Quantum Electronics

Prof. K. Thyagarajan

Department of Physics

Indian Institute of Technology, Delhi

Module No. # 01

Brief review of electromagnetic waves;

Light propagation through anisotropic media

Lecture No. # 02

Anisotropic Media

So, this is the second lecture; do you have any questions for what we covered yesterday? So, yesterday, we just looked at the basic Maxwell's equations, and also looked at plane wave solutions. And we found the relationship between intensity and the electric field, and showed the relationship between the direction of k vector, E vector, H vector, and so on.

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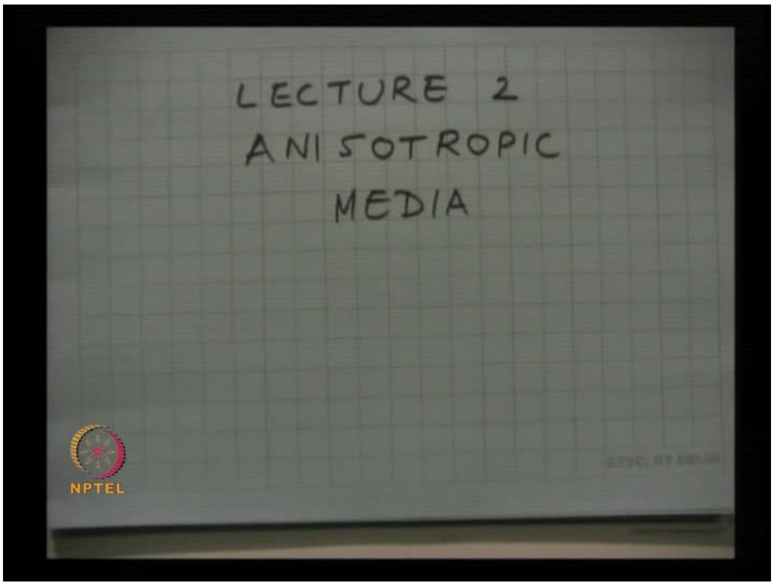

So, today, what we will primarily look at is an anisotropic media, but before that let us recall a bit of diffraction; although we will not be using too much of diffraction in the course, but I think we should be aware. Because remember, yesterday, we had seen that the intensity is proportional to the square of the electric field.

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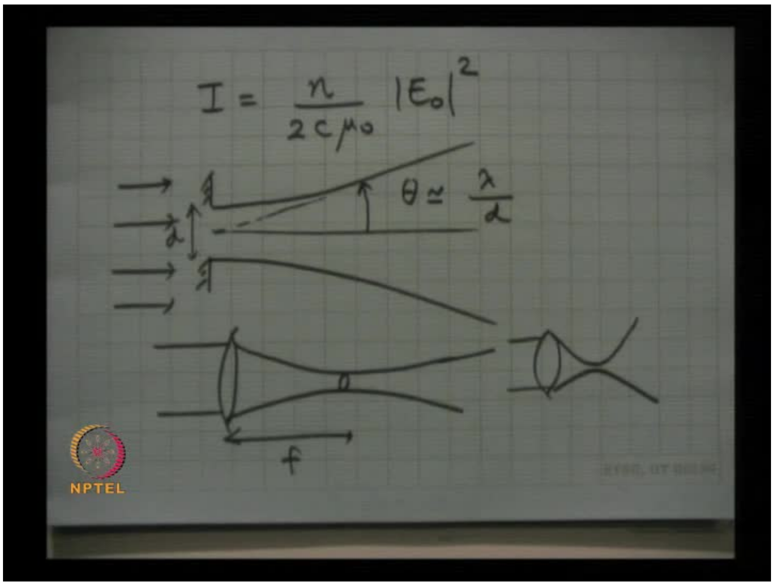

So, intensity we had shown yesterday is, I is equal to n by 2 c mu 0 mod E 0 square. The non-linear effects depend on the strength of electric field; the stronger the electric field the stronger are the non-linear effects. So, to increase the effects of nonlinearity, you have to increase the electric field, which means increase intensity. So, for a given power you can increase the intensity by focusing the beam more, because intensity is power per unit area.

So, if you take a certain power and focus it to a smaller cross section, you can increase the intensity and hence increase the electric field. So, if you focus a laser beam into a medium, you can increase the electric fields and hence the intensity, and hence the non-linear effects. But the problem is, if you try to focus a beam, then it will not maintain the cross section for a long distance and that is controlled by diffraction effects.

So, for example, if you take where diffraction comes in; so if I take a slit and send a plane wave into the slit, the beam actually comes out and diffracts like this; this angle is approximately lambda by d, the width of the slit. So, this angle lambda by d is called the angle of diffraction; the smaller the value of d, the larger is angle of diffraction. So, it is kind of an uncertainty principle between the width of the beam and the angle of diffraction; if you try to reduce the width, the beam reacts by diverging mode. So, if you take a beam - for example, so, how do you increase the intensity? You can focus it by a lens; so I take a lens and you have focus on the beam by a certain focal length lens.

So, I take a certain focal length lens, this must be the approximately the focal length here, and it gets focused. So, if I want to reduce the cross section of the beam, I will take a lens of a smaller focal length; so if I take a lens, which is a smaller focal length lens, then you can see that this focuses like this, and defocuses very quickly. This cross section, if you want to maintain a certain cross section over long distance, diffraction plays a role here, and does not allow you to keep the same cross section for a long distance. If you try to reduce the cross section, you pay a price by increasing the divergence.

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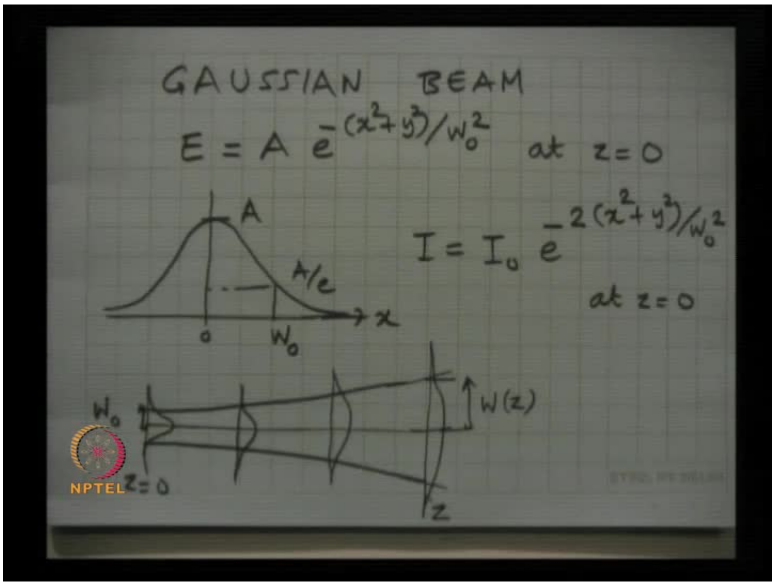

So, as an example, let me look at a Gaussian beam, which is the intensity distribution that is coming out of a laser is usually a Gaussian distribution. So, let me look at a Gaussian beam; so the electric field of a Gaussian beam is some amplitude exponential minus x square plus y square by w 0 square.

So, if you plot the electric field as a function of distance something like this, and this is A here; so this is a function of x and the distance at which it becomes A by e is called as w 0 here. This value w 0 is the distance from the axis, where the amplitude of the electric field decreases by a factor of 1 by e, and because intensity is proportional to square of electric field at a distance w 0 from the axis, the intensity goes down by a factor of 1 by e square; so this w 0 is called spot size of the Gaussian beam.

So, typically in a laser, when the laser beam comes out, w 0 is of the order of 1 millimeter; half a millimeter, 1 millimeter. So, the beam comes out and then this electric field actually is at one plane; so let me call this at z is equal to 0, this electric field does not maintain itself forever, the same x and y distribution will not maintain itself, because of diffraction.

So, we will not study diffraction in detail, but let me tell you here; let me just give you the formula, which gives me how the intensity varies with distance as the beam propagates. So, the intensity distribution of this Gaussian beam is given by I 0 exponential minus twice x square plus y square by w 0 square, where I 0 is the intensity on the axis; at x is equal to 0, y is equal to 0, the exponential factor is 1 and the intensity is I naught. And at x square plus y square is equal w 0 square, the intensity falls down by a factor of 1 by e square; so this is at z is equal to 0; so this is the Gaussian beam. So, let me plot here; so this is the Gaussian beam at z is equal to 0 here; so this is z is equal to 0 here, as the beam propagates it expands like this. So, here for example, the Gaussian beam will become like this, here it is slightly smaller here, that even bigger; the Gaussian beam keeps on expanding as it propagates and this is called diffraction divergence.

