Quantum Electronics
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Module # 03
Second Order Effects
Lecture # 18
Non - Linear Optics (Contd.)

We continue with our discussions on optical parameter oscillators.

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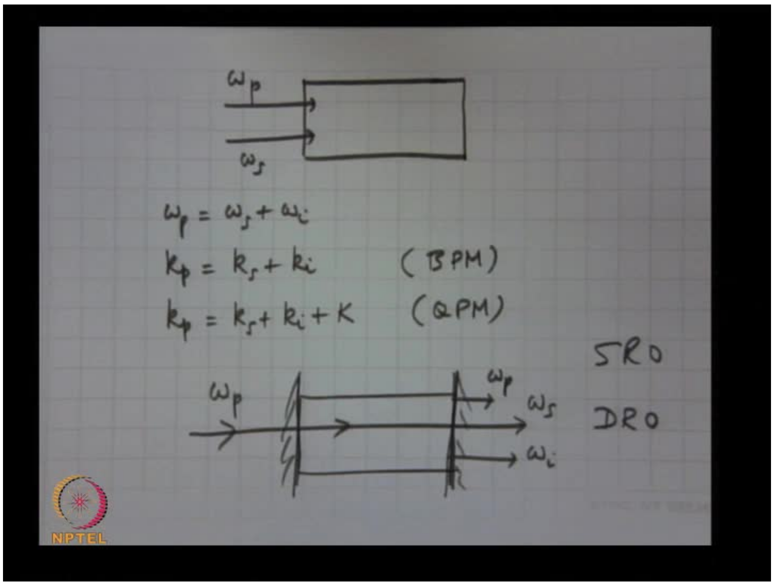

Remember what we have seen is if you take a non-linear medium processing a finite chi 2, if you launch a frequency omega p pump and a signal omega s with omega p is equal to omega s plus omega i. Then it is possible to amplify the signal at frequency omega s by the process of parametric down conversion. That means this particular device acts like an amplifier and also we have seen that we did satisfy the condition k p is equal to k s plus k i or k p is equal to k s plus k i plus k this is Q P M, this is Birefringence Phase Matching.

It is possible to tune the frequency at which this amplifier would work by adjusting this condition. So, for a given pump and signal frequency you have to satisfy the energy conservation equation and the momentum conservation equation either the first one or the second one if there is periodic domain reversal. This is a very interesting device because this amplifies signals at frequency omega s and the value of omega s can be changed by changing the orientation of the crystal for example. So that, birefringence phase matching is satisfied by different sets of wavelengths, signal and idler or by choosing different k values that the periodic domain reversals. So that, you can have a tunable.

Virtue of an amplifier, you can actually put the amplifier within a pair of mirrors and convert the amplifier to an oscillator and oscillator it is a device which is a source of radiation. If I put this amplifier between a pair of mirrors and if I launch lay it and omega p into this crystal, let me assume a mirrors have no refraction at the pump frequency. So, the mirrors are completely transmit at to get the at the pump frequency.

When pump enter the crystal the first thing that happen is it can spontaneously down convert to a pair of frequencies omega s and omega i that satisfy the pair of energy conversion, and momentum conservation conditions. One there happens, then I can have these 2 mirrors. So that, there highly reflecting for example, at the frequency omega s. So, the omega s slides when it propagates through this it gets amplify once after having generated spontaneously the signal photons. The signal photons get amplified when they reach the second mirror here at this point if the reflectivity at the signal frequency is high part of that signal is transmitted and majority of it is reflected back into the cavity. As I propagate here in the reverse direction I do not satisfy the phase matching condition. So, the signal just propagates and gets reflected from the first mirror, and then again it propagates the forward direction getting amplify. What is going to happen is if the refractivity's of the mirrors are very high most of the energy of the signal is reflected back and forth into inside the cavity, and it grows in magnitude. I can have a situation where the mirrors are transmitting at the idler frequency.

Omega i actually it does not reflect back and forth it escapes from the cavity but, the omega s signal is completely is sort of reflected within the cavity and has a very high energy density inside the cavity. This is called as a singly resonant oscillator where you are resonating only one frequency omega s, what will happen is if I put in a pump omega p the gain inside the crystal depends on the intensity of the light at omega p.

Remember the gain coefficient depends on the non-linear coefficient of a crystal and it also depends on the intensity of the pump wave. If my intensity of pump wave is sufficiently high what can happen is the gain suffered by the signal in one round trip can be compensated by or can compensate the loss suffered by the signal in one round trip. The condition for oscillation is as the signal does one round trip inside the crystal and that means it starts from the mirror goes to the second mirror reflects, comes back reflects. So, that is one complete round trip.

If in one round trip the loss is compensated by the gain then I will have a strong signal coming out from here and of course,, omega i will also be coming out, and unused omega p will also be coming out. So, I can actually make a source of radiation at omega s by taking this non-linear crystal putting it inside a cavity, the cavity will resonate one frequency omega s and what will happen is just like a laser actually, it is a coherent source in a standard laser like helium neon laser or ruby laser or, (( )) laser. The amplification is brought out by population inversion and that depends on energy levels, available energy levels of the atoms.

This amplification is a non-linear process and by orienting the crystal in different directions I can actually change the frequency at which it will amplify. So, this is a tunable source of radiation and it can have a very broad tuning range independent of the energy levels, there is no absorption by the crystal simply at non-linear conversion process.

Just like a laser I need to satisfy some conditions of round trip gain and also the frequencies which you will oscillate here are determined by the standing wave pattern since have the cavity, not all frequencies are allowed because only those frequencies in which the phase change in one round trip is exactly equal to multiple of 2 pi can cancel or build up inside the cavity. I can also have a situation where not only the signal is reflected but, also the idler is reflected back into the cavity in which case I will have to have standing wave conditions for omega s and standing waves of conditions of omega i, and again the round trip gain must be equal to loss..

