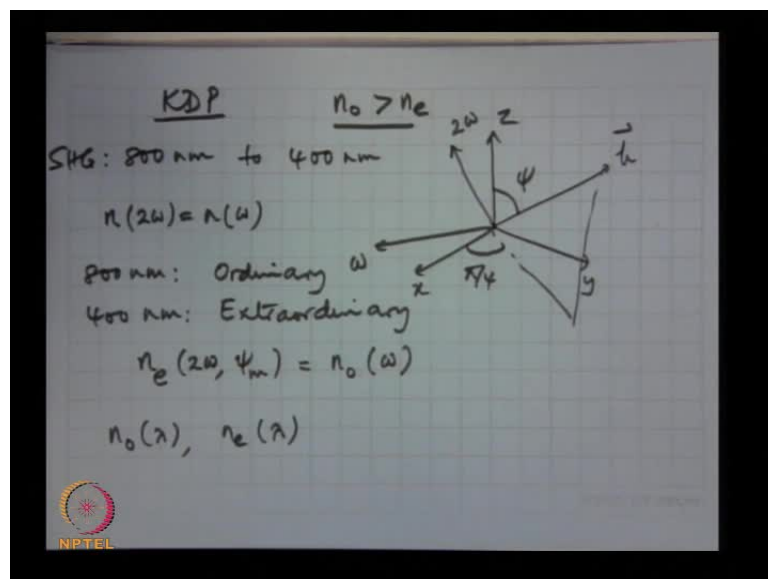


Quantum Electronics
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Module No. # 03
Second Order Effects
Lecture No. # 16
Non - Linear Optics (Contd.)

So, today what I want to do is, discuss some examples. The first example what I to discuss, is tuning; because, we will need this when we go into parametric amplifiers. Now, what is the meaning of tuning? Now, let me go back to KDP, which we have discussed sometime back in second harmonic generation.

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So remember, in KDP, potassium di-hydrogen phosphate, n_o is greater than n_e ; and, for second harmonic generation, what we had looked at was, if x, y, z, is like this, my propagation angle is in some direction making an angle ψ with the optic axis, to maximize the, **non-linear**, effective non-linear coefficient, what should this angle be?

Pi by $\frac{\pi}{4}$, 45 degrees, so that, the effective non-linear coefficient maximizes; otherwise, as we found, there is a $\sin \phi$ term coming and that becomes 0, if you propagate in the x z plane or the y z plane.

So, we have looked at second harmonic, **so what was the...** So, there are two wavelengths, **one is the...** For example, let me take second harmonic from 800 nanometers to 400 nanometers, **so what should be the...** For second harmonic generation, which wavelength is incident? 800 nanometers is incident and that generates 400 nanometers. So, what is the polarization state of the 800 nanometers that I choose? Remember this condition.

We have to choose ordinary because, as the frequency increases, or as the wavelength decreases, the refractive index increases; so, if I need to achieve, what is the condition that we have obtained? The refractive index of the wave at 2ω must be equal to the refractive index of the medium at wave at the frequency ω . So, we have seen that this will happen, provided I choose my ω like this; and then, I **get a 2ω** in this direction, polarization. So, the 800 nanometers will be ordinary, and the 400 nanometers that comes out will be extraordinary.

By choosing the polarization states and the direction of propagation ψ , making a particular angle, so what I will have to have is, the extraordinary index at 2ω of the wave propagating along a direction ψ , must be equal to the ordinary refractive index at frequency ω , which is independent of the direction of propagation.

So, for second harmonic, I input waves at 800 nanometers oriented as an ordinary wave, propagating at an angle, making ψ with the optic axis; in order to maximize the effective non-linear coefficient, I choose the propagation plane containing k and z axis to be inclined at 45 degrees to the x axis. And so, for example, for a given medium, I know how n_o depends on wavelength and how n_e depends on wavelength; this is Sellmeier coefficients, Sellmeier **equation tell me**. So, I use these numbers to calculate the angle at which I need to propagate so that I get phase-matching.

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$$\sin^2 \psi_m = \frac{\frac{1}{n_o^2(\omega)} - \frac{1}{n_o^2(2\omega)}}{\frac{1}{n_e^2(2\omega)} - \frac{1}{n_o^2(2\omega)}}$$

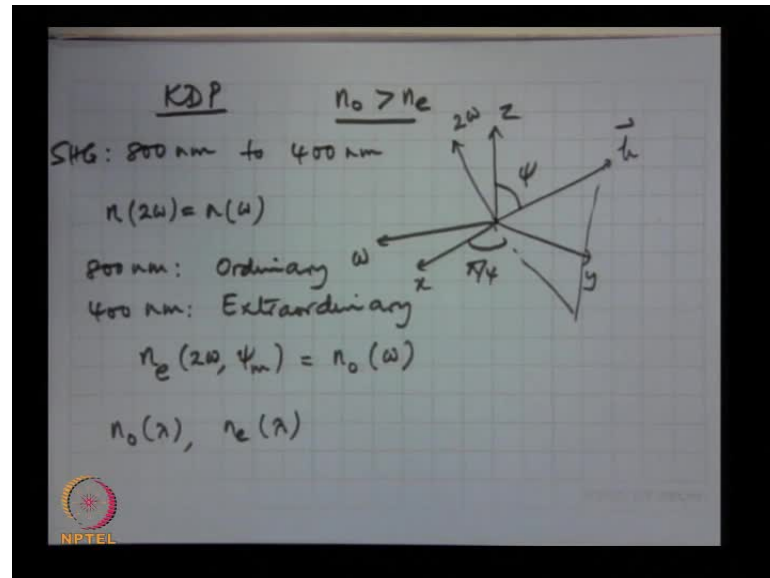
$\omega \rightarrow 800 \text{ nm}$
 $2\omega \rightarrow 400 \text{ nm}$

$$n_o(\omega) = 1.5015, \quad n_o(2\omega) = 1.5238$$
$$n_e(\omega) = 1.4633, \quad n_e(2\omega) = 1.4797$$
$$\psi_m = 44.69^\circ$$

So, let me take an example, so here, **what I do is...** Last time, we had done for ruby laser, but let me take another set of wavelengths; so, I choose **so** 800 nanometers is one wavelength and a second harmonic is 400 nanometers. So, remember this equation we have obtained, sin square psi m must be equal to 1 by n o square of omega minus 1 by n o square of 2 omega divided by 1 by n e square of 2 omega minus 1 by n o square of 2 omega.