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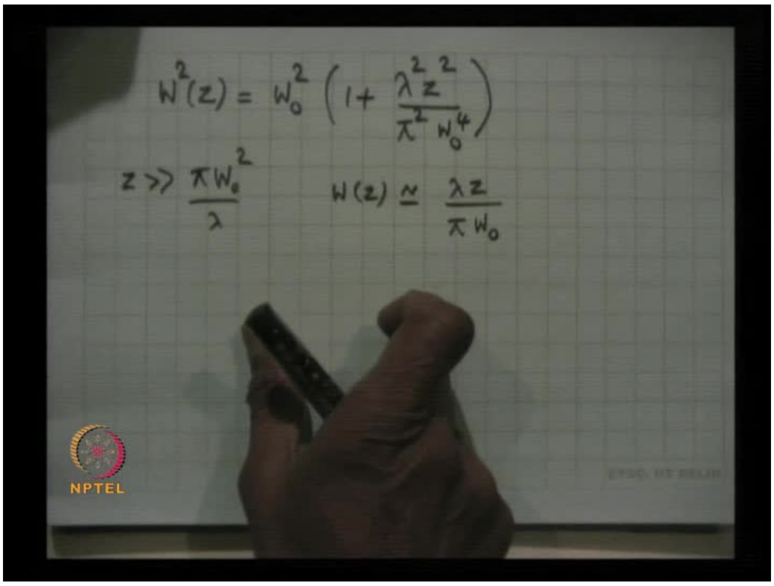

So, if this is w 0 here, suppose, this plane I call z, and this distance is w of z, then there is a relationship between w z and w 0, which is obtained by analyzing diffraction. So, let me give you the expression here; so w of z w square of z is equal to w 0 square into 1 plus lambda square z square by pi square w 0 4, the width of the Gaussian beam, the spot size of the Gaussian beam increases with z according to this formula; lambda is a wavelength of light in the medium in which the beam is propagating and z is the distance at which you are measuring the spot size. For large z, for z much greater than pi w 0 square by lambda, the second quantity in the bracket becomes much more than 1, and w of z can be approximated as lambda z by pi w 0.

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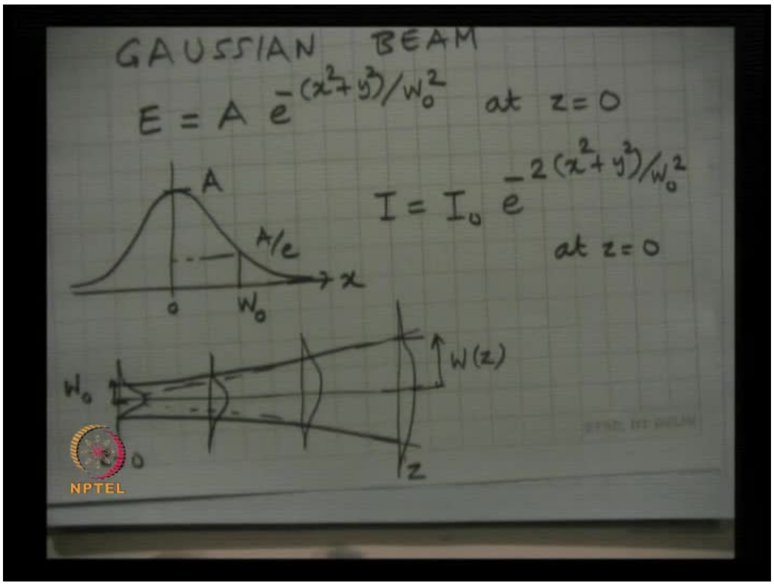

I can neglect one in comparison to lambda square z square by pi square w 0 4, and w of z becomes lambda z by pi w 0. So, the spot size increases linearly with z for large distances; for small distances the spot size does not increase linearly with z; it is a formula, which 1 plus lambda square z square by pi square w 0 4. This is why, I plotted here; it starts almost like this and then starts to expand, and then what is called as the far field in the Fraunhofer region, this spot size increases linearly with z.

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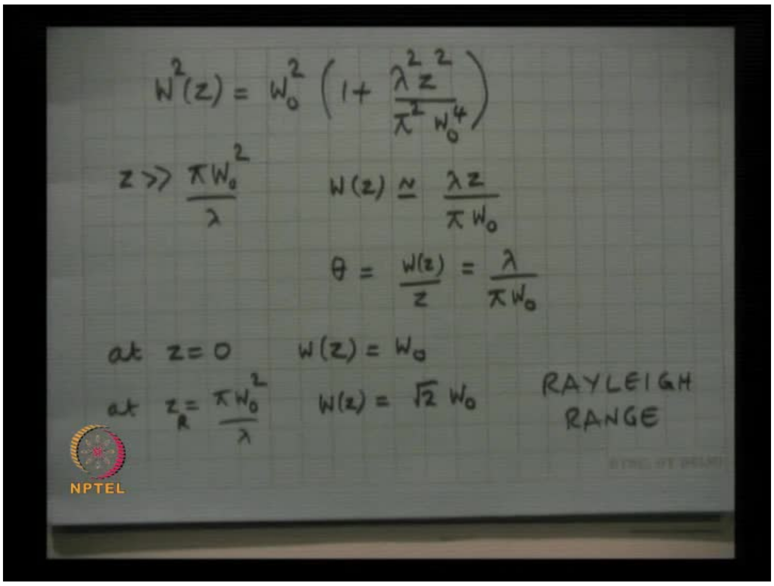

So, I can define the angular diffraction here, is equal to w of z by z, which is equal to lambda by pi w 0, is of the order of wavelength divided by the size of the Gaussian beam. The spot size lambda by d is a very approximate formula depending on the shape of the beam; if it is Gaussian, it is lambda by pi w 0. If you take circular aperture, it is 1.22 lambda by the diameter of the circular aperture, and so on.

So, these are factors come in because of more precise estimations, but the angular diffraction is approximately lambda by the width of the beam. So, you see this particular expression tells me that… So, let me calculate, what is the distance at which the spot size becomes root 2 times the input? So, at z is equal to 0; for example, your w of z is w 0; so at z is equal to pi w 0 square by lambda, what will be the value of w z?

So, this factor becomes 1; so that is root 2 times w 0; so the spot size will increase by a factor of root 2 in this distance, and let me put a subscript here, this is called the Rayleigh range. The distance from the plane z is equal to 0, at which the spot size increases by a factor of root 2; please note here that z is equal to 0 is special, because that is, this position, where w z is minimum; for z positive and negative, the spot size is more. So, this z is equal to 0 plane is also called as the waist of the Gaussian beam. So, usually in a laser, the waist of a Gaussian beam could be inside the laser at the mirror or outside the laser, depending on the laser construction the waist of the Gaussian beam could be inside the laser cavity or the mirror on one of the mirrors of the cavity or outside the mirror outside the laser, but depending on the w 0 value. So, w 0 is the spot size at z is equal to 0, which is the minimum spot size called the waist of a Gaussian beam. So, this tells me, this is a distance over which your spot size will increase by a factor of root 2; smaller the w 0, smaller is the Rayleigh range. See, if you try to restrict the beam to small diameter, you will diffract much faster.

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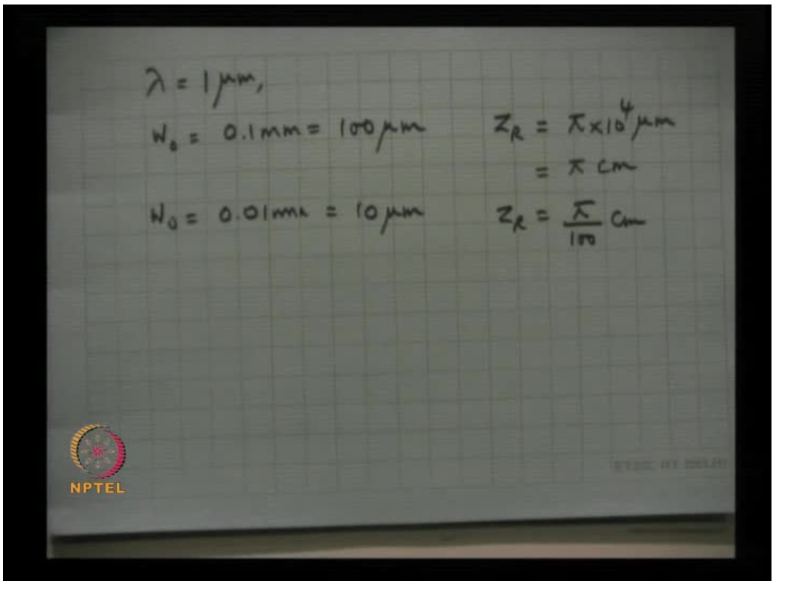

So, for example, if you take a wavelength of 1 micron, lambda is equal to 1 micrometer; so let me take w 0 is equal to 0.1 millimeter, which is 100 microns. So, the Z R comes out to be pi into 10 to the power 4 micrometers, which is pi centimeter. So, if you start from the x from z is equal to 0 over a distance of pi centimeters, which about 3.1 centimeters, the spot size becomes root 2 times larger than at the axis. If you reduce w 0 to 0.01 millimeter, which is 10 micrometers, because you have decreased w 0 by a factor of 10, Z R will get reduced by factor of 100; so Z R becomes pi by 100 centimeter.

