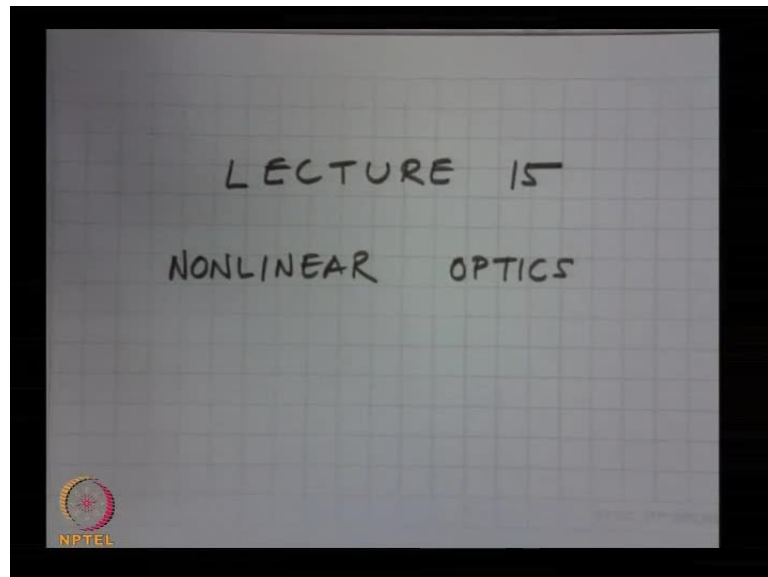


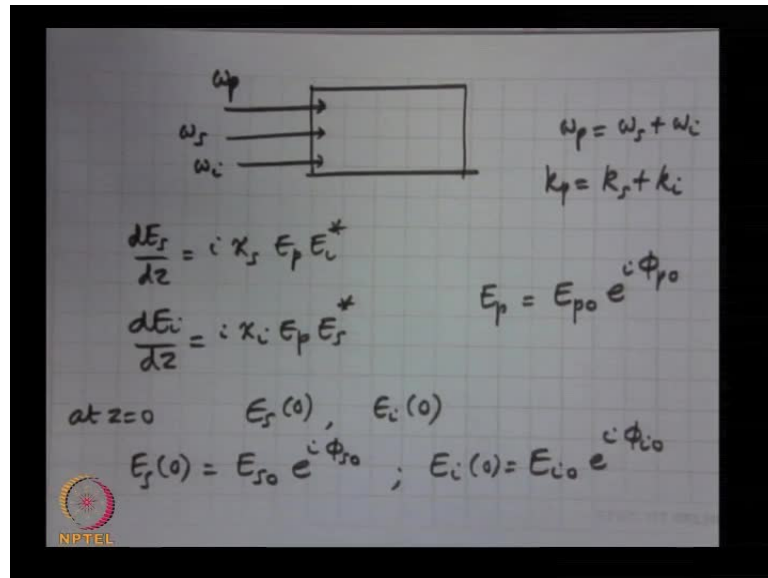
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**Module No. # 03**  
**Second Order Effects**  
**Lecture No. # 15**  
**Non – Linear Optics (Contd.)**

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So, we will continue with discussions on the parametric amplifier. Let me recall, we are looking at a following situation, where there is this non-linear crystal, you have a pump at omega p, you have signal at omega s and idler at omega i coming in.

We showed earlier that, if you had only the pump and signal input, then the signal always gets amplified, irrespective of the phase of the signal; that is called the phase insensitive amplifier. If in addition to signal, I also put an idler with this frequency omega p becoming equal to omega s plus omega I; and we are also assuming k p is equal to k s plus k I, that is phase matching to be present.

So, if I have both omega s and omega i at the input, so we have these two equations if you try to solve yesterday, dE s by dz is equal to i kappa s E p E i star; and dE i by dz is equal to i kappa i E p E s star.

And at z is equal to 0, E s of 0 - the fields are E s 0 - and E i 0. So, E s 0 and E i 0 are complex fields; we will actually replace E s 0 by some amplitude and a phase. Similarly, the idler at the input is a complex electric field is, E i 0 exponential i phi i0; and we also assume before that the pump is given by E p0 exponential i phi p0.

