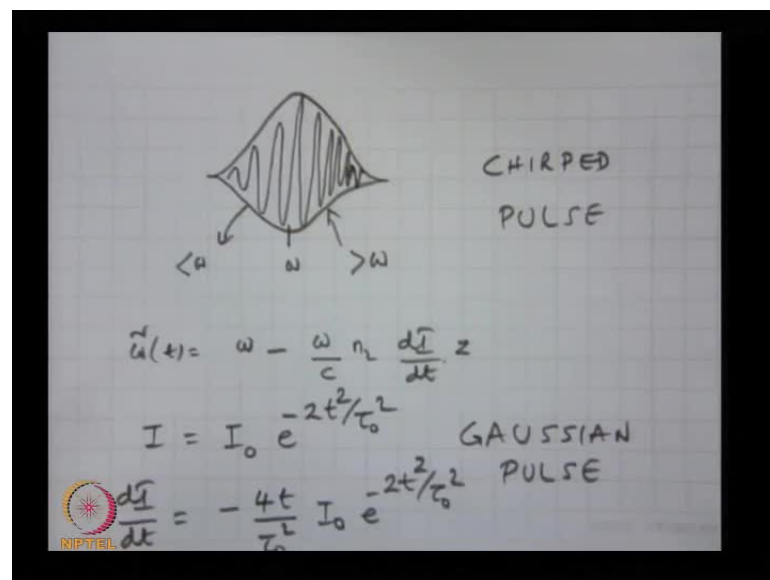
$$\delta\nu = 2 \times 10^{12} \text{ Hz} \Rightarrow \delta\omega \approx 12 \times 10^{12} \text{ rad/s}$$

$$\tau_{\text{min}} \sim \frac{1}{\delta\omega} = \frac{1}{12} \times 10^{-12} \text{ s} \approx 0.1 \text{ ps}$$

Logo: NIPTEIL

So,  $\Delta\lambda$  is equal to  $\lambda$  by  $\nu$  into  $\Delta\nu$ , which is equal to  $\lambda^2$  by  $c$  into  $\Delta\nu$ , this is  $10^{-12}$  by  $3 \times 10^8$  into  $2 \times 10^{12}$ , that is about 0.6 nano meters, what is the minimum width to which this pulse can be suppose, I assume that this is a spectral width of the pulse 2 terahertz, if I have a pulse with  $\Delta\nu$  is equal to  $2 \times 10^{12}$  hertz this corresponds to  $\Delta\omega$  of the order of  $2\pi$  times this, that is  $12 \times 10^{12}$  radians per seconds, what is the minimum pulse width to which this pulse can be compressed is of the order of  $1/\Delta\omega$ , which is  $1/12 \times 10^{12}$  seconds which is about 0.1 picosecond, what is actually happening is I start with the 1 picosecond pulse propagate through this medium, it remains 1 picosecond approximately but, its spectral width increases because of this surface modulation because of nonlinear effect the 1 picosecond pulse, which was launched has increased its spectral width  $\Delta\omega$  the output is more than  $\Delta\omega$  the input. Now once I have a broaden spectral width in principle actually, I can compress it 2 of it of the order of 0.1 picosecond, this is a very standard technique used in pulse compression, you start with a pulse use Some effect nonlinear effect to increase the spectral width of the pulse and then compress it.

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Now how do I compress the pulse like this?, look at this pulse this pulse has let me show the other figure this pulse has a frequency spectrum like this, leading like this, say your pulse coming in leading edge has lower frequencies trailing edge has higher frequency, let me draw these lines, this is higher frequency, the pulse is going like this in space this

is lower frequency higher frequency leading edge is lower frequency trailing edge is higher frequency. Now how do I compare this pulse? I must pass it through a medium in which the higher frequencies travel faster than the lower frequencies, what will happen is this will get compressed this portion of the pulse should travel slower compared to this portion of the pulse, leading edge is slowed down slightly the trailing edge is made faster and the pulse will compress, I need a medium in which there is a strong dispersion, I can use a medium in which higher frequencies travel faster than lower frequencies and appropriately, if I chose the length of the medium or whatever it is that dispersion because if the velocity of the light depends on frequency it is dispersion, I need to achieve a dispersion which can help me to compress this, pulse to the smallest width possible, people use a pair of gratings appropriately chosen gratings to compress a pulse which has a higher spectral width, you have a nonlinear medium first increasing the spectral width and then you use a dispersive element to compress the pulse.

So, what you are doing is essentially? you are not generating energy here, all you are doing is redistributing and making it compressed, the peak order will increase if there is no loss the peak order will increase the spectral width will decrease, in many applications you need pulses which are tens of femtosecond long this is 100 femtoseconds, 0.1 picosecond is 100 femtoseconds, actually you can use this pulse compression technique if you have an appropriate medium which introduces a lot of these nonlinear effects, I increase the spectral width by using a nonlinear effect and then, I use dispersion to compress the pulse, this is one standard technique which is used for pulse compression and that is based on the surface modulations

Now, what we will not discuss is more details of surface modulation which actually lead to the concept of what are called as Solitons, I left it as a possible term paper for all of you can actually use this nonlinear effect to generate pulses of light which will not disperse as they propagate through the medium normally, if you launch a light pulse into a medium because the light pulse contains many frequencies and because each frequency travels with a different speed the pulse which is launched actually, becomes broader in time, if you launch a one picosecond pulse it may become 2 picoseconds, 5 picoseconds as it propagates through the medium because this 1 picosecond contains a spectrum of frequencies, Some frequencies travel faster, Some frequencies travel slower, effectively the pulse gets broadened, this is called dispersion this is a very important effect in optical