So, you cannot have a 10 micron beam over a large distance, because by the distance of pi by 100 centimeters the beam has increased its cross section, the diameter by root 2 factor. So, when you try to focus a beam to increase the intensity, you have to be conscious of the fact that intensity is not maintained over long distance, the beam will diverge very quickly; so there is the relationship between the distance over which you can maintain the cross sectional dimension of a beam approximately to within a certain cross section and the cross section area itself. So, it is not always that you can increase the focusing, to increase the intensity and electric field, but then you will lose in terms of how much length you can keep that intensity alive. So, many times what happens is, people use waveguides to maintain a cross section over long distances, because in a waveguide like an optical fiber there is no diffraction; you have total internal reflection, which is not allowing the beam to diffract, and a cross section can be maintained over a very long distances.

So, for non-linear interactions also, people work with waveguides; these are waveguides with small lengths a few 5 centimeters or 10 centimeters and a cross sectional area could be as small as the few micrometer square; diffraction will not play any role there, because the wave is guided by total reflection and so the beam cross section does not change at all over the entire length.

So, you can have much more increased non-linear effects; if you maintain the cross section over longer distances and that is achievable in the form of waveguides. So, there are a number of people who work on waveguides to have a strong non-linear interaction especially in optical fibers. For example, you have very strong interactions, in-fact, the non-linear effects in optical fibers restrict the overall communication link that you have, how much of communication you can achieve?

So, diffraction will play a significant role in all optics experiments on and techniques and one has to be conscious of this fact, whenever we desire experiments on, when we look at numbers I cannot assume. I have an area of a diameter of 10 microns and take a length of interaction of 5 centimeters it is not possible, unless I have a waveguide, unless I know that I can restrict this. If I have free space propagation, if I have homogeneous medium, I cannot maintain this.

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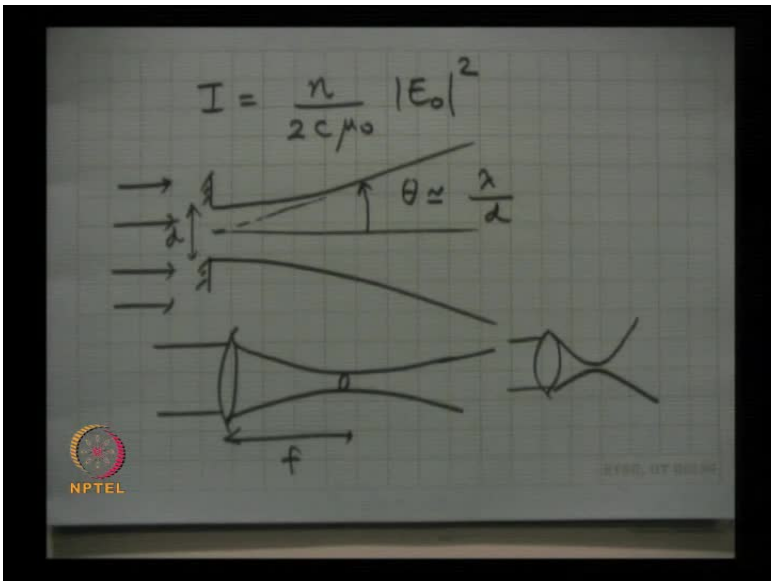

So that I have to be little conscious of, in-fact, what I called as uncertainty relationship between the angular diffraction and the width of the beam is kind of a Fourier transform relationship. It is a Fourier transform relationship, the angle of diffraction, the divergence and the width here, are Fourier transform pairs; Just like, you have frequency and time; if you have a short pulse, you have a large spectrum; if you have long pulse, you can have a short spectrum.

Similarly, if you have a narrow beam, you have large angle diffraction; if you have a wide beam, you can have small angle of diffraction; you cannot beat this. So, this diffraction is present in all systems, in all situations and whenever we design experiments or when we think of numbers, we need to be aware of this concept.

So, I would not go much beyond this in diffraction, because we will not discuss more of details of this, but as I was mentioning, we need to be conscious of this. So, we will not touch upon diffraction anymore, except that when we look at the intensity levels in non-linear effects, we will at that time bring back diffraction at that time.

So, now, what I am going to start is essentially a discussion on anisotropic media. Now, as we will see some of the non-linear effects require certain conditions for efficient interactions and those conditions require us to use media, which are anisotropic. Now, what is the meaning of anisotropic medium?

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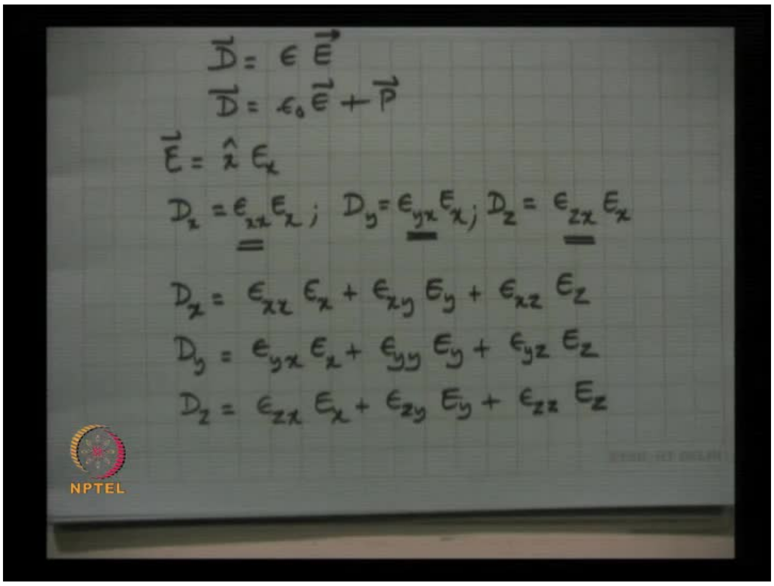

You see, yesterday, we wrote this equation D is equal to epsilon E, and I said that epsilon is a scalar; so this means that no matter, which direction I apply an electric field, the D vector is parallel to the electric field. Now, in many crystals this does not happen; if I apply an electric field like this, the displacement vector could be different orientation and that comes in, because remember D is nothing but epsilon 0 E plus P, polarization. And what is polarization? Polarization is the dipole moment per unit volume; so when an electric field is applied to an atom, there is a displacement of positive, negative charge centers that make dipoles and usually the dipole is oriented along the direction of the electric field. But if the structure is not symmetric, you may apply an electric field like this and the dipole gets generated in some other orientation, because it has restoring forces from all directions, which may not be all symmetric.

So, in that case, what will happen is, P is no more in the direction of the electric field and such media are called anisotropic. So, P and E are in general not parallel in these media; so this depends on the crystal structure and there are many crystal structures in which this will happen; there are crystal structures in which this does not happen, because of symmetry, but a majority of crystals possess anisotropy.

So, in general, which means that suppose, I take a medium and apply an electric field, which is along the x direction; suppose, like this is x cap E x, what you will find is, you will find a finite D x, a finite D y, and a finite D z in general. So, let me put a constant here, epsilon x x E x; epsilon y x E x; epsilon z x E x; so, these are three constants. So, I apply an electric field along a direction x, some arbitrary direction, which I call x and I get displacement vectors along all three directions, that means the displacement vector has all three components D x, D y, and D z. Similarly, if I plan in the y direction, I will have another three components D x, D y, D z and along the z direction. So, in general, I can write; if I have a general electric field, I can have epsilon x x E x plus epsilon x y E y plus epsilon x z E z. So, this is the contribution to D x from E x, contribution to D x from E y, contribution to D x from E z. Similarly, D y will have epsilon y x E x plus epsilon y y E y plus epsilon y z E z, and D z is equal to epsilon z x E x plus epsilon z y E y plus epsilon z z E z. So, these equations tell me that, if I have some arbitrary coordinate system x, y, z, I apply an electric field, which has components E x, E y, and E z; this electric field can in general generate all three components of a displacement vector D x, D y, D z all of them depend on all the three components of the electric field.