Yes, Mohith

Sir, omega s and omega i are determined by the orientation of the crystal.

Yes.

Now, after this the further condition should also be that it should be the standing wave condition should also be imposed.

Yes

Now, in case when omega i and omega s satisfy the orientation of the crystal and do not satisfy the standing wave conditions.

So, I come to these numbers, what is first of all one thing is that it is not that amplification can take place only at one frequency. We have seen that there is one frequency for which this condition is exactly satisfied, ok. That is the maximum gain if my signal frequency deviates from this condition, delta k become a finite gain drops.

It is a dropping, there is dropping gain. There is still gain as you change that. So, there is a range of signal frequencies over which I will get gain from the crystal for a given orientation of a crystal. Once I fix the crystal direction there is one pair omega s, omega i which exactly satisfy this condition for that why delta k is equal to 0. I get maximum gain if omega s is slightly different from this number delta k is not equal to 0 but, there is still gain.

Until, remember we had obtained a condition until g square is greater than delta k square by 2 by 4 I have gain and after that the hyperbolic functions change into sine and cosine functions rather than the hyperbolic functions and the gain drops. I will give you some numbers later on that the range of frequencies over which I can achieve gain, it is called the gain band width and that will contain many modes many standing wave patterns of this cavities.

Sir but, in order to use at, as you know in for a laser.

Yeah

We need to ensure that the omega is said be 1.

Yeah

The cavity length should be in accordance with that omega s.

Yeah but, these omega is differences are very small will gets some numbers even a standard, in a standard laser like a helium neon lasers you are not getting one wave frequency usually you are getting 3 or 4 frequencies coming out which are very closely like it will reach each other.

And then peak. So, intensity peak that for single frequency.

Yes. But, the intensity difference will not be significantly different I mean there are intensities are not significantly different. I can have typically in a laser I will not have a single frequency oscillation, there are multiple frequencies coming out I need to do something special to make the laser single frequency laser. I need to put a device inside the cavity like a (( )) to make sure that there is one frequency that comes out of the laser. Which means it will have a large coherence lines, delta lambda becomes very small unless it will not become that small unless I do something special on inside the laser cavity.

Apart from that there is also a problem that frequency can keep fluctuating the laser can be oscillating at the single frequency but, that frequency cab be easily moving back and forth here and there slightly because at the temperature changes the length of that cavity will change. And that change in length of the cavity means the frequencies of oscillation will change. So, there is a small fluctuation, if I need to have a frequency stabilized laser, I need to give a feed back and make sure that the frequency at which the laser is oscillating remains fixed. Even standard lasers do not give me single frequency unless I do special.

This will give me for 1 frequency which will be the largest gains but, usually there it is a single frequency output that comes out from here, the advantage is it is tunable and over a large range of wave lengths independent of the energy levels of the crystal here, there is no absorption unlike a standard laser, there is population inversion created by absorption and emission.

I can have a situation there both are oscillating and that is called a doubly resonate oscillator. So, you have a singly resonant oscillator and a doubly resonant oscillator. In a doubly resonant oscillator both the signal and the idler are resonating inside the crystal both are forming sanding wave patterns inside the cavity. The mirrors have high reflectivity's both at omega s and at omega i, and I will show you that this imposes severe conditions on the stability of a cavity because if the cavity has a slight changes then the laser becomes unstable, the frequencies are shifting back and forth very violently as if you do not maintain the cavity when I precise condition.

I have also shown you that the power required for a D R O is much less than power required for an S R O, what is the pump power? So, the question is how much pump power do I need to make the laser oscillates. I take a crystal say lithium niobate coated inside a pair of mirror say 98 percent of reflecting mirrors, certain length of a cavity 5 centimeters. So, what is the power required at omega p for this to start lasing, to start emitting.

I must make sure that the round trip loss becomes equal to round trip gain. The round trip loss is fixed by the reflectivities of the mirrors and any other loss that takes place inside the cavity like absorption scattering the gain is determined by the intensity of the pump. So, for a given loss inside the cavity I will have a threshold pump power, I will have a threshold pumping intensity or a threshold pump power at which this will start to oscillate.

It is like any other laser in any other also I need a certain minimum current in the helium neon cavity or a minimum pumping power before this laser starts to oscillate or in a semiconductor laser I need a minimum current to make the laser starts to emit because I need to overcome losses in the cavity. Also note that the oscillator starts by spontaneous parametric down conversion, parametric fluorescence that is the noise that starts the laser from to oscillate a noise is required for any oscillator start. So, first what we will do is look at a singly resonant oscillator discuss a singly resonant oscillator and then look at D R O.

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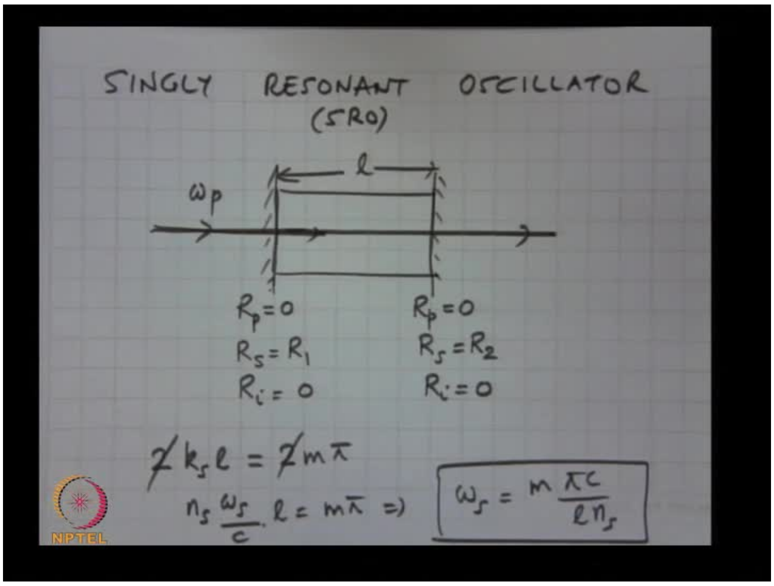

Singly Resonant Oscillator, let me draw the system here is my crystal of length l. Let me assume that the crystal fills the entire cavity. So, I have light at omega p frequency coming from here. The reflectivity of the mirror at pump is 0 R p is the energy reflectivity of the mirror at the pump frequency p omega p, R s the reflectivity the signal frequency. Let me assume it is R 1 and this is R 2 they could be different they could be equal.

