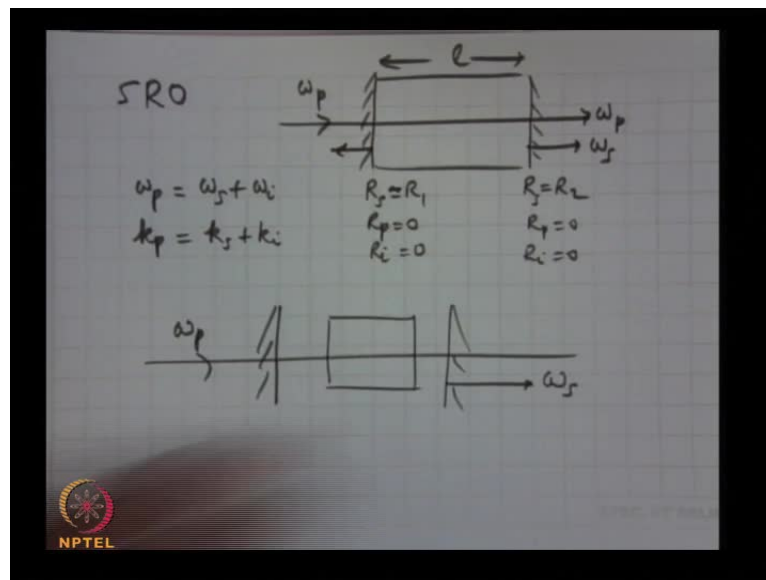


**Quantum Electronics**  
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**Module No. # 03**  
**Second Order Effects**  
**Lecture No. # 19**  
**Non- Linear Optics (Contd.)**

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We continue our discussion we have had on parametric oscillation. What we had discussed last time what is what is called as singly resonant oscillator. Remember we had this crystal here there are 2 mirrors on either side this is the length of a crystal here so, I launch a high energy pump wave  $\omega_p$  and assume that the crystal satisfies this phase matching condition corresponding to a pair of frequencies  $\omega_s$  and  $\omega_i$  this or the corresponding quasi phase matching condition.

In a singly resonant oscillator the reflectivity of the mirrors at the signal frequency is quite high. While at the pump frequency and the idler frequency  $R_s$ ,  $R_p$  and  $R_i$  they are close to 0, that means pump wave gets transmitted through the crystal the idler wave also escape from the cavity while the signal is trapped within the cavity by mirrors of reflectivities  $R_1$  and  $R_2$  and these are close to one these are high reflectivity mirrors. **So** because it is a resonant cavity for only the signal only one of the frequencies between signal and idler is called singly resonant oscillator. **So** what we found out is that because of this cavity the signal frequency should form standing waves inside the cavity.

**So**, at that it remains the frequencies  $\omega_s$  that are possible to exist within the cavity. **So**, once  $\omega_p$  and  $\omega_s$  gets fixed  $\omega_i$  also gets fixed. We also calculated what is the threshold power or threshold pump intensity required for overcoming the losses in the cavity and that tells me what kind of power levels I need to feed in at the pump frequency for the oscillation to begin.

When the losses at the signal frequency inside the cavity are exactly compensated by the gain provided by the parametric amplification process, the signal then oscillates continuously within the cavity and I get a coherent source of light at signal frequency. This is  $\omega_p$  coming out and newly generated  $\omega_s$  coming out if the 2 reflectivities are unequal or not equal to one neither of them equal to one I have signal coming out from both sides and the idler is escaping the cavity.

We also saw the relative bandwidths the amplification bandwidth the inter mode spacing and the width of each mode. **So**, if you make a singly resonant oscillator cavity and if you pump with a corresponding pump here ensure that the cavity satisfies the conditions for phase matching and the energy conservation that you get the oscillations begin as soon as you reach threshold pump.

**So**, actually what you are converting from  $\omega_p$  to  $\omega_s$  and this the laser coming out the coherent source at  $\omega_s$  is by drawing energy from  $\omega_p$  and as I mentioned before when you try when you start to draw a lot of energy from  $\omega_s$  I cannot use the original equations where I assumed no pump depletion. I need to solve all the equations simultaneously, I need to assume I need to consider a fact that the electric field at the pump will also decay within the cavity because of conversion to  $\omega_s$ .

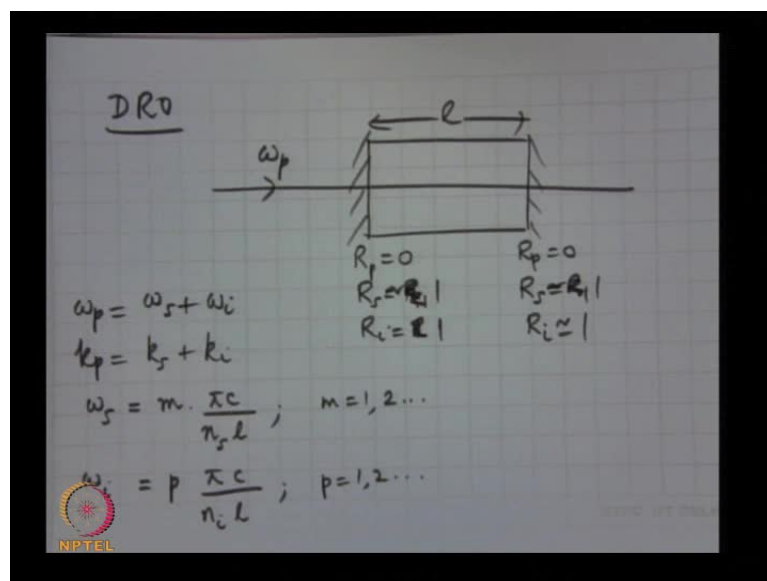
**So**, this is a singly resonant oscillator and as I showed you earlier also I can tune this oscillator by changing the pairs of frequencies at which phase matching condition is generated is satisfied. If you rotate the crystal for example, suppose their mirrors were not touching the cavity the they were outside. I could actually rotate the crystal within the cavity or the direction of propagation of the pump within the cavity, **so** I could have a situation like this for example, I have a mirror here I have a mirror here and I have the crystal separately inside.

**So**, I could actually rotate the crystal within the cavity change the angle of propagation of the pump with respect to the optic axis within the crystal and use birefringence phase matching to pick up pairs of frequencies  $\omega_s$   $\omega_i$  where I will satisfy the phase matching condition and the oscillation condition.

**So**, as I rotate my crystal here the frequency coming out from here  $\omega_s$  frequency coming out will change for a fixed  $\omega_p$  because the frequency of maximum gain will change because, as you rotate the crystal using birefringence phase matching the frequency pair  $\omega_s$   $\omega_i$  which satisfies both these equations will change. And **so** I will be able to tune the oscillator I can do that or I can change the temperature of the crystal which will change the refractive indices and hence change this condition or I can do other things.

**So**, it is a tunable oscillator and, you can actually tune it over very large range of wavelengths you are not using any energy levels unlike a laser based on population inversion and hence there is a huge tunability that is possible in such an oscillator. You also estimated the powers required 15 watts 20 watts of power is required to start the laser to start this coherent source. **So**, the other oscillator which we will discuss today is the doubly resonant oscillator D R O.

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Here what you have is use the same geometry again let me assume the crystal is filling the cavity I have a mirror here a mirror here this is the length of a cavity I again launch a pump from here. Now what I have is  $R_p$  is equal to 0,  $R_p$  is equal to 0,  $R_s$  is equal to  $R_1$ . Let me assume the 2 mirrors have the same reflectivity and  $R_i$  is equal to  $R_2$  let me keep a  $R_s$ ,  $R_s$ ,  $R_s$ ,  $R_s$  let me this is close to 1 this is close to 1 and this is  $R_i$  close to 1  $R_i$  is also close to 1. The 2 reflectivities are equal I am assuming a separate special case that two reflectivities is equal I could have different reflectivities but, let me assume the energy reflectivity as the signal frequency for both mirrors are almost equal and equal to  $R_s$  which is close to 1 similarly, the reflectivities of the mirrors at the idler frequencies are  $R_i$  and both close to 1.

**So**, in this case what is going to happen is now, both the signal and the idler will form standing wave patterns inside the cavity. The signal generated inside the cavity and the idler generated inside the cavity will oscillate back and forth within the cavity forming a standing wave pattern inside the cavity and because I am resonating both the signal and the idler within the cavity this is called a doubly resonant oscillator.

**So**, what are the conditions I need to satisfying for  $\omega_s$  and  $\omega_i$ . First thing is  $\omega_p$  is equal to  $\omega_s$  plus  $\omega_i$ ,  $k_p$  is equal to  $k_s$  plus  $k_i$  and, then Resonating pitches and elements.

Resonant modes so what is what will be the values of  $\omega_s$  we had calculated last time  $M$  times.  $M$  times  $\frac{2\pi c}{\lambda_s}$  by  $\frac{2\pi c}{\lambda_i}$  into  $L$   $N$  is a integer. Similarly  $\omega_i$  also has to satisfy a similar condition with a different integer **so** let me  $\frac{2\pi c}{\lambda_s}$  by  $\frac{2\pi c}{\lambda_i}$  into  $L$ .

In the singly resonant oscillator the second equation was not there was no condition on  $\omega_i$ . **So**, in addition to  $\omega_s$  being resonant inside the cavity I also have to have a condition that  $\omega_i$  which are allowed inside the cavity are only these and, please note that because  $n_s$  and  $n_i$  are different the frequency spacing of the signal and idler frequencies are different. And I will show you later on thus this leads to a lot of problems in the instability of the cavity. So, let me assume I can find a set of  $\omega_s$   $\omega_i$  which satisfies all these 4 equations.

**So**, what will happen is when I will launch  $\omega_p$  into the cavity I will first generate spontaneous the down converted photons because, of these two conditions being satisfied

there will be pair of photons  $\omega_s$   $\omega_i$  over a range certain range. Remember that even if  $\Delta k$  is not equal to 0 there is a finite probability of there is an amplification or there is a generation that takes place. Now out of those frequencies which are generated the noise generated those frequencies will satisfy these condition also will then start to resonate inside the cavity and increase in energy.

**So**, what will happen is I will have a strong signal and a strong idler coming out of the cavity, two new frequent two frequencies coming out which satisfy the pair satisfies all these four equations simultaneously.