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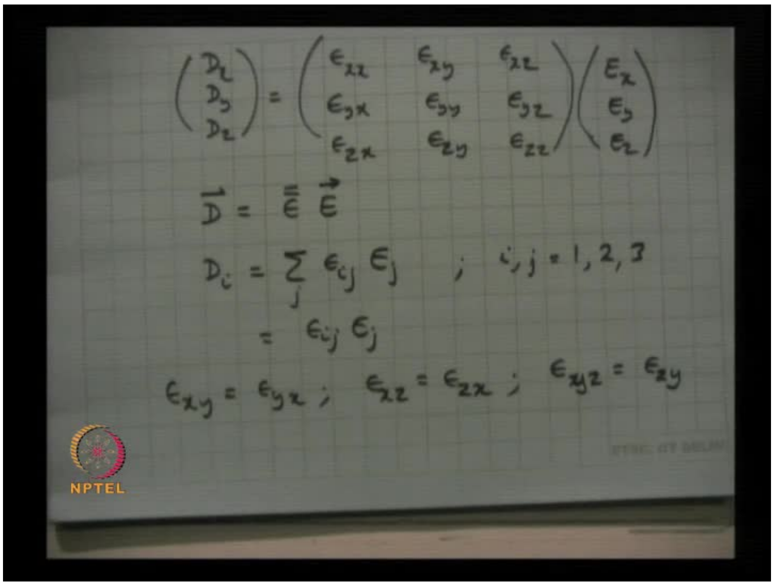

So, in-fact, I can write this in a matrix form; this will become D x D y D z is equal to epsilon x x epsilon x y epsilon x z, epsilon y x epsilon y y epsilon y z, epsilon z x epsilon z y epsilon z z into E x E y E z in a matrix form; this is the same three equations. So, in-fact, I can write this as this is a D vector is equal to epsilon into E vector, where epsilon is now, this 3 by 3 matrix; this column vector is the D vector; this column vector is the E vector, and that is the epsilon tensor. So, in component form, this is actually D i is equal to sigma j epsilon i j E j. So, here i and j, they can take values 1, 2, and 3. So, many time this is simply written as epsilon i j E j assuming that repeated indices are summed over. So, this epsilon is the permittivity tensor, it has in general nine components. Now, one can show through arguments of energy conservation that is epsilon tensor will be symmetric; this is metric tensor.

So, you have epsilon x y is equal to epsilon y x, epsilon x z is equal to epsilon z x, and epsilon y z is equal to epsilon z y; so there are only actually six elements in the epsilon matrix. So, this matrix is symmetric, but in general all elements are nonzero. Now, it is possible to rotate the coordinate system in such a fashion that this epsilon matrix becomes diagonal; you can always diagonalize this matrix and you will get in one coordinate system, which is called the principle axis system or the medium in which the epsilon matrix becomes symmetric.

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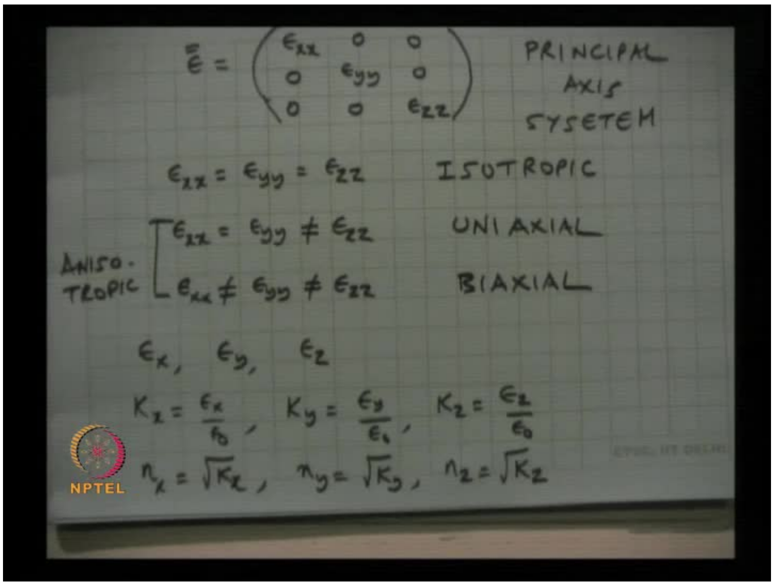

So, I will have epsilon x x 0 0, 0 epsilon y y 0, 0 0 epsilon z z; so this is called the principle axis system; the x y z principle axis, the x y z coordinate system in which epsilon is diagonal is called the principle axis system and this orientations of x, y, and z depend on the symmetries of a crystal. So, if I know the crystal with its symmetry properties, I can find out which are the three principle axis directions of the crystal.

So these are three special directions in the crystal; so if I write epsilon matrix in that coordinate system, where x y z correspond to the principle axis, then epsilon is diagonal.

If I use any other coordinate system, I will generate of diagonal elements. Just like, you have transformation property in the vectors from one coordinate system to another coordinate system; you also have transformation properties of tensors from one coordinate system to another coordinate system.

If epsilon matrix is this in one x y z coordinate system, if I rotate by an angle theta about the z axis, what will be the new epsilon matrix in the new coordinate system? I can calculate from here; there are some transformation relationships, which tell me what the new epsilon should be. So, in most discussions, we will always be using the principle axis system, because that is the easiest system to employ; unless, otherwise stated. If I find an epsilon, which is not diagonal, then I know that I am not using the principle axis system there; I have to now, rotate and find out that principle axis system in which my epsilon matrix becomes diagonal. But most of the time in the course, we will be always using this principle axis system and all quantities given in literature in terms of the epsilon values or non-linear coefficient values, they are all in the principle axis system.

Now, I can have three situations; I can have a situation, when epsilon x x is equal to epsilon y y is equal to epsilon z z. In this case, this epsilon tensor becomes epsilon times unit tensor and then if you go back and look at the equations, it becomes a scalar essentially, the equation becomes D is equal to epsilon E and epsilon becomes a scalar. (Refer Slide Time: 28:33). So, this is called, this corresponds to isotropic medium; if the three elements principal so these are called the principal dielectric permittivities. If all the three principal dielectric permittivities are equal, it is called isotropic. If two of them are equal and it is not equal to the third one, this is called uniaxial. If all three are unequal, this is called biaxial. So, these two are anisotropic. So, if all the three elements are equal, it is isotropic; if two of the elements are equal, it becomes uniaxial; if all the three elements are unequal, it is called biaxial.

And by convention, the two equal components are kept as epsilon x x and y y that is by convention. So, the two equal components will correspond to x and y, and the third one is the z one, and similarly, for the biaxial all three elements are unequal. So, most materials like glass is isotropic; you can have materials like lithium niobate, quartz crystal these are all uniaxial and there are elements like mica, mica is the biaxial medium it has all three elements unequal. So, these elements, these anisotropic crystals provide us with additional degrees of freedom to achieve efficient non-linear interactions as we will go through we will see. So, what we want to do is to understand, how light propagates in this media, what is the velocity of this wave? I have now, in-fact, three quantities here; so let me redefine now here.

So, I can define; now, because I am going to use the principle axis system. Let me just write epsilon x, epsilon y, and epsilon z instead of, epsilon x x y y z z and but remember, these are not the x and y, z components of vector; these are three elements of the tensor the three diagonal elements. I can define dielectric constants as epsilon x by epsilon 0, k y is equal to epsilon y by epsilon 0, and k z as epsilon z by epsilon 0, these are called the three principle dielectric constants.

Three principle dielectric permittivity's, three principle dielectric constants, and n x is equal square root of K x, n y is equal to square root of K y, and n z is equal to square root of K z, the three principle refractive indices; permittivity's, dielectric constants and three principle refractive indices. So, in isotropic all three are equal, in uniaxial two of them are equal the x and y parts, and then for biaxial all three are unequal.