For example I could have this to be 1 and this to be some finite value 98 percent, I could have both finite numbers whatever it is and then finally, R i is equal to 0 for both the mirrors. So, the mirrors are highly reflecting at the signal frequency and completely transmitting at the pump and the idler frequency, the pump enters and just goes through the crystals. The way I will analyze the problem is 2 things, one is I need to find out what is the frequencies of oscillation of the cavity and secondly what is the threshold power required for the laser to start oscillating. First thing is what are the frequencies of oscillation of this cavity remember the phase shape in one complete round trip must be in integral multiple of 2 pi.

So, what is the condition, I will get 2 times k s times l is equal to I am assuming in writing this condition that the mirrors are not introducing any phase changes and the phase changes primarily because of propagation through the crystal. If there were phase changes suffered because of the mirrors I have to add those phase change to 2 k s l. So, k s times l is the phase change is going from here to here, another k s times l is the phase change is going from here to here. So, the total phase change in one round trip is 2 times k s l and that must be analytical multiple of 2 pi.

This gives me this goes out 2 goes off. So, omega s by c into l is equal to m pi or omega s is equal to m into pi c by this is omega by c omega n s. Sorry there must be frequency, a refractive index her, n s is the refractive index of the medium at the frequency omega s, the propagation constant inside the medium is omega by c times of the refractive index. The frequency divided by c times n s into l is equal to m pi. So, the frequencies of oscillation are m into pi c y by l n s.

This is a discrete frequencies and what is the difference we can calculate what is the frequency spacing between the modes. These are the modes of oscillation of this cavity that means the modes that can survive within the cavity which from a standing wave pattern inside the cavity.

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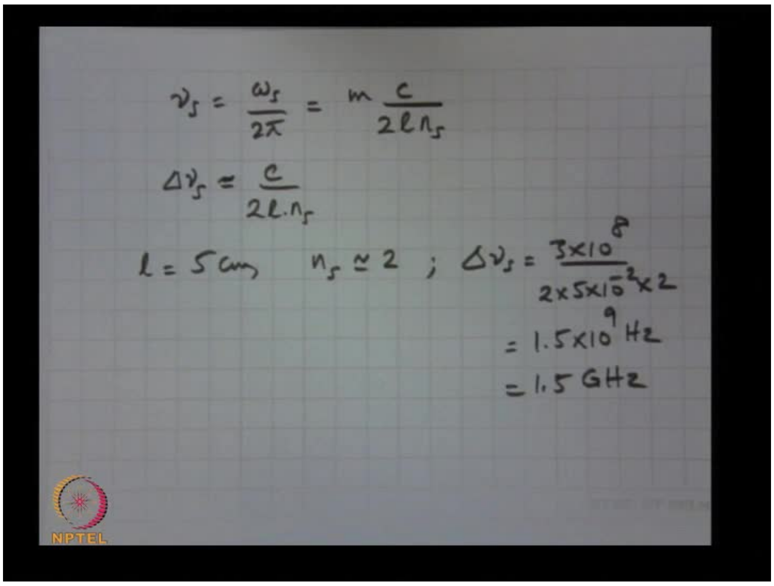

So, in terms of nu, let me write this is nu s is equal to omega s by 2 pi which is equal to m into c by 2 times l times n s, this is the angular frequency omega s and that is a frequency of oscillation. Now, assuming that the frequencies are not very far spaced which means I am assuming that n s does not change much, as I change my frequency. The inter mode frequency will be how much inter mode spacing c by 2 time l times n s.

Let me put some numbers here, let me take a cavity of 5 centimeters length, what is the value of n s approximately 2 for lithium niobate is 2 other wise 1.8 for some crystals, is of the order of 2. So, delta nu s comes out to be 3 10 to the power 8 by 2 into 5 10 to the minus 2 into 2. Which is equal to, how much is that?

1.5 into 10 to the power 9.

1.5 into 10 to the power 9 hertz. So, that is 1.5 gigahertz the frequency spacing is about 1.5 gigahertz, every 1.5 gigahertz there is a possible frequency of oscillation within the cavity. There is no condition on omega I except that this frequency omega s plus omega i must be equal to omega p.

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Omega s is fixed by the cavity, these are only allowed frequencies, omega p is what I have chosen. So, omega i automatically gets fixed knowing omega p and this omega i can calculate what are the omega i's which will be coming out of the crystal.