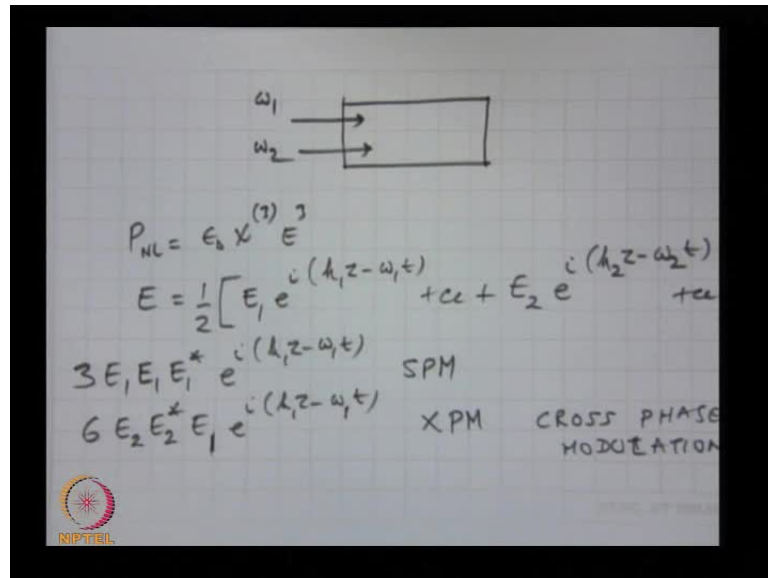
fiber communication because in communication you use digital signals and those signals start to overlap there is interference between adjacent pulses.

You can use this nonlinear effect to overcome this dispersion this chirping is playing a very important part there, I will not discuss that, these pulses which do not disperse as they propagate are called Solitons and this is a very important area of research in many in communication and also, there are Soliton lasers, actually you can also use this to overcome diffraction, if you launch a light beam it deflects because of diffraction you can have sufficient power in this to use surface modulation to compensate for this diffraction, Soliton a temporal Soliton is a pulse of light which propagates without dispersion in time a spatial Soliton is a beam of light which does not suffer diffraction in space it is a same problem one is in time domain and one is in spatial domain

So, diffraction in spatial domain is Something like dispersion in the temporal domain the pulse broadens in temporal domain the beam diffracts becomes wider in the spatial domain, you can use this effect to overcome the broadening in time domain leading to what are called as temporal Solitons you can use this to overcome diffraction in spatial domain which is called as spatial Soliton, these are very interesting effects which we will not go through in detail in this course but, this with this background that I have given here it should be possible for you to use this background to understand some of these features

So, do you any question on this otherwise i will just introduce what we are going to talk next and then which we will discuss in more details in the next class ok?

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So, next what we will going to do is instead of one frequency I want to put in 2 frequencies, what will be effect of this, P nonlinear is epsilon 0 **chi** 3 into the total electric field cube E cube, what is E corresponds to now E 1, E 2 the power i, k 1, z minus omega 1 t plus, complex conjugate pulse E 2 exponential i, k 2, z minus omega 2 t plus, complex conjugate this is the electric field of the omega 1 wave this is the electric field of the omega 2 wave now, can you tell me, if I substitute this into this equation what are the terms I will generate what frequencies

For example this into this, that is this square into this what will that be omega term which will be surface modulation, it will be of that form E 1, E 1 star, into you have exponential i, k 1 z minus omega 1 t, this is self phase modulation which you have just now discussed it does not depend on second frequency at all. What will be the factor multiplying out 3 a square b a plus, b plus, c plus, d. whole cube there is the 3 a square b, now I can also have, is there any other term at exponential i, k 1, z minus omega 1 t?, that is all right,

Yes So, i will have E 2, E 2 stare, i, k 1, z minus omega 1 t because you have canceled off by E 2 E 2 star ,you cancelled off the time and space dependence and all you are left with is this space and time dependence of E 1, what will be the factor 6, a, b, c , this lead to a nonlinear polarization at frequency omega 1 because of the presence of omega 2, the light at omega 2 Is changing the refractive index of the medium at frequency omega 1 of

this light wave and this is called cross phase modulation it is called cross phase modulation because  $\omega_2$  frequency the presence of  $\omega_2$  alter the propagation of  $\omega_1$ ,  $\omega_2$  is leading to a change in refractive index as seen by the  $\omega_1$  wave the first term is the  $\omega_1$ , wave sees a change in refractive index because of  $\omega_1$  wave self phase modulation this is the term in which the  $\omega_1$  wave sees a change in refractive index, because of the presence of  $\omega_2$  wave and that is called cross phase modulation.

So, if you launch 2, pulse 2 light waves into the medium you can actually alter the propagation constant  $\omega_1$  by, changing  $\omega_2$ , if I launch for example, a very weak beam at  $\omega_1$  and if I change the power of  $\omega_2$ , I will change the phase of  $\omega_1$  because, the refractory index as seen by  $\omega_1$ , now also will contain a  $\omega_2^2$  term So, i can actually modify the refractive index of the medium at a particular frequency by using another wave at a different frequency and this is used in optical switching. I can use this effect to switch light at  $\omega_1$  from one port to another port by using interference effects.

We will stop here i have also come out with another term here which is a very interesting term which is called the four wave mixing term and, we will not discuss much more detail of cross phase modulation but, the 4 wave mixing we will discuss a little more detail because, as you can see here this is already self phase matched right the 4 wave mixing term, we will see is not phase matched automatically and I need to do something to phase match that term and we will discuss that because that has also important application in quantum information because, people use this effect this 4 wave mixing effect to generate thumb classical states of light using this process ok?.

Thank you.