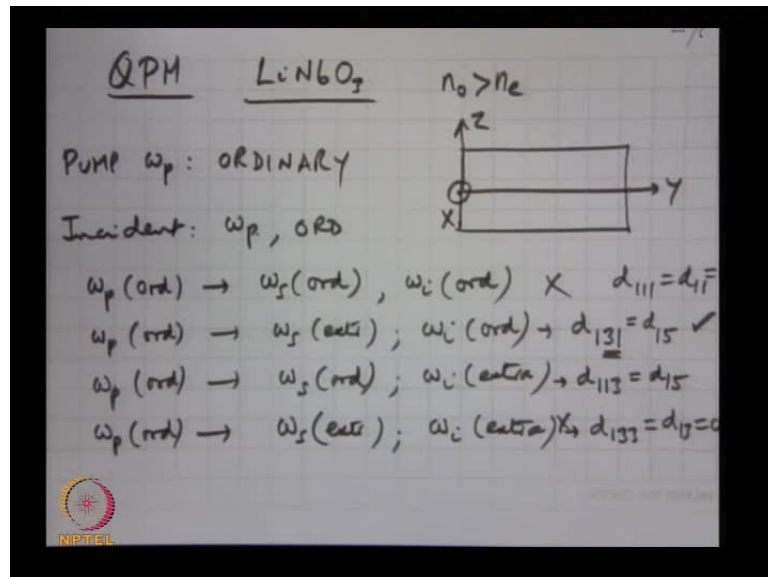


Quantum Electronics
Prof. K. Thyagarajan
Department of Physics
Indian Institute of Technology, Delhi

Module No. # 03
Second Order Effects
Lecture No. # 17
Non - Linear Optics (Contd.)

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Let me recall what we were discussing last time. We started looking at the following situation, where the input pump is an ordinary wave, always **along** the x direction, and propagating along the y-axis in lithium niobate. Then, we found out the possibility of generating a signal and idler pair which are ordinary ordinary or extraordinary ordinary, ordinary extraordinary and extraordinary extraordinary; because of Quasi-phase-matching, I can always ensure that phase matching is possible through a Quasi-phase searching phenomenon; and hence, in principle, I should be able to generate any of these pairs.

So, for lithium niobate, we have found that to generate the ordinary ordinary from an ordinary pump, you need a d 11 element and that is 0 in lithium niobate; so, this process

will not take place. The omega pump which is ordinary is getting converted to extraordinary ordinary pair or the ordinary extraordinary pair, can happen because of the d_{15} which is non-zero; and then, to convert the ordinary pump into an extraordinary pair of signal and idler photons requires a d_{13} , which is also 0. So, the only possibility is, if I launch an omega pump which is ordinary wave, I can generate an orthogonal pair of signal idler photons or like, which is, **that means**, I can have either this signal as an extraordinary wave and an idler as an ordinary wave, or the signal as an ordinary wave and the idler as an extraordinary wave.

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$$P_i = 2 \epsilon_0 d_{ijk} E_j E_k$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$

$$\omega_p(o), \omega_s(e), \omega_i(o) \quad k_p - k_s - k_i = K$$

$$\frac{2\pi}{\lambda_p} \cdot n_o(\lambda_p) - \frac{2\pi}{\lambda_s} \cdot n_e(\lambda_s) - \frac{2\pi}{\lambda_i} \cdot n_o(\lambda_i) = \frac{2\pi}{\lambda_1}$$

Because, these two processes are different, we need two different phase-matching conditions, and I have written down the phase-matching conditions corresponding to the two processes. The first 1, required this condition - the ordinary pump, extraordinary signal, ordinary idler and you need a capital lambda 1 periodicity for achieving Quasi-phase-matching.

So, if I generate a medium with this lambda 1 period, calculated from this equation, then an ordinary **pump** photon can down convert to an extraordinary signal photon and an ordinary idler photon; that is parametric fluorescence which we have not yet derived or obtained, that will come from quantum-mechanical analysis. But, at the same time, **we have** we can see, that if I launch an ordinary pump photon light and an extraordinary signal light, the extraordinary signal can be amplified by the same process.

Similarly, if I launch at λ_i wavelength, an ordinary wave and a λ_p wavelength, ordinary wave, the λ_i will get amplified and λ_s will get generated in the process. The polarization states of the output are automatically defined by the phase-matching condition and the non-zero element of the d tensor. It is also possible to generate a pair of ordinary signal photon and extraordinary idler photon, provided I have another period, capital λ_2 , because it depends now on the ordinary index at λ_s and the extraordinary index at λ_i ; this λ_1 depended on extraordinary index at λ_s and an ordinary index at λ_i .

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| λ | n_o | n_e |
|-------------------------------|-------|------------------------|
| $\lambda_p = 0.6 \mu\text{m}$ | 2.296 | 2.211 |
| $\lambda_s = 1.0 \mu\text{m}$ | 2.236 | 2.160 2.160 |
| $\lambda_i = 1.5 \mu\text{m}$ | 2.213 | 2.140 2.140 |

$\lambda_1 \approx 5.23 \mu\text{m}$
 $\lambda_2 \approx 6.10 \mu\text{m}$

So, as an example, let me take lithium niobate. Again, let me go back to lithium niobate and let me look at these following wavelengths, $\lambda_n o n e$; let me take a 0.6 micron wavelength, the ordinary index is 2.296 and extraordinary index is 2.211; at 1 micron wavelength, the ordinary index is 2.236 and the extraordinary index is 2.160, sorry 2.160. So, if this is the pump and this is signal, what will be the idler wavelength? Can you calculate? What will be the wavelength of the idler? How much?

0.67? No.

How much is it?

1.5 micrometer.

Please be careful with the calculations. 1.5 micrometer; so, that 1.5 micrometers, the ordinary index is 2.213 and extraordinary index is 2.140. So, I have actually calculated this from the Sellmeier equation for the ordinary and extraordinary indices of lithium niobate. So, these are estimated values so, I can use these equations to calculate λ_1 , because I know ordinary extraordinary indices at the three wavelengths; and similarly, I can also use the second equation to calculate λ_2 ; and what I find is, λ_1 comes out to be 5.23 micrometers and λ_2 comes out to be 6.1 micrometer; not very close but... They are different, they are different periods. The periods are different, because the refractive index at the wavelength is different, and the polarization states are different.