And, because we are assuming that there is no depletion the pump, the pump fields and a phase remain constant as the fields propagate. So, I have a pump coming in at a phase phi p0, a signal as phi s0, an idler at phi i0, the corresponding amplitudes; and E p

supposed to be large, which means that the pump frequency  $\omega_p$  is a strong wave and so the electric field  $E_p$  is independent of  $z$ .

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$$E_i(z) = i \sqrt{\frac{\omega_i n_s}{\omega_s n_i}} e^{i\phi_{p0}} E_{s0} e^{-i\phi_{s0}} \sinh gz + E_{i0} e^{i\phi_{i0}} \cosh gz$$

$$E_s^*(z) = E_{s0} e^{-i\phi_{s0}} \cosh gz - i \sqrt{\frac{\omega_s n_i}{\omega_i n_s}} e^{-i\phi_{p0}} E_{i0} e^{i\phi_{i0}} \sinh gz$$

So, then, we solve these two equations; so, we actually differentiate this equation, substitute from the second equation and got the solution. So, let me write down the solutions again. Solution we got was,  $E_i$  of  $z$  is equal to  $i$  square root of  $\omega_i n_s$  by  $\omega_s n_i$  exponential  $i\phi_{p0}$ . And then you had,  $E_{s0} - E_{s0}^*$  yesterday I had written - so which is  $E_{s0}$  exponential minus  $i\phi_{s0}$  sin hyperbolic  $gz$  plus  $E_{i0}$  exponential  $i\phi_{i0}$  cos hyperbolic  $gz$ .

The solution I had written yesterday was terms of this  $E_{s0}^*$ ,  $E_{i0}$ , and  $E_{s0}$  and  $E_{i0}$ . So, these are the complex fields; at  $z$  is equal to 0, so I am substituting the values of the complex electric fields in terms of amplitude and phase.

And similarly,  $E_s^*$  of  $z$  is equal to  $E_{s0}$  exponential minus  $i\phi_{s0}$  cos hyperbolic  $gz$  minus  $i$  square root of  $\omega_s n_i$  by  $\omega_i n_s$  exponential minus  $i\phi_{p0}$ , and then you have  $E_{i0}$  there, so which is  $E_{i0}$  exponential  $i\phi_{i0}$  sin hyperbolic  $gz$ .

So, this is the complex electric field of the signal, at  $z$  is equal to 0 - actually the complex conjugate of that - and  $E_{i0}$  exponential  $i\phi_{i0}$  is the complex electric field of the idler, at  $z$  is equal to 0.

Now, so these are general relations, which will tell you how the signal and idler go as  $z$  increases. Now, I want to consider one specific case to show you the phase sensitive nature of the amplifier. I want to consider the case, where the number of photons of signal and the number of photons of idler which are incident, are equal.

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$$P_s(z) = \frac{n_s}{2c\mu_0} |E_s(z)|^2 S = \frac{n_s}{2c\mu_0} E_{s0}^2 S$$

$$P_i(z) = \frac{n_i}{2c\mu_0} |E_i(z)|^2 S = \frac{n_i}{2c\mu_0} E_{i0}^2 S$$

$$N_{s0} = \frac{P_s(z)}{h\omega_s} = \frac{n_s}{2c\mu_0} \frac{1}{h\omega_s} E_{s0}^2 S$$

$$N_{i0} = \frac{P_i(z)}{h\omega_i} = \frac{n_i}{2c\mu_0} \frac{1}{h\omega_i} E_{i0}^2 S$$

$$\frac{n_s}{\omega_s} E_{s0}^2 = \frac{n_i}{\omega_i} E_{i0}^2 \Rightarrow E_{i0} = \sqrt{\frac{\omega_i n_s}{\omega_s n_i}} E_{s0}$$

So, that there powers are not equal, I am putting the numbers equal. So, the number of photons of idler and number of signal photons are equal at the input. So, what is that mean? Let me try to find out, what is it mean in terms of the amplitudes,  $E_{i0}$  and  $E_{s0}$ . So, the power at the input of the signal is  $n_s$  by  $2c\mu_0$  mod  $E_{s0}$  square into area, which is nothing but  $n_s$  by  $2c\mu_0$   $E_{s0}$  square into  $S$ , because  $E_{s0}$  is  $E_{s0}$  exponential  $i\phi_i$  and that is a mod square here.