So, what I do is, **I am given**, for example, so, omega corresponds to 800 nanometers and 2 omega corresponds to 400 nanometers; so, I have these numbers n o at omega is equal to 1.5015, n o at 2 omega is equal to 1.5238, n e at omega is equal to 1.4633, and n e at 2 omega is equal to 1.4797 So, I have calculated these **from a set of equations**, and what I get is, the angle psi m comes out to be 44.69 degrees. So, this angle is different from the angle we had obtained earlier, because **that was**, wavelengths were different; that was 694.3 nanometers, ruby laser input wavelength, second harmonic of that, and we have got 50.5 degrees **or so**. So, the angle depends, of course, on the numbers here, refractive indices

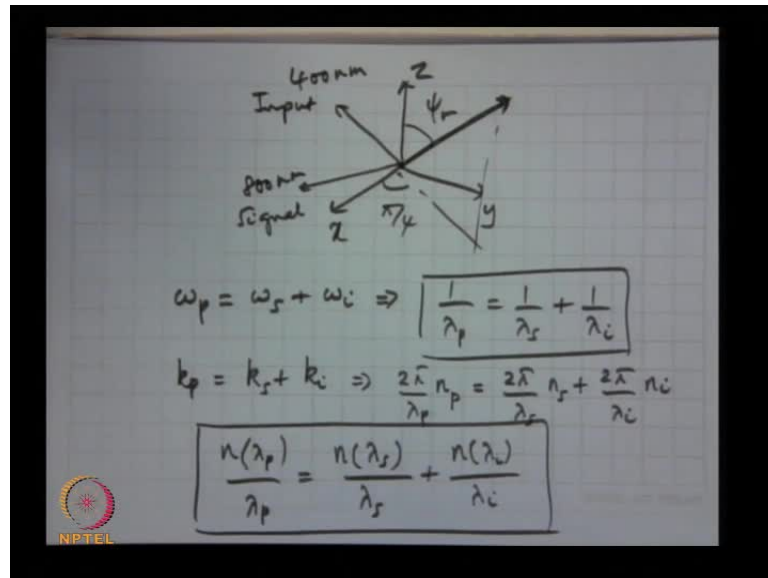
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So, if I take a KDP crystal, propagate at an angle making 44.69 degrees with the optic axis and launch 800 nanometer of the ordinary wave then, I will satisfy phase-matching; I will have a finite coupling coefficient and I will get second harmonic coming out at 400 nanometers along this direction; the k vector is along this direction and I can calculate the s vector direction etcetera, from here. And, I have the formulae to calculate what is the efficiency. Remember, we had an expression of efficiency in terms of the non-linear coefficient, in terms of length, in terms of the power, the fundamental etcetera everything.

Now, the problem you are looking at, is inverse; so, what I now do is, if I want to look at parametric amplification, what should I do? The same problem, I want you to look as a parametric amplifier. So, my input should be 2 omega and an input also at omega, which is a weak signal. So, if I choose the same direction, because I am satisfying the phase-matching condition, whether I launch omega and generate 2 omega or launch 2 omega and generate omega, the phase-matching condition is the same; because, it is simply saying that $k_{2\omega}$ is equal to $2k_{\omega}$, the phase-matching condition is same, the set of equations are the same, delta k is the same; whether, **I** if I launch omega and generate 2 omega, the input, the initial conditions are different from launching 2 omega and omega simultaneously; that is all, there is only difference of initial conditions.

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So, to study this as the parametric amplifier, what I should do is, I launch 2 omega, that means, I launch a 400 nanometer wave as an extraordinary wave propagating at an angle of 44.69 degrees with the optic axis, so what do I expect? So, I take this the KDP crystal, again go back to KDP crystal, and my propagation direction makes an angle psi m with the optic axis, and I launch a 400 nanometers wave like this, input; so, this is again making an angle of pi by 4 with the x axis so that, I get maximum non-linear coefficient.

And, if I simultaneously launch a 800 nanometer signal, so **this is a**, this will be a high power, 400 nanometers will be much higher power, 800 nanometer will be weak signal. So, along this direction, when they propagate, the 800 nanometer signal will get amplified or attenuated depending on the phase difference, phi 2 minus 2 phi 1; if it is minus pi by 2, I get amplification; if phi 2 minus 2 phi 1 is plus phi by 2, I get attenuation of the signal; that means, either if the phase is at right, I go from 400 to 800 nanometer, or if the phases are, such that, phi 2 minus 2 phi 1 is plus pi by 2, the 800 nanometer signal goes to 400 nanometer. So, either splitting of 400 nanometer photons into 200 nanometer photons or joining a 400 nanometer photons to form one 400 nanometer photon, depends on the phase of relationships between the 2 signals which are coming in.

So, I can have an amplifier; the same problem I can look at as an amplification, and this is that degenerate parametric amplifier; because, the 2 wavelengths which are interacting

with the pump, have the same value; there are two signals, one 800 nanometer photon, another 800 nanometer photon interacting with the 400 nanometer. So, this is an example of $\omega_p = \omega_s + \omega_i$, where ω_s and ω_i are equal. There is only one signal coming at 800 but, because this wavelength is double the wavelength, the frequency, is half this frequency, so I have 2 ω by 2 photons coming in and interacting with this ω photon, or this is a higher frequency photon; these are lower frequency photons.

In spontaneous parametric down conversion, which we have not yet discussed, I do not even launch this; I just launch 400 nanometer photons into this crystal, and what I expect to get, 800 nanometer light coming out; that is, spontaneous generation of the lower frequency waves.

Direction should be all the same.

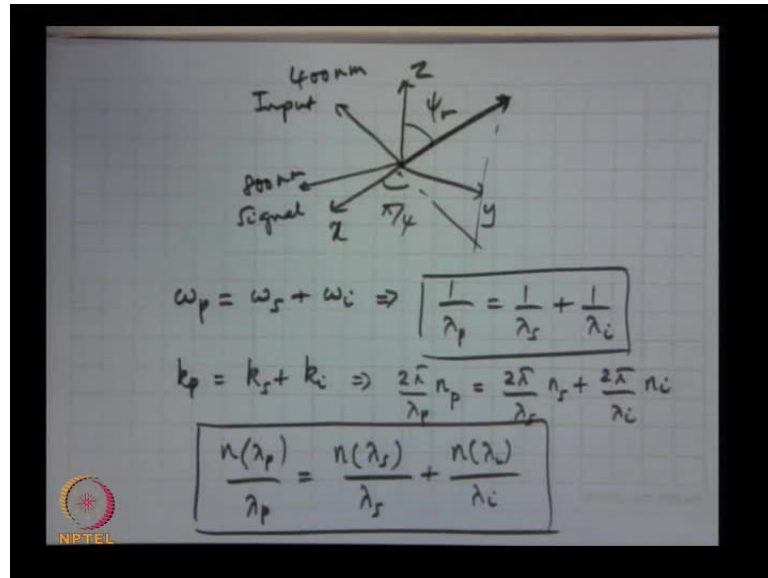
Because, that is the direction in which phase-matching is taking place. If I choose some other direction, for 400 nanometers, I will not satisfy the phase-matching condition; I will come to this problem now.

So, I can actually generate 800 nanometer **photons, pairs of photon** that 800 nanometer, by shining light at 400 nanometers, along this direction; so, there will be certain efficiency of generation which you will calculate later, but this is what is called as spontaneous parametric down conversion or parametric fluorescence, so I launch 400 nanometer, and out comes 800 nanometer light from the crystal. So, **every**, I mean, there are the 800 nanometer signal photon that is coming from splitting a 400 nanometer photons into pairs of photons.