Now our objective now is to find out what is the power or the what is the intensity required at the pump frequency to satisfy the condition that the overall gain becomes equal to overall loss. That means in one round trip what is the loss suffered the net loss suffered by the by the mean must be equal to the net loss must be 0 there must be no loss the gain must compensate the loss there is loss because, as the signal bounces back and forth it loses a certain fraction  $1 - R_s$  here and  $1 - R_s$  here the idler loses has  $1 - R_i$  and  $1 - R_i$  here and so the inside the cavity the energy in one round trip gets reduced because of finite transmission of the mirrors but, it gets amplified because of parametric amplification process and when the amplification gain the when the gain becomes is equal to loss at this threshold and the oscillation will start.

Now I have is in the  $R = 0$  case remember I did not write any equation for the idler. Now I need I cannot do that I have to write the complete equation. **So**, remember what we had done was we had solved the equations for a general case **so**, you see when you when you write the condition remember here that at the starting point there is both  $E_s$  and  $E_i$  both  $E_s$  and  $E_i$  propagate over a length  $l$ , both  $E_s$  and  $E_i$  get reflected propagate back, both  $E_s$  and  $E_i$  get reflected and that is one round trip.

**So**, when the beam goes from the left mirror to the right mirror there is amplification. And that amplification will now depend on both  $E_s$  and  $E_i$  the initial conditions for my amplification problem is now  $E_s$  at 0 is equal to is not equal to 0 and  $E_i$  is at 0 is not also equal to 0 **so**, let me recall that two equations we have derived earlier.

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$$E_s(z) = E_s(0) \cosh(gz) + i \sqrt{\frac{\omega_s n_i}{\omega_i n_s}} E_i^*(0) \sinh(gz)$$

$$E_i^*(z) = -i \sqrt{\frac{\omega_i n_s}{\omega_s n_i}} E_s(0) \sinh(gz) + E_i^*(0) \cosh(gz)$$

$$g^2 = \kappa_s \kappa_i |E_p|^2$$

$$\Delta k = 0$$

So, if you have the situation like this. So, this is  $z$  is equal to 0 this is  $z$  is equal to  $L$  so  $E_s$  at  $z$  at some value of  $z$  is actually  $E_s$  at 0 because, hyperbolic  $g z$  plus  $i$  times square root of  $\omega_s n_i$  by  $\omega_i n_s$   $E_i^*(0)$  sine hyperbolic  $g z$  and  $E_i^*$  of  $z$  is equal to minus  $i$  under root  $\omega_i n_s$  by  $\omega_s n_i$   $E_s(0)$  sine hyperbolic  $g z$  plus  $E_i^*(0)$  because hyperbolic  $g z$ .  $G$  is the gain coefficient  $g$  square is  $\kappa_s \kappa_i \text{ mod } E_p$  square again, these equations have been written under no pump depletion approximation.

So, to calculate threshold I can use these pair of equations because just as threshold the conversion is below threshold the conversion is not very efficient. At threshold after threshold the conversion is very efficient you generate a lot of signal and idler from the pump but, the threshold calculation I can assume that I can estimate the threshold by assuming no pump depletion.

The earlier the  $s R 0$  case was when  $E_i(0)$  is equal to 0 so, you can already see that  $E_s$  of  $z$  is given by  $E_s(0)$  because hyperbolic  $g z$  which is what we had used. And remember this also assumes  $\Delta k$  is equal to 0 complete phase matching. Because, otherwise I have some other coefficients inside hyperbolic functions and solutions are different the gain coefficient is less if you get out of phase matching. So, this is the  $E_s$  and  $E_i^*$  at any value of  $z$  given  $E_s$  and  $E_i^*$  at  $z$  is equal to 0.

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$$\begin{pmatrix} E_s(z) \\ E_i^*(z) \end{pmatrix} = \begin{pmatrix} \cosh gz & i \frac{\sqrt{\omega_s N_i}}{\omega_i N_s} \sinh gz \\ -i \frac{\sqrt{\omega_i N_s}}{\omega_s N_i} \sinh gz & \cosh gz \end{pmatrix} \begin{pmatrix} E_s(0) \\ E_i^*(0) \end{pmatrix}$$

$$= \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} E_s(0) \\ E_i^*(0) \end{pmatrix}$$

**So**, now the problem becomes very simple let me write this equation as follows. Let me write this as  $E_s(z)$  and  $E_i^*(z)$  is equal to  $\cosh(gz)$  times  $E_s(0)$  plus  $i \frac{\sqrt{\omega_s N_i}}{\omega_i N_s} \sinh(gz)$  times  $E_i^*(0)$  minus  $i \frac{\sqrt{\omega_i N_s}}{\omega_s N_i} \sinh(gz)$  times  $E_s(0)$  plus  $\cosh(gz)$  times  $E_i^*(0)$ . I have written these pair of equations in a matrix form.

**So**, let me call this some  $A$   $B$   $C$   $D$ . where  $A$  is because  $\cosh(gz)$ ,  $B$  is the coefficient here,  $C$  is this quantity and  $D$  is because  $\cosh(gz)$ . **So**, this matrix  $A$   $B$   $C$   $D$  matrix actually connects the signal idler pair electric fields at  $z$  and  $z$  is equal to 0.

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$$\begin{pmatrix} E_s(z) \\ E_v^*(z) \end{pmatrix} = \begin{pmatrix} \cosh gz & i \sqrt{\frac{\omega_s \mu_i}{\omega_i \mu_s}} \sinh gz \\ -i \sqrt{\frac{\omega_i \mu_s}{\omega_s \mu_i}} \sinh gz & \cosh gz \end{pmatrix} \begin{pmatrix} E_s(0) \\ E_v^*(0) \end{pmatrix}$$

DRD

$\omega_p \rightarrow$

$R_p = 0$        $R_p = 0$   
 $R_s \approx R_{s1}$        $R_s \approx R_{s1}$   
 $R_i \approx 1$        $R_i \approx 1$

$\omega_s = \omega_r + \omega_i$   
 $k_s = k_r + k_i$

NPTEL

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$$\begin{pmatrix} E_s(z) \\ E_v^*(z) \end{pmatrix} = \begin{pmatrix} \cosh gz & i \sqrt{\frac{\omega_s \mu_i}{\omega_i \mu_s}} \sinh gz \\ -i \sqrt{\frac{\omega_i \mu_s}{\omega_s \mu_i}} \sinh gz & \cosh gz \end{pmatrix} \begin{pmatrix} E_s(0) \\ E_v^*(0) \end{pmatrix}$$

$$= \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} E_s(0) \\ E_v^*(0) \end{pmatrix}$$

$$\begin{pmatrix} E_s(mL) \\ E_v^*(mL) \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} E_s(mL) \\ E_v^*(mL) \end{pmatrix}$$

Zoom

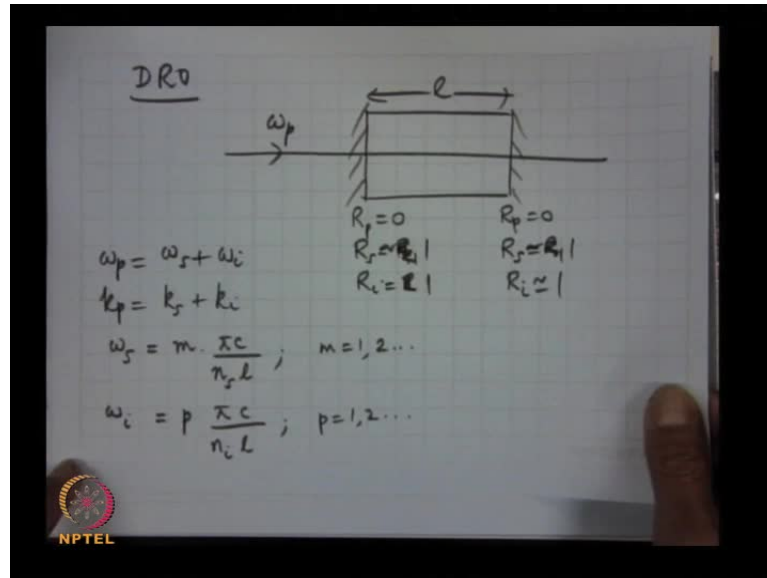
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Now how do I write a matrix for reflection. **So**, this is the matrix represent the transformation from the plane  $z$  is equal to 0  $z$  is equal to 1. Please remember for one round trip what I need to do is start with a pair of  $E_s$  and  $E_i$  here propagate from  $z$  is equal to 0 to  $L$  let it reflect propagate from  $z$  is equal to 1 to  $z$  is equal to 0, let it reflect and I get a pair of  $E_s$  and yes  $E_i^*$  and, one round trip represents this entire process. And the  $E_s$   $E_i^*$  that I get after one round trip must be equal to the one which I started with then, I will have an oscillation condition threshold condition. **So**, this matrix



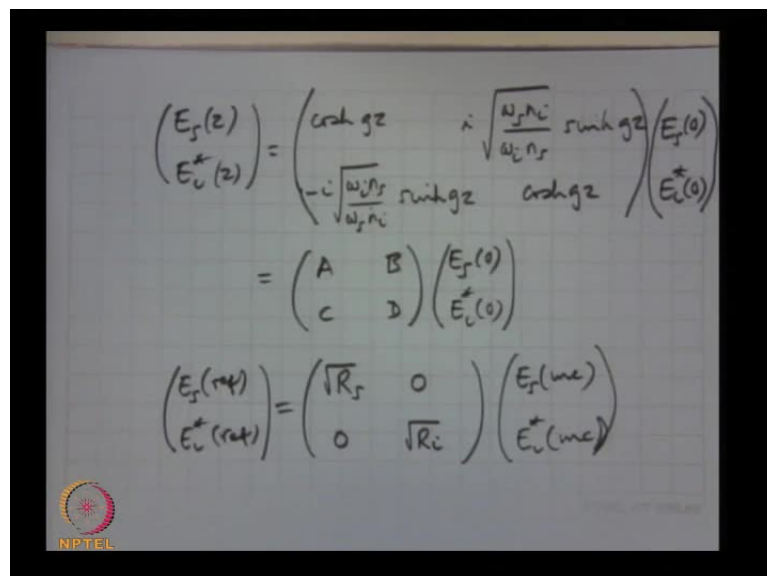
represents the change in  $E_s$  and  $E_i^*$  as they go from here to here. Now what about the reflection how do I represent in terms of the matrix in terms of reflection.

