Quantum Electronics
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Module No. # 04
Third Order Effects
Lecture No. # 20
Third Order Non - Linear Effects

So, what I would like to do today is to start the discussion on third order non-linear effects, which are very briefly introduced in the last class.

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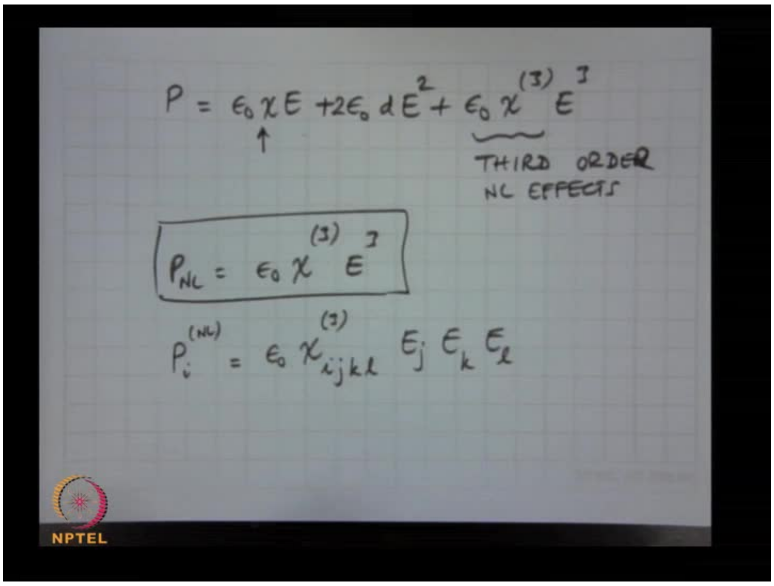

Remember, the general expression for polarization is epsilon 0 chi E plus 2 epsilon 0 d E square plus epsilon 0 chi 3 E cube. This is the linear term; this is the second order term; this is the third order term. (Refer Slide Time: 01:03) This term is absent in media processing a center of inversion symmetry. So, those media, the first non-linear term is the E cube term, and this is the third order non-linear effect term, non-linear effects.

So, if you take, for example, any liquid, glass, then it does not possess the second order effect; all media possess a third. So, even the glass that is used in optical fibers for example, will possess this effect. As I was mentioning, this causes problems in communicating through light using optical fibers.

So, what I would like to do is to discuss some of the effect that this leads to; we will not go into too much details, but I will present you some effects like self wave modulation or four wave mixing that are apparent from this, this term here.

Some of the application which I gave first - the phase conjugation is actually coming from this term. So, there are many other terms; optical bi-stability can be used, for, from this term and so on. There are topics which I have given you, which are effects of this in an optical system. So, Let us start with considering, so, the non-linear polarization term that we are going to look at is epsilon 0 chi 3 E cube.

Now, I am using a scalar notation here, but if were to write in the complete vector form, I would have to write like this - P i non-linear is equal to epsilon 0 chi 3 i j k l E j E k E l. This was 2 epsilon 0 d i j d i j k E j E k. Now that is a chi i j k l E j E k E l. So, that is a 3 cross 3 cross 3 cross 3 matrix. It is a tensor.

And, so this, at least some elements of this matrix are nonzero for every medium. So, you can have, for example, the chi matrix for isotropic media for crystals, for whatever it is, for different media, the chi matrix has some elements which are 0 because if we have a completely isotropic medium - a medium which has complete symmetry in all directions, then that symmetry should be represented in this matrix. Whether I choose x y z in this orientation or x y z in this orientation or any other orientation, the chi matrix should be independent of my x y z orientation, if the material is completely a homogeneous and isotropic and it has no symmetry problems axis at all. So, by using these symmetry arguments, actually one can calculate the chi i j k l matrix and exactly like we did - what we did for the second order non-linear effects. We would have to use the chi i j k l matrix along with the orientation of electric vectors to calculate what are the non-linear polarization generated. And, so, depending on the material and the orientations, there will be polarization generated in the medium.