So, now, suppose I change this angle; I keep the 400 nanometer fixed and I change the angle; as I change my angle, remember, what are the conditions I need to satisfy? In general, I will have $\omega_p = \omega_s + \omega_i$. If $\omega_s = \omega_i$, then it is corresponding to second harmonic, otherwise, not; and then, what is the other condition? So, in terms of wavelength, what does this give me? If I write this equation in terms of wavelengths, not in frequency wavelength, in free space; this is $2\pi c/\lambda_p = 2\pi c/\lambda_s + 2\pi c/\lambda_i$, so, $2\pi c$ will cancel of, so, $1/\lambda_p = 1/\lambda_s + 1/\lambda_i$

by λ_i ; because, this is $2\pi c$ by λ_p , $2\pi c$ by λ_s and $2\pi c$ by λ_i ; all wavelengths are wavelengths in free space.

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The other condition is phase-matching, so, k_p is equal to k_s plus k_i . So, in terms of wavelengths, this will be 2π by λ_p into refractive index, let me call this n_p , refractive index at the pump frequency is equal to 2π by λ_s n_s plus 2π by λ_i n_i , so, which is nothing, but say, this is one equation and **the other equation is...** Let me write this explicitly, n of λ_p by λ_p is equal to n at λ_s by λ_s plus n at λ_i by λ_i .

I have not written right now, whether they are extraordinary or ordinary indices; but I know, that if I take this example of KDP, these two must be ordinary and this must be extraordinary. Because, the higher frequency wave is extraordinary and the lower frequency waves are ordinary waves; this is an example, **so this could be a...** This is a situation where λ_s is equal to 800, λ_i is equal to 800, and I get λ_p is 400 nanometers and corresponding refractive indices, these two wavelengths are equal; so, these two refractive indices are equal, and this equation just tells me n at λ_p is equal to n at λ_s is equal to n at λ_i .

Now, if I change my direction, that means, if I have, **so, if have** the crystal **which is** in which I am propagating at 44.69 degrees, now, initially; I change my angle, slightly;

when I change my angle slightly, lambda p is fixed; I am still launching 400 nanometer, I have not changed 400 nanometer. This refractive index will now change because, this is an extraordinary wave. So, let me write this explicitly now, in our case.

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Handwritten equations on a grid background:

$$\frac{n_e(\lambda_p, \psi)}{\lambda_p} = \frac{n_o(\lambda_s)}{\lambda_s} + \frac{n_o(\lambda_i)}{\lambda_i}$$

$$\frac{1}{\lambda_p} = \frac{1}{\lambda_s} + \frac{1}{\lambda_i}$$

$\lambda_p = 400 \text{ nm}, \quad \psi = 44.69^\circ \quad \lambda_s = \lambda_i = 800 \text{ nm}$
 $\psi = 44.2^\circ, \quad \lambda_p = 400 \text{ nm}$
 $\lambda_s = 700 \text{ nm}, \quad \lambda_i = 933 \text{ nm}$

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In our case, because, the pump is an extraordinary wave, the extraordinary refractive index at lambda p, psi, some angle psi by lambda p is equal to the ordinary index at lambda s by lambda s plus the ordinary index of lambda i by lambda i. Psi is some angle now; if I choose psi is equal to psi m 44.69 degrees, the solution of these two equations, this and this equation, **simultaneously gives me...** So, if I choose lambda p is equal to 400 nanometers, and if I choose psi is equal to 44.69 degrees, these two equations' solution gives me lambda s is equal to lambda i **is equal to...** Which is what I have done, I am just giving the same result in a slightly different fashion.

Please note these two equations; for a given lambda p, I can use this second equation to replace lambda i by lambda p and lambda s. So, I can eliminate lambda i from these two equations, so, I will have one equation containing lambda p and lambda s; and of course, the angle and the corresponding refractive indices. So, for a given value of lambda p and psi, I need to solve these two equations simultaneously, to find what is the set of lambda s lambda i, **which satisfies both these equation;** there will be only one set. Because, if I replace lambda i in this equation, this equation will contain lambda p psi and lambda s

only; and for a given λ_p and ψ , this will give me a one value of λ_s ; and that value of λ_s substituted here, will give me a λ_i value.

So, what I need to do is, if I change my angle, I need to resolve these two sets of equations, for which I need to know - how n_o varies with wavelength and how n_e wave varies with wavelength with Sellmeier equations; and I solve these two equations simultaneously, and get the new pair of wavelengths, λ_s λ_i , which satisfies this equation. So, what I did was, I took some arbitrary angle; so, if I choose now ψ is equal to 44.2 degrees instead of 44.69 degrees, and I keep λ_p as 400 nanometers; the pair of wavelength which satisfy these equations, comes out to be λ_s is equal to 700 nanometers and λ_i is equal to 933 nanometer.

These are approximate in the sense, that there may be plus with some additional point, some wavelength, but this is just to give you an idea, that if I choose my wave, if I choose my angle of propagation instead of 44.69 to 44.2 degrees and I launch the same pump wavelength 400 nanometer. Now, the pair of wavelengths of signal and idler, which satisfy these two equation simultaneously, the energy conservation and momentum conservation equation is another pair of λ_s λ_i ; so, what does it mean, if I take this KDP crystal and if I launch 400 nanometer wavelength at 44.2 degrees, I will generate a signal photon at 700 nanometers and an idler photon at 933 nanometers, which is spontaneous fluorescence or parametric fluorescence.

It will not generate 800 nanometer now, because 800 nanometer wavelengths do not satisfy the phase-matching condition for this propagation direction; there is another pair of wavelengths, λ_s λ_i , which satisfies the energy conservation and momentum conservation equation; and this pair of wavelengths comes out to be around 700 nanometer and 933 nanometer.

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$$\frac{n_e(\lambda_p, \psi)}{\lambda_p} = \frac{n_o(\lambda_s)}{\lambda_s} + \frac{n_o(\lambda_i)}{\lambda_i}$$
$$\frac{1}{\lambda_p} = \frac{1}{\lambda_s} + \frac{1}{\lambda_i}$$

$\lambda_p = 400 \text{ nm}, \quad \psi = 44.69^\circ, \quad \lambda_s = \lambda_i = 800 \text{ nm}$

$\psi = 44.2^\circ, \quad \lambda_p = 400 \text{ nm}$

$\lambda_s = 700 \text{ nm}, \quad \lambda_i = 933 \text{ nm}$

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One will be less than 800, the other will be more than 800 because of the equation here; and, this is called the signal, the lower wavelength is called the signal higher frequency, and the other one is called the idler. So, what it implies is, if in addition to 400 nanometers, if I launch a 700 nanometer signal at this angle 44.2, I will amplify this 700 nanometer signal; I will not amplify 800 nanometers now, I will only amplify 700 nanometers; the wavelength at which the amplifier works, has changed because, the phase-matching condition in that direction has changed.

So, if I take this KDP crystal, launch light at 44.69, it will be an amplifier for 800 nanometer wavelength; if I choose my angle at 44.2 degrees, it will amplify either the 700 nanometer or 933 nanometer; it can amplify either of them. It is my nomenclature to call one, signal and other, idler; there is no difference between the signal and idler, they are two different wavelengths. By nomenclature, I call the signal as the higher frequency and the idler as the lower frequency wave; there is complete symmetry. If I change that, if I replace lambda s by lambda i and lambda i by lambda s, they are the same set of equation; there is no difference. I am just calling one of them signal, the other as idler.