So, if I make a grating of 5.23 micron and launch an ordinary polarized light at 0.6 micron, in the crystal; so, ω_p I am launching, so, if the incident light at ω_p is ordinary polarization, then, and of the grating here, this is a Quasi-phase-match grating here, corresponds to a period of 5.23 microns, then, I will convert the 0.6 micron to the two wavelengths 1 micron and 1.5 microns wavelengths; and their polarization states will be automatically determined by the fact, that the first one corresponds to a signal which is extraordinary; this one will give me an extraordinary signal and an ordinary idler; this one will give me an ordinary signal and an extraordinary idler pair.

So, as an amplifier, I can use this, this particular period, to amplify an extraordinary polarized signal, and in the process, generate an ordinary polarization idler; this one I can use, this period I can use to amplify an ordinary wave at 1.0 micron and generate an extraordinary wave at 1.5 microns. So, the crystal, the directional propagation and the polarization state of the pump, determines what is possible and what kind of periods I need to achieve this interaction process. Now, suppose I were to launch light at 0.6 micron, and imagine a situation where I have in this port crystal here, I could generate somehow both these periods.

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The image shows handwritten notes on a grid background. At the top left, there is a small diagram of a grating with a vertical arrow labeled 'Z' and a horizontal arrow labeled 'y'. Below the diagram, there is a table with four columns: λ , n_o , and n_e . The first column is labeled λ and has three entries: $0.6 \mu\text{m}$, $1.0 \mu\text{m}$, and $1.5 \mu\text{m}$. The second column is labeled n_o and has three entries: 2.296, 2.236, and 2.213. The third column is labeled n_e and has three entries: 2.211, ~~2.160~~, and ~~2.140~~. Below the table, there are two equations: $\lambda_1 \approx 5.23 \mu\text{m}$ and $\lambda_2 \approx 6.10 \mu\text{m}$. In the bottom left corner, there is a small circular logo with the text 'NIPTEIL' below it.

| λ | n_o | n_e |
|-------------------|-------|------------------|
| $0.6 \mu\text{m}$ | 2.296 | 2.211 |
| $1.0 \mu\text{m}$ | 2.236 | 2.160 |
| $1.5 \mu\text{m}$ | 2.213 | 2.140 |

$\lambda_1 \approx 5.23 \mu\text{m}$
 $\lambda_2 \approx 6.10 \mu\text{m}$

Remember, I do not have to have a sinusoidal function, **I can...** I do not have to have one period; I can have a functional dependent which has more than one period - need not be periodic; if I have multiple periodicities in a function, I can have multiple spatial frequencies, I can have a function of time which has frequencies ω_1 and ω_2 simultaneously; $\cos \omega_1 t$ plus $\cos \omega_2 t$, a function of this type has both frequencies.

Similarly, I can have a grating, a spatial variation in which both periods are present simultaneously; I will come to this problem later when I discuss the quantum-mechanical aspect. So, if I both getting simultaneously, what will happen if I launch a pump which is ordinary? So, in principle, both wavelengths **will have both the pumps**, will have both the polarization states.

So, let me launch light pump light, so the pump photon which comes in, could interact with this grating and generate an extraordinary idler and an ordinary signal pair; it could, at the same time, have interacted with this grating and generated an ordinary signal and an extraordinary idler pair; both are possible. Classically, I will say that the output consists of either an extraordinary signal and an ordinary idler or an ordinary signal and an extraordinary idler. What I will show you is, when I look at the quantum-mechanical picture of this interaction process, this is incomplete. I will show you that the output

polarization state of the signal and idler are undefined; they will get defined by the process of your measurement.

There is a complete difference in terms of pictures which I can generate from classical and quantum-mechanical analysis; and, the photons that will come out, the pair of photons that will come out here, have this property of what is called as an entanglement, polarization entanglement. And this, I will come to a little later when we discuss the quantum picture; but please note here, that I could have structures which has multiple Quasi-phase-matching periods; I showed you this is a part of domain engineering. I can have a domain reversal, **which is**, whose period is changing with position called chirped grating, I could have all kind of functional dependence; this functional dependence is my choice. And depending on the choice, I can generate, in the functional dependence, multiple periods and those multiple periods will then be responsible for interaction with this input pump photon to generate pairs of signal and idler or, idler photons.

So, that is a very interesting picture that will develop when we do the quantum-mechanical analysis, because the quantum-mechanical analysis is not just **read** trying to calculate a spontaneous efficiency and so on; the picture is completely different. The predictions from there have no classic counterparts; the properties of the generated photons here which are coming out cannot be explained classically. There are there are certain properties which have no classical explanation, and that needs a purely quantum-mechanical treatment; and, that is what we will do after we finish up the classical discussions on non-linear optics; and, one of them is the property of entanglement where the light coming out from here, now; the signal and idler photons which are coming out are said to be entangled in polarization states, that means, the state of polarization of the output here are undefined. The only thing I know for sure is that, signal and idler photons are orthogonally polarized.

Classically, I will say it is either the horizontal vertical pair of signal idler or the vertical horizontal pair of signal idler; if signal is like this, idler will be like this or, if idler is like this, signal is like this; this is the only conclusion I get from here. But what I will find out when I doing the quantum-mechanical analysis is, that, this is more than this; it has **some properties or**, I cannot even define the polarization state of the output light, **or the signal** or the idler; it is undefined.

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| $\frac{LN}{\lambda}$ | λ | n_o | n_e |
|----------------------|-------------------|-------|------------------|
| λ_1 | $0.6 \mu\text{m}$ | 2.296 | 2.211 |
| λ_2 | $1.0 \mu\text{m}$ | 2.236 | 2.160 |
| λ_3 | $1.5 \mu\text{m}$ | 2.213 | 2.140 |

$\lambda_1 \approx 5.23 \mu\text{m}$
 $\lambda_2 \approx 6.10 \mu\text{m}$

Diagram: A rectangular grating structure is shown with a vertical Z-axis and a horizontal y-axis. The wave vector k_p is indicated by an arrow pointing along the y-axis.

The polarization state will get defined, the moment I do a measurement of the polarization state; and what I will find is, whatever measurement I do on the signal photon, influences the result on the measurement of the idler photon, irrespective of the distance separating these two photons. So this, I will explain again later, but this is an interesting feature that will come out from purely quantum-mechanical argument; I cannot show this through a classical arrangement.

For a given period, let us say capital lambda 1, then also, quantum-mechanically, both the polarization states are possible; if you have a grating, first one into only period lambda 1.

Yes, I will only generate a signal which is extraordinary and an idler which is ordinary.