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So, for example, let me let me write suppose I want to write  $E_s$  reflected and  $E_i^*$  reflected in terms of a matrix and  $E_s$  incident and  $E_i^*$  incident. I am assuming that the mirrors do not introduce any phase changes please remember in writing these equations I have assumed that the mirrors do not introduce any phase changes this is only the propagation phase change.

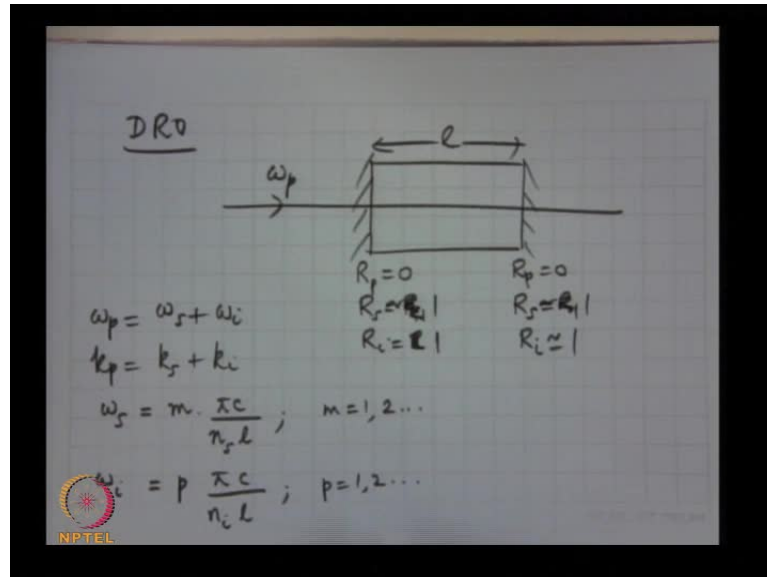
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So, the mirrors are assumed to be mirrors which do not at a given any phase changes but, they only reflect with energy reflection coefficients of  $R_s$  and  $R_i$  so, what is matrix  $i$  must use.


Root  $R_1$  is 0.

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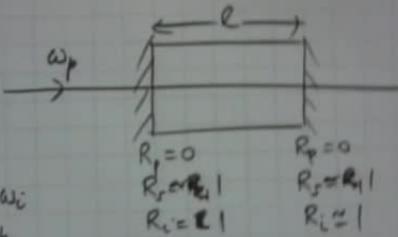

Root  $R_s$  0. Because  $E_s$  reflected is root  $R_s$  times  $E_s$  incident  $E_i$  star reflected is root  $R_i$  times  $E_i$  star incident. This is assuming no phase change otherwise I would have add exponential  $i \phi_s$  and  $i \phi_i$  sitting here. And what is the matrix so, this is the matrix in going from left to right, what is the matrix in going from right to left same. So, what matrix same means 1 0 ratio No Multiply and divide reflection matrix.

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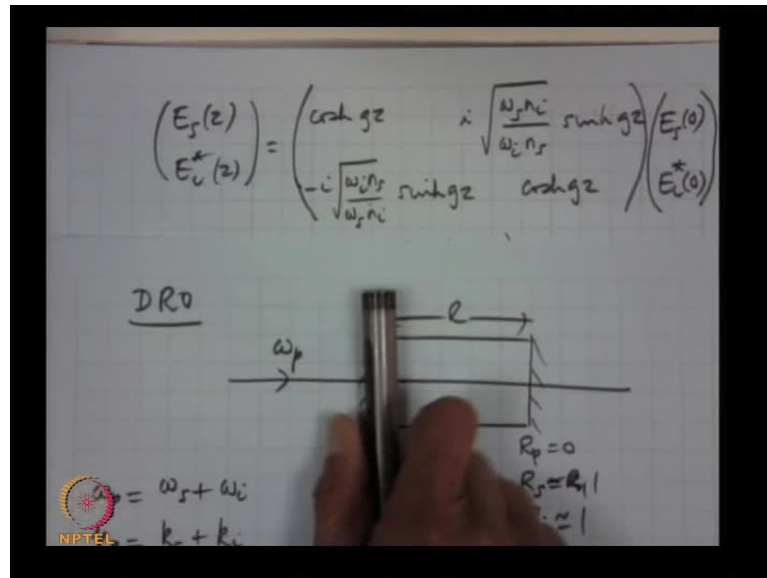
$$\begin{pmatrix} E_s(z) \\ E_v^*(z) \end{pmatrix} = \begin{pmatrix} \cosh gz & i \sqrt{\frac{\omega_s n_i}{\omega_i n_s}} \sinh gz \\ -i \sqrt{\frac{\omega_s n_s}{\omega_i n_i}} \sinh gz & \cosh gz \end{pmatrix} \begin{pmatrix} E_s(0) \\ E_v^*(0) \end{pmatrix}$$
$$= \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} E_s(0) \\ E_v^*(0) \end{pmatrix}$$
$$\begin{pmatrix} E_s(ml) \\ E_v^*(ml) \end{pmatrix} = \begin{pmatrix} \sqrt{R_s} & 0 \\ 0 & \sqrt{R_i} \end{pmatrix} \begin{pmatrix} E_s(mc) \\ E_v^*(mc) \end{pmatrix}$$


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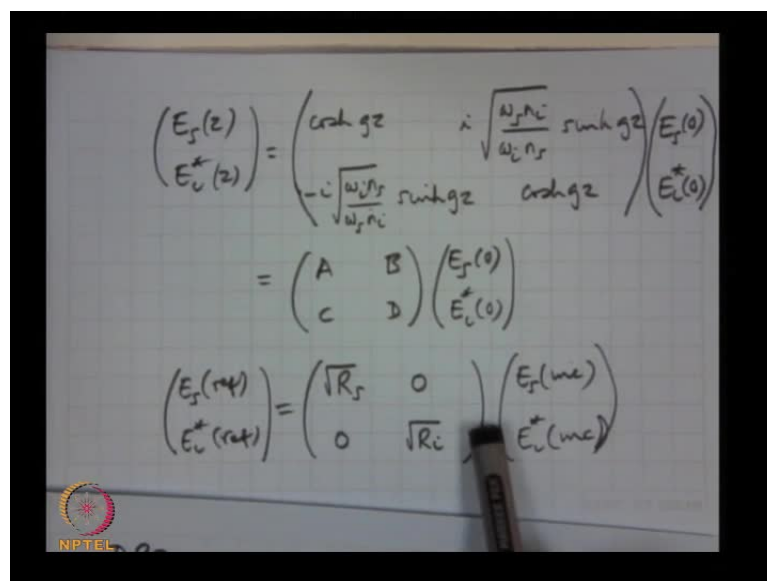
DRD


$$\omega_p = \omega_s + \omega_i$$
$$k_p = k_s + k_i$$
$$\omega_s = m \cdot \frac{\pi c}{n_s l}; \quad m = 1, 2, \dots$$
$$\omega_i = p \cdot \frac{\pi c}{n_i l}; \quad p = 1, 2, \dots$$


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No this is reflection matrix. **So**, I start from I start from z is equal to 0 at z is equal to l I will obtain by multiplying the field here field pair here by this matrix then I will get the reflected wave by multiplying by this matrix, then the wave propagates from this mirror to this mirror **so**, whatever what is matrix if there is no change in Unit matrix.

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$$\begin{pmatrix} E_s(t) \\ E_i^*(t) \end{pmatrix} = \begin{pmatrix} \sqrt{R_s} & 0 \\ 0 & \sqrt{R_i} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{R_s} & 0 \\ 0 & \sqrt{R_i} \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} E_s(0) \\ E_i^*(0) \end{pmatrix}$$

$$= \begin{pmatrix} R_s A & R_s B \\ R_i C & R_i D \end{pmatrix} \begin{pmatrix} E_s(0) \\ E_i^*(0) \end{pmatrix}$$

$$\begin{pmatrix} E_s(t) \\ E_i^*(t) \end{pmatrix} = \begin{pmatrix} E_s(0) \\ E_i^*(0) \end{pmatrix}$$

NPTEL

And then another reflection which is the same as this because I am assuming the mirrors to be identical. **So**, I can actually complete one round trip **so** what will happen is let me write this as so  $E_s$  after one round trip  $E_i^*$  after one round trip is equal to **so** this will be  $\sqrt{R_s} \sqrt{R_i}$  into a unit matrix into  $\sqrt{R_s} \sqrt{R_i}$  into  $A B C D$  matrix into  $E_s(0) E_i^*(0)$ . **So**, this is starting this is propagation from left to right reflection by the mirror propagation from right to left reflection by the mirror.

**So**, if I open this up the product of these two is simply  $R_s R_i$ . **So**, I will have  $R_s A$  and then  $R_s B$ .  $R_i C$  and  $R_i D$  is that right it is  $R_i C$  and  $R_i D$ . **So**, what is the condition I impose now. Homogenous this is a unit matrix.

No my condition is this must be equal this may not be unit matrix this has to be equal to this. What is my condition that the signal and idler fields after one round trip must repeat themselves. **So**, this  $E_s$  after one round trip and  $E_i^*$  after one round trip must be equal to  $E_s(0)$  and  $E_i^*(0)$ .