So, this is called angle tuning. The wavelength at which the amplifier will work, changes as I change my orientation of the crystal. So, as I mentioned, we will put this amplifier within a pair of mirrors and make an oscillator, a coherent source of light; the wavelengths coming out of this coherent source of light will be the wavelengths for

which this crystal will amplify. So, if I choose the crystal to be oriented at 44.69 degrees within the mirror pair, I expected 800 nanometer coming out; if I rotate the crystal slightly to 44.2 degrees then, I will get 700 nanometer or 933, or **both of them coming out**, both of them coming out.

So, I can actually **take** a tunable laser; it does not depend on energy levels. Remember, helium-neon lasers, ruby lasers - all these depend on energy levels of atoms and molecules; these are fixed. But here, I can tune; all I need to do is the change orientation of the crystal; and now, this is an amplifier for another pair of wavelengths, and hence, I can make it oscillate for another pair of wavelengths and I get the laser, which is tunable. So, I will come to this tuning, and the oscillator etcetera later, which is called the optical parametric oscillator.

When we change the orientation of this 2 omega wave with the optic axis, then we need to ensure that accordingly, the bilateral are exogenous direction; because, just changing the propagation, the directional ways, gets polarized; **it also has to be taken care of vertical...**

This is the direction of polarization of the d vector of the wave.

It does not... It also does not remain fixed as we change the side.

No, it is like this; **I have** this is the pump coming, my crystal is sitting here, and I rotate the crystal; this polarization is fixed. The optic axis, I am **I am** changing the orientation the optic axis of the crystal by rotating crystal. So, automatically, because of the orientation, it is always an extraordinary wave. Because, I am only rotating in that, **in that**, plane; I am not rotating in any other direction, I am just changing the value of psi m keeping pi by 4 fixed; everything else is fixed.

Pi by 4 angles are adjusted.

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$$\frac{n_e(\lambda_p, \psi)}{\lambda_p} = \frac{n_o(\lambda_s)}{\lambda_s} + \frac{n_o(\lambda_i)}{\lambda_i}$$
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$\lambda_p = 400 \text{ nm}, \quad \psi = 44.69^\circ, \quad \lambda_s = \lambda_i = 800 \text{ nm}$

$\psi = 44.2^\circ, \quad \lambda_p = 400 \text{ nm}$

$\lambda_s = 700 \text{ nm}, \quad \lambda_i = 933 \text{ nm}$

Yes, automatically, this wavelength becomes an extraordinary wave, now propagating in a different direction, and hence its refractive index will change; and because its refractive index is changed, the pair of wavelength will satisfy the... Simultaneously, energy conservation and momentum conservation equation has changed.

Sir, initially, what we said, that was depending on the initial phase, will either get a signal amplification or signal integration?

For degenerate, for the degenerate case. In the non-degenerate case, I showed you, that if I only put signal and pump, I will always have amplification, cos hyperbolic square g z; if I put signal and idler and pump, then it will depend on the phase of the signal and idler with respect to the pump; otherwise, no. In the degeneration case, I do not have a phase insensitive situation. Because, when I say, when I put an 800 nanometer, I am already putting signal and idler simultaneously. They are not separate but in this case, if I only put this, or if I only put this - with the pump, I have, always amplification, always amplification.

Why is not the new wavelength has been created? Why cannot it be an extraordinary wave, like, we are not considering the acceleration?

Can somebody tell me? Why cannot these wavelengths be extraordinary?

Because then, then this energy, sorry, momentum...

I will not be able to satisfy this equation; if this is extraordinary, this extraordinary, this is also extraordinary; this will be not satisfied.

Sir, but there it is to be only for negative case.

This, for one particular crystal, I am looking at KDP; if I take another crystal, there will be situation, will be different.

Spontaneous; that can be major size will tiny, small, not as large as the that size; it can half some angle...

No, for example, suppose I were to check at 800 nanometers itself; if all were extraordinary, is it not possible?

It can be at different angles there, so the psi can be written; they do not have to be...

The direction of propagation, you mean?

Please remember, I am writing a scalar equation for momentum conservation; but actually, it is a vector equation. So, if what I can have is, possibly the pump photons are coming like this, signal photons are going here, idler photon will going here; I must satisfy all components of momentum to be conserved.

So, I have not discussed here the general situation, where the pump is going like this, signal is going like this, idler is going in another different angle, completely different angles; it is possible. There are situation where this happens very nicely; and I will come to this example later on, because this is used to generate entangled photon pairs, this kind of a process in which the directions are all different. It is not necessarily that this is a collinear case, where all the signal, the pump idler and pump, everything is travelling in the same direction; but you are perfectly right, it can happen in any other direction; but then, I need to satisfy with the corresponding vector phase-matching condition.

Do not they need to travel the same direction for application to take place?

Yes, that is another problem. But suppose, I am looking at spontaneous parametric generation, there is no amplification, it generates automatically; it will generate automatically along all those directions in which I satisfy phase-matching condition; and there, many more than one direction, and this I will come to an example later on. There is possible that the momentum conservation is equation satisfied for directions of propagations of signal and idler, more than one pair of directions; and so, I will get light in all those directions in which I am satisfying a phase-matching condition.

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$$\frac{n_e(\lambda_p, \psi)}{\lambda_p} = \frac{n_o(\lambda_s)}{\lambda_s} + \frac{n_o(\lambda_i)}{\lambda_i}$$

$$\frac{1}{\lambda_p} = \frac{1}{\lambda_s} + \frac{1}{\lambda_i}$$

$\lambda_p = 400 \text{ nm}, \quad \psi = 44.69^\circ \quad \lambda_s = \lambda_i = 800 \text{ nm}$
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$$\frac{n_e(\lambda_p, \psi)}{\lambda_p} = \frac{n_o(\lambda_s)}{\lambda_s} + \frac{n_o(\lambda_i)}{\lambda_i}$$

$$\frac{1}{\lambda_p} = \frac{1}{\lambda_s} + \frac{1}{\lambda_i}$$

$\psi = 44.2^\circ \quad \lambda_p = 390 \text{ nm}$
 $\lambda_p = 400 \text{ nm}, \quad \psi = 44.69^\circ \quad \lambda_s = \lambda_i = 800 \text{ nm}$
 $\psi = 44.2^\circ, \quad \lambda_p = 400 \text{ nm}$

So, this is one interesting example where I can actually keep the pump wavelength fixed, and I change my angle of propagation, and I tune. Now, suppose I keep this angle and change my pump wavelength, what is going to happen? So, suppose I keep the angle at 44.2 and change λ_p to 390 nanometers, instead of 400 nanometer.

Now, again, I need to satisfy both these conditions - ψ is known 44.2, λ_p is 390, the refractive index has **not changed**. Because, the refractive index at 400 nanometers, at 44.2 degrees, **what**, is different from the extraordinary index at 390 nanometers at 44.2 degrees. Remember, one by any square of ψ is equal $\cos^2 \psi$ by n_o^2 plus $\sin^2 \psi$ by n_e^2 . So, as you change the wavelength, the ordinary and extraordinary index changes because of dispersion; and so, the index seen by the 390 nanometer wavelength, as it propagates along a direction making 44.2 degrees with the optic axis, has now changed. If this changes, this pair of equations have to be resolved again; and what I have found is, I got the solution now, for λ_s comes out to be 725 nanometers **and λ_i is...** So, I can actually tune the pair of wavelengths by not rotating the crystal, but changing your pump wavelength. If I change my pump wavelength, automatically I change the conditions of phase-matching, analogy conservation, and I will find a new pair of wavelengths which satisfy both the condition simultaneously.