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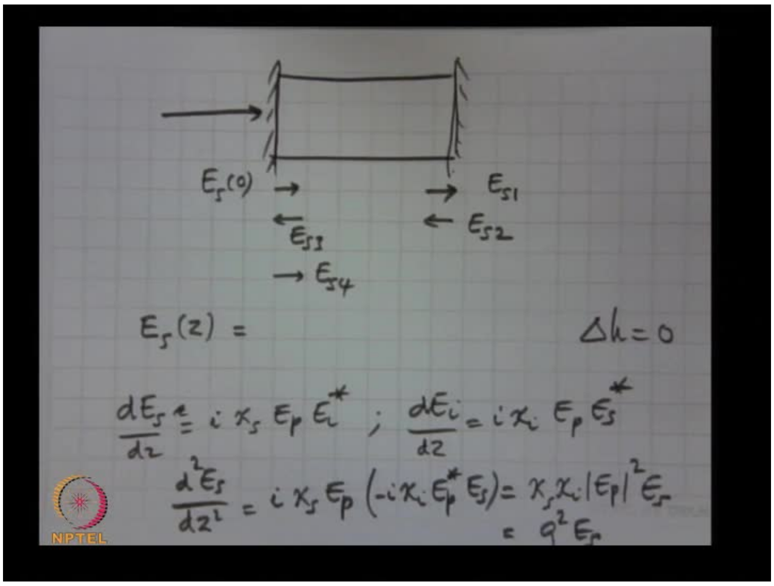

Now, I need to calculate the threshold condition. What is the power required or what is the intensity of the pump required for the oscillator to start? So what do I do, this is my crystal again that is a mirror here there is another mirror here. Let me start with amplitude E s 0. At this mirror the amplitude incident is E s 1, the amplitude reflected is

E s 2, the amplitude incident here is E s 3 and the amplitude reflected is E s 4 that is one round trip starts from here gets reflected comes back and comes out. So, what is the condition for oscillation E s 4 must be equal to E s 0. If that happens then every time you have a round trip whatever you have lost you have gain and this continuous forever.

Sir, this is in the steady state

Steady state, yes. We are looking at steady state situation where your continuous pump coming in from here and the signal will adjust itself to a value such that there is a complete compensation of loss by the gain. So, now to simplify analysis let me assume the only lost mechanism is the 2 mirrors which have a finite refractivity of r 1 and r 2.

Sir, in this case when the omega s is getting directed.

Yeah

Here is only omega s, it confidently as spontaneous convergence from omega s to omega i, suppose.

If omega s and omega i are strong enough both of them it is also possible, back conversion to omega p it is possible surely it is possible but, in the first analysis to get the threshold value I do not have to worry about it.

Because, at threshold the intensity it is a signal and idler are not, are very high beyond threshold. Once it starts to oscillate there is a lot of power at omega s inside the cavity there is omega p and there is also omega i because it is getting generated inside the cavity but, getting out of the cavity I need to look at back conversion also.

Which means, when I actually have to solve those equations to get the signal power that is coming out finally, we are not going to do that here what I am going to do is what is the pump power require to start the laser to start the oscillator. When the oscillator starts the signal powers are not very high it is just how we calculating threshold. At the threshold the signal powers are not that high and we can estimate a very well what is the pump electric field required or pumping intensity required to satisfy the condition that loss is equal to gain.

So, Sir, in that case we need to, this process will also be a process of loss.

Yes. For the signal, yes exactly the back conversion to omega p is another loss mechanism for this pump, for the signal. Yes, ok.

So, remember we had obtained an expression for gain can you recall E s of z, what is the expression for the gain with delta k is equal to 0 and only the signal incident.

Let me write down the equations d E s by d z is equal to i kappa s E p E i star d E i by d z is equal to i kappa i E p E s star i differentiate the first equation d square E s by d z square is equal to i kappa s E p d E i star by d z which is minus i kappa i E p star E s which is equal to kappa s kappa i mod E p square E s.

Now, can you tell me what the expression is, we call this g square E s, what is the solution of this equation?

(())

And cos hyperbolic, right? So, what is the condition I should put?

For example, the solution of this equation is E s of z is equal to A sin hyperbolic g z plus B cos hyperbolic g z. These are the 2 differential equations, I am assuming pump to remain constant no pump depletion then I differentiate, I get this equation.

(()) 0, z equal to 0, signal is 0.

No, signal is finite.

E s 0, yes.

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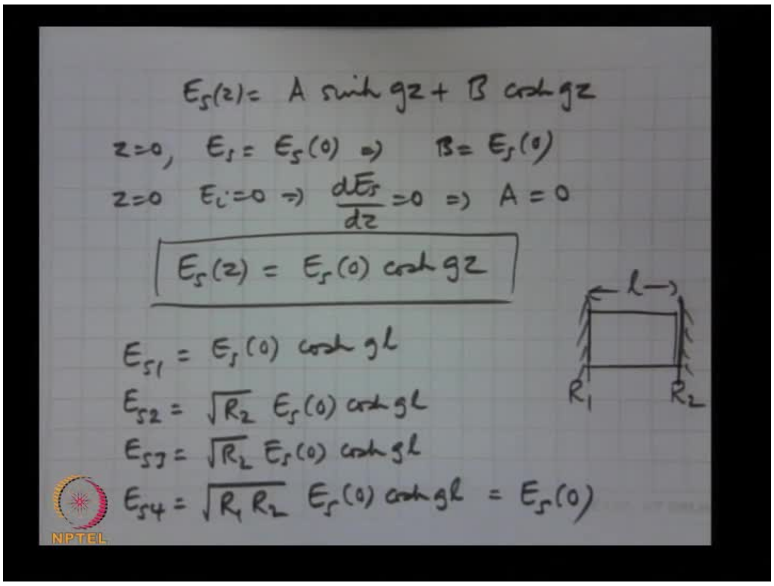

So, z equal to 0, E s is equal to E s 0 this implies B is equal to

E s 0

E s 0, how do I find A?