Now, we will not look at the details of this. But we will use this equation (Refer Slide Time: 04:59) - this scalar equation - assuming that the chi 3, that I am using here is an effective chi 3 coming from one of the elements are a combination of this; just like we did for the second order effects and E is the corresponding electric field and P non-linear is the polarization that is generated.

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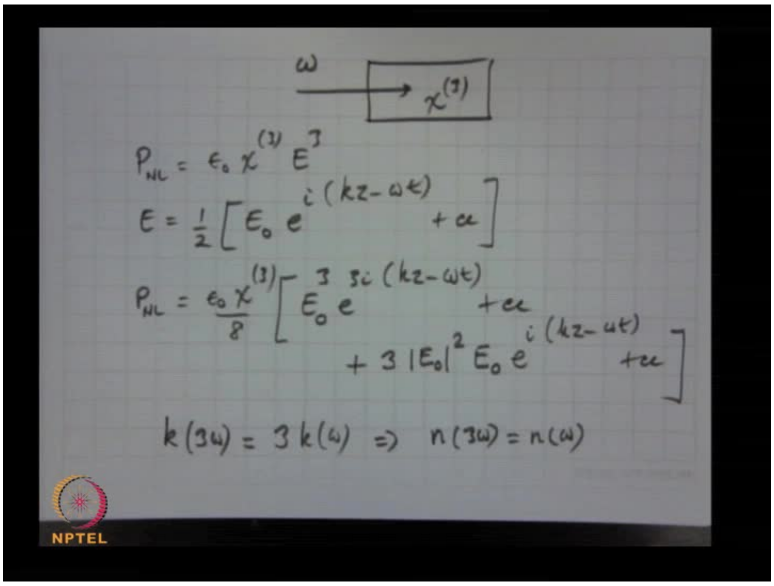

So, that will simplify the analysis significantly and we will get to understand what the primary effects are and what does it lead to. So, the first thing, let us start by again just like we did for second harmonic. Let us start (( )) a medium and launch a wave or frequency omega; so, this is the medium possessing chi 3, there is no chi 2, there is no d element here.

So, the non-linear polarization is epsilon 0 chi 3 E cube. So, E is electric field which is half E 0 exponential i k z minus omega t plus complex conjugate. In the other case, it was epsilon 0, 2 epsilon 0 d E square. Now, it is simply epsilon 0 chi 3 E cube; that is the only difference. So, what are the terms that I will get in the non-linear polarization? Epsilon 0 chi 3 by 8, so, I will have, so, it is a plus b whole cube. So, I have E cube E 0 cube exponential 3 i k z minus omega t plus complex conjugate plus - what is the term I will get?

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3 mod of E 0 square into E naught cos k z 2 by k z minus

a cube plus b cube plus 3 a square b plus 3 ab square. (Refer Slide Time: 07:15) This is the term that will lead to third harmonic generation. What is a phase matching condition? What condition would I have to satisfy, if I want this term to generate third harmonic?

k is equal to 2 c

The propagation vector of 3 omega term must be equal to 3 times propagation vector of the omega term. So, can you tell me what this is in terms of refractive indices?

N at 3 omega is equal to n of omega. So, if I want to use this process for third harmonic generation, I can do so provided, I have this conditions satisfied.

Now, usually in media like glass, liquids, this is never satisfied, because n of g omega is not equal to n of omega, because of dispersion. So, usually, this term does not lead to the generation of third harmonic; there is polarization, there is non-linear polarization, but it does not lead to the generation of the electromagnetic wave at that frequency, because you are not phase matched; please note: individual dipoles are oscillating at the frequency 3 omega. They are all trying to radiate at frequency 3 omega, but the radiation coming from all these collection of dipoles is cancelling of each other. And so, there is no electromagnetic field at frequency 3 omega. So, this term usually does not lead to any generation, because normally I will not be satisfying this condition. So, there is no problem, that is a third harmonic term. What about this term? What is the frequency of this? It is omega.