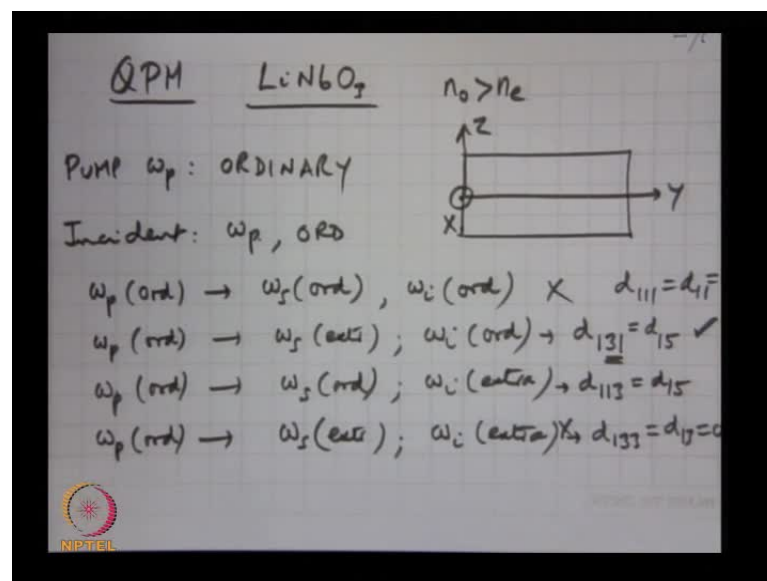
This is in agreement with quantum mechanics and but, if we have both the periods, then the entanglement property is there.

Yes, because, when I have both periods, both processes are possible; and the output is not simply either this or this; it is more than that; and that will come out when we do the quantum mechanical-analysis.

And classically, both kind of polarization of both the ladles are there, if you have double...

Yes, that is what I am trying to show you; classical explanation tells me that, with this grating, if... Because, when the photons comes in, if it interacts with this grating, it will generate an extraordinary signal ordinary idler pair; if it is this grating which affects it, it will generate the other pair. So, the output is much more than this classical interpretation of what is coming out. So, given a crystal, and given a crystal means, given the d tensor of the crystal, I can find out what are the possible orientations of the pump and signal and idler, which can interact through the non-zero elements of the d tensor; and knowing that, I can also calculate what are the grating periods required for this interaction to phase-match or Quasi-phase-match and so on. So, all this is contained in the energy conservation equation, the momentum conservation equation and the d tensor of the crystal.

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So, what we have done in the class right now is, just discussed one example of incident line at omega p being an ordinary wave, and with the possibility of generating orthogonal polarization states of this signal and idler.

Sir, when we have two gratings and we will be having externally the polarization of both the signal and the idler, so there will be like interference of this; we have extraordinary signal and extraordinary idler, so they will...

No, they are two different frequencies anyway.

Which kind of signal?

Yeah, or the pitch are at such a high frequency, that you cannot normally observe them; unless the frequencies are very close, your detector will not, your detector will respond to your detector responds to e_1 plus e_2 mode square, which is e_1 square plus e_2 square plus $2 e_1 e_2$ into $\cos \phi$, between the two electric fields; but, because the frequencies are different, the interference term is varying so fast like beats, exactly like beats, that normally you will not observe.

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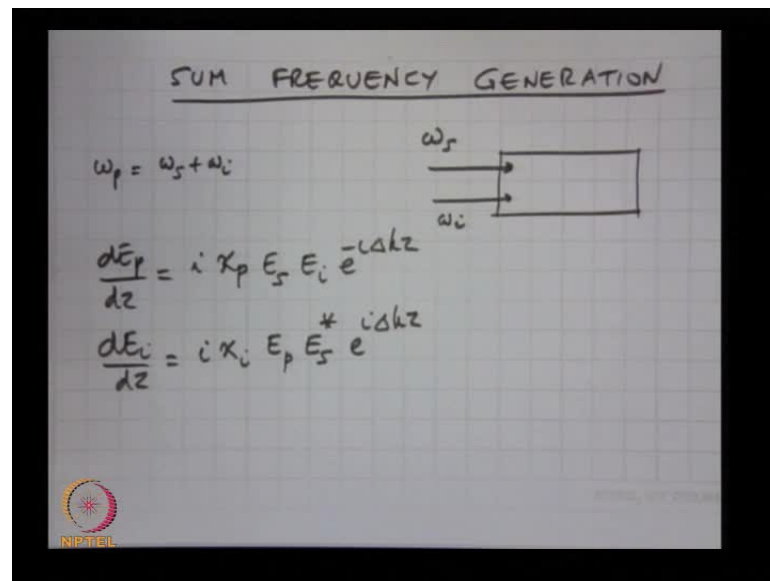
| λ | n_o | n_e |
|-------------------------------|-------|------------------|
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$\lambda_1 \approx 5.23 \mu\text{m}$
 $\lambda_2 \approx 6.10 \mu\text{m}$

If the frequencies are close by a few kilohertz or few megahertz, then the detector can respond and tell you that there are beats coming in; that is possible, surely possible. So, these frequencies which were...; these wavelengths are very far apart. Frequency difference is so huge, that the detectors do not normally pick up these beats, but otherwise, you are perfectly right. Anything else?

So, what I wanted to do before we move into an oscillator problem is to discuss the sum frequency generation. So, what we will do first is to look at the following problem, that now, I have, I want to look at sum frequency.

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So, tell me what I should consider as input? ω_s and ω_i . Because, I want to generate an ω_p , which is equal to the sum of the input frequencies; I could have called it ω_1 , ω_2 and the output of ω_3 ; it is the same. But I want to use the same set of equations, so I will use the same equation for the frequencies, so, ω_p is equal to ω_s plus ω_i . So, what pair of equations do I pick up to solve and what approximation should I make?

I have 3 equations; one for ω_p , one for ω_s and one for ω_i . So, out of this, for example, remember, in difference frequency generation, we had ω_p and ω_s coming in; and I assumed ω_p is a strong light beam, intense light beam, and ω_s is weak, so, I amplify ω_s and I generate ω_i . Here, I want to do some frequency generation, so one of these beams has to be high power; so, let me assume that to be ω_s , and ω_i is a weak beam. So, let me give you a typical example; remember, ω_i is smaller than ω_s , so, λ_i is larger than λ_s . So, suppose I have light coming at 1.8 microns wavelength, there are no efficient detector there for this light; and secondly, the detectors which are available are quite noisy.