From E i

E i is equal to 0

Differentiate E s

No, I do not have, here E i is equal to 0. Yeah, at z is equal to 0. So, E s by d z is equal to 0 at z equal to 0 that means what is the value of A, A is 0. So, I will simply get, at z is equal to 0, E i is equal to 0 implies d E s by d z is equal to 0 this implies A is equal to 0. So, the solution is E S of z for large g z it becomes exponential but, otherwise it is cos hyperbolic. This gives me the expression for the signal field in, going from let me call this mirror M 1 to mirror M 2. From M 1 to M 2 the signal field increases gets multiplied by factor cos hyperbolic g z while going from M 2 to M 1, there is no multiplication factor because in the reverse direction delta k is not equal to 0. In fact delta k is very large because delta k, what will be the delta k for reverse direction you will get k p plus k s plus k i or minus k i, you can do. But, it will not be able to satisfy this condition. So, the pump going in this direction, signal going in this direction there is no amplification. Because, there is no pump in the reverse direction, there is no amplification and E s remains the same.

Let me calculate, I need to calculate from E S 0, what is E s 1, E s 2, E s 3 and finally, E s 4. Now can you tell me what is E s 1, E s 1 is equal to E s 0, cos hyperbolic g times l the length of a crystal is l. Let me draw the figure here again, these mirrors have reflectivity's R 1 and R 2. So, what is E s 2, square root of R 2 times this, remember this r 2 is the energy reflectivity and these are electric fields. The energy reflectivity is R the amplitude of reflectivity is square root of R assuming no phase change in reflection. Otherwise I will have an exponential, some phase factor. It will be square root of R 2, E s 0, cos hyperbolic g l, E s 3 is in this back in the first mirror which is the same as E s 2 and then E s 4, and another multiplication by square root of R 1 the fields starts from here gets amplified and reaching here gets reflected by a factor root of R 2. Then it does not get amplified, it just propagates as such and then it get reflected by the mirror M 1 with the reflectivity square root of R 1 amplitude reflectivity square root of R 1 and that is the field after 1 round trip. And this must be equal to E s 0.

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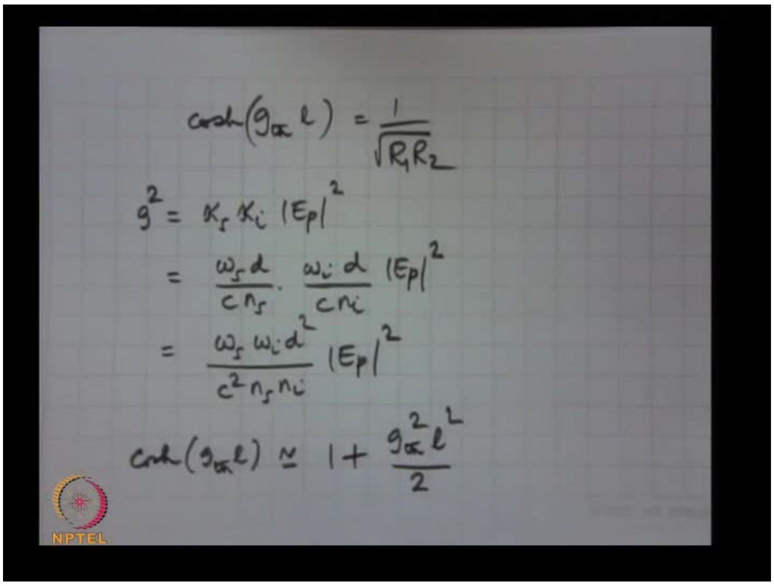

I get a condition that cos hyperbolic, let me call this cos hyperbolic if it is a subscript threshold g t h l is equal to 1 by square root of R 1 R 2. And g, remember g square was equal to kappa s kappa i mod E p square and this is omega s d by c n s omega i d by c n i mod E p square which is equal to omega s omega i d square by c square n s n i mod E p square. So, the gain coefficient g depends on the non-linear coefficient d and the electric field of the pump frequency, and the electric field at the pump frequency depends on the intensity of the pump inside the crystal and that will depend on the power of the pump that I am feeding in and what I am doing to focus if I am focusing with the same power the intensity will increase. So, it will increase if I focus the beam inside the cavity.

So, depending on the refractivity's of the mirrors and the length of the cavity there is a value threshold value of mod E p square and that means a threshold intensity of the pump that will be required for the oscillator to start. If you do not put enough pump power into the crystal if you do not have enough pump power coming from here it will not oscillate because the gain is less than the loss. As you increase the pump power inside into the crystal there will be a critical point at which this becomes equal to the left hand side becomes equal to right hand side, and then it starts to oscillate and if you increase the power beyond this pump power, beyond this the signal will increase just like a laser.

Now, R 1 and R 2 are very close to 1, 98 percent 99 percent because I need to have a good resonator with which I need to reflect most of the light inside the cavity. So, that the intensity of the signal inside is high. The right hand side is very close to 1, cos hyperbolic x is close to 1. So, what is the, what do I do, I can expand to get an approximate expression for g threshold I can expand cos hyperbolic g threshold l knowing that that value is close to 1. So, what is expansion of cos hyperbolic x for x close to 1, can you find out?

Hyperbolic x

Yeah, cos hyperbolic g threshold l when it is close to 1 because I know R 1and R 2 are close to 1. Yes. So, cos hyperbolic, g threshold l is approximately equal to 1 plus at x is equal to 0 cos hyperbolic is 1, see it has to 1 plus something. So, I can substitute this into this equation for the threshold and get an approximate equation for g threshold because R 1 and R 2 are close to 1.