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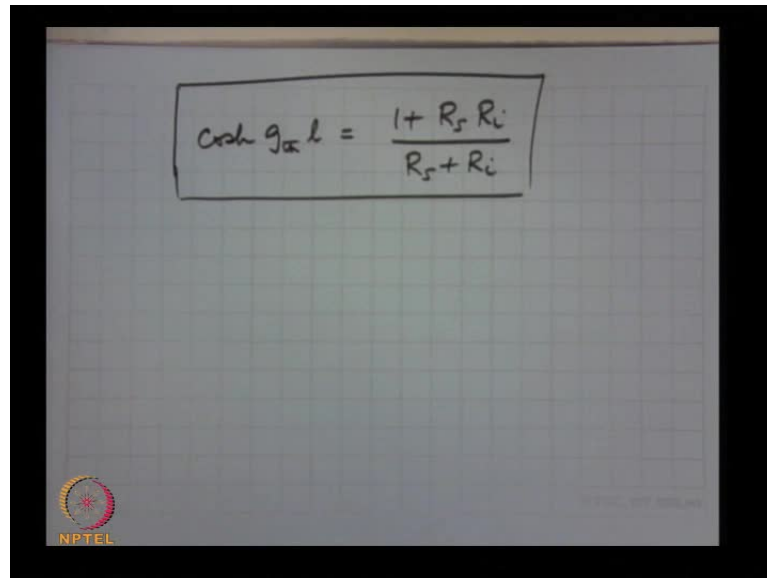
$$\begin{pmatrix} R_s A & R_s B \\ R_i C & R_i D \end{pmatrix} \begin{pmatrix} E_s(0) \\ E_i^*(0) \end{pmatrix} - \begin{pmatrix} E_s(0) \\ E_i^*(0) \end{pmatrix} = 0$$
$$\begin{pmatrix} R_s A - I & R_s B \\ R_i C & R_i D - I \end{pmatrix} \begin{pmatrix} E_s(0) \\ E_i^*(0) \end{pmatrix} = 0$$
$$\begin{vmatrix} R_s A - I & R_s B \\ R_i C & R_i D - I \end{vmatrix} = 0$$

No we got an equation right let me substitute here. **So**, I will get an equation which is simply  $R_s A R_s B R_i C R_i D$  into  $E_s(0) E_i^*(0)$  minus  $E_s(0) E_i^*(0)$  is equal to 0.

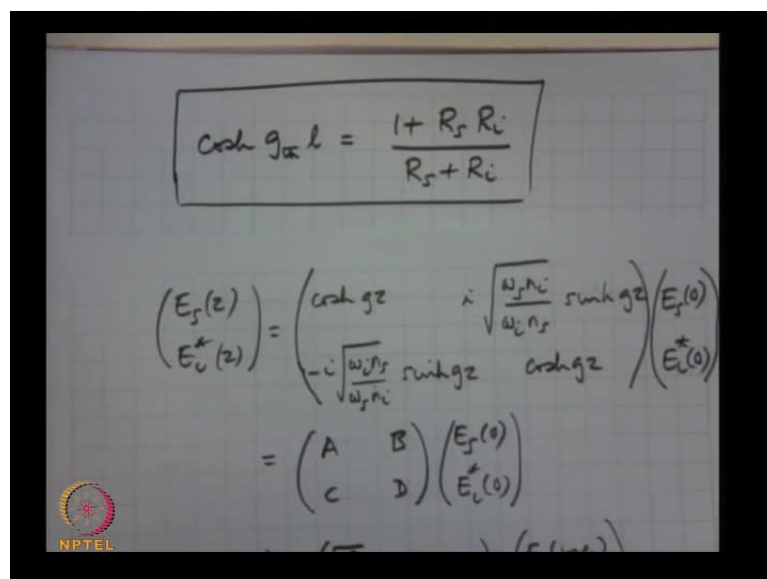
**So**, this is nothing but,  $R_s A$  minus  $I R_s B R_i C R_i D$  minus  $I$  into  $E_s(0) E_i^*(0)$  is equal to 0 because this is unit matrix into this quantity. **So**, actually this equation is simply this so next what do I do.

For a non trivial solution the determinant of this equation this matrix must be 0 so, I get a condition that determinant of  $R_s A$  minus  $I R_s B$ ,  $R_i C$ ,  $R_i D$  minus  $I$  must be equal to 0. **So**, let me leave this simplification to you have expressions for  $A B C$  and  $D$  **so**, what you will get is the following.

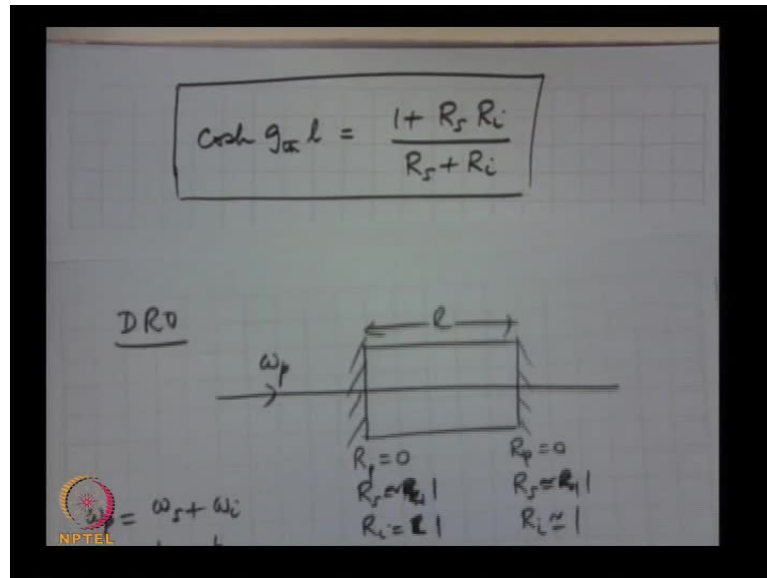
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$$\cosh g_{\alpha} l = \frac{1 + R_s R_i}{R_s + R_i}$$

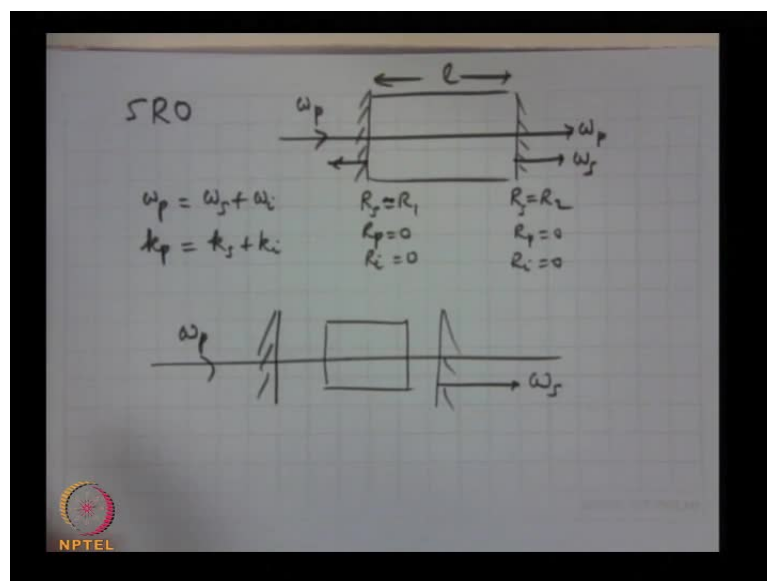
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$$\cosh g_{\alpha} l = \frac{1 + R_s R_i}{R_s + R_i}$$
$$\begin{pmatrix} E_s(z) \\ E_o^*(z) \end{pmatrix} = \begin{pmatrix} \cosh gz & i \sqrt{\frac{\omega_s n_i}{\omega_o n_s}} \sinh gz \\ -i \sqrt{\frac{\omega_o n_s}{\omega_s n_i}} \sinh gz & \cosh gz \end{pmatrix} \begin{pmatrix} E_s(0) \\ E_o^*(0) \end{pmatrix}$$
$$= \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} E_s(0) \\ E_o^*(0) \end{pmatrix}$$

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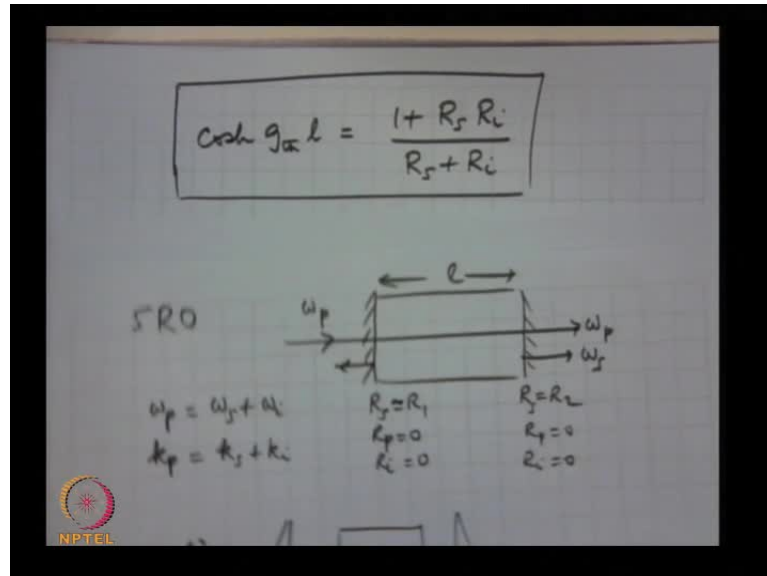


Cos hyperbolic gain threshold  $L$  is equal to  $1 + R_s R_i$  in a B C D I need to use  $z$  is equal to  $L$ . Remember this is any  $z$  this A B C D which appears here has  $z$  is equal to  $1$  in the equation because I propagate from left to right over a length  $L$  the crystal has a length  $L$ . And I am assuming here that the crystal fills the entire cavity if it does not happen the gain will be only happening in a part of the cavity please remember if the situation is like this if the situation like this the gain happens only in this part of cavity. **So**, that length  $L$  which appears here must be this length only and, the frequencies are determined by this



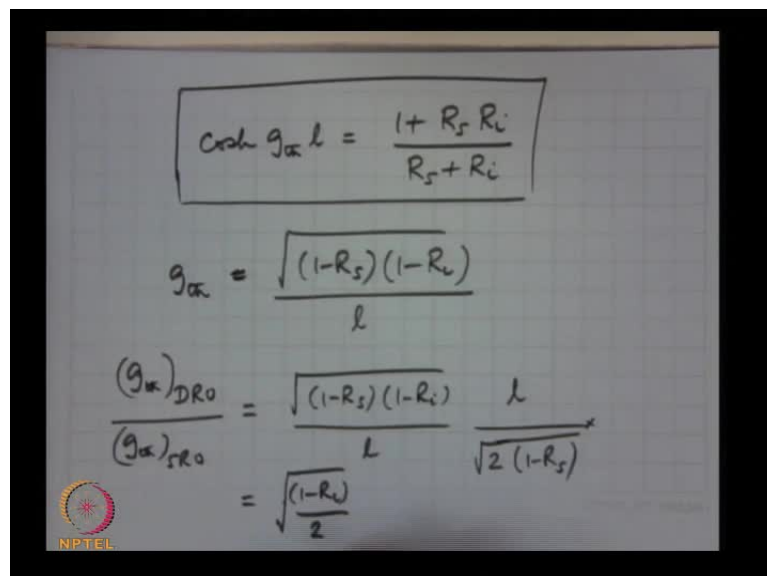
entire length and the refractive index of the crystal and the refractive index of outside everything ok.