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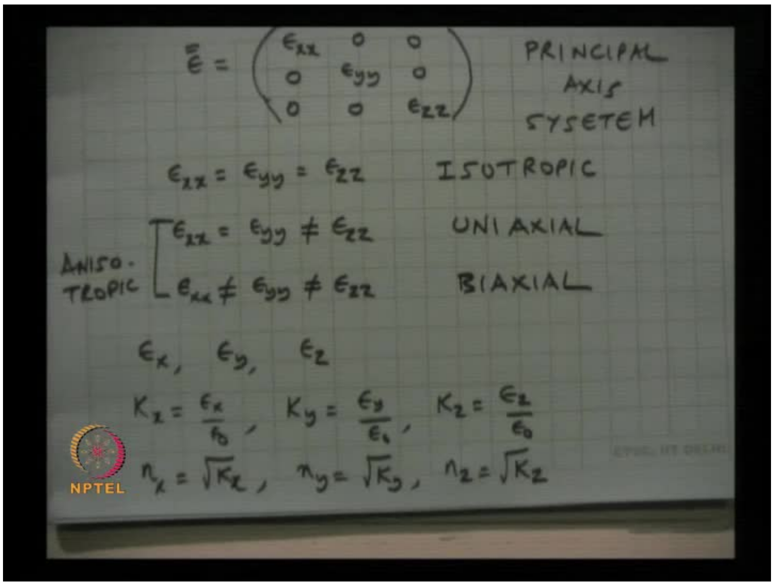

So, what we are going to understand is, suppose, I send a light beam through a medium which has three principle indices n x, n y, and n z what is the velocity of the light of that light beam and does it maintain its polarization state as it propagates, does it changes polarization state, what happens, how do I analyze this? So, as all such phenomena are contained in Maxwell's equations; Maxwell's equation describes all electromagnetic phenomena.

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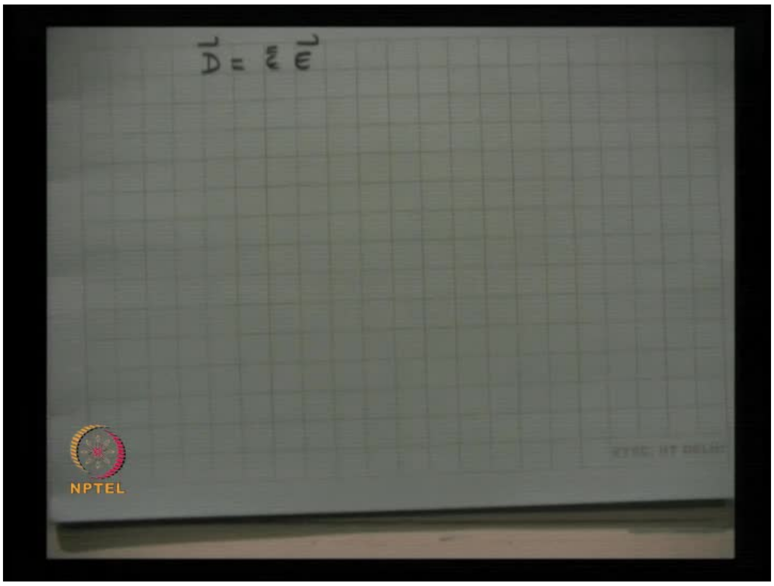

So, what we need to do is to analyze Maxwell's equations with under this condition, that D is equal to epsilon E; I am just drawing a double line on top of epsilon to remind you that it is a tensor, it is not a scalar. So, in media in which D and E are not parallel in general, I would like to understand, how life propagates. Now, please note, there is one special thing about the principle axis, what will happen if I apply an electric field along one of the principle axis? Let me go back to this equation.

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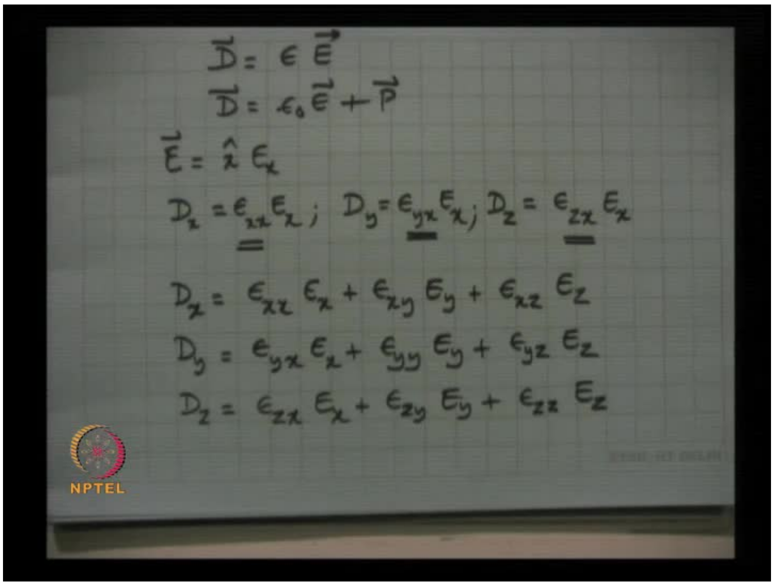

So, if I apply an electric field along the y direction, which happens to be the principle axis, what happens? So, E y is finite, E x and E z are 0, and as principle axis, so what is D x?

D will be y direction

D will be only the y direction, because remember, in the principle axis system these are all 0; the half diagonal elements are all 0. So, you will have D x is equal to epsilon x x E x, D y is equal to epsilon y y E y, D z is epsilon z z E z. So, if I apply an electric field along E y D x and D z are 0, only D y survives; so principle axis directions are three special directions in which if you apply an electric field, displacement vector is parallel to it. Yes, Mohit.

All are the principle access are necessarily perpendicular to each other; I mean, if the symmetry properties - they can be such that three access that guarantee the half diagonal components to be 0 may be is not exactly perpendicular to each other.

No. So, the question is all the three principle axis directions always perpendicular to each other? Yes, they are always perpendicular to each other, because you will always find a coordinate system, which is a orthogonal coordinate system in which your epsilon matrix becomes diagonal; so it is a orthogonal system and so these are three special directions and we must remember this.

So, if I apply an electric field along x, I will only generate D x and similarly, for the y and z if you apply an electric field along any other direction, you will generate in general all three components D x, D y, D z.

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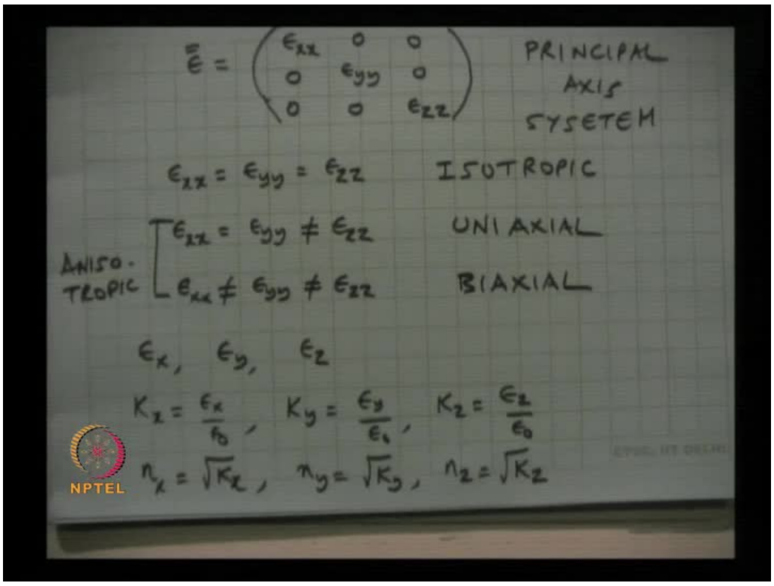

This is I am just extending the concept of refractive index; I define the principle dielectric constant as epsilon x by epsilon 0, and I define the principle refractive index as this. Now, this does not mean, I have still not told you where this refractive index will play a role as far as the velocity is concerned; I will not find that out; so I will relate the velocity of the light wave in the medium to n x, n y, and n z.

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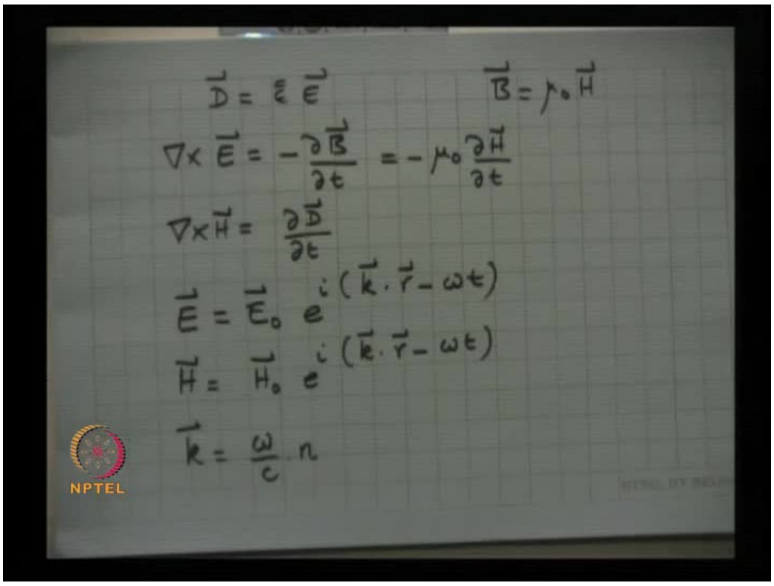

So, I have defined three principle refractive indices, three principle dielectric constants, three principle dielectric permittivity's. So, I will now use; I will analyze the problem and I will obtain the velocity of the wave in terms of n x, n y, and n z. So, we will continue to use this equation B is equal to mu naught H; so we will assume the medium to be nonmagnetic; so B is equal to mu naught H. So, let me write the Maxwell's equations again, so del cross E, the two Maxwell's equation del cross E is minus del B by del t is equal to minus mu naught del H by del t, and del cross H is equal to del D by del t.