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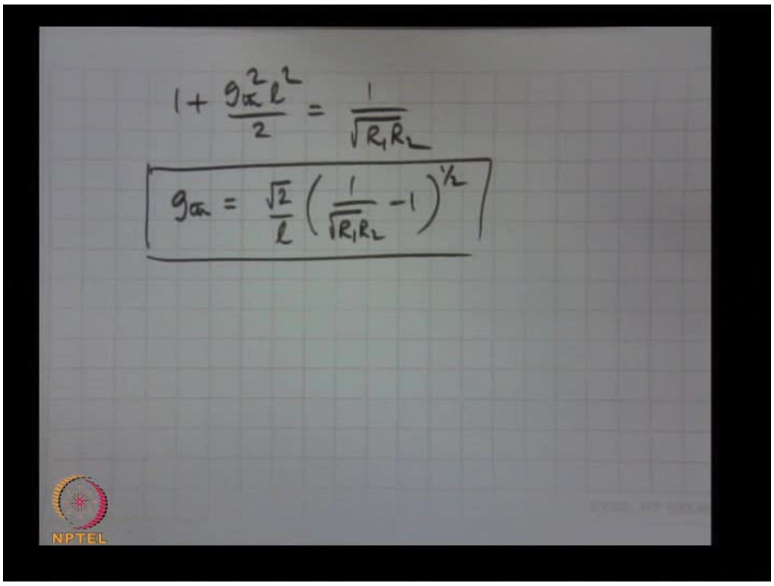

And if I assume for example, this gives me 1 plus g threshold square l square by 2 is equal to 1 by square root of R 1 R 2 or g threshold is equal to, this is 1 by square root of R 1 R 2 minus 1 into square root of 2 by l raised by half. So, how do I relate this to the power of the pump?

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So, g square depends on E p square like this, what do I do? How do I calculate I p or p p? I must replace mod E p square by the intensity at the pump.

How are they related?

P g square by (( ))

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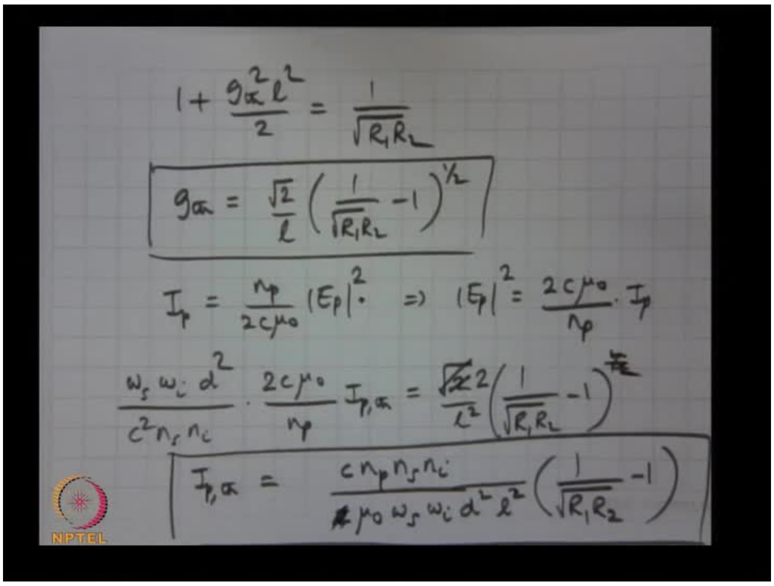

Yes. I have I p is equal to n p by 2 c mu 0 mod E p square into that is all. So, this implies mod E p square 2 c mu 0 by n p into I p. So, g threshold becomes omega s omega i d square by c square n s n i into 2 c mu 0 by n p I p threshold is equal to square root of 2 by l 1 by square root of R 1 R 2. That gives me an I p threshold, once c goes off and I get c, n p, n s, n i by.

Sir this is the left side is a square of g (( )) the equation that you have written

Yeah, that is g square.

To right side

Yeah. So, I must remove this. Sorry, yeah, this is 2 by l square and this goes off. Yeah, thank you.

So, I p threshold c n p and n s n i by 2 mu 0 omega s omega i d square. So, that factor of 2 goes out from here l square 1 by square root of R 1 R 2 minus 1. So, let me estimate some numbers. Let us put some numbers and calculate what is the threshold intensity required. We have an idea added in gigwatts per square meter or is it megawatts per square meter or is a watts per square meter. We have to be clear whether it is a toll feasible, let me substitute the numbers now we do a quick calculation.

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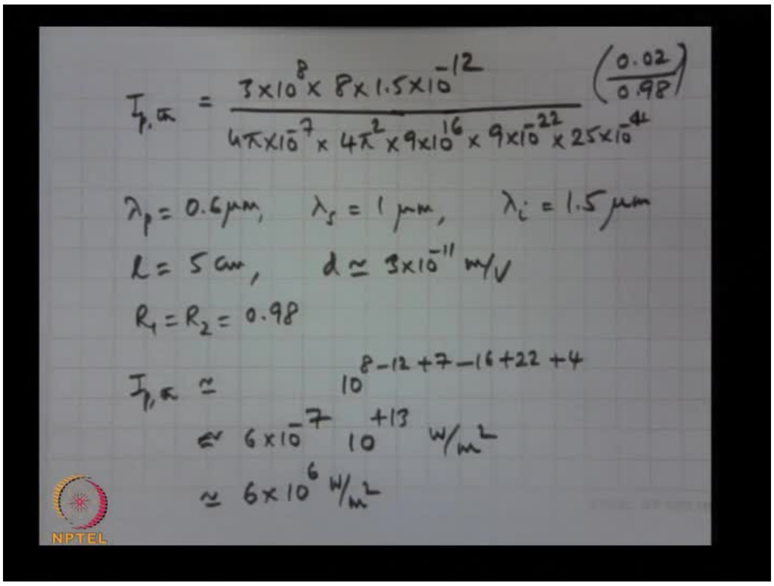

Now, I p threshold. So, this is c which is 3 10 to the power 8 meter per second. What do I substitute approximately for n p, n s, n i approximately 2 is 2 is 2 say that is 2.1 2. Etcetera. Let me substitute this as 8 divided by 4 pi 10 to the minus 7 is mu 0. Now omega s is 2 pi c by lambda s there is no refractive index coming in omega s, it is a frequency. I will have 4 pi square, c square by lambda s lambda i. I will substitute lambda s lambda i. So, let me assume lambda p is equal to 0.6 micron this is same number we had used last time lambda s is 1 micro meter and lambda i comes out to be 1.5 micro meters. And let me take a length of 5 centimeter and what is approximate value of d 30 10 to the minus 12 for lithium niobate. So, 3 10 to the power minus 11 meters per volt and let me assume R 1 is equal to R 2 is equal to 0.98. So, 4 pi square c square by lambda square which is 1.5 10 to the minus 12 into d square which is 9 into 10 to the minus 22 into l square 25 10 to the minus 4 into 1 by R minus 1, 1 by 0.98 minus 1 which is 0.02 by 0.98.