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Handwritten notes on a grid background, likely from a video lecture. The text includes:

- $\psi = 44.2^\circ$ $\lambda_p = 390 \text{ nm}$
- $\lambda_s = 725 \text{ nm}$ $\lambda_i = 844 \text{ nm}$

A diagram shows a coordinate system with x, y, and z axes. A vector labeled "Input" is in the x-z plane at an angle ψ to the z-axis. A vector labeled "Signal" is in the x-y plane. A vector labeled "Pump" is in the y-z plane. The angle between the Input and Signal vectors is $\pi/4$.

Below the diagram, the following equations are written:

$$\omega_p = \omega_s + \omega_i \Rightarrow \frac{1}{\lambda_p} = \frac{1}{\lambda_s} + \frac{1}{\lambda_i}$$

$$k_p = k_s + k_i \Rightarrow \frac{2\pi}{\lambda_p} n_p = \frac{2\pi}{\lambda_s} n_s + \frac{2\pi}{\lambda_i} n_i$$

The NPTEL logo is visible in the bottom left corner.

So, if I start from 44.69 degrees and launch 400 nanometers, I will get output light at 800 nanometers, or it will act like an amplifier for the 800 nanometer. Let me look at parametric fluorescence, so, I launch 400 nanometers at 44.69 degrees with the optic axis, I get 800 nanometer light coming out; and the polarization state at 800 nanometer will be ordinary, because it is the ordinary index; it is the ordinary index at λ_s and λ_i , which satisfies this equation with the pump being extraordinary.

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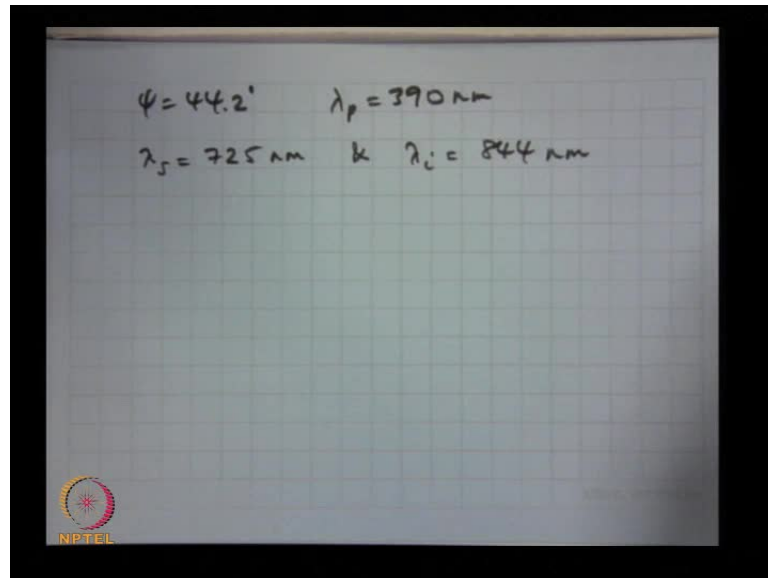
$$\frac{n_e(\lambda_p, \psi)}{\lambda_p} = \frac{n_o(\lambda_s)}{\lambda_s} + \frac{n_o(\lambda_i)}{\lambda_i}$$

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 $\psi = 44.2^\circ, \quad \lambda_p = 400 \text{ nm}$
 $\lambda_s = 700 \text{ nm}, \quad \lambda_i = 933 \text{ nm}$

If I change my angle to 44.2 degrees, keep the pump wavelength fixed, the output light now is at 700 nanometers and 930 nanometers; so, same pump photon at 400 nanometers, now splits into a different pair of wavelengths. As I told you, this equation has infinite number of solutions; if I fix λ_p and I ask you, what is λ_s and λ_i , it can be anything as long as this equation is satisfied. But I need to satisfy also this equation, and then, I get only one pair of λ_s λ_i , which satisfies energy conservation and momentum conservation simultaneously. So, if I change my angle to 44.2, keeping the pump wavelength fixed, I get a new pair of wavelengths coming out, signal and idler.

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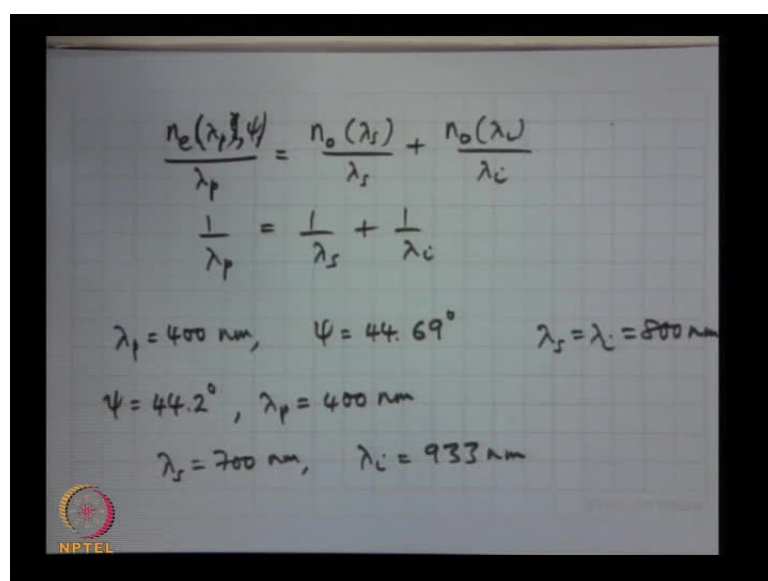
Handwritten notes on a grid background showing phase-matching parameters:

$$\psi = 44.2' \quad \lambda_p = 390 \text{ nm}$$
$$\lambda_s = 725 \text{ nm} \quad \text{or} \quad \lambda_i = 844 \text{ nm}$$

The NPTEL logo is visible in the bottom left corner.

Both are ordinary again; if I now change the pump wavelength to 390 nanometers, then again the wavelengths change because, again, the phase-matching condition is now different for another set of pair of idler and single wavelengths. So, this is actually tuning; I can tune the amplifier or the oscillator when I generate an oscillator by either tuning - angle tuning, or by changing the pump wavelength; there is also another mechanism which is called temperature tuning.