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So please be careful that you need to calculate properly the frequencies of oscillation and this amplification will only depend on the length of the crystal not the length of a cavity. In my example which I am considering the cavity is equal to length of the length of the cavity is equal to the length of the crystal the crystal is filling the entire cavity otherwise, this length  $L$  is the length of the crystal within the cavity.

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If you put  $R_i$  is equal to 0 you essentially get back to the old expression that we had. Now I can now again  $R_s$  and  $R_i$  are usually very close to unity. So, this is almost close to unity  $1 + \frac{1}{2}$  this is almost close to unity so, I can again make a an expansion of because hyperbolic function and I get the following expression for  $g$  threshold this is assuming approximately. This is assuming that the reflection coefficients and the signal and idler are very close to unity otherwise, this is the expression. I can approximate this equation provided the signal and idler reflection coefficients are close to unity and I can have expression for this.

**So**, can you tell me what is the ratio of this so,  $g$  threshold doubly resonant oscillator to  $g$  threshold singly resonant oscillator what is the ratio.

Let me write here  $1 - R_s$  into  $1 - R_i$  by 1 and what was the  $g$  threshold  $S_{R0}$  singly resonant oscillator we had done last time.  $\sqrt{2}$  by  $\sqrt{1 - R_s}$  under  $\sqrt{1 - R_i}$  or  $\sqrt{2}$  No under  $\sqrt{1 - R_i}$  by Divide by  $\sqrt{2}$  times so  $\sqrt{2}$  comes in numerator because this  $g$  threshold is in the denominator. Now u can give me the numerator 1 by.

$\sqrt{2}$  1 by  $\sqrt{2}$  into 1 upon into 1 upon under root of  $R_1 R_2 - 1$  raised to power minus 1 that is always there only 1 by  $\sqrt{R_1 R_2}$ .

Put  $R_1$  is equal to  $R_2$  is equal to  $R_s$ . Because there you have both mirrors of the same reflectivity equal to  $R_s$  so, what do I get? Under  $\sqrt{1 - R_s}$   $R$  by  $\sqrt{2}$   $R_s$  is close to 1. So I am neglecting that  $R_s$ .  $R$  by  $\sqrt{2}$ , Is this right. No it is in numerator or upon  $R$ . And  $R_s$  is equal to 1 I am assuming that  $R_s$  I am neglecting because  $R_s$  is close to 1 I am throwing away that  $R_s$  is this right. Then it should come one by Is this right. So, are we assuming? And it is very close to 1.

But, close to one but, I cannot make it 0. See I am making a factor  $R_s$  I am just putting at 1 but,  $1 - R_s$  is close to 0. **So** I cannot put as 0 then I get 0 **so** obviously I know that a threshold  $g$  for  $D_{R0}$  is not 0.

Now from here can I estimate the ratio of the intensities required for the  $D_{R0}$  and  $S_{R0}$  what is  $g$  threshold what is  $g$  given by.

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$$g^2 = \kappa_r \kappa_i |E_p|^2$$

$$= \kappa_r \kappa_i \cdot \frac{2c\mu_0}{n_p} I_p$$

$$I_T = \frac{n_p}{2c\mu_0} |E_p|^2$$

$$R = \frac{T (I_{T,\alpha})_{DR0}}{(I_{T,\alpha})_{SR0}} = \frac{(g_{\alpha})_{DR0}^2}{(g_{\alpha})_{SR0}^2} = \frac{1-R_i}{2}$$

$$R_i = 0.98 ; T = 0.01$$

Remember  $g$  square is  $\kappa_r \kappa_i |E_p|^2$ . **So** this is  $\kappa_r \kappa_i |E_p|^2$  so  $I_p$  is equal to  $\frac{n_p}{2c\mu_0} |E_p|^2$  so this into  $\frac{2c\mu_0}{n_p}$  into  $I_p$ .

**So**  $I_p$  is proportional to  $g$  square and, this ratio is in terms of so  $I_p$  threshold  $D R 0$  by  $I_p$  threshold  $S R 0$  will be equal to  $g$  threshold  $D R$  square  $D R 0$  by  $g$  threshold square  $S R 0$  which is equal to  $1 - R_i$  by  $2$ .

Now if  $R_i$  is close to one say  $R_i$  if I take at 98 percent of reflection if  $R_i$  is close to point 0.98 this ratio becomes if I call this  $R$  is a ratio  $R$  not  $R$  some ratio  $T$  so  $T$  becomes 0.01.

The threshold intensity required is a factor of 100 now. **So**, we are talking of if 10 watts there say 100 milliwatts here 100 times down and that is very interesting because then you have **you do not have you do not** need so much power and you can almost have a continuous wave oscillation of this parametric oscillators.

**So**, a doubly resonant oscillator requires much lower powers to operate than a singly resonant oscillator because, here both the signal and idler are having high intensity within the cavity and both of them are responsible for drawing energy from the pump.

And of the start you need much lower threshold pump powers for a doubly resonant oscillator as compared to singly resonant oscillator.

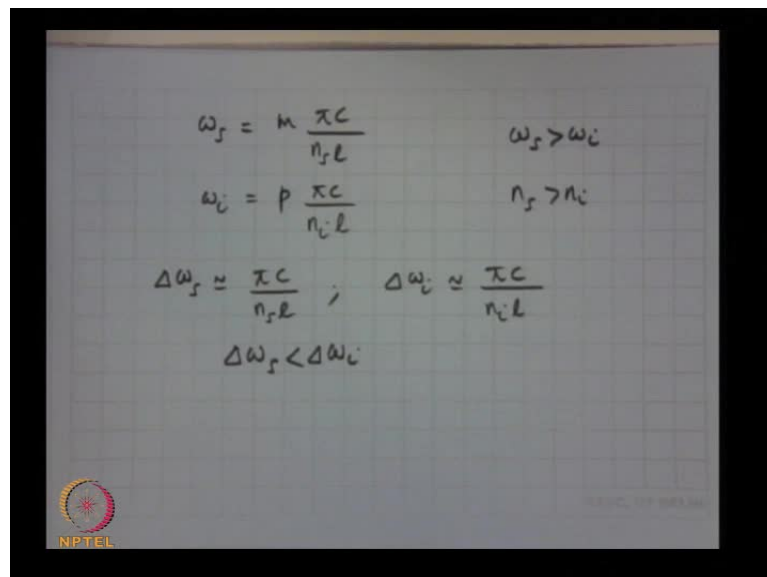
Threshold pump powers of doubly resonant oscillator is low and we are assuming that no pump with depletion approximation for.

No so, I cannot use any pump depletion approximation to calculate the signal and idler powers coming out but, as I mentioned I am neglecting pump depletion only to estimate the threshold value.

Because I know just before threshold the depletion is very little just above threshold there is a strong depletion. So, I am approximating the position of the threshold by assuming a no pump depletion approximation and calculating the threshold.

**So**, this has been estimate a very good estimate of the threshold pump power required to start the oscillator. Now me try to show you schematically what the problem is in terms of stability of this oscillator.

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The image shows a slide with handwritten mathematical equations on a grid background. The equations are:

$$\omega_s = m \frac{\pi c}{n_s l} \quad \omega_s > \omega_i$$
$$\omega_i = p \frac{\pi c}{n_i l} \quad n_s > n_i$$
$$\Delta \omega_s \approx \frac{\pi c}{n_s l}, \quad \Delta \omega_i \approx \frac{\pi c}{n_i l}$$
$$\Delta \omega_s < \Delta \omega_i$$

In the bottom left corner, there is a logo for NPTEL (National Programme on Technology Enhanced Learning) and the text "NPTEL".

Now let me go back to those equations which we have written requiring the standing wave at the signal and idler frequencies. Remember these 2 equations so, let me write them again here so,  $\omega_s$  must be  $m\pi c$  by  $n_s l$  and  $\omega_i$  is equal to  $p\pi c$  by  $n_i l$ .

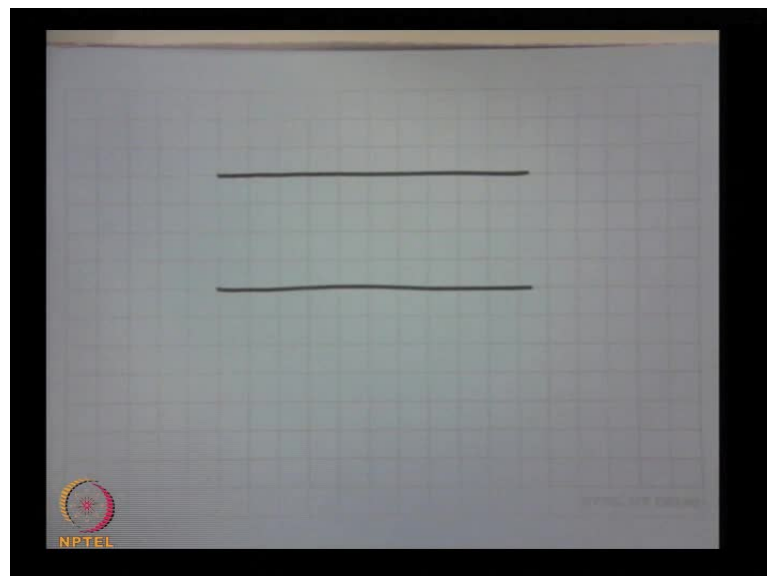
Now I am assuming  $\omega_s$  is greater than  $\omega_i$ . The signal frequency is higher than the idler frequency. Now from here  $\Delta\omega_s$  is approximately  $\pi c$  by  $N_s l$  and  $\Delta\omega_i$  is approximately  $\pi c$  by  $N_i l$ .