So, what I would like to do is to convert the light signal at 1.8 micron into less than 1 micron, where I can use silicon detectors to detect light and process signal; it is a very efficient method. So, I have 1.8 micron, I put a light at a lower frequency, say, 1 micron;

the sum of these two, you can calculate; it comes to below 1 micron. So, I can actually convert light at higher wavelengths to light at smaller wavelengths by using the sum frequency generation process; it is very similar to second harmonics. Second harmonic is ω_s is equal to ω_i , here they are different. So, now tell me which equations should I take? ω_i and ω_p equation. And assume E_s is almost a constant, so let me write down the equation; so, dE_p/dz is equal to $i\kappa_p$. What are the two terms I will get here? $E_s E_i \exp(-i\delta k z)$, and then, dE_i/dz is equal to $i\kappa_i E_p E_s^* \exp(i\delta k z)$.

So, I have a strong signal coming in; ω_s is a strong light wave, ω_i is a weak light wave and it can be weak or strong; it does not matter. But ω_i is assumed to be strong, so that, I neglect the depletion of the ω_s wave. Obviously, I cannot generate ω_p , unless I deplete the ω_s , so, as an approximation. So, now, **first let me look at...** again I know that when δk is equal to 0, I will have maximum efficiency of this process.

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$$\frac{dE_p}{dz} = i\kappa_p E_s E_i$$

$$\frac{dE_i}{dz} = i\kappa_i E_p E_s^*$$

$$\frac{d^2E_p}{dz^2} = i\kappa_p E_s \left(\frac{dE_i}{dz}\right) = i\kappa_p E_s (i\kappa_i E_s^* E_p)$$

$$= -\kappa_i \kappa_p |E_s|^2 E_p$$

$$= -\delta^2 E_p$$

$$E_p(z) = A \cos \delta z + B \sin \delta z$$

So, let me look at these equations for δk is equal to 0, that is, phase matched case; so, I will get dE_p/dz is equal to $i\kappa_p E_s E_i$; and dE_i/dz is equal to $i\kappa_i E_p E_s^*$. So, let me differentiate the first equation, so I get d^2E_p/dz^2 is equal to $i\kappa_p$ - E_s is assumed to be constant - into dE_i/dz which is

equal to $i \kappa_p E_s$ into $i \kappa_p E_s^* E_p$, which is equal to $-\kappa_p i \kappa_p \text{mod } E_s^2 E_p$.

What is the difference between this equation and the equation we had got for different frequency generation?

(0)

Yes, it is a negative sign here, so its solutions will be oscillatory; so, let me call this minus of delta square or something $-E_p$, so, what are the solutions? E_p of z is equal to $A \cos \delta z$ plus $B \sin$ oscillatory solutions.

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$$\begin{aligned}
 E_p(z=0) = 0 &\Rightarrow A = 0 \\
 E_p &= B \sin \delta z \\
 E_i(z=0) &= E_i(0) \\
 B \delta &= i \kappa_p E_s E_i(0) \\
 B &= \frac{i \kappa_p E_s E_i(0)}{\delta} = \frac{i \kappa_p E_s E_i(0)}{\sqrt{\kappa_i \kappa_p |E_s|^2}} \\
 &= i \sqrt{\frac{\kappa_p}{\kappa_i}} E_i(0)
 \end{aligned}$$

So, how do I find out the constants A and B ? I have the initial conditions E_p of z is equal to 0 , because I am only coming in with the signal and the idler. So, E_p is equal to 0 at z is equal to 0 , and so, E_p at z is equal to 0 , is equal to 0 , implies A is equal to 0 . So, the solution is E_p is equal to $b \sin \delta z$. So, also, let me assume that E_i at z is equal to 0 , is equal to $E_i(0)$; that is the other condition that I am coming in with the signal and idler; so, there is no pump, so I use this equation and I write, $d E_p$ by $d z$ is $B \delta$ $d E_p$ by $d z$ at z is equal to 0 , is equal to $i \kappa_p E_s E_i$ at z is equal to 0 , so, this gives me $B \delta$ is equal to $i \kappa_p E_s E_i(0)$.

Let me write this as E_i of 0, let me just keep the same notation E_i of 0; so, b is equal to $i \kappa_p E_s E_i$ of 0 divided by δ . And δ is, from here, so this is $i \kappa_p E_s E_i$ of 0 - **divided by delta** - is square root of $\kappa_p i \kappa_p \text{mod } E_s$ square; so, this is equal to i times square root of κ_p by κ_i . Let me assume E_s is real, let me assume the phase of the signal to be 0, so $E_s, \text{mod } E_s$ square is equal to E_s , and I get into E_i of 0.

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$$E_p(z) = i \sqrt{\frac{\kappa_p}{\kappa_i}} E_i(0) \sin \delta z$$

$$E_i(z) = \frac{1}{i \kappa_p E_s} \cdot i \sqrt{\frac{\kappa_p}{\kappa_i}} E_i(0) \delta \cos \delta z$$

$$E_i(z) = E_i(0) \cos \delta z$$

$$P_p(z) = \frac{\kappa_p}{2c\mu_0} |E_p|^2 \cdot S$$

$$= \frac{\kappa_p}{2c\mu_0} \cdot \frac{\kappa_p}{\kappa_i} |E_i(0)|^2 \sin^2 \delta z$$

$$=$$

So, the solution **I get for E p...** Let me write the two solutions for E_p of z , E_p of z becomes i times square root of κ_p by κ_i E_i of 0 $\sin \delta z$; and, how will E_i of z vary? E_i of z , I get from this equation, so E_i of z will be 1 by $i \kappa_p E_s$ into $d E_p$ by $d z$, which is, square root of κ_p by κ_i E_i of 0 $\delta \cos \delta z$. E_i of z is 1 by $i \kappa_p E_s$ times $d E_p$ by $d z$, so, $d E_p$ by $d z$ I substitute from here; and all these factors you can show will cancel off, and I get E_i of 0 $\cos \delta z$. They have to cancel, because, if z is equal to 0, E_i of 0 is equal to E_i of 0, so, these factors you can substitute back for E_s and δ and κ_p ; all these factors will **this** just cancel off and I will finally get this, so E_i of z . So, I am going to calculate - what is the power in the signal in the pump, or in the converted frequency. n_p by $2 c \mu_0 \text{mod } E_p$ square into area, which is equal to n_p by $2 c \mu_0 \kappa_p$ by **kappa s** $\kappa_i \text{mod } E_i$ 0 square \sin square δz , which is **equal to...**, let me substitute all the quantities.