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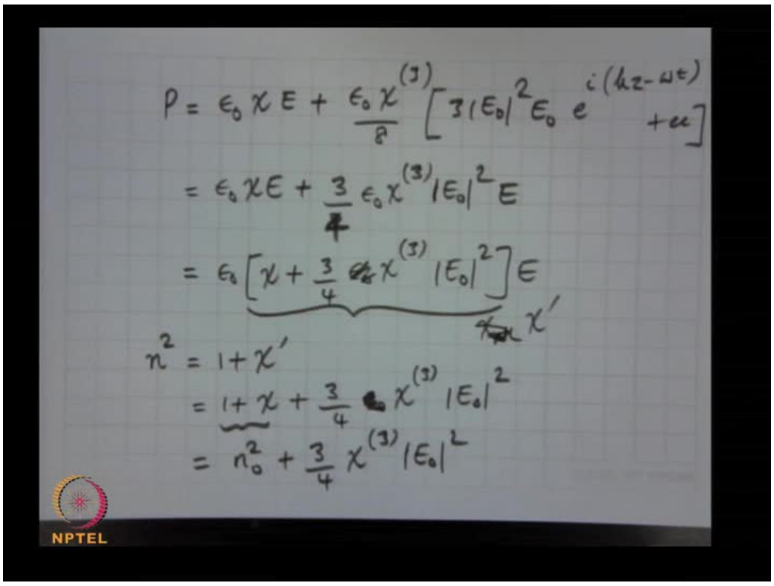

So, this is the non-linear polarization at omega. Now, what is the total polarization frequency omega? So, the polarization at frequency omega is epsilon 0 chi E plus epsilon 0 chi 3 by 8 into 3 mod E naught square E naught exponential i k z minus omega t plus complex conjugate.

Now 3 mod E naught square is real, so, it has the same term in the complex conjugate term. Actually, this second term is plus 3 by 8 epsilon 0 chi 3 mod E 0 square into (Refer Slide TIME: 00:00) So, if I write 3 by 4 here, what is the remaining term? Half E 0 exponential i k z minus omega t plus complex conjugate which is E. E is half E naught exponential i k z minus omega t plus complex conjugate 3 mod E naught square is real, so, I can take it out; it is common between this term and the complex conjugate term and the remaining factor with the half is nothing but E.

Now, first note that this is a non-linear polarization at frequency omega and propagating at a velocity omega by k. So, what wave will it try to generate? It will try to generate wave at omega frequency, because that is a non-linear polarization at frequency omega and it has a velocity omega by k, which is exactly equal to the velocity of the electromagnetic field at omega; it is automatically phase matched.

This effect will always contribute, because even if I do not do anything, the velocity of this non-linear polarization term is exactly the same as the velocity of the electromagnetic field that it is trying to generate and it is completely phase matched.

I can write this equation as epsilon 0 chi plus 3 by 4 epsilon 0 chi 3 mod E naught square into E. So, in the presence of this term, epsilon 0 chi 3 E square E cube term, it looks as if the susceptibility had got modified; the susceptibility was chi; for, if I neglect the third order effect, otherwise is this. And how is susceptibility related to the refractive index? So, this is the effective susceptibility in the presence of nonlinearity.

How is the refractive index related to chi epsilon the susceptibility?

n square is equal to 1 plus chi. (Refer Slide Time: 13:10)

So, let me, if I call this chi effect, chi, chi nonlinear, no, if I call this chi prime, so, this is the one this medium. Now, as a refractive index n square which is 1 plus chi prime, which is 1 plus chi plus 3 by 4 epsilon 0 chi 3 mod E naught square.

What is 1 plus chi, the refractive index of the medium, if you neglect nonlinearity?