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Handwritten notes on a grid background showing the phase-matching equation and numerical examples:

$$\frac{n_e(\lambda_p, \psi)}{\lambda_p} = \frac{n_o(\lambda_s)}{\lambda_s} + \frac{n_o(\lambda_i)}{\lambda_i}$$
$$\frac{1}{\lambda_p} = \frac{1}{\lambda_s} + \frac{1}{\lambda_i}$$

Examples:

$$\lambda_p = 400 \text{ nm}, \quad \psi = 44.69^\circ, \quad \lambda_s = \lambda_i = 800 \text{ nm}$$
$$\psi = 44.2^\circ, \quad \lambda_p = 400 \text{ nm}$$
$$\lambda_s = 700 \text{ nm}, \quad \lambda_i = 933 \text{ nm}$$

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If I change the temperature of the crystal, all the refractive indices will change because the refractive index of a medium depends on temperature. So, by changing the temperature, from room temperature to 100 degree centigrade or 150 degree centigrade, I can actually change the indices and I can change these wavelengths coming out; that is called temperature tuning. So, I can have temperature tuning, angle tuning, or I can change the pump wavelength and tune.

So, this tuning mechanism is very interesting, is important, because **I can**, as I mentioned, I can make a coherent source of light like a laser; starting from one laser at 400 nanometer, I can generate a tunable laser which emits wavelengths; and I can tune the wavelengths by adjusting the orientation of the crystal, or the temperature, or whatever it is. So, this is one example which I wanted to discuss, to tell you the flexibility that is available; and also note, that the polarization states and the wavelengths of the signal and idler coming out are automatically decided by the phase-matching condition and by the energy conservation condition.

Now, it is possible that you may not be able to find any pair of wavelength, λ_s , λ_i , for which, both these n gets satisfied; it is possible. And then, there is no down conversion; I need to satisfy phase-matching to be able to generate the two new wavelengths, the longer wavelengths photons that are coming out. Tell me, if I launch 700 nanometers and 933 nanometers as ordinary waves along this angle, what will it generate? I launch 700 nanometers light, 933 nanometers light at 44 degrees, 44.2 degrees, and these are both ordinary waves. What will I generate?

400 nanometers

400 nanometers - this is same process, and that is called **sum** frequency generation. The frequency of this plus frequency of this is equal to frequency of this; and it is a same set of three equations. Only thing is, it depends on the pair of equations which I take to solve and initial conditions; if I launch only ω_p , it is parametric fluorescence; if I launch ω_p and ω_s , it is amplifier for a λ_s and generating a different frequency idler; if I launch λ_s and λ_i , I generate λ_p , which is sum frequency generation. So, those three equations contain all the three wave interaction processes; this is called three-wave-interaction because, there are three wavelengths which are

interacting with each other to exchange energy through the non-linear coefficient of a crystal. So, do you have any questions in this?

So, I thought this was an interesting example that can help us understand - how it is possible to tune the pairs of wavelengths, and what kind of frequencies and wavelengths and polarization state that will come out from parametric down conversion?

Signal will always be amplified irrespective of the phase of the signal incomes.

Yes, which I showed you in a class.

(Refer Slide Time: 34:40)

$$\frac{n_e(\lambda_p, \psi)}{\lambda_p} = \frac{n_o(\lambda_s)}{\lambda_s} + \frac{n_o(\lambda_i)}{\lambda_i}$$
$$\frac{1}{\lambda_p} = \frac{1}{\lambda_s} + \frac{1}{\lambda_i}$$

$\lambda_p = 400 \text{ nm}, \quad \psi = 44.69^\circ \quad \lambda_s = \lambda_i = 800 \text{ nm}$

$\psi = 44.2^\circ, \quad \lambda_p = 400 \text{ nm}$

$\lambda_s = 700 \text{ nm}, \quad \lambda_i = 933 \text{ nm}$

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If it is non-degenerate, that means, if these two wavelengths are different, then, if I launch, only this end pump or only this end pump, then this wavelength will always get amplified, irrespective of the phase of the pump and the signal; it always goes as $\cosh(\gamma z)$ or $g z$. If I launch all three waves, then the phase of relationship starts to play very important role - which direction energy is flowing? Either from the lower frequency wave to higher frequency waves or higher frequency waves to lower frequency waves.

Sir, when we have signal and idler incident; under these conditions, it would be like some frequency generation. Then, what should be, what set of conditions should be there

for difference? You can say (()), how did we differ, and what would be the conditions (())?

I am considering three frequencies which satisfy the equation, ω_p is equal to ω_s plus ω_i . So, if I want to study difference frequency generation, I will assume that the two waves which are incident, are ω_p and ω_s , so that, it generates ω_i is equal to ω_p minus ω_s . If I want to study some frequency generation, I will assume that my input is ω_s and ω_i , which generates ω_p .

So, then ω_i - 700 nanometer will be like ω_p ; then, if in case of difference, you can see with this.

Not, ω_p . ω_p is here, the highest frequency.

No, this is not been generated - right? If I am talking these two wavelengths are incident 700 nanometer and 933.

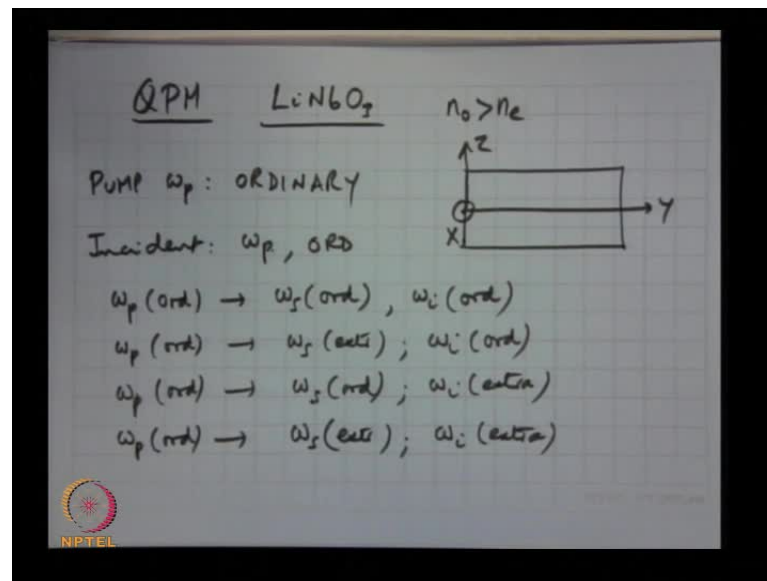
Yes

And I want difference frequency generation between these two.

Then one of them has to be called ω_p and the other one be called ω_s .

But, what I am saying is, if I am studying some frequency, I will call the waves so that, the same set of equation... Otherwise, I have to keep track of, what if, what is the higher frequency, lower frequency, everything; but here, just by calling ω_p is equal to ω_s plus ω_i , and getting those three equations, I will call the frequencies by the corresponding names depending on whether I am looking for sum frequency or difference frequency or parametric down conversion etcetera.

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So, this is one example, which I wanted to discuss. The second one which I want to see is the following. I want to look at in terms of Quasi-phase-matching, and let me take Lithium Niobate as the example. Now, Lithium Niobate has again $n_o > n_e$; and let me look at the following situation, I have the crystal like this and this is y axis z axis. Now, I launch a pump as an ordinary wave; so, what is the polarization state at the input? So, pump is ω_p and this is ordinary so, what is the polarization state? x. yes, this direction because, if I propagate along this direction, the z direction is the extraordinary. Now, these are the principle axis of the crystal, this is not the x y z laboratory coordinate system. This is the principle axis x y z so, this is x axis so, ω_p has x polarization. Now, I want to find out whether **this polarization...** So, what possibilities can exist? So, ω_p is the incident so, incident is ω_p and ordinary.