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$$= \frac{\omega_p}{2c\mu_0} \cdot \frac{\omega_p}{\omega_i} \cdot \frac{2c\mu_0 P_i(0)}{\omega_i} \sin^2 \delta z$$

$$P_p(z) = \frac{\omega_p}{\omega_i} P_i(0) \sin^2 \delta z$$

Let me substitute all the quantities here, is equal to n_p by $2 c \mu_0 \kappa_p$ is ω_p d by $c n_p \kappa_i$ is ω_i d by $c n_i$; and I want to replace E_i^2 by the power at the ω_i frequency. So, how is the power related to this? P_i of 0 is equal to n_i by $2 c \mu_0$ mod E_i^2 square into area; so, this is $2 c \mu_0 P_i$ of 0 by n_i into area into sine square δz ; this is n_p by $2 c \mu_0 \kappa_p$ by κ_i into E_i^2 mod square into sine square δz , $2 c \mu_0$ goes off from here, **n_i, n_i there is...** There has to be an s here, area, please write this s here; there is an area here which cuts off, $d c n_p \omega_p$ by ω_i p i of 0. Why is this factor ω_p by ω_i coming?

It is a photon number problem. The maximum power I can generate at the pump is ω_p by ω_i times P_i of 0; P_i of 0 h cross ω_i is the number of photons entering per unit time into the crystal at ω_i frequency, this implies, I have completely converted all those photons. If I convert all the photons at ω_i frequency to ω_p frequency, how many photons will I generate? P_i of 0 by h cross ω_i , because, P_i of 0 is the input power at ω_i frequency, P_i of 0 by h cross ω_i is the number of photons at ω_i entering per unit time at ω_i ; If I can convert all that into the ω_p frequency, the number of photons which will be coming out per unit time at ω_p is P_i of 0 by h cross ω_i ; and so, the power at the pump ω_p will be this multiplied by ω_p , which is simply ω_p by ω_i into P_i of 0 sine square δz . So, this now, is completely different solution compared to what we got for some frequency; so here what happens is, P_p of z will go like this.

So, this is a function of z , this is p_p ; and p_i where its amplitude would be as much as this, or it will be smaller or larger? **Smaller**. Smaller, because ω_i is smaller than ω_p ; the number of photons finally are equal, so this idler will have to go like this. When **the** it starts with full power in idler and **no pump**, no ω_p , it converts everything to ω_p and then, back to ω_i , back to ω_p ; it is oscillatory. This solution is very different from the solution for difference frequency generation; this is sum frequency generation. So, you can actually convert all the power from the idler, the ω_i frequency to ω_p frequency, provided you have phase matching condition.

So, if you launch a certain number of idler photons, in principle, if you choose a length **which is...** How much is the length I must choose? Δz must be equal to π by 2. I must choose a length, such that, the sin function becomes 1; and, if I choose that length **of the** of the crystal, then at the end of the crystal, I would convert all the idler photons at ω_i frequency to ω_p frequency at the output. Of course, if the ω_i frequency signal is weak, I will still have a less number of photons coming at the pump, but they are at a different wavelength.

Where is this extra energy coming from? ω_s . This here, the number of photons are equal, but you have also taken out exactly the same number of photons from ω_s light and converted to ω_p . Every time you generate an ω_p photon, you have consumed an ω_i photon and an ω_s photon; you cannot generate ω_p from only ω_i , you need ω_s also.

So, the ω_s is absolutely required, and **the**, you can actually calculate what is the decrease in power of the ω_i at this point. Here, at this point, what will be the content of the crystal? ω_p and ω_s . After this, ω_p down converts to ω_s and ω_i .

From here, there was incident and ω_i and ω_s ; ω_i got converted to ω_p , so, at this point, you had ω_p and ω_s . Beyond this point, the power flows from ω_p to ω_s , and ω_p becomes 0, generating ω_i and ω_s and it is just an oscillatory function of distance.

We have assumed that E_s is not depleting, so E_s is constant.

In this calculation?

Should also reflect this, or there is some inconsistency in the equations. Because, if you are saying that the energy you get taking some photons from s to have (ω) then, we are assuming that there is some depletions, there is some depletion going on, that is,...

No, that depletion is not apparent in my simulation; here, I am assuming p_s is constant. So, there is an inconsistency, because, the some of the power conservation is not being maintained in my simulation, because I cannot have p_p , p_i and p_s , the way I have done. But the interpretation that p_s must have decreased is by my logical argument; that is all. This analysis, I have assumed p_s is constant and that is not correct.

So, this is an approximate equation which I have got assuming the ω_s ; ω_p is constant, but I know, that when I solve all the equations exactly, I will be able to convert from ω_i to ω_p , and I would have also taken up some energy from ω_s , in this process. Because, I know this process interpretation as a merging of photons at ω_s and ω_i to generate ω_p . So, every time I lose a pump photon at ω_i , I should have lost a photon at ω_s also, simultaneously. So, that is an important thing, but, so, this equation right now is inconsistent, because I have only assumed p_p is a function of z , and p_i is a function of z , and p_s is independent of z ; this is not correct. We have not satisfied this set of equations; they will approximately satisfy this set of equation, not correct.

(ω)

You will not be, because, you will see the p_p of z plus p_i of z plus p_s of z is not a constant. Because p is constant, and you will not find p_p plus p_i ; it is not constant. You see, p_i is $p_i 0 \sin^2 \cos^2 \Delta z$, and this plus this not constant, I need the other one to make it a consistent. So, the sum frequency has applications in converting light from a longer wavelength to shorter wavelengths; and essentially, you can achieve this by using non-linear effects, which means, effectively, you can convert light at a infrared wavelengths to into the visible regional spectrum.

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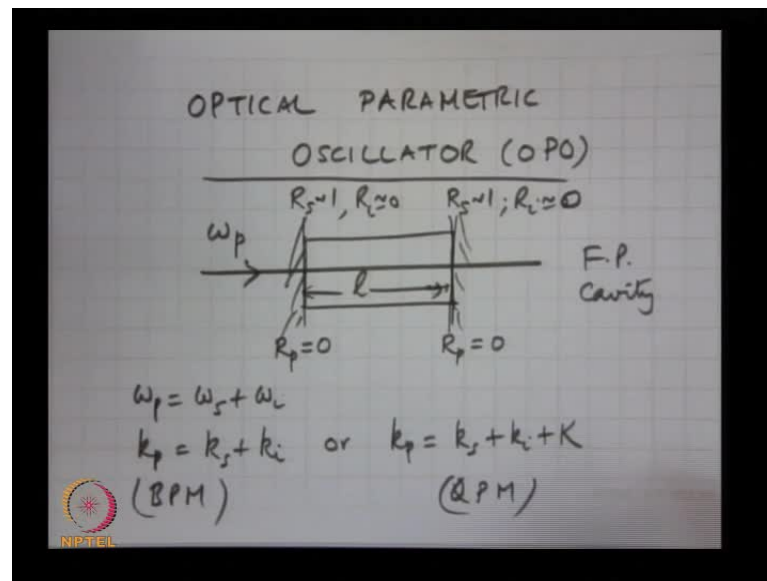
$$= \frac{\kappa_p}{2\omega_p} \cdot \frac{\omega_p \kappa}{\kappa_p} \cdot \frac{\kappa \omega_i}{\omega_i \kappa} \cdot \frac{2\omega_p P_i(0)}{\kappa} \sin^2 \delta z$$