Similarly, the power in the idler is  $n_i$  by  $2c\mu_0$  mod  $E_{i0}$  square into  $S$ , which is equal to  $n_i$  by  $2c\mu_0$   $E_{i0}$  square into  $S$ . so, what is the number of photons at the signal incident per unit time at the input? Let me call this  $N_{s0}$ , this is  $P_{s0}$  by  $h$  cross  $\omega_s$ ;  $P_{s0}$  is the power of the signal at the input. So, if I divide by the energy per photon, I get the number of photons entering at the signal frequency per unit time, which is equal to  $n_s$  by  $2c\mu_0$   $1$  by  $h$  cross  $\omega_s$   $E_{s0}$  star into  $S$ .

And what is the number of idler photons incident per unit time?  $P_{i0}$  by  $h$  cross  $\omega_i$ , which is equal to  $n_i$  by  $2c\mu_0$   $1$  by  $h$  cross  $\omega_i$   $E_{i0}$  square into  $S$ . This is a

number of photons at the signal frequency entering per unit time into a crystal; this is the number of photons at the idler frequency entering per unit time in the crystal (Refer Slide Time: 08.11).

And I want to consider the situation, where these two numbers are equal. What is that mean? This means that  $n_s \omega_s E_{s0}^2$  is equal to  $n_i \omega_i E_{i0}^2$ ; you the other factors are common;  $1/2 c \mu_0 h \omega$  and  $s$  are all common in between these two. So, if this condition is satisfied, then I have equal number of signal photons or idler photons. So, this implies that I must take an electric field amplitude at the idler, which is equal to  $\omega_i n_s \omega_s n_i E_{s0}$ .

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Handwritten mathematical derivation on a grid background:

$$E_{s0} = \sqrt{\frac{\omega_s n_i}{\omega_i n_s}} E_{i0}$$

$$E_s^*(z) = E_{s0} e^{-i\phi_{s0}} \cosh g z - i E_{s0} e^{-i(\phi_{p0} - \phi_{i0})} \sinh g z$$

$$= E_{s0} e^{-i\phi_{s0}} \left[ \cosh g z + e^{-i(\phi_{p0} - \phi_{i0} - \phi_{s0} + \frac{\pi}{2})} \sinh g z \right]$$

$\phi_{p0} - \phi_{s0} - \phi_{i0} = -\frac{\pi}{2}$ $E_s^*(z) = E_{s0} e^{-i\phi_{s0}} e^{g z}$ <p>AMPLIFICATION</p>	$\phi_{p0} - \phi_{s0} - \phi_{i0} = +\frac{\pi}{2}$ $E_s^*(z) = E_{s0} e^{-i\phi_{s0}} e^{-g z}$ <p>ATTENUATION</p>
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So, if I take this electric field amplitude of the idler, then I have equal numbers of signal in idler photons and trigger crystal. So, what are these imply in my solution? Look at this factor,  $E_{i0}^2$  square root of  $\omega_s n_i \omega_i n_s$ . So, this actually this implies that, **let me write it the other way around**, this actually implies,  $E_{s0}$  is equal to square root of  $\omega_s n_i \omega_i n_s$  into  $E_{i0}$ .

So, if I choose this condition, then  $E_{i0}$  under root  $\omega_s n_i \omega_i n_s$  becomes  $E_{s0}$ . So, let me look at the signal field only, so this quantity, this square root multiplied by  $E_{i0}$  is nothing but  $E_{s0}$ . So, this  $E_s^*$  of  $z$  will be  $E_{s0} e^{-i\phi_{s0}}$

cos hyperbolic  $gz$  minus  $i E_{s0}$ , because this  $E_{i0}$  multiplied by this is  $E_{s0}$ , into at exponential minus  $i$  by  $p_0$  minus  $\phi_{i0}$  sin hyperbolic  $gz$ .