Can simply multiply all the numbers and just I leave the factors out. So, I have something into there is, 10 raise to power 8 from here minus 12 plus 7 minus 16 plus 22 plus 4. I have 10 to the power how much is, yeah plus minus 13 right, plus 13 and what then what is this factors. So, the remaining factors if we multiply.

3.6 and 10 to the power 10 to the power 8.

Forget about all the factors, I forgetting about the 10 to the power.

Yeah. So, I am telling the answer.

You are getting the total answer, what is the answer?

8.6 into 10 to the power 8.

You should get something like 6 10 to the minus 7 you will get something like 6 megawatts per square meter.

Anyway we just calculate multiply and it comes out to be of the order of 6 megawatts per square meter. So, what is a power required? How do I estimate, I must have some area, what area do I take, 1 centimeter square that is large usually laser beams are much smaller in size few millimeter.

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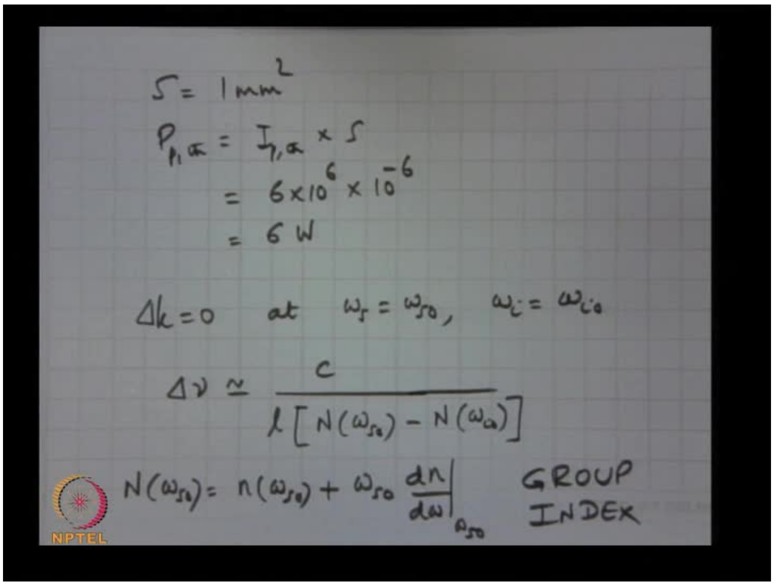

Let me take 1 millimeter square area let me assume an area of 1 millimeter square. The pump power threshold is I p threshold into s which is 6 10 to power 6 into 10 to the minus 6 watts.

May be 8 6 to 10 watts of power is required for the oscillator start. So, that is a lot of power

7.3 mega watt

7.3

So, I said between 6 and 8, 6 and 10. It is of the order of about 10 watt of power is required for the oscillatory to start and if your thresh, if your pump is, power is less than 6 watts. In this example the oscillator will not start because there is not enough gain provided by the crystal for overcoming the loss in the cavity which is because of here finite reflectivity in a mirrors. In fact the loss will be slightly higher because there is also scattering loss. There are refraction losses, there are other kinds of losses in the cavity which we have not taken into account here just took an estimate of the numbers involved. Because I will show you that in a doubly resonant oscillator the power requirements are much lower about a factor of 100 down compare to this.

Now we have a quiz

Sir, no sir, next time

Next time, when?

Wednesday

Wednesday

Ok

Now, I need to estimate 2 things I have already estimated one thing the frequency spacing between modes 1.5 gigahertz. I need to know, what is the frequency region over which this oscillator will provide gain? Is it of the order of 1.5 gigahertz, has much larger or much smaller, I need to have this number. Now, I had left a problem to you for calculating a bandwidth of a second harmony generation. I need to do very similar thing here, I need to calculate. So, for example, I have delta k is equal to 0 at some signal frequency omega s is equal to omega s 0 and the corresponding idler frequency omega i is equal omega i 0. The question essential is this amplifier provides amplification over what range of signal frequencies.

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If I take this crystal I do not put in inside the cavity but, I look at it in a amplifier. So, if I take this crystal and I launch pump, and signal then at one particular frequency pair this maximum gain where I satisfy delta k is equal to 0. Now if I change my omega s keeping omega p constant I will not have delta k is equal to 0 and the gain will drop, I cannot use the solution that we have obtained earlier because now delta k is not equal to 0 in fact we have done this in the class if delta k finite we had obtain in effective gain which was square root of g square minus delta k square by 4. So, as delta k increases the effective gain starts to fall and there is a certain range of frequencies over which the amplifier will provide gain.

Now, let me give you an expression for this and I will leave it as an a exercise for you, please do it if you have difficulties we will discuss in this class but, please raise this issue if you have problem in deriving this expression we will come back and calculate here. This is under sum approximation, I leave for you to find out what the approximations are you need to make a Taylor series expansion and so on. But, the frequency range is approximately c by l times, I will tell you what this coefficients are N at omega s 0 is N at omega s 0 plus omega s 0, d n by d omega at omega 0.

What is this quantity? Have you seen this before?

Expansion of

Yeah but, what is this quantity N, group index. The velocity at which the group velocity of the wave in the medium is determined by just number, this value of index and not the refractive index n defines the phase velocity, N defines the group velocities.