$$P_p(z) = \frac{\omega_p}{\omega_i} P_i(0) \sin^2 \delta z$$

The efficiency, etcetera, depends, of course, on the non-linear coefficient which is contained here and the kind of lengths required; I mean, delta depends on the non-linearity; you see here, delta depends on kappa i kappa p; and, kappa i and kappa p, both are functions of non-linearity, so, delta square is kappa i kappa p mod E s square. So, it depends on the signal power and also the non-linearity, which is contained in the de-coefficients inside the kappa. So, how much is the length required to do this, etcetera, is a function of the crystal.

Of course, in all these analysis, we have assumed that there is no other last mechanism of this light waves; we are assuming the crystals are completely transparent. So, in principle, I need to take that into account, if the lens becomes longer and longer; I cannot neglect the fact that light could be lost by scattering processes or absorption by other mechanisms and so on. So, that finishes sum frequency generation from two input low frequency signals to generate a higher frequency signal at the output. And, as I was mentioning, the second harmonic generation is one special case of the sum frequency, where 2 frequencies are equal; omega s is equal to omega i, and I can generate a 2 omega light at the output. Now, I want to discuss optical parametric oscillators, also called OPO.

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So, I have shown, that if you launch light at ω_p and ω_s and satisfy the phase matching condition, you can amplify ω_s light; the power at ω_s frequency goes up as $\cosh(\gamma_s)$ or something. So, that is an optical amplifier; this amplification is very similar to what you can amplify light by using population inversion. In lasers, you have population inversion, which means, you put more atoms in the excited state compared to the ground state, **and then**, because of most emissions, the input light gets amplified, and you make an optical amplifier.

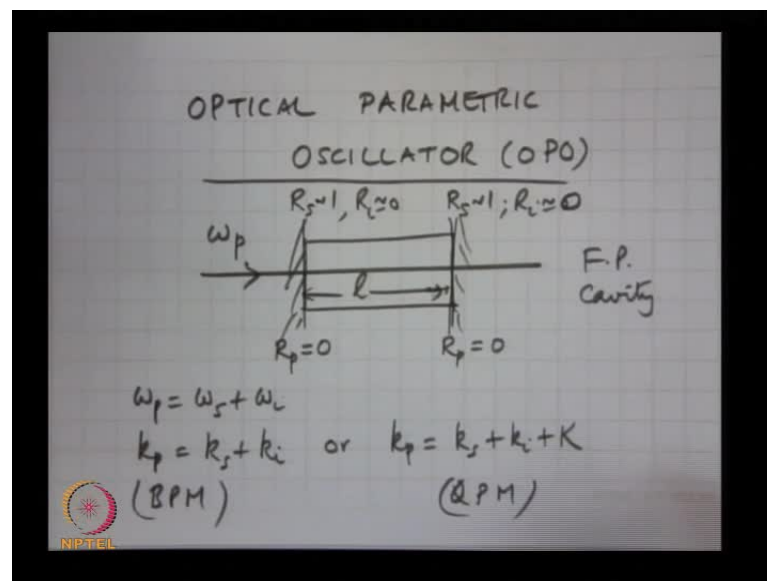
So, this is another kind of amplifier; the great advantage here, is this amplifier does not depend on the existence of certain energy levels at certain frequencies. Because, as long as the crystal has non-linearity and it is transparent, I can generate any pairs of ω_s , ω_i , ω_p ; I can have any combination as long as I can satisfy the phase matching condition. And, **d-tensor** is finite; there is a de-tensor element which gives you a coupling.

So, these are optical amplifiers; and I can convert an amplifier to an oscillator, that means, a source of radiation; an oscillatory is a source. An amplifier is only amplifying for an amplifier; you input the signal, it gets amplified as it comes out. A source, you just give it energy, and it generates radiation at a certain frequency.

So, just, I can convert an amplifier into an oscillator by putting this amplifier within a pair of mirrors. So, let me take a pair of mirrors, let me assume that I have... This is ω_p coming in; now, usually, I will choose the reflectivity of this mirror at the pump frequency to be 0. I can make mirrors having reflectivity at certain wavelengths and complete transmitting in other wavelengths; this is possible by using... What do I do? Thin film dielectric coatings; I can have a coating, and by interference effects, I can have very strong reflectivity at certain wavelengths and very weak reflectivity at other wavelengths.

So, theoretically I am assuming that these two mirrors have 0 reflectivity, the pump just goes through. So, now let me assume, that I have bi-mechanism, either by Birefringence-phase-matching, and or by Quasi-phase-matching, I have I am satisfying the phase matching condition for one pair of frequencies, ω_s and ω_i .

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k_p is equal to k_s plus k_i or k_p is equal to k_s plus k_i plus k ; this is birefringence phase-matching; this is Quasi-phase-matching. By some arrangement, I can achieve phase matching. So, now, I launch a pump light into the crystal. Can you tell me an electronic oscillator, how does it start?

I have an electronic oscillator, which means, a function generator - instrument which generates electric waves at a certain frequency. How do they generate? How do they

start? And what frequency will they emit? So, I am just feeding power into the system; they may emit a resonant frequency. But, why does it generate at all? Why does it start to generate? How does it **start from...** Where does it start to generate?

It is only an amplifier, remember.

I have a circuit in which I keep on feeding energy, and that circuit has a resonant **mode, which is...**

No, **but**, your energy is fed into different frequency. So, how does it generate a new frequency?

(O)

Noise. If there was no noise, there is no oscillation; there must be some noise.

In a laser, what is the noise? Spontaneous emission. The moment you take atoms in the excited state, they will start to jump down, and without any stimulation, and, that light which generates, generated by the spontaneous emission process, is the noise that starts the laser. If the spontaneous emission did not take place, there is no laser; so, spontaneous emission, like noise, is absolutely required for the start of the oscillator. What is the noise here?

Modes are only frequencies which can exist in a system; it does not mean they will exist.

What **noise? From** where do I get **omega s that first?**

All the frequencies are present at all times.

Now, here, what is the process which will start this laser, like spontaneous emission incident?