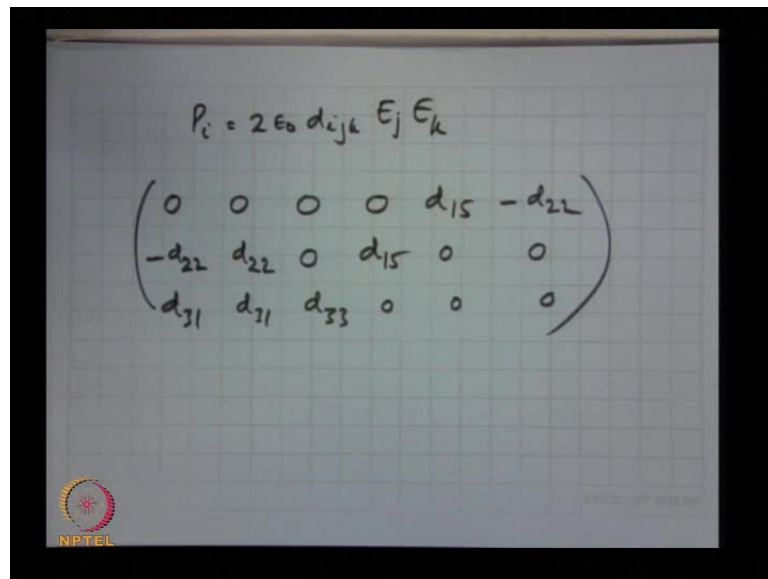
Now, so, I am looking at down conversion. So, there are three possibilities so, ω_p ordinary leading to ω_s ordinary and ω_i ordinary. Now, please remember, now **I can**, I am using Quasi-phase-matching so, I can choose an appropriate capital K vector **to match** to satisfy the phase-matching condition. So, I am not restricted now to using Birefringence phase-matching so, I am taking the propagation to be along one of the principle axis of the crystal; in KDP, I could not do that.

The other possibility is ω_p ordinary giving ω_s extraordinary and ω_i ordinary; ω_p ordinary giving ω_s ordinary and ω_i extraordinary, which is

the same as this, except that the names are different now; and finally ω_p ordinary can give me ω_s extraordinary and ω_i extraordinary, four possibilities.

How do I find out which one can at all take place, and under what conditions? Remember, I need to satisfy two conditions, one is phase-matching, and there should be a non-linear coefficient coupling the polarization states; there must be an effective non-linear coefficient, must not be 0. Please note, I must satisfy phase-matching condition which I can always choose by appropriately choosing the capital K vector, the special frequency of the non-linear grating. But, I must also have a finite non-linear coefficient coupling these states. So, now, let me look at the first process; so, the first process is, what is a polarization state of the ω_p ? ω_p is x polarized because, it is ordinary; ω_s is also x polarized and ω_i is also x polarized.

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The image shows a handwritten equation and a tensor matrix on a grid background. The equation is $P_i = 2\epsilon_0 d_{ijk} E_j E_k$. Below it is a 3x6 matrix representing the non-linear tensor d_{ijk} :

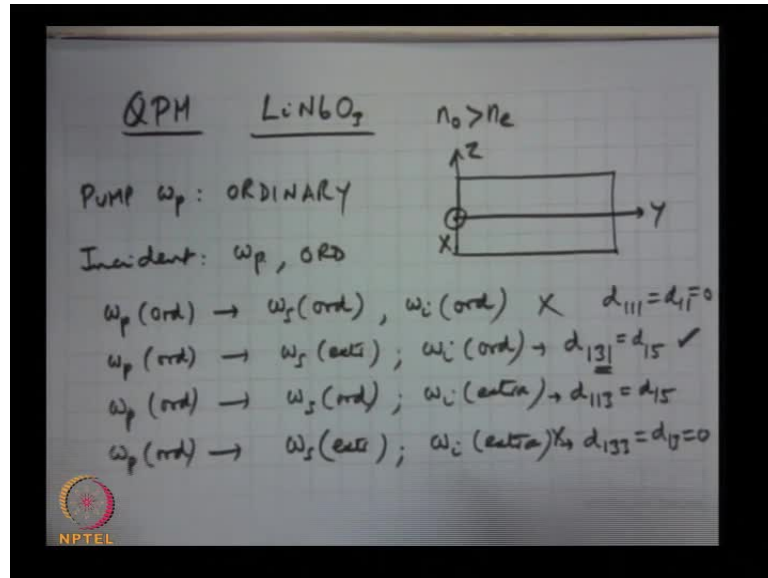
$$\begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$

An NPTEL logo is visible in the bottom left corner of the slide.

So, if you look at this equation which we added in earlier, the polarization p_i is equal to $2\epsilon_0 d_{ijk} E_j E_k$; these are total electric fields, total polarization. So, I have to substitute E_j as the sum of electric field of the pump signal and idler. Similarly, E_k pump signal plus idler, and I will find out what are the polarizations which I have state generated. Now, this process I am looking at is coupling an x polarization of the pump, an x polarization of the signal and an x polarization of the idler so, what is the non-linear tensor element that will be responsible? d_{111} .

Which is d_{11} ? Now, let me write for Lithium niobate, 0, 0, 0, 0, d_{15} , minus d_{22} , minus d_{22} , d_{22} , 0... Please just verify, I am configuring the state; Yeah, that is fine.

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Can the first process take place? d_{111} is d_{11} ; this last two 1s are contracted to a single 1 so. d_{111} is actually d_{11} , and that is 0; so, this cannot take place, I mean, satisfy phase-matching, but there is no non-linear coefficient coupling, the ordinary polarization of the pump, and ordinary polarization signal, and ordinary polarization of idler. What about the second process? So, the second one is... So, ω_p is ordinary so, let me write this; so, this requires d_{111} which is equal to $d_{11} = 0$. What does this require? So, this is $d_{1\omega_s}$ extraordinary so, d_{131} which is equal to d_{15} . d_{131} is how much? d_{15} . d_{131} is 3; this contraction is 5 and that is not 0; even 5 is finite. So, I will calculate now, what is the period required. But, before that, what about this one? d_{113} , which is d_{15} ; d_{131} and d_{113} are the same; and what about this d_{133} , $d_{133} = 0$? So, these two process is cannot take place.

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$$P_i = 2 \epsilon_0 d_{ijk} E_j E_k$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$

QPM LiNbO₃ $n_o > n_e$

PUMP ω_p : ORDINARY

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A diagram shows a rectangular crystal with a coordinate system where the z-axis is vertical, the y-axis is horizontal to the right, and the x-axis is a dot pointing out of the page. A circle with a plus sign is on the z-axis, representing a pump wave.

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QPM LiNbO₃ $n_o > n_e$

PUMP ω_p : ORDINARY

Incident: ω_p , ORD

NPTEL

A diagram shows a rectangular crystal with a coordinate system where the z-axis is vertical, the y-axis is horizontal to the right, and the x-axis is a dot pointing out of the page. A circle with a plus sign is on the z-axis, representing a pump wave.