Sir, E naught in that.

Yes sorry E. So, 1 plus chi, I call it n 0 square; n 0 is the refractive index of the medium for low intensities. If E naught is very small, this term is negligible and the refractive index is n naught. So, n naught is the refractive index of the medium at low intensities and as you increase the intensity, the refractive index becomes the function of the intensity, and this is given by this equation.

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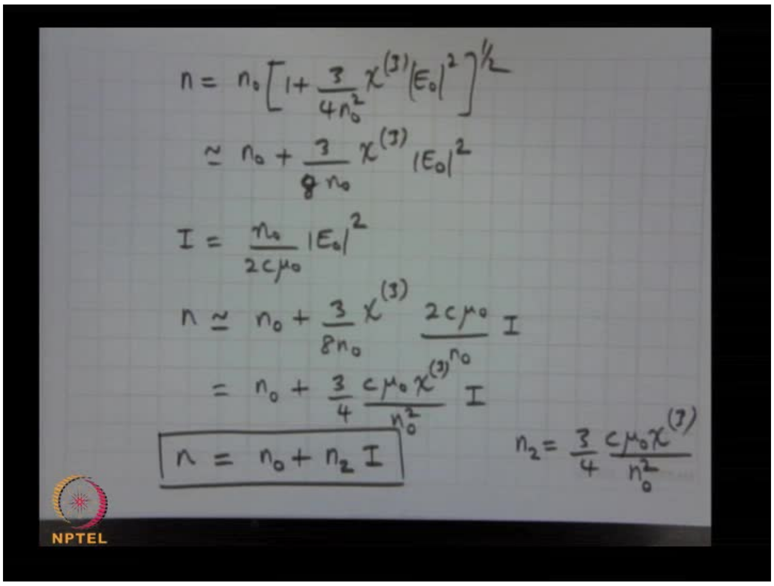

Now, usually this term chi 3 is very small, and so, this term is much smaller than the refractive index of medium. So, I can make an expansion and write this as n is equal to n 0 into 1 plus 3 by 4 n 0 square chi 3 E naught mod square this per half; so, which is approximately n 0 plus 3 by 4 n 0 3 by 8 n 0 chi 3 mod E naught square, because this term chi 3 is very small, chi 3 time mod E naught square is very small, I will give you some numbers; so, this is an expansion and I can actually relate E naught to the intensity of the wave. Intensity of the wave is nothing but n naught by 2 c mu 0 mod E naught square.

Here I am using n naught, I am not using the modified refractive index, because the change is actually extremely small. So, n is equal to is approximately equal to n naught plus 3 by 8 n 0 chi 3 into 2 c mu 0 by n naught into I, which is equal to n naught plus 3 by 4 c mu 0 chi 3 by n naught square. And, I, usually, this is written as n 0 plus where n 2 is this term here. So, n 2 is equal to 3 by 4 c mu 0 chi 3 chi n 0 square.

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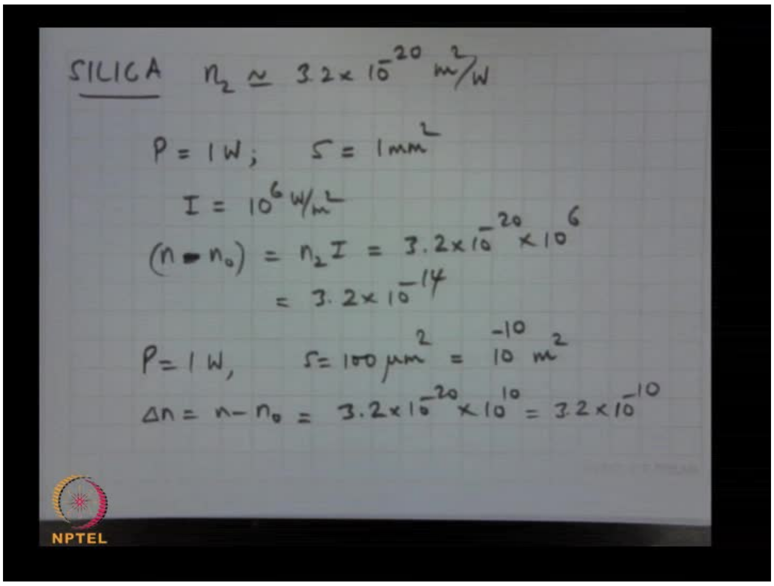