We have omega p incident and omega s will be there by the surrounding noise, that is

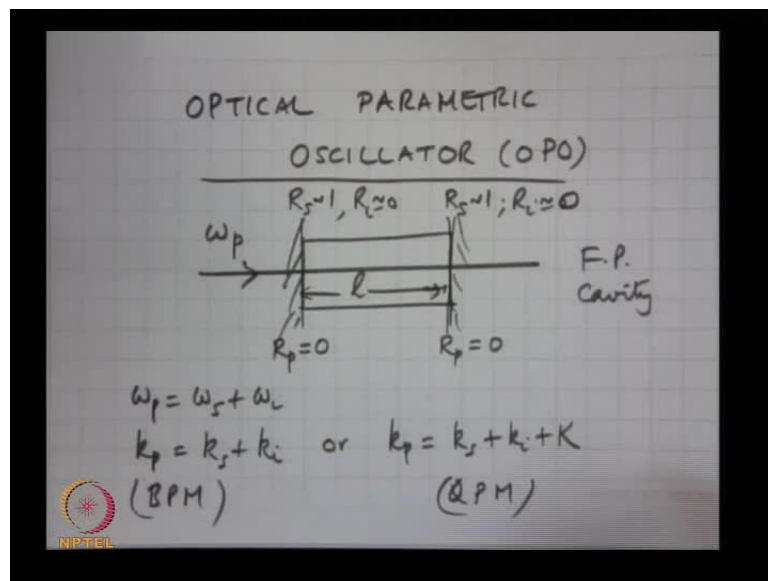
Quantum fluctuating noise? which **results in what first?**

Generation of omega i and omega s. From what process?

Spontaneous parametric fluorescence or down conversions. ω_p photon comes in, it interacts with the crystal and spontaneously down-converts to a pair of ω_s and ω_i photons; that down conversion is brought about by the vacuum fluctuation just like spontaneous emission is brought about by the vacuum fluctuation. So, this ω_p photon comes in or light comes in and spontaneously split into ω_s ω_i photons, some of them.

Now, I can have, for example, this is the reflectivity of the signal wavelength is close to 1 and reflectivity at the signal wavelength is also close to 1, for example., I make the mirrors which are transmitting at pump, but very high reflectivity at signal wavelengths, so, what is going to happen is... And, let me assume that R_i is also 0 and R_i is also 0; the reflectivity at the idler wavelengths are 0.

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So, the spontaneously generated i signal photons, which direction will they be generated? In the forward direction. So, when they come here, part of them will get reflected back, some of them will transmit; suppose 99 percent, 1 percent gets reflected back; and as it propagates to the crystal, does it get amplified?

No.

Why?

Because, the k vector is going in the opposite direction, so, it will not be phase matched.

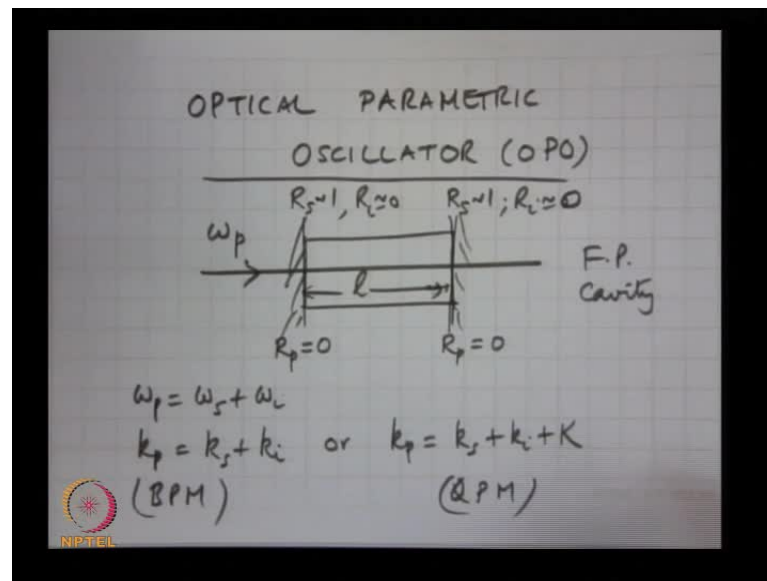
k_p is like this and k_s is like this, you will not satisfy the phase matching, that is, you will require a negative k_s here; this will not satisfy either of these conditions. So, the signal light comes back without amplification; in a laser, **it gets** in a normal population inversion laser, it gets amplified in the reverse direction also. It uses the same population inversion that gets amplified in the reverse direction; it comes here, then again, partly transparent, partly reflected; and once it starts again, now it uses the pump and gets amplified; because, I have shown you that the signal at this point, now, there is a signal and pump; and the signal will now draw energy from the pump and get amplified as it propagates; and, this will go back and forth and will increase this signal inside. And of course, the level of idler **will have to idler** is not increasing inside; it is escaping from the cavity. Because, you are not holding it on inside the cavity; this is a Fabry-Perot cavity. You studied Fabry-Perot in a course; this is a Fabry-Perot cavity. So, the signal light is trapped inside the cavity with little bit of escaping from both sides; and as the signal increases in intensity, what should happen? It cannot go on increasing infinitely.

No, but amplification is always there.

Gain saturation. Any amplifier has to have a gain saturation; because, **start** you start with a loop gain more than 1. So, the amount of loss is less, then gain, **so** it increases in amplitude; if you have all the time gain, more than loss; this will keep on increasing to infinity. But what is going to happen is, as the signal becomes larger and larger, it will bring down the gain; and it will bring down the gain to a value, such that, gain is equal to loss; that is gain saturation.

So, what is going to happen is, as the signal power increases, the pump power will start to drop; our assumption that the pump is non-depleted will fail, at that point. I cannot keep assuming pump is un-depleted when the signal becomes so strong, and so much of energy being drawn out from the pump. So, the pump power will drop down; and once the pump power drops, the gain will drop, because, the gain depends on the pump power.

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And I will reach a gain saturation where the signal will be, such that, the round trip gain is equal to round trip loss, and the light will come out at both ω_s and ω_i . **Omega i** ω_s are very strong wave because, it is a fraction of what is contained inside, but there is a lot of power at ω_i also coming out of the cavity.

What would contribute to the loss?

The loss is the finite reflectivity; suppose 99 percent- suppose 99 percent, every time I go through one round trip, I lose 2 percent of that light; so, I must gain the 2 percent. So, the pump power will adjust itself to give me a gain of 2 percent for every loss of 2 percent in one round trip.

(O)

Gain. For any oscillator, that is the place where the oscillation will become steady, and you will get a continuous signal coming out.