And I will get approximately because please remember  $N_s$  and  $N_i$  are also functions of frequency I am neglecting that frequency dependence in writing this equation. Otherwise I must take into account the fact that if  $\omega_s$  changes  $n_s$  also changes significantly so I am neglecting that.

So, which is larger  $\Delta\omega_s$  or  $\Delta\omega_i$ . So is  $N_s$  larger or  $N_i$  larger as the frequency increases refractive index increases.

So,  $N_s$  is bigger than  $N_i$ . So  $\Delta\omega_s$  is less than  $\Delta\omega_i$  frequency spacing's are not the same. Now let me draw you a figure which sort of shows this problem because of this condition.

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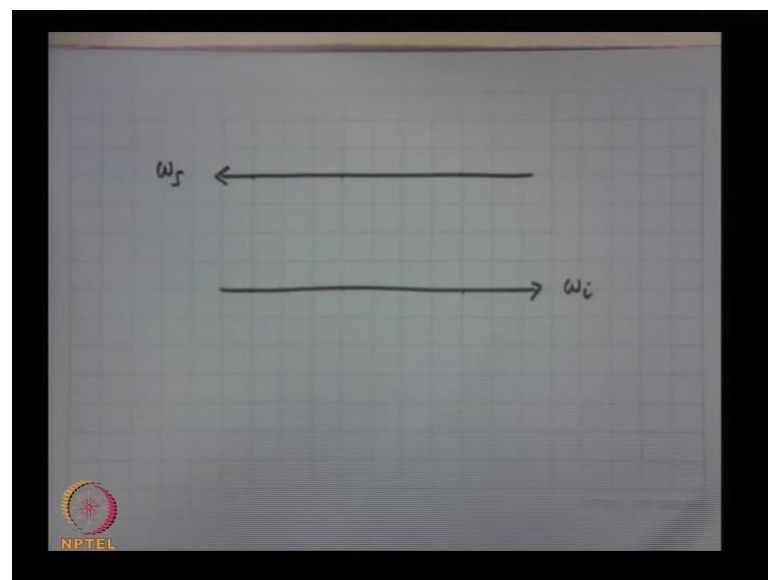


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$$\omega_s = m \frac{\pi c}{n_s l} \quad \omega_s > \omega_i$$
$$\omega_i = p \frac{\pi c}{n_i l} \quad n_s > n_i$$
$$\Delta \omega_s \approx \frac{\pi c}{n_s l} \quad ; \quad \Delta \omega_i \approx \frac{\pi c}{n_i l}$$
$$\Delta \omega_s < \Delta \omega_i$$
$$\omega_s + \omega_i = \omega_p$$

Get the figure which is a very interesting figure which I draw. Please remember that the sum of these 2 frequencies must be  $\omega_p$ .

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Delta omega 0 spacing need not be the same must this is spacing now what I draw is the figure like this. Omega s increases in this direction and omega i increases in this direction. Such that at any value the sum of these two is constant please note this figure the axis is reverse of each other. So, the sum of these two values is the same at the sum of these two values the same as sum of these two values. I can have a scale which is increasing this direction of one and increasing the other direction in the other scale.

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$$\omega_s = m \frac{\pi c}{n_s l} \quad \omega_s > \omega_i$$

$$\omega_i = p \frac{\pi c}{n_i l} \quad n_s > n_i$$

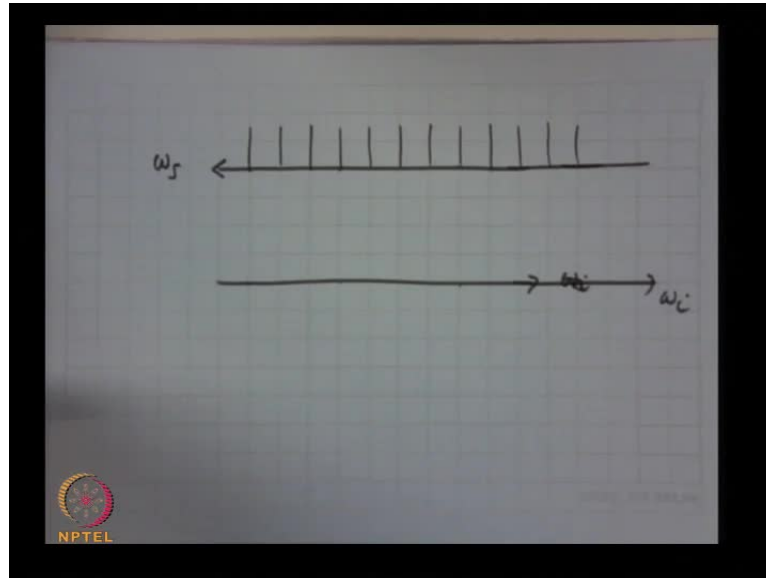
$$\Delta \omega_s \approx \frac{\pi c}{n_s l} \quad ; \quad \Delta \omega_i \approx \frac{\pi c}{n_i l}$$

$$\Delta \omega_s < \Delta \omega_i$$

$$\omega_s + \omega_i = \omega_p$$

These two figures I have scaled such that omega s plus omega i at any value is omega p. Now in these figure I let me draw the lines corresponding to this omega s and this omega I remembering that this spacing's are given by these.

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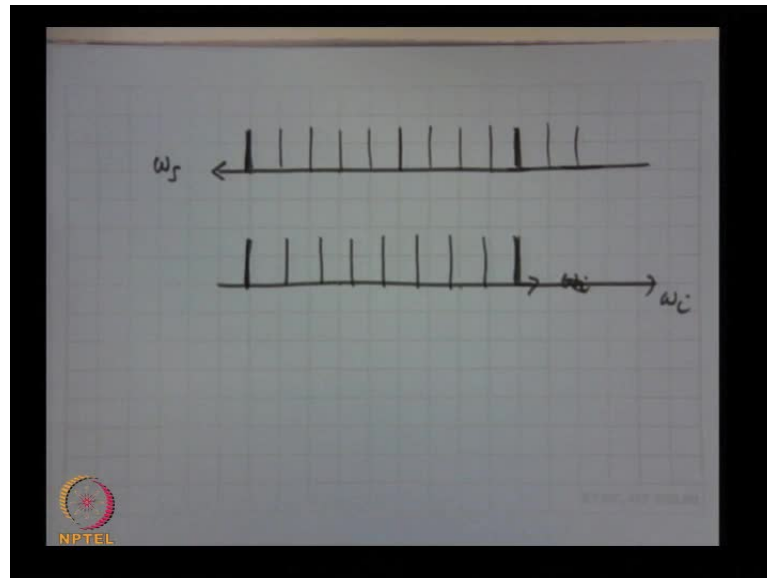
And that delta omega s is less than delta omega i. So, first let me draw for example, the spacing here. Some arbitrary number let me draw let me pull this up here so the way I have drawn this figure is the spacing delta omega s is one of the this squares.

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$$\omega_s = m \frac{\pi c}{n_s l} \quad \omega_s > \omega_i$$
$$\omega_i = p \frac{\pi c}{n_i l} \quad n_s > n_i$$
$$\Delta \omega_s \approx \frac{\pi c}{n_s l} \quad ; \quad \Delta \omega_i \approx \frac{\pi c}{n_i l}$$
$$\Delta \omega_s < \Delta \omega_i$$
$$\omega_s + \omega_i =$$



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Now please note that because of this condition is like a vernier because suppose let me assume that this is a value of  $\omega_i$  where this is the value of  $\omega_p$  and  $\omega_s$   $\omega_i$  where I satisfy this condition both of this suppose.

This is  $\Delta\omega_s$   $\Delta\omega_i$  is larger. So the next one will not appear here it will appear somewhere here the next allowed  $\omega_i$  frequency is not on this line corresponding to this  $\omega_s$  slightly different. So, what will happen is this come here then here then here then here and matched again matched right.

It's like a vernier you see this is 1, 2, 3, 4, 5, 6, 7, 8, 9 divisions and 1, 2, 3, 4, 5, 6, 7, 8 divisions.

**So** there is a match here and there is a match here. So, either this pair will oscillate or this pair will oscillate this these pairs cannot oscillates because corresponding to this  $\omega_s$  there is no corresponding  $\omega_i$  which can form a standing wave pattern inside the cavity.

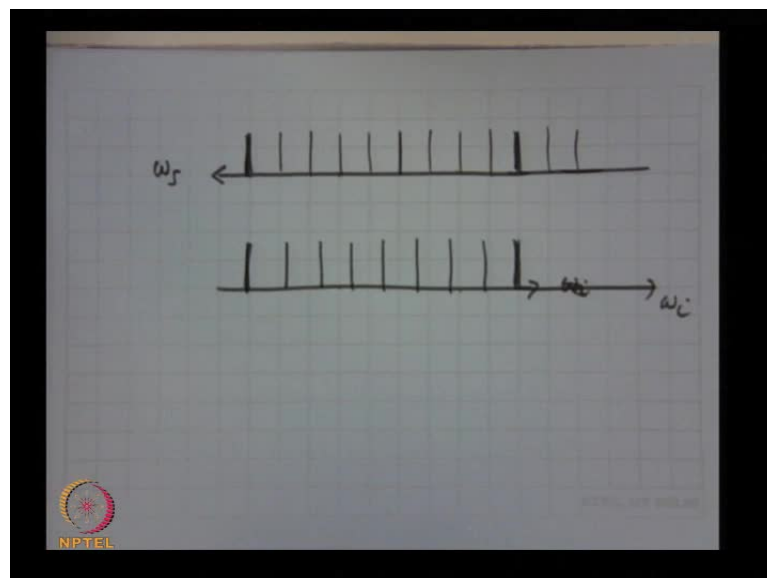
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$$\omega_s = m \frac{\pi c}{n_s l}$$
$$\omega_i = p \frac{\pi c}{n_i l}$$
$$\Delta \omega_s \approx \frac{\pi c}{n_s l}$$
$$\Delta \omega_i \approx \frac{\pi c}{n_i l}$$
$$\omega_s + \omega_i = \omega$$

$\omega_s > \omega_i$   
 $n_s > n_i$

So, either this pair will oscillate or if not then this pair has oscillate or there will be another pair coming like this every range of frequency there will be pair of frequencies which will satisfies both these conditions simultaneously.