If you go back and look at wave propagation medium the phase velocity, the rate at which the phase is propagating is determined by n, c by n the velocity at which the wave packet is propagating is determined by c by N. This is the group index at signal frequency and that is the group index at the idler frequency. Similarly, I will have N of omega i 0 is equal to n s, n of omega i 0 plus omega i 0 d n by d omega at omega i 0.

So, given a refractive index of the medium I can actually calculate the group index at the signal frequency and at the idler frequency. Approximately the range of signals over which the amplifiers can provide gain is determined by this condition, the first thing you notice is that if you go to a position of degeneracy, what is the meaning of degeneracy?

Omega is equal to 0

Omega i 0 is equal to omega i 0 the bandwidth becomes infinite according to this formula. But, I cannot use this formula, I must go to the next higher order term in my Taylor series expansion. It does not mean infinite bandwidth it means that the bandwidth become very large. If you want a large bandwidth you need to operate near degeneracy of the parametric amplifier then you have a large bandwidth otherwise if bandwidth is reduced because to the difference in this numbers.

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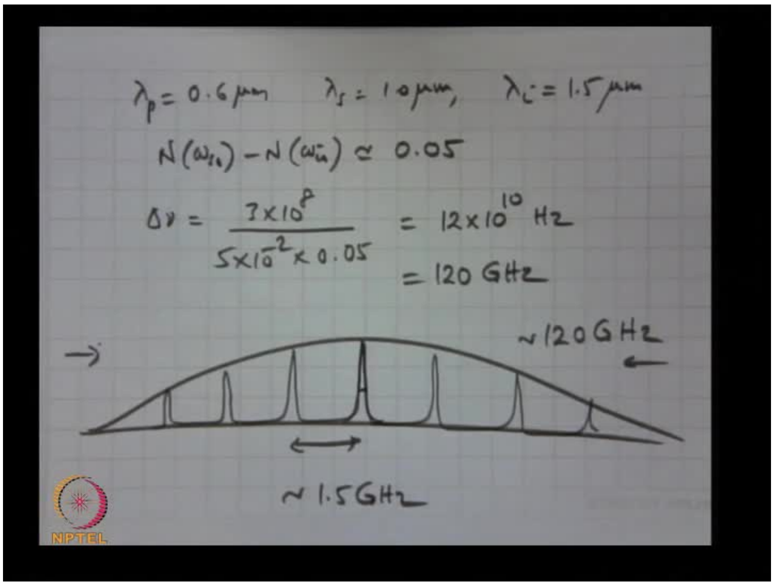

Let me just give you some typical numbers this group index difference for the case which we have considered as lambda p is equal to 0.6 microns lambda, s is equal to 1.0 microns and lambda i is equal to 1.5 microns, the group index difference delta omega s 0 minus n of, sorry omega i 0 is approximately 0.05 and delta nu comes out to be 3 10 to the power 8 by 5 10 to the minus 2 into 0.05.

How much is that? So, 3 by 25 10 to the power 12, how much is that?

12 into, 120 gigahertz much larger than 1.5 gigahertz get calculated. It looks like this you have a gain is provided like this and you have the modes this spacing is about 1.5

gigahertz and this width is about there are large number of frequencies within the entire gain bandwidth. It is not, I am not restricted by the fact that the gain bandwidth is smaller than the interest mode spacing the mode spacing is much smaller than the entire gain bandwidth.

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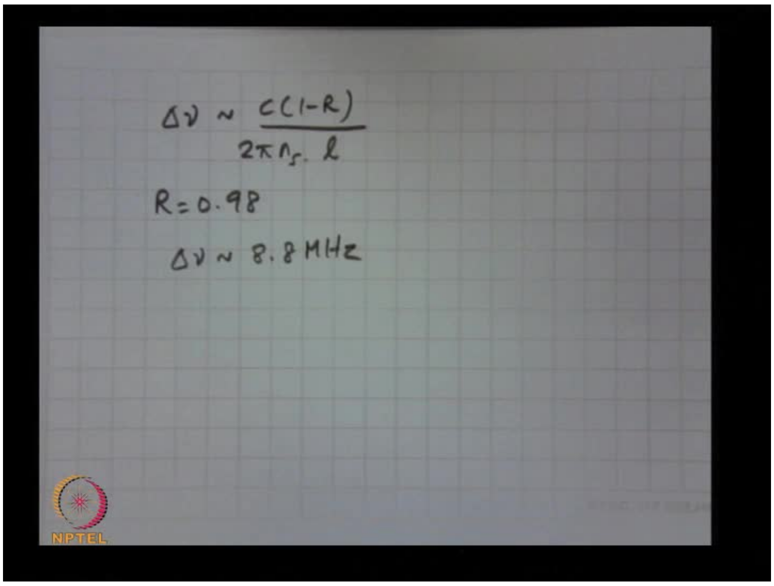

The other thing I need to estimate is this the way I have drawn it is as if they are well resolved frequencies, what determines the width of this each line, have you studied fabry perot into parameter, what determine the width of this lines, reflectivity of the mirrors the coefficient of finesse of the resonator and let me do the expression if you have forgotten please go back and read fabry perot, the effect of each mode is approximately c into 1 minus R by 2 pi n s into l.

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And if you take R of 0.98 delta nu comes out to be 8.8 megahertz. So, that is why I have drawn the width of these lines to be much smaller than the spacing between these lines. So, this width is about 9 megahertz approximately, inter mode spacing of 1.5 gigahertz and the overall gain bandwidth of 200 gigahertz on 1.5 gigahertz.

I think, I we stop here. I will continue with this little bit of discussion on this modes here and then we move on to w resonant oscillator, and again we need to calculate what is the threshold power required for w resonant oscillator.