Now, I want to look for solutions of plane wave; plane wave solutions, how will a plane wave propagate in such a media? So, let me substitute a solution of this type E is equal to E naught E to the power i k dot r minus omega t, a monochromatic wave, a plane wave propagating along some direction defined by k vector and having an electric field E naught vector. The corresponding H vector will be some H naught E to the power i k dot r minus omega t.

Now, you see, when I substitute solutions of this type, I am looking for someone special class of solutions called Eigen modes. I assume, E naught vector and H naught vector to be independent of x y z, which means as the wave propagates, I want the polarization state not to change; I am looking for special solutions called the Eigen modes of the propagation, which are such that as the plane wave propagates, E naught vector remains fixed, it does not change with as it propagates. So, usually state of polarization does not change. Yes.

Sir all the three states are homogeneous

Yes, we are assuming that all medias are homogenous; I have only introduced an isotropy; so, there is the medium is homogenous and linear. Because of this equation, there is linear and it is homogenous, which means that all the components epsilon x, epsilon y, epsilon z that they are all independent of x y z.

All the three are three different indices, what is the value of a?

Now that we have three refractive indices, what is the magnitude of k? We will find that out; we have to write for example, k vector is equal to omega by c into some refractive index n, I have to find this out. Because n will define the velocity of the wave; c by n will give me the velocity phase velocity of that wave; I do not know the value of n, how n is related to n x n y and n z, I will find out.

So, I know the principle refractive indices, but I do not know yet, how the speed of the wave will be determined by the three principle refractive indices; so that is my problem. So, k is still undefined in terms of n, I know it is omega by c into some refractive index, because the phase velocity is given by c by the refractive index and I still do not know the value of n, I will find it out as the solution to my problem.

So, the problem which I have is, I do not know the value of n and I do not know what should be the direction of E naught vector, which will satisfy the condition that this will be a mode of propagation. This is a very important point; it is like, when you look at oscillations of a string fixed at two ends, you first look for modes; you look for modes of oscillation similarly, because you can analyze any problem of oscillation of a string as a super position of the modes, modes form a complete set of solutions.

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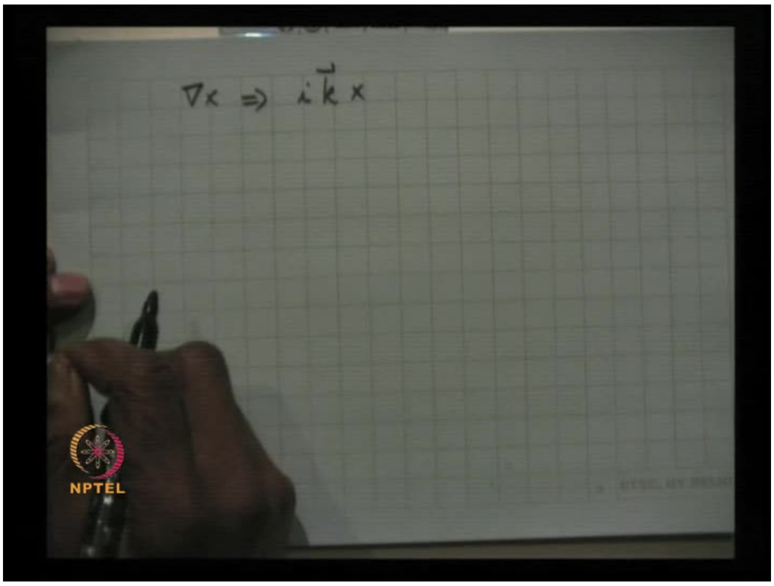

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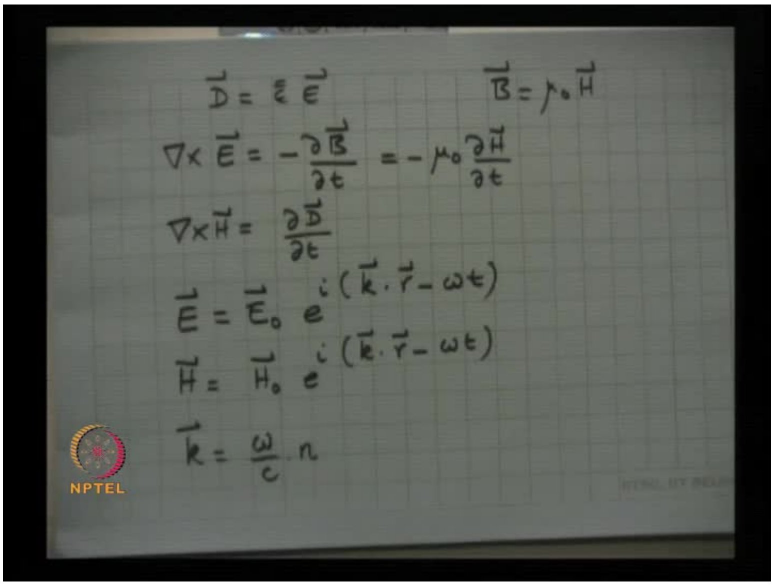

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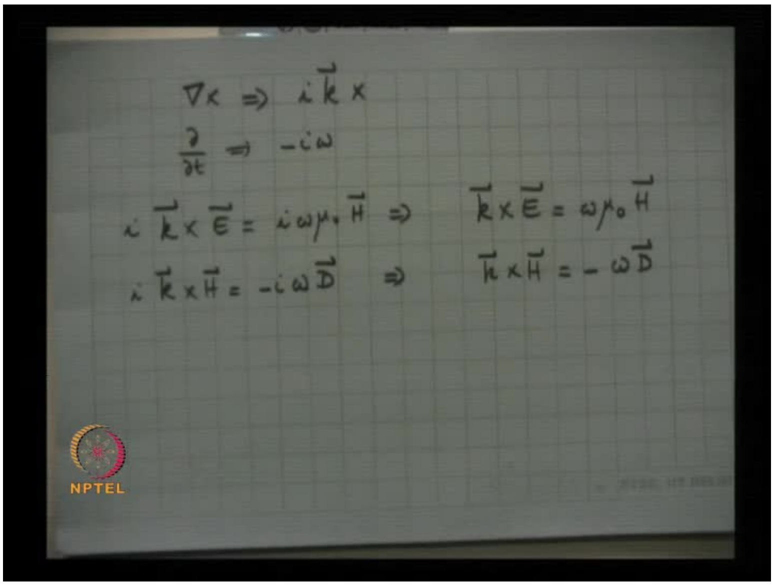

Similarly, here, I am looking for modes that I am looking for the polarization states, which will propagate unchanged and as a plane wave, because I am assuming E naught and H naught to be independent of x y and z. So, let me substitute this into the two curl equations; so please remember that because of the form of this here, I will have a del cross becomes i k cross, wherever you get del cross it will just become i k cross, because of this phase dependents of this form. And del by del t will get replaced by minus i omega, del dot would have replaced by i k dot, because of the form exponential i k dot are minus omega t.

So, the first equations becomes i k cross E i omega mu naught H, which tells me k cross E is equal to omega mu naught H; second equation becomes i k cross H is equal to minus i omega D, this implies k cross H is equal to minus omega D.

Could you explain the inclusion by which we are saying that these Eigen modes will form completed? I will know, we can define all other propagation modes will this gives Eigen modes.

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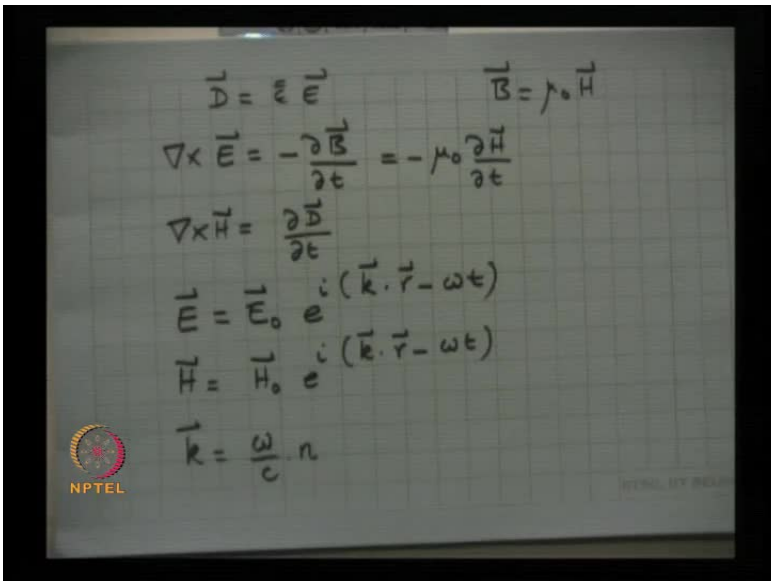

This is something like an Eigen value problem; now, what we will find is, we will find two independent polarization states, which will propagate unchanged for any given direction of propagation and I know that any other polarization state will be written as a super position of these two problem, these two components, these two polarization states because this wave is the transfers electromagnetic wave, there only two independent solutions for any propagation direction.