$\omega_p(\text{ord}) \rightarrow \omega_s(\text{ord}), \omega_i(\text{ord}) \times d_{111} = d_{11} = 0$
 $\omega_p(\text{ord}) \rightarrow \omega_s(\text{ext}); \omega_i(\text{ord}) \rightarrow d_{131} = d_{15} \checkmark$
 $\omega_p(\text{ord}) \rightarrow \omega_s(\text{ord}); \omega_i(\text{extra}) \rightarrow d_{113} = d_{15}$
 $\omega_p(\text{ord}) \rightarrow \omega_s(\text{ext}); \omega_i(\text{extra}) \times d_{133} = d_{13} = 0$

So, if I launch a pump ω_p , which is ordinarily polarized along the y axis of a crystal, propagating along the y axis, I can either generate an extraordinary polarization of signal and ordinary polarization of the idler, or an ordinary polarization of the signal and extraordinary polarization of the idler. What will be the phase-matching condition for the first process?

(Refer Slide Time: 45:24)

$$P_i = 2 \epsilon_0 d_{ijk} E_j E_k$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$

$$\omega_p(\omega), \omega_s(\omega), \omega_i(\omega) \quad k_p - k_s - k_i = K$$

$$\frac{2\pi}{\lambda_p} \cdot n_o(\lambda_p) - \frac{2\pi}{\lambda_s} \cdot n_e(\lambda_s) - \frac{2\pi}{\lambda_i} \cdot n_o(\lambda_i) = \frac{2\pi}{\lambda_1}$$

So, I am looking at omega p ordinary, omega s extraordinary and omega i ordinary, what will be the phase-matching condition? So, I am, I am looking at Quasi-phase-matching so, remember the general phase-matching, Quasi-phase-matching condition is $k_p - k_s - k_i = K$; so, this is 2π by lambda p into n p, sorry, n - it is an ordinary index because, the pump is ordinary, ordinary index at lambda p minus 2π by lambda s into the signal is extraordinary. So, extraordinary index at lambda s minus 2π by lambda i into ordinary index at lambda i is equal to 2π by lambda. The k_p corresponds to 2π by lambda p into refractive index of the wave at the pump frequency which because it is ordinary wave, it is n_o of lambda p; signal is an extraordinary wave so, minus 2π by lambda s n e of lambda s; idler is an ordinary wave so, 2π by lambda i n o of lambda I; so, let me call this lambda 1.

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$$P_i = 2 \epsilon_0 d_{ijk} E_j E_k$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$

$$\omega_p(\omega), \omega_s(\omega), \omega_i(\omega) \quad k_p - k_s - k_i = K$$

$$\frac{2\pi}{\lambda_p} n_o(\lambda_p) - \frac{2\pi}{\lambda_s} n_e(\lambda_s) - \frac{2\pi}{\lambda_i} n_o(\lambda_i) = \frac{2\pi}{\lambda_1}$$

What about this last one? What about this other one? Where the other one is omega p ordinary, omega s ordinary so, **omega p**, omega p ordinary, omega s ordinary and omega i extraordinary; so, again I have 2 pi by lambda p n o at lambda p minus 2 pi by lambda s n o at lambda s minus 2 pi by lambda i n e at lambda i is equal to 2 pi by lambda 2. So, if I make a Quasi-phase-match grating, satisfying this equation, lambda 1; so, I know, if I give you the pump wavelength, I can choose a pair of lambda s lambda i, which satisfy the energy conservation equation; for those lambda s lambda i, I calculate the capital lambda 1 required.

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$\omega_p(o), \omega_s(o), \omega_i(e)$

$$\frac{2\hbar}{\lambda_p} n_o(\lambda_p) - \frac{2\hbar}{\lambda_s} n_o(\lambda_s) - \frac{2\hbar}{\lambda_i} n_e(\lambda_i) = \frac{2\hbar}{\lambda_2}$$

$$P_i = 2\epsilon_0 d_{ijk} E_j E_k$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$

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$P_i = 2\epsilon_0 d_{ijk} E_j E_k$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$
 $\omega_p(o), \omega_s(e), \omega_i(o) \quad k_p - k_s - k_i = K$

$$\frac{2\hbar}{\lambda_p} n_o(\lambda_p) - \frac{2\hbar}{\lambda_s} n_e(\lambda_s) - \frac{2\hbar}{\lambda_i} n_o(\lambda_i) = \frac{2\hbar}{\lambda_1}$$

Similarly, I can also calculate the lambda s, the n o of lambda s and n e of lambda I, and calculate the lambda 2 required. So, if I make this grating of lambda 1, the input ordinary wave at omega p, the input ordinary wave which comes and polarize like this, will split into an extraordinary wave signal photon and an ordinary wave idler photon. So, what will come out is, the signal will be z polarized and the idler will be x polarized.

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QPM LiNbO₃ $n_o > n_e$

PUMP ω_p : ORDINARY

Incident: ω_p , ORD

$\omega_p(\text{ord}) \rightarrow \omega_s(\text{ord}), \omega_i(\text{ord}) \times d_{111} = d_{11} = 0$

$\omega_p(\text{ord}) \rightarrow \omega_s(\text{extra}); \omega_i(\text{ord}) \rightarrow d_{131} = d_{15} \checkmark$

$\omega_p(\text{ord}) \rightarrow \omega_s(\text{ord}); \omega_i(\text{extra}) \rightarrow d_{113} = d_{15}$

$\omega_p(\text{ord}) \rightarrow \omega_s(\text{extra}); \omega_i(\text{extra}) \times d_{133} = d_{15} = 0$

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The extraordinary wave is z polarized, ordinary wave is x polarized; if I choose this period, the same pump photon at λ_p splits into a signal which is ordinarily polarized and an idler which is extraordinarily polarized, and I will continue with this example in next class, we will calculate this λ_s required, approximately.

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$$P_i = 2 \epsilon_0 d_{ijk} E_j E_k$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & d_{15} & -d_{22} \\ -d_{22} & d_{22} & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{pmatrix}$$

$\omega_p(o), \omega_s(e), \omega_i(o) \quad k_p - k_s - k_i = K$

$$\frac{2\pi}{\lambda_p} n_o(\lambda_p) - \frac{2\pi}{\lambda_s} n_e(\lambda_s) - \frac{2\pi}{\lambda_i} n_o(\lambda_i) = \frac{2\pi}{\Lambda_1}$$

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And so, the interesting feature of this problem is, I will again tell you what it leads to a quantum-mechanical picture, and it leads to some very interesting observations which I

will again discuss later on when I discuss quantum-mechanical principles. Any questions?

Thank you.

Sir, in this process we can choose the polarization of the output wavelength body.

Yes, provided, the corresponding non-linear coefficient is nonzero.

From which ends two and three?

Yes, we can choose. For example, I cannot have an ordinary wave coming into the pump, and I want the ordinary signal and ordinary idler; it is not possible because, that is not possible.

Sir, which amongst which are possible?

Because, that is the advantage of Quasi-matching now, because, I can appropriately choose the lambdas required to achieve that down conversion process. In fact, I can choose the wavelengths, I can choose the p, I can choose the polarization states also; wavelengths also, I can choose because, then, the corresponding lambdas will change, that is all.