This leads to what is termed as intensity dependent refractive index. The refractive index of the medium gets changed, if you change the intensity, and that comes in because of the chi 3 term. Now, let me give you some values for n 2, for silica, the value of n 2 is about 3 into, so, n 2 for silica. What will be the unit of n 2?

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Square by per joule meter square per

Meter square per what? 3.2 into 10 to the minus 20 - see extremely small. Let me calculate the change in refractive index that will take place. So, what value intensity should I choose? Let me assume certain power focused into a certain area; so, let me take a power of 1 watt and what area should I choose? 1 millimeter squares - good. So, the intensity will be, What is the intensity in watts per meter square?

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So, n will be n minus n naught. The change in refractive index because of nonlinearity is n 2 times I, which is 3.2 10 to the minus 20 into 10 power 6, which is, that is a ridiculously small value change, but let me assume that I have this change taking place over a certain length of medium and that is where the optical fiber comes into picture. I can have light propagating over a long length in an optical fiber without getting diffraction, without undergoing diffraction.

For example, if I put 1 watt of power into an optical fiber, what is the area I will take? For an optical fiber, the area is instead of 1 millimeter square, typically 100 micrometer square, even less than that. So, let me take an area of, so, power is 1 watt, an area of 100 micro meter square. So, this is 10 to the power minus 10 meters square.

So, delta n is equal to n minus n naught is equal to 3.2 10 to the power minus 20 into 10 to the power 10, intensity is 1 by 10 to the power minus 10; so, that is 3.2 into 10 to the the minus 10 not much bigger but is of this order.

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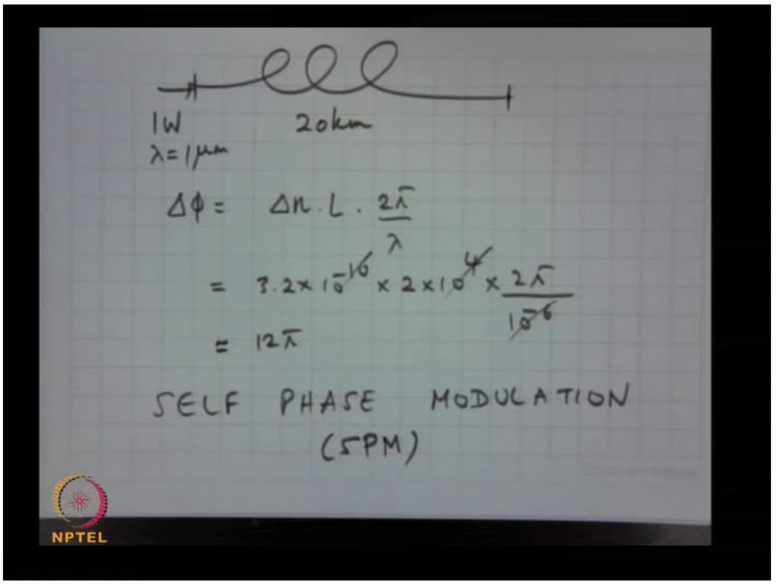

Now, in an optical fiber the light can propagate over long lengths. Let me take a typical length of 20 kilometer. I take an optical fiber, I launch 1 watt and this length is 20 kilometer. Now, let me neglect the attenuation the fiber, let me neglect the fact that the power is decreasing as it propagates; if I neglect that, so what will happen, suppose, I will start with 1 milli watt of power, look at the output light, and I go to 1 watt of power. What will happen here?