So, this remains the same; the second factor simply becomes this  $E_{i0}$  times square root of  $\omega_s n_i$  by  $\omega_i n_s$  becomes  $E_{s0}$ . So, this tells me, this is  $E_{s0}$  exponential minus  $i \phi_{s0}$  let me take out common, cos hyperbolic  $gz$ , now let me write this as, plus exponential minus  $i \phi_{p0}$  minus  $\phi_{i0}$ , I have taken this factor out, so I must multiplied by exponential plus  $i \phi_{s0}$ ; so I get minus  $\phi_{s0}$  here. **And then, this minus  $i$  becomes...** into sin hyperbolic; this minus  $i$  is exponential minus  $i \pi$  by 2 and then I have taken this factor out. So, I get this expression for  $E_s$  star of  $z$ .

So, now, you see here, if  $\phi_{p0}$  minus  $\phi_{s0}$  minus  $\phi_{i0}$  is equal to minus  $\pi$  by 2, then  $E_s$  star of  $z$  will be  $E_{s0}$  exponential minus  $i \phi_{s0}$  into exponential  $gz$ . If the phase of the pump signal and idler are such that,  $\phi_{p0}$  minus  $\phi_{s0}$  minus  $\phi_{i0}$  is minus  $\pi$  by 2, then the signal gets amplified.

If  $\phi_{p0}$  minus  $\phi_{s0}$  minus  $\phi_{i0}$  is equal to plus  $\pi$  by 2, then  $E_s$  of  $z$   $E_s$  star of  $z$  is equal to  $E_{s0}$  exponential minus  $i \phi_{s0}$  into - so this is plus  $\pi$  by 2 plus  $\pi$  by 2 is  $\pi$  - so that becomes exponential minus  $gz$ . So, this is Amplification; this is Attenuation (Refer Slide Time: 13.34).

So, there it is apparent that, depending on the phase between the signal and the idler, and the pump, either the signal gets amplified or it gets de amplified. So, as I mentioned before, amplification means that pump photons are down converting and generating signal and idler photons.

If it is attenuation, what it implies? A signal and idler photons are combined to generate pump photons. Energy as to be conserved; so, I cannot increase the signal photons by, I can increase the number of signal photon - that amplified signal - only if I down convert from pump.

So, compare to the earlier case, where there was no idler incident. In this case, the signal gets amplified or attenuated, depending on the phase of the signal. For a given pump and idler, the phase of the signal will determine whether it gets amplified or attenuated; that

is a phase sensitive amplification and that is very interesting, because we will come back to this.

In fact, we will come back to the degenerate case, where remember, in the degenerate case, if I had  $2\omega$  and  $\omega$  incident, it is automatically a phase sensitive amplifier. If I do not put  $\omega$  at the input, classically there is no generation  $\omega$ ; but quantum mechanically, we will see that, even if you do not put any  $\omega$  at the input, the moment I put  $2\omega$  into the crystal, there will be generation of  $\omega$ , because of what is called as spontaneous parametric down conversion.

So, that has to be explained quantum mechanically; we will come to this little later after we start discussing on quantization of electromagnetic fields. But, all these explanation is classical and this actually takes forward into quantum mechanical also; the only problem is, classically I do not predict spontaneous down conversion, while that comes out in quantum mechanics; this amplification process is exactly the same whether it is classic or quantum mechanical.

So, this we will come back to this later. In fact, as I said we will come back to the amplification process with the degenerate case. But before I continue with that problem, let me go back to these equations and find out... suppose I do not say satisfy phase matching condition, do I get amplification? Remember I have solve this equations and assuming phase matching condition. Under phase matching condition, I have shown that signal will get amplified, if I have only signal and pump. And I can still have amplification provided, I have a proper phase between idler signal and pump.

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$$\frac{dE_s}{dz} = i \chi_s E_p E_i^* e^{i \delta k z}$$

$$\frac{dE_i}{dz} = i \chi_i E_p E_s^* e^{i \delta k z}$$

$$\frac{d^2 E_s}{dz^2} = i \chi_s E_p E_i^* (i \delta k) e^{i \delta k z} + i \chi_s E_p \frac{dE_i^*}{dz} e^{i \delta k z}$$

$$= i \delta k \frac{dE_s}{dz} + i \chi_s E_p (-i \chi_i E_p^* E_s e^{-i \delta k z}) e^{i \delta k z}$$

$$= i \delta k \frac{dE_s}{dz} + \chi_s \chi_i |E_p|^2 E_s$$

So, to find out whether amplification is still possible, let me go back to those equations without putting  $\delta k$  is equal to 0. So, I have  $dE_s$  by  $dz$  is equal to  $i \chi_s E_p E_i^* e^{i \delta k z}$  and  $dE_i$  by  $dz$  is  $i \chi_i E_p E_s^* e^{i \delta k z}$ .