I do not know whether the two independent solutions are linearly polarize solutions or circularly polarize solutions or elliptically polarize solutions. So, in Faraday effect, I will find that the two linear Eigen modes, so two Eigen modes are circularly polarized, which means when you have a right circular will remain a right circular; a left circular remains left circular as they propagates here; a linear will remain linear, another linear will remain linear, but which linear? There are so many infinite number.

So, I have to find out; so if I give you a anisotropic crystal and I say, I propagate like this in some direction with respect of principle axis, which polarization should I choose here, so that it propagates unchanged; I am trying to find the solution and what polarization and with what velocity, at what speed will it propagate?

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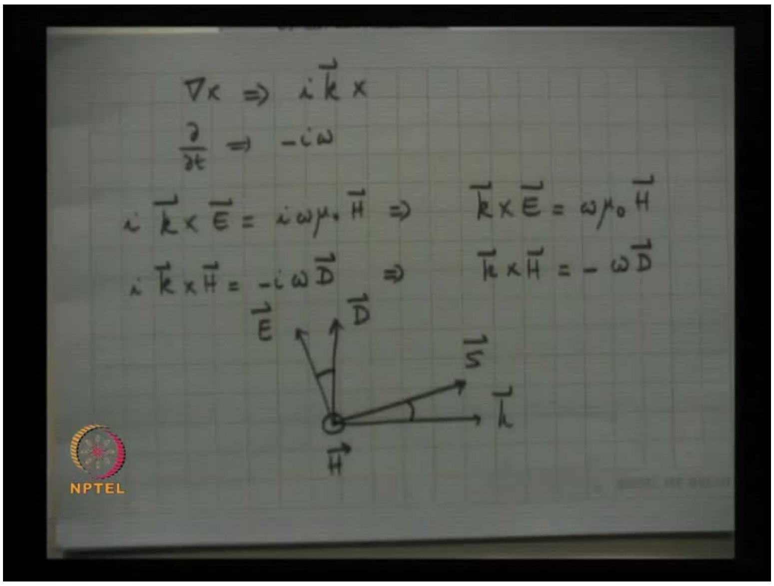

So, all this is related to the principle axis system and n x, n y, and n z and because of its transfers nature we know that there are two independent Eigen modes I will get. I do not know, what polarization states are those Eigen modes yes yet so let me put this back here Now, please remember, D and E are in general not parallel; so let me take a direction of propagation. So, this is k; so I am not plotting the x y z axis, here in some arbitrary direction of propagation k vector. The second equation tells me D is perpendicular to k. So, let me draw a D here; the second equation also tells me H is perpendicular to D and H is also perpendicular to k from the first equation; so it has to be out of this claim, k cross H is minus D.

So, I am just plotting the H vector towards me; so k is perpendicular to D from here; H is perpendicular to D from here, and H is also perpendicular to k from the first equation. Hence, now, D and E are parallel to each other, and E is also perpendicular to H from the first equation; so E must lie in this plane and in general, it will not be parallel to D.

So, let me draw an E vector like this; I do not know whether it is on this side or that side, I just draw at some general E vector, and pointing vector E cross H will be like this. Now, this angle will be equal to this angle; so in general, the pointing vector and the propagation vector are not parallel, because D and E are not parallel.

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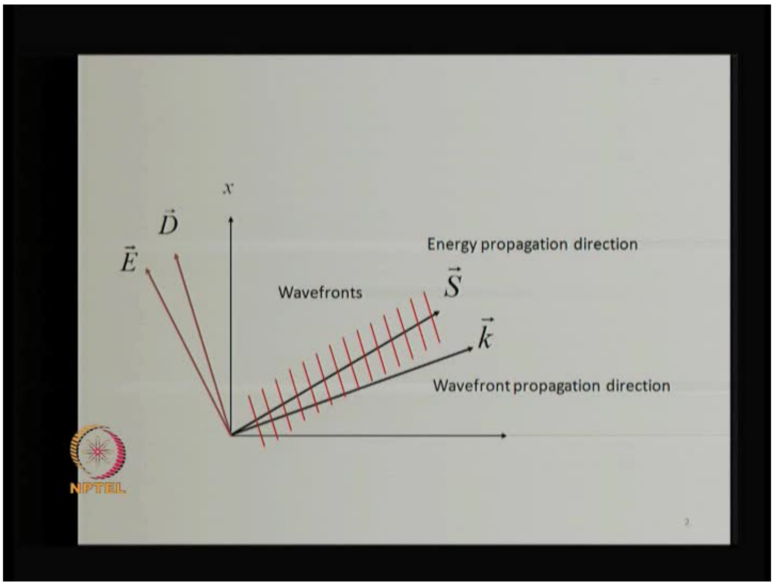

So, let me show you a slide, which is an interesting slide, which shows you this. So, here is the figure, which shows the respective directions of k D E and S, and the red lines which I am plotting there are actually the wave fronts; k vector is always perpendicular to wave front, because by definition k dot r is constant on the phase front; so on the phase front is like this. So, k vector is like this, but is propagating like this, the direction.

So, I had a beam like this; if I had a finite cross section beam, it will go like this, it will be facing straight, but it will move at an angle. So that is what I am trying to draw here, S vector shows me the direction of propagation of the energy; the red lines are not perpendicular to S vector, and they are perpendicular to k vector.

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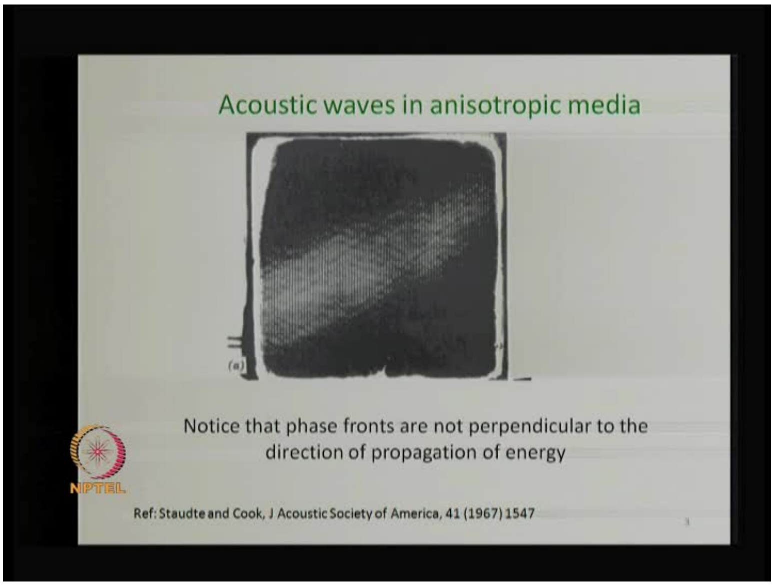

And I found a nice picture here of acoustic waves in anisotropic medium; the anisotropy is an acoustic anisotropy not optical anisotropy and you can see here, it is a photograph taken of the wave fronts, which are those lines and the beam is propagating at an angle exactly the way I have drawn in the figure.