I launch, 1 milli watt either light wave coming out, instead of 1 milli watt, I make it 1 watt. What change will I see here?

More than 1000 times.

The power is more, anything else I see? Numerical aperture is, you see, numerical aperture does not depend in principle on the power that I launch; that is fixed on the fiber.

Properties.

So, what is the power doing? It is changing a refractive index. So, what will it do at the output? Something will change.

It will increase, it is not changed.

It will increase the refractive index, so what will happen?

(( ))

No that change in refractive index. Please remember it is 10 power minus 10, so that cannot change the spot size much because the refractive index is 1.45 plus 10 power minus 10.

(( ))

Exactely.

If you have 1 milli watt of power, the light comes out at the certain phase change; if you make it 1 watt, the output phase will change. What is the change in phase, that I will see? How does the phase depend at the output, depend on power now?

Delta n into length; no, that is not change in phase, something is missing.

(( ))

Into right (( )). So, let me assume lambda 1 micrometer. So, let me calculate this, 3.2 10 to the minus 10 into 2 into 10 to the power 4 into 2 pi by 10 to the minus 6. So, about 12 pi, huge change.

Remember pi phase change you can go from maximum interference to minimum in interference and 12 pi is a huge phase change. And that is happening because of this factor here. This factor is 10 power 10; L by lambda is 10 to the power 10, approximately. Because the wavelength is very short compared to the length of propagation, the cumulative effect of this phase, of this change in refractive index can be very significant.

So the output, the light at the output the phase of that high light output can be changed by changing the input power. What is the light doing? The light is changing its own phase, it is called self phase modulation. The light enters the medium, changes its refractive index and hence gets effected by the change of refractive index. The phase at the output will change as to change input power; it is called self phase modulation - SPM. And that is a very important effect in optical fibers that you can change the phase of the output light, by changing the power. Normally, you change the phase by changing the temperature of big medium or putting a strain on the medium or changing the path length, changing the wavelength. Here, I am not doing anything, I am just changing the power level, the output phase changes.

For example, if I were to use this in a part of an interferometer, I can go from constructed to destructive interference, simply by changing the power of the light wave; which means, I can switch a light from one path to another path; if it is a part of a interferometer, if it is constructive interference, it comes out here; if it is destructive it comes out here.

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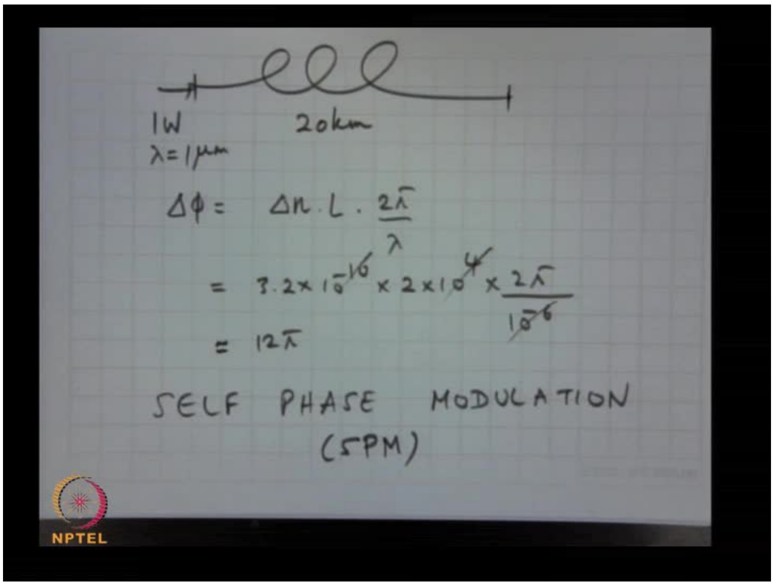

So, I can go from constructive to destructive, simply by changing the power level; that is an example of all optical switching. I can switch the light beam from one port to the other port, simply by changing the power level of the light. So, I am not doing any other switching; and that switching takes place extremely fast, because the changes that we are talking off are extremely rapid. These are electronic transitions, electronic effects and these are extremely fast effects.