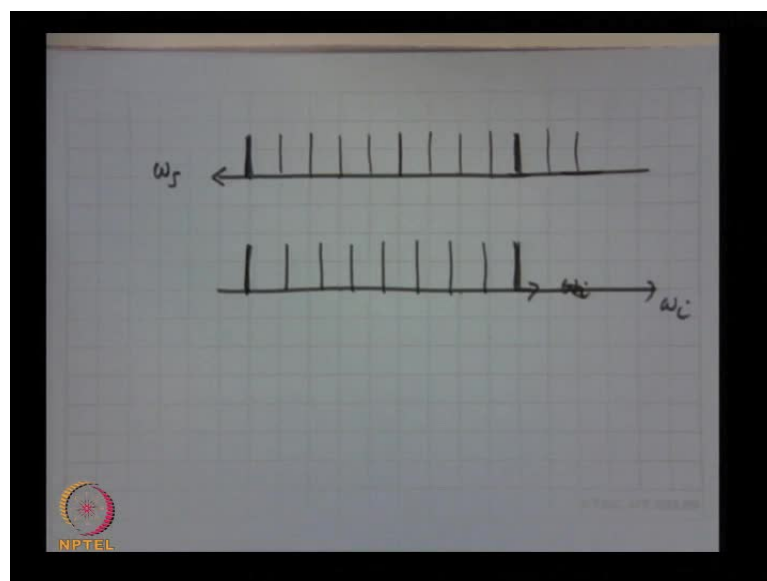
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$$\omega_s = m \frac{\pi c}{n_s l} \quad \omega_s > \omega_i$$
$$\omega_i = p \frac{\pi c}{n_i l} \quad n_s > n_i$$
$$\Delta \omega_s \approx \frac{\pi c}{n_s l} \quad \Delta \omega_i \approx \frac{\pi c}{n_i l}$$
$$\Delta \omega_s < \Delta \omega_i$$
$$\omega_s + \omega_i = \omega_p$$

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That case of problem because suppose for some reason temperature changes or length changes both move both these combs move because, the refractive index depends on temperature or the  $\omega_s$  and  $\omega_i$  values depend on temperature on the length of a cavity any fluctuation in the length of the cavity or the temperature is going to move both these combs. And what will happen is the oscillator will adjust itself to a pair of frequencies which satisfies the conditions at they are exactly overlapping both frequencies must be allowed in the cavity.

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DRO

$\omega_p = \omega_s + \omega_i$   
 $k_p = k_s + k_i$   
 $\omega_s = m \cdot \frac{\pi c}{n_s L}; \quad m = 1, 2, \dots$   
 $\omega_i = p \cdot \frac{\pi c}{n_i L}; \quad p = 1, 2, \dots$

$R_p = 0$        $R_p = 0$   
 $R_s \approx R_1$        $R_s \approx R_1$   
 $R_i \approx 1$        $R_i \approx 1$

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$\omega_s$  ←

→  $\omega_i$

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**So**, what is actually happening is there are too many constraints on the pair of frequencies  $\omega_s$  and  $\omega_i$  as there are too many constraints. And this leads to a problems of problems of instability of the oscillator if as you suddenly see one pair of frequencies coming suddenly that shifts to some other pair and then it shifts back to another pair it keeps on fluctuating randomly.

Because depending on the fluctuations in the cavity the pairs which satisfy the condition that the sum of these two equal to  $\omega_p$  phase matching condition and also that they are both modes of the cavity is putting too much constraints on the frequency pair and these leads to lot of instability in the parametric oscillator.

**So**, D R 0's are by nature not very stable. **So**, you need to take very extensive you need to put extensive efforts to make the cavity as stable as possible. This does not happen in singly resonant oscillator because you need to satisfy only one of these conditions.

In case we have I mean  $l$  is not the same for the both of them will  $x$   $z$  be different for idler and signal  $w$  s. **So**, we have  $\Delta\omega_s \Delta\omega_i$  will be 0  $\Delta\omega$  in this probably will not be there.

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$$\omega_s = m \frac{\pi c}{n_s l} \quad \omega_s > \omega_i$$

$$\omega_i = p \frac{\pi c}{n_i l} \quad n_s > n_i$$

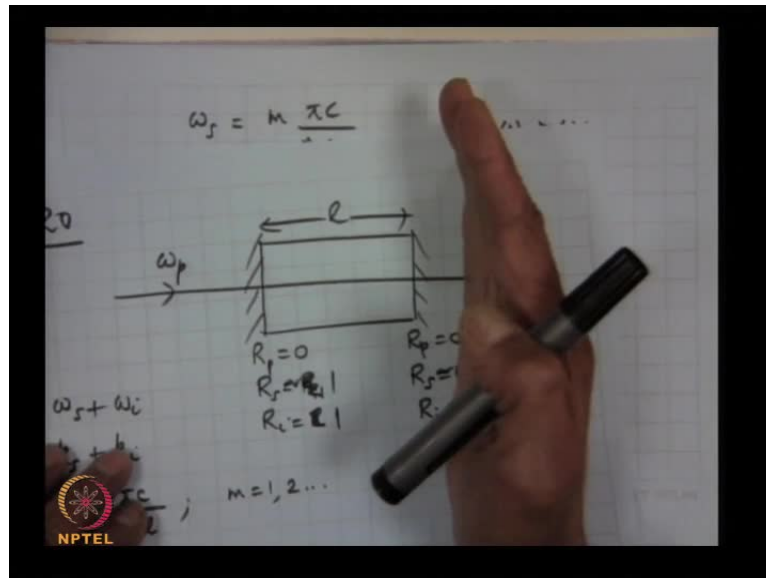
$$\Delta\omega_s \approx \frac{\pi c}{n_s l} \quad ; \quad \Delta\omega_i \approx \frac{\pi c}{n_i l}$$

$$\Delta\omega_s < \Delta\omega_i$$

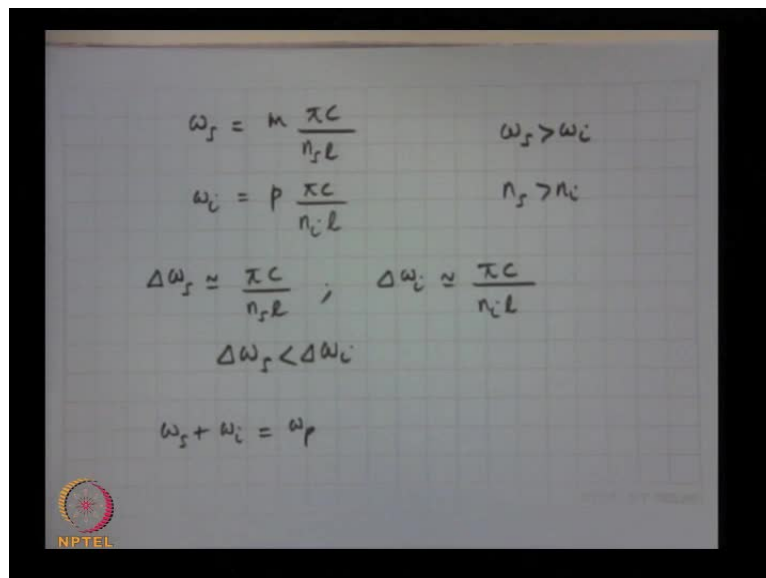
$$\omega_s + \omega_i = \omega_p$$

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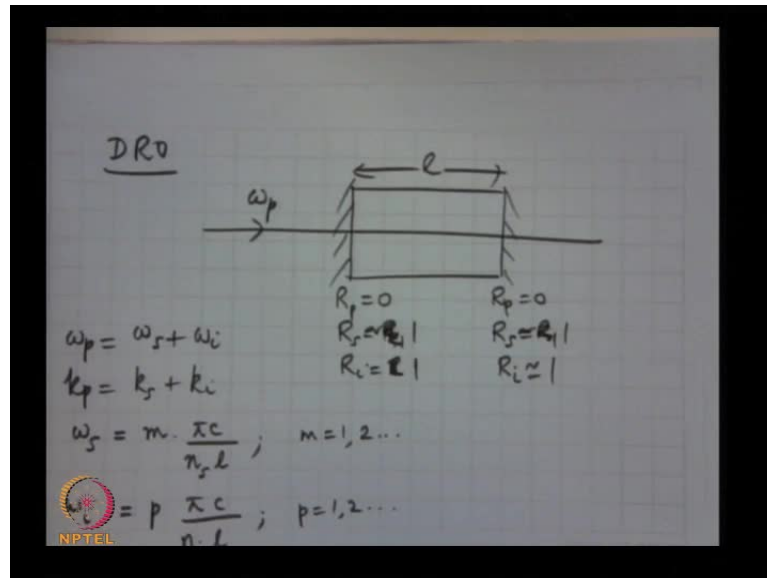
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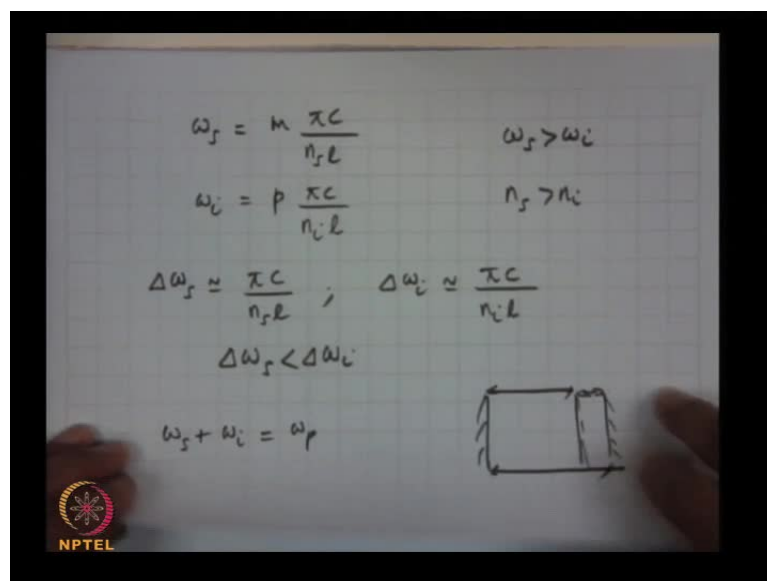
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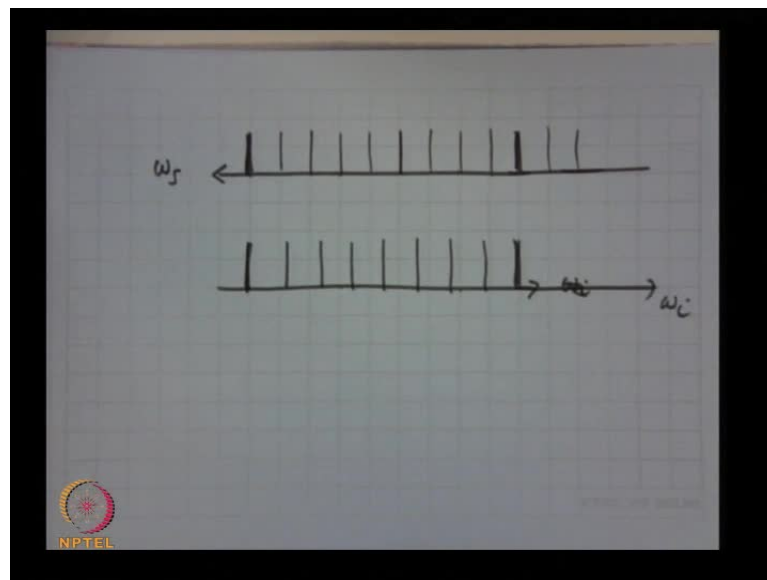