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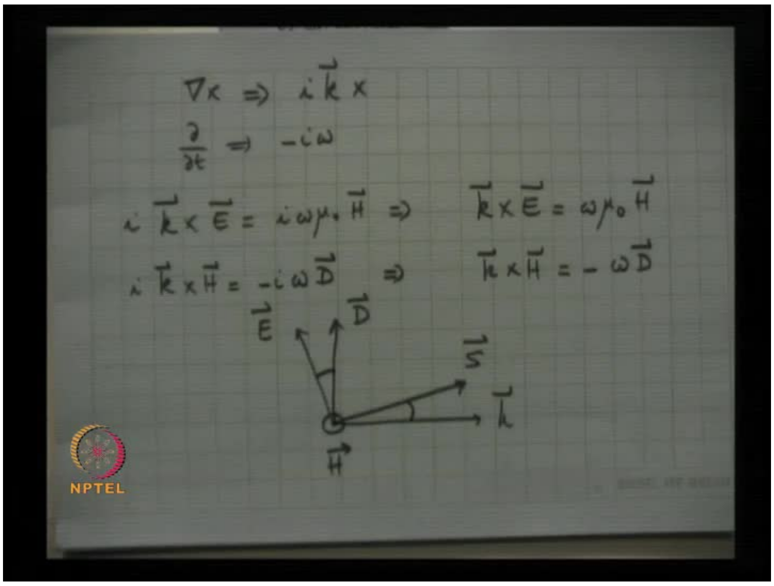

So, it is a beam which starts like this is a finite beam; it does not go like this, it goes like this at an angle. So, this is very characteristic of anisotropic medium, that the direction of propagation of the energy and the direction of propagation of the wave front are not parallel to each other, they are at an angle. We will get to know this numbers of this angle, this angle is not very large, it is a few degrees usually, but it is finite, it is not 0. And it plays a role in many interactions, because the energy plays its own role in terms of conversion efficiencies; the k vector plays its role in terms of the efficiency and so this is important; this knowledge that this k and S are not parallel is important to know.

Now, can you tell me is there any direction in which if I direct k vector? S vector will also be in a same direction, what should be the k vector direction, so that S vector is parallel to k vector, is it possible in anisotropic medium, which direction?

If t is in the direction of one of the principle axis.

But only k is in my control, what direction should I propagate? k should be one of the principle axis; for example, if I choose k along x, and if the polarization D is along y, because D is perpendicular to k. So, if I choose my k to be along x and D to be along y, E will be along y because D is along principle axis and E is along y, S is parallel to k. So, if I propagate in anyone of the principle axis directions, the k vector and S vector become parallel to each other.

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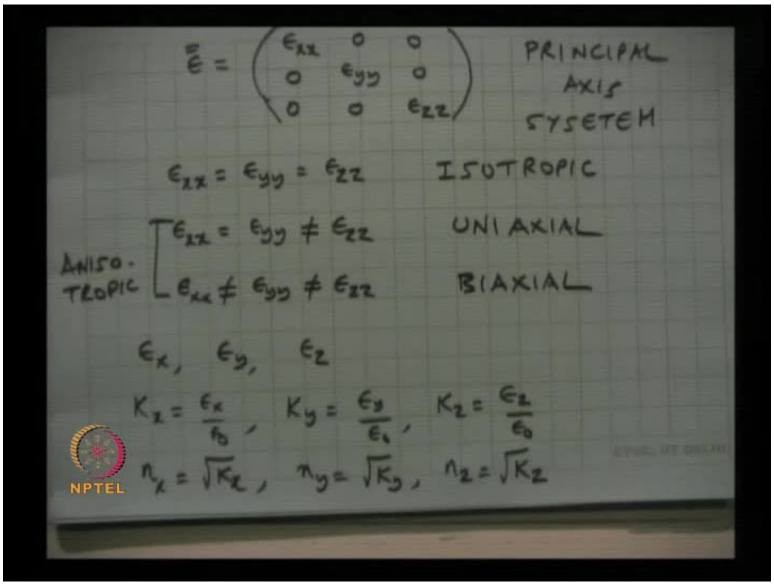

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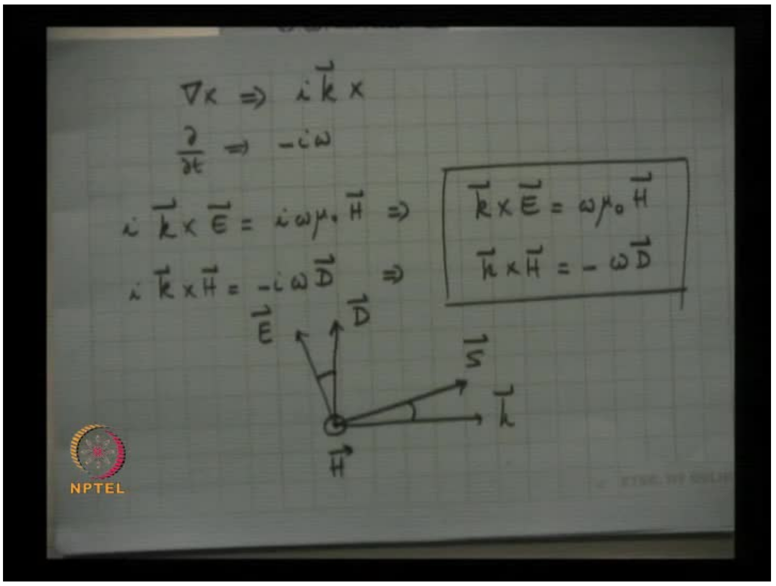

Now, there is a classification which I told you, we will see this uniaxial, the name comes from the fact that in media, which are which have this property there is one special direction; so I will come to this little later. So, in general k and S are not parallel, and so let me try to find out; so from these two equations I have got this fact that k and E are parallel to perpendicular to each other; E and S are perpendicular to each other; k and S are in general not parallel, and similarly, E and D are in general not parallel.

So, what is our problem? Our problem now, is to find out from these equations; I need to solve these equations to find out, what is the velocity of propagation of the wave and what is that direction of polarization that I must choose, so that it remains unchanged?

Now, let me tell you that because D is perpendicular to k vector in anisotropic media, the polarization state is defined by the D vector and not by the E vector. See, if I say linearly polarized along certain direction, it is the direction of D vector, then it defines the direction of k vector. So, D vector defines the orientation, the polarization rather than the E vector in an anisotropic media. In isotropic media it does not matter; so D vector represents polarization state and in general E and D are not parallel to each other.

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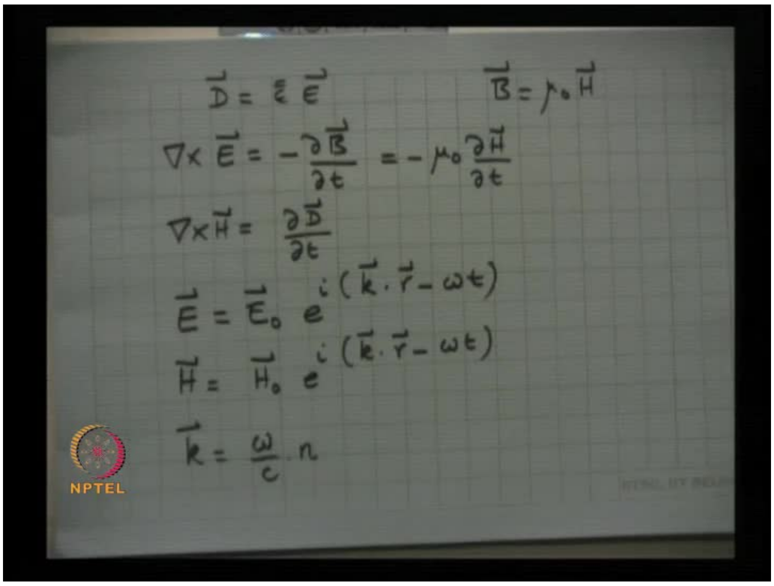

So, what we will do in the next class, is to solve this two equations; simplify these two equations and find out what is the direction of this E 0, what should be, what should I take, what is the E 0 direction that I will have to choose?

If I give you a propagation direction, what should be the E 0 vector direction and what will be the corresponding value n? So, this n will give me the Eigen value, and E 0 vector will give me the Eigen vector, just a kind of a Eigen value problem and so we will just start from these two equations in the next class and get the solutions. So, do you have any questions?

Could you explain? Again, if I choose, if I send the wave yes, in the direction of one of the principal axes, D can be anywhere in the perpendicular plane; so do I need to ensure as this, the order D it also along one of the other principal axes.

No, let me tell you this; if I choose one principle axis direction, I will show that the Eigen modes are D vectors along the other two principal axis directions. So, if I choose my direction of propagation along x, the D vector along y will propagate as unchanged, D vector along z will propagate unchanged, but the velocities of these two waves will be different.

So, if I choose this Eigen mode, then S vector is parallel to k vector; if I choose this Eigen mode S vector is parallel to k vector, if I choose any other state it has to be written as a super position of these two, but in both cases, S vector is parallel to k vector; the only difference would be the velocities of these two waves are different.

So, the phase there will be a phase change; there will be a phase difference accumulated, but the S vector still will be parallel to k vector. If you choose a D vector like this, it has been written as component this and component this, this component travels with a certain speed this component travels with a certain speed, but both components have their S vectors parallel to k vector and the energy propagation in the same direction. So, whenever you propagate along anyone of the principal axis, the energy propagation direction is parallel to the k vector direction.