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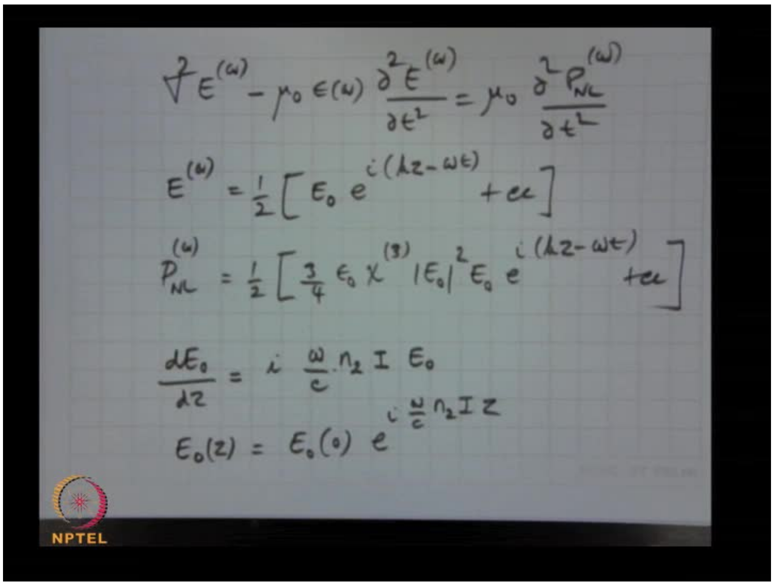

There is no restriction here of the speed. It is very extremely high speed. This is a very important effect and this for example, this change in refractive index, I could have also done in the following way, for example, what would I have normally done? I would have solved this equation, del square E of omega minus mu naught epsilon of omega del square E of omega by del t square is equal to mu naught del square P non-linear at omega by del t square.

Remember this equation we had derived - right at the beginning - where the electromagnetic field at a frequency and the non-linear polarization at the frequency, satisfy these equations. So, in my case, E omega is equal to half E naught E to the power i k z minus omega t plus complex conjugate and P non-linear at omega is equal to half 3 by 4 epsilon 0 chi 3 mod E naught square E naught exponential i. (Refer Slide Time: 27:09)

This is from this equation, which we had written just now. So, the non-linear polarization at frequency omega is in the second term here, and that is what I am writing with a factor of half outside. So, I use the same procedure as I did for study in second harmonic generation or parametric amplification, I substitute E omega on the left hand side, I substitute P non-linear omega on the right hand side, neglect the second derivative of E naught with the f into z, solve the equation.

Let me give you the equation that I will get, if I do this procedure. What I will get is this will lead to this. So, I leave this substitution and simplification to you so I get d E naught by dz is equal to i omega by c n 2 intensity into E naught. You just substitute into this equation, use the definition of n 2. What is the solution of this equation? If I is independent of z, if I assume no attenuation, what is solution of this equation? E naught 0 into (()) right; just I take the E naught on the left hand side and integrate omega c n 2 I are constants. So, get this integral and so the E omega, I can write expression now, I substitute this into this equation. This is the way E naught behaves as a function of z; please note here the amplitude is remaining constant the phases changing, depending on the intensity.