Sir, when the three signal idler and pumper instead, so in this case can be a studies management in such case.

Sure, because for example in the KDP, I showed you that  $\omega$  into  $\omega$  phase matching is possible. So, it is also possible that, instead of  $\omega_s + \omega_i = 2\omega_p$ , I can also  $\omega_s + \omega_i = \omega_p$ ; I can use the same birefringence, where they I had, for example, the  $\omega$  wave was ordinary and the  $2\omega$  was extraordinary. So, here, I can have an  $\omega_p$  which is extraordinary, generating an  $\omega_s$  and an  $\omega_i$ , which are both ordinary. Remember, the  $\omega$  was ordinary and  $2\omega$  was extraordinary; so, the higher frequency there is extraordinary.

So,  $\omega_p$  will be extraordinary, and  $\omega_s$  and  $\omega_i$  will be ordinary. So, it is possible, so I birefringence phase matching in all cases, but of course, depending on the frequencies and the crystals and the refractive indices. It is possible to have birefringence phase matching or otherwise quasi phase matching is always possible, because I can always have a periodic variation and get rid of the  $k \delta k$  term.



So, now, what do I do? How I solve this problem to find out whether I can get amplification? So, little differentiate the first equation. So,  $d^2 E_s$  by  $dz^2$  is equal to  $i \kappa_s E_p E_i^* \exp(i \Delta k z) + i \kappa_s E_p \frac{dE_i^*}{dz} \exp(i \Delta k z)$ ; I am not differentiate  $E_p$ , because I am assuming low pump depletion and  $E_p$  is assumed be a constant.

So,  $i \kappa_s E_p E_i^* \exp(i \Delta k z)$  is nothing but  $dE_s$  by  $dz$ . So, I get first term is  $i \Delta k \frac{dE_s}{dz}$ ; and second term contains  $\frac{dE_i^*}{dz}$  - I use the second equation to substitute - so, I get plus  $i \kappa_s E_p \frac{dE_i^*}{dz}$  is minus  $i \kappa_p E_i^* \exp(i \Delta k z)$ .

So, this becomes  $i \Delta k \frac{dE_s}{dz} + \kappa_s \kappa_p |E_p|^2 E_s$ ; I mean additional term coming in the differential equation, because of  $\Delta k$  naught being 0. If I put  $\Delta k$  is equal to 0, I get back the original equation that we had got. Remember, this we are called as  $g$  square.

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$$\frac{d^2 E_s}{dz^2} - i \Delta k \frac{dE_s}{dz} - g^2 E_s = 0$$

$$D^2 - i \Delta k D - g^2 = 0$$

$$D = \frac{i \Delta k \pm \sqrt{-(\Delta k)^2 + 4g^2}}{2}$$

$$E_s(z) = A e^{\frac{i \Delta k z}{2}} e^{\left(\frac{\sqrt{g^2 - \Delta k^2 / 4}}{2}\right) z} + B e^{\frac{i \Delta k z}{2}} e^{-\left(\frac{\sqrt{g^2 - \Delta k^2 / 4}}{2}\right) z}$$

So, this is an equation - second order differential ordinary differential equation. So, I get  $d^2 E_s$  by  $dz^2$  minus  $i \Delta k \frac{dE_s}{dz}$  minus  $g^2 E_s$  is equal to 0. How do I solve this equation - ordinary second order differential equation? So, let me recall, so I replace the differentials by  $D$  minus  $i \Delta k D$  minus  $g^2$  is equal to 0; the roots are  $D$  is equal to  $i \Delta k$  plus minus  $\Delta k^2$  plus  $4g^2$  by 2.

You have studied differential equation, so you should write. So, the solution will have,  $E$  s of  $z$  is  $A$ , the first solution is  $i \Delta k$  by 2 exponential square root of  $g$  square minus  $\Delta k$  square 4 into  $z$  plus  $B$  times exponential  $i \Delta k z$  by 2 into exponential minus square root of  $g$  square minus  $\Delta k$  square