No but, you see please remember that if the it depends also an N s and N i which are functions of frequency. **So**, they are not uniformly spaced actually the way I have written here this for the small delta omegas I have assumed these are frequency independent but, even if you choose this the different lines here n s and n i will be frequency dependent and I will not be exactly able to match at every frequency both the counts this can lead to a problem but, **off course** may be if I choose different cavity lines which means I put another mirror here outside and the because of frequencies above I do not know what

kind of different lines are required. And then if the mirrors are not precisely satisfying the condition of 0 reflectivity at certain wavelengths you will form another standing wave pattern cavity inside see you will have three cavities now interacting with each other you will have a situation like this you have 1 2 and 3 still one mirror here another mirror here another mirror here.

So, signal may be oscillating like this idler may be oscillating like this but, there is another cavity sitting here in between which can lead to problems. So, these are coupled cavity systems so there are people who do coupled cavities but, I do not know whether that can resolve some of these issues.

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So, a D R O has this problem but, as you can see here the D R O has a much lower threshold requirement than compared to a singly resonant oscillator and it is very interesting from that point of view.

So, with this we will finish discussion on the second order non-linear effects. And so if you have any questions on the entire topic of second order non-linear effects we can have a short discussion right now otherwise, I will introduce to you the next higher order non-linear effect and which we will discuss not in detail but, briefly before we move onto a quantum mechanical interpretations of the processes, anything that you would like to ask or raise.



So, what we have essentially seen using second order effects we have second harmonic generation which generate twice the frequency, then we looked at the reverse process of generating  $\omega$  from  $2\omega$  and I showed you that classically that is a impossible process but, it does takes place and that is a quantum mechanical explanation for it. At the same time if I launch  $\omega$  and  $2\omega$  simultaneously I can amplify or attenuate the  $\omega$  frequency and this is a phase sensitive parametric amplifier.

So, the energy can either transfer from  $\omega$  to  $2\omega$  or  $2\omega$  to  $\omega$  depending on the phase of the two inputs at the input of the crystal. We then shifted onto a process in which the three waves are all having different frequencies  $\omega_p$   $\omega_s$   $\omega_i$ . So, I can use this so that is the more general second harmonic generation is a special case of this where I have  $\omega_s$  is equal to  $\omega_i$  but, I can have all three frequencies are different, So I can use this process a three way mixing process they are all three way mixing processes to generate some frequency by launching  $\omega_s$  and  $\omega_i$  or difference frequency by launching  $\omega_p$  and  $\omega_s$  and the difference frequency generation process I showed you leads to an amplification of the signal.

So, I can have in this case I can have either phase insensitive amplification by launching  $\omega_p$  and  $\omega_s$  only or a phase sensitive amplification if I launch all the 3 waves simultaneously and this phase sensitive amplification is very interesting because I will show you later on in quantum mechanical analysis that this can lead to very interesting output from these amplifiers. And once we have an amplifier for a frequency here we can actually put that inside a cavity an optical cavity which is a pair of mirrors and convert the amplifier to an oscillator and that is the optical parametric oscillator.

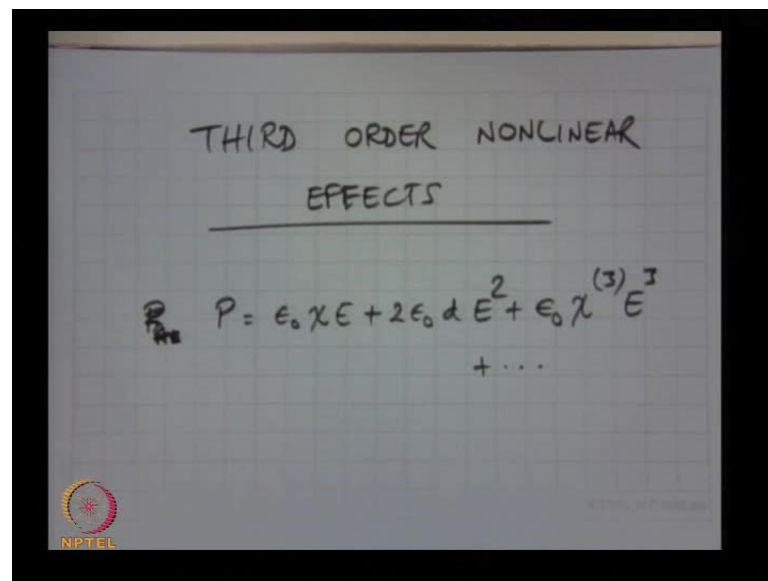
And so these are commercially available devices o p which can be used for tuning a over a broad range of wavelengths primarily in infrared and ah these are many of them are pulsed parametric oscillators where the pump is not a continuous wave but, a pulse wave and we have not discussed the pulsed operation of this of these lasers or of these oscillators but, what we have seen essentially is that what are the conditions needed for the oscillation to begin what kinds of pump levels are required what kind of pump power levels are required for crossing the threshold and when will the oscillation start. And the most interesting aspect of these o p o s is the tunability where you can actually tune that the frequency of oscillation because, it is a non-linear process and not dependent on the energy levels of the atomic system there is no absorption here this is a parametric process

where there is no absorption unlike, a laser built on population inversion a pump frequency is absorbed energy is absorbed the atoms go from one level to other level then they drop down to the third level somewhere which is the metastable level and then there is a population inversion generated and then there is amplification.

**So**, there is an energy loss in this cavity here because you are losing some energy of the pump when the atoms transmit from one level to another level here. This is a completely parametric process wherein principle there is no energy loss.

If you note if you neglect all the other scattering losses this is a system in which the input energy is equal to output energy and **you can you can** have very interesting properties of these amplifiers based on non-linear process.

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THIRD ORDER NONLINEAR  
EFFECTS

$$P = \epsilon_0 \chi E + 2\epsilon_0 d E^2 + \epsilon_0 \chi^{(3)} E^3 + \dots$$

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**So**, we will just spend a few minutes in the next topic which is third order non-linear effects. If you recall we are written  $p$  is equal to  $p$  is equal to  $\epsilon_0 E$  plus  $2\epsilon_0 d E^2$  plus  $\epsilon_0 \chi^{(3)} E^3$  and so on. I am running a scalar equation just for simplicity this is the linear part these are the second order non-linear effects proportional to  $E^2$  this is the third order non-linear effects proportional to  $E^3$ . This one is present only in crystals which do not possess a center of inversion symmetry **so**, if you take a medium like glass which has a completely random matrix this coefficient is absent

**So** the first non-linear term that comes in is  $\epsilon_0 \chi^3 E^3$  and, those are responsible for third order effects.

What I will show you is some of these effects are automatically phase matched, that means the non-linear polarization and the wave it is trying to generate are actually travelling at the same speed they are automatically phase matched. Some of these processes need you to take steps to phase match them. You can build amplifiers based on this just like you can build amplifiers based on this these are also called parametric amplifiers.

This particular one actually I will show you leads to what is called as an intensely dependent refractive index. The refractive index of the medium depends starts to depend on the intensity of the light wave because of this  $E^3$  term and this effect leads to applications in all optical switching where you can switch light from one port to another by using non-linear optical effects and so on. **So** this is the very interesting effect and this particular term is extremely important it has been studied widely because of optical fiber communication. Optical fibers are used in communication they are based on glass silica glass and this is not present this is the non-linear term that comes into picture. And this leads to just like other non-linear term it leads to mixing of light at different frequencies, if you launch light at two frequencies you will start to generate new frequencies if you satisfy phase matching conditions.

**So**, it starts to generate new frequencies and that creates a problem because at those new frequencies you may be actually having a communication going on. So, if you launch  $\omega_1$   $\omega_2$  and  $\omega_3$  the waves at  $\omega_1$  and  $\omega_2$  can mix together to create  $\omega_3$ . **So**, the person who is using  $\omega_3$  to communicate will start to have cross talk from  $\omega_1$  and  $\omega_2$  the other two people who are talking. So, this is a serious problem in communication and so people have found a method to reduce this effect of this and that is by killing phase matching. **If you if you** allow if you do not if you ensure that there is no phase matching this process become very weak and you sort of gain in that in that process.

I can actually use this phase matching to use this process to generate new frequencies. So, actually this non-linear effect can be used for amplification or for generating new frequencies or it can play a role in which you do not want non-linear effects. And that happens primarily in communication systems where because light waves are propagating through 1000 kilometers of fiber so, that is a thousand kilometer length is a huge length in which these non-linear effects keep on building up and they lead to problems and one of the methods which I will briefly discuss is to destroy phase matching and, once you destroy that the efficiency goes down significantly and you have almost gotten rid of the problem of course some of them you still are not able to get rid of them.

So, I think we will stop here. So next time what we will do is we will start to analyze the effects of this term we will not spend too many lectures on this but, some brief discussion and then we will move on to quantum mechanical analysis ok thank you.