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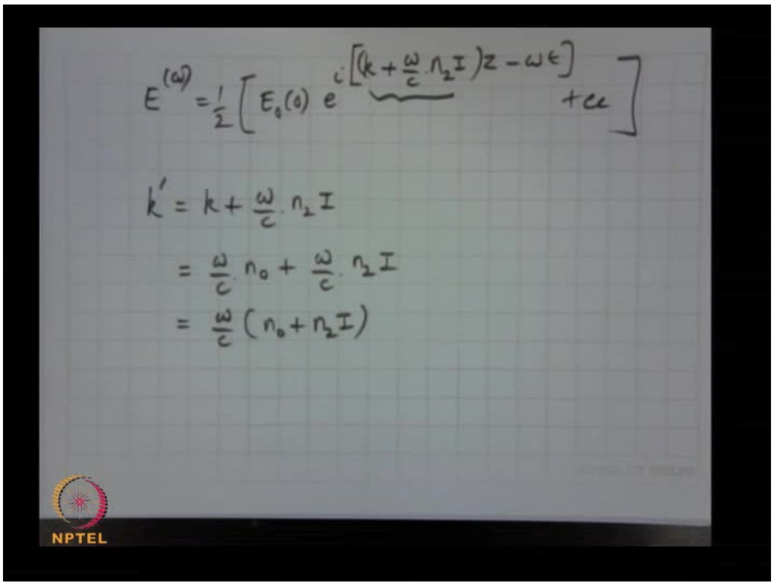

So, I substitute this term into this equation and write the total electric field expression E of omega is equal to half E naught of 0 exponential i. Now, I have k plus omega by c n 2 I into z minus omega t plus complex conjugate. This E naught has an additional term here z dependent term, I add that with the k z term and what do I get a propagation constant which depends on intensity.

So, if you define this as the propagation constant in the process of the chi 3 term, you can see that this lead as you see k omega by c is k naught. The propagation constant is free space; so the actual the propagation constant in the presence of this nonlinearity is k plus omega by c into n 2 I. k is the propagation constant for low intensities, which is omega by c into n naught plus omega by c into n 2 into I. This is nothing but omega by c into n naught plus n 2 I exactly what I showed you just now; that effectively the refractive index of the medium changes in the presence of this non-linear.

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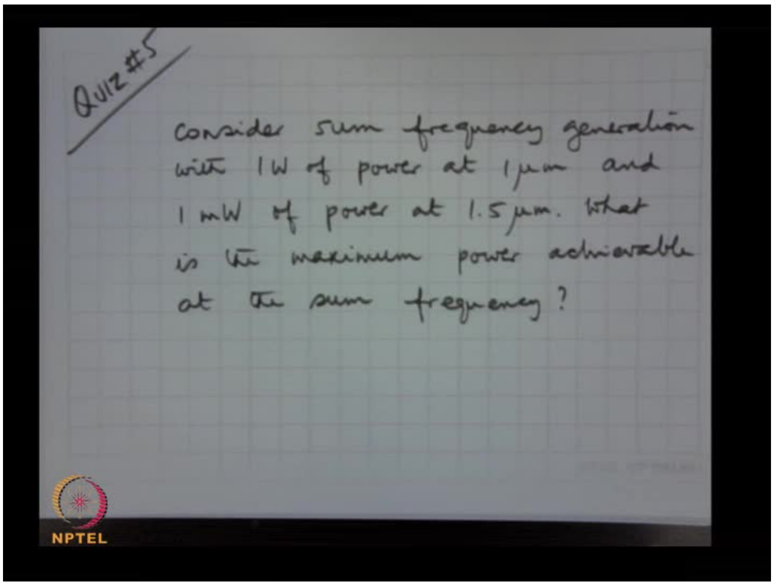

So, that is a very interesting self phase modulation effect and this particular effect leads to couple of topics which I gave you, self-focusing, it will leads to spatial solutions, it leads to temporal solutions, it leads to many effects. We will not discuss all this in the class, but I leave it as a topic for some of you to take up and read on your own and it is simply uses the fact that the refractive index of the medium now depends on the intensity. (( )).

This is a problem on sum frequency generation, this 1 watt of power at 1 micro meter and 1 milli watt of power at 1.5 micrometer incident. The process that takes place is sum frequency generation and what is the maximum power that is achievable at the sum frequency?