But the amount the power of the signal that comes out, it is only like 1 or 2 percent of the ω_p ; maximum if we say that all the power ω_p is converted to ω_s , let us say, I mean...

It is only a fraction of what is contained inside the cavity at ω_s , ω_p is still coming out; I am not converting all ω_p to ω_s . If this is 99 percent, and 99 percent, what I am getting is, 1 percent of one either side of what is contained inside the cavity.

Whatever is contained inside of the cavity, so, it has to be of the order of power of ω_p ; and the output we are getting very less power of ω_s , because it is only 1 percent of the maximum that is achieved inside the cavity.

Yes.

That could be in watts; I can put 10 watts of power at ω_p and I can get a watt of power at ω_s . We will put some numbers and calculate. When we actually calculate with these, the oscillation condition. So, what we need to do is, we need to have an oscillation condition. What is the pump power required by the laser, for the oscillator to start and what will be the frequency? The frequency is given by this, is, there any other condition on ω_s . It has to be a resonant mode inside the cavity; so, what is the frequency which will be resonant inside this cavity?

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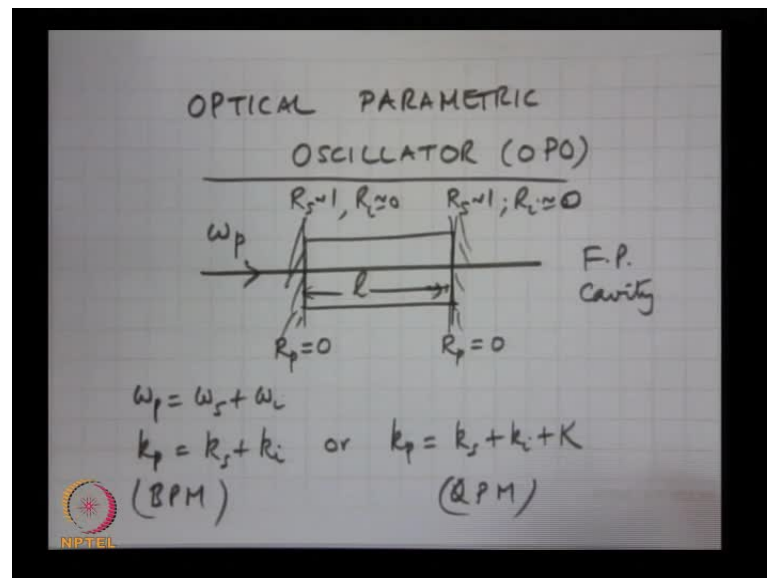
$$l = m \frac{\lambda_s}{2n_s} = \frac{m}{2} \frac{\lambda_s}{n_s}$$
$$\lambda_s = \frac{2n_s l}{m} = \frac{c}{\nu_s}$$
$$\Rightarrow \nu_s = \frac{m c}{2n_s l}$$

The image shows a handwritten derivation on a grid background. The first equation is $l = m \frac{\lambda_s}{2n_s} = \frac{m}{2} \frac{\lambda_s}{n_s}$. The second equation is $\lambda_s = \frac{2n_s l}{m} = \frac{c}{\nu_s}$. The third equation is $\Rightarrow \nu_s = \frac{m c}{2n_s l}$, which is enclosed in a hand-drawn box. In the bottom left corner, there is a small circular logo with the text 'NPTEL' below it.

Yes. So, if the length, if I call the length as l , l is equal to $n \lambda_s$ by 2 and let me call it m ; and this is m by 2, please remember, this n is, I must write refractive index. Because, this is usually the wavelength which I am using, are free space wavelengths, so, the

wavelengths inside the cavity is λ_s minus. So, the wavelength which will come out, which will oscillate, **is in terms...** in fact, I can write in terms of frequency. So, this is c by ν_s , so, this implies ν_s is **equal to...** The frequency has to satisfy this condition as well as this condition of ω_p is equal to ω_s plus ω_i and this condition of k_p is equal to k_s plus k_i .

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So, this ω_s will actually adjust itself to satisfy these conditions and what will come out is, **suppose** to be light at this frequency ω_s ; and, of course, ω_i light is also coming out of the cavity. This is what is called as a singly resonant oscillator, SRO, which means, only the signal is resonating inside the cavity; the idler is not resonating, because the mirrors have 0 reflectivity at the idler. I can also have a situation where R_i is also close to 1, then it is called a doubly resonant oscillator because then, the signal and idler will both resonate inside the cavity.

I can also have a situation where pump also resonates inside the cavity, and I will have a triply resonant oscillator. We will discuss the doubly resonant and the singly resonant oscillator in the class, and I will show you that the powers required for operation of the doubly resonant are much lower than required for the singly resonant oscillator, but at some price, and the price is in terms of instability of the laser system itself.

So, we will stop here now, so, what we will do is discuss and calculate what is the threshold pump power required for the laser for the oscillator to start, for the singly resonant case, for the doubly resonant case, and how does it depend on the reflectivity of the mirrors, the length of the cavity etcetera. Do you have any questions?

Sir, omega s is trapping, so, how will you...

No, not fully trapped, close to 1, I am writing; 99 percent may be, reflectivity, like a laser; the laser has mirrors. If you have mirrors of 100 percent reflectivity, nothing comes out its oscillating, but nothing comes out, but I want something output. So, I need to have a something, at least one of the mirrors to be having reflectivity less than 1, and that will be transmitting that fraction. So, this mirror has a finite reflectivity at the resonant omega s frequency, and so, a little light at omega s will be coming out, and that is quite strong already. The mirrors are not perfectly reflecting at all wavelengths, then you cannot see what is inside; nothing comes out from inside; this is completely opaque for you.

In general, in the lasers, when we do second harmonic generation, so, in that also, do we have oscillators, such that, the power coming out at the second harmonic...

No, you can have a laser and the second harmonic generator outside, or you can put the second harmonic generator crystal inside the cavity.

Intra-cavity second harmonic generation.

Light in pulse lasers, usually, when they are used for the laser emission processes, there is a second harmonic which is coming out, so inside, there is some oscillator of some kind....

There is a crystal. So, the mirrors are 100 percent reflecting at the omega frequency and partially transmitting at 2 omega frequency.

So, we have such a system inside.

Not such a system, just a crystal; it is exactly the same as a laser, where the population inversion...

For any laser, there are mirrors; there are mirrors, there is a laser cavity and there is a crystal. Inside that crystal, actually, helps to convert the oscillating ω to 2ω , and what comes out is 2ω and not ω . Anything else?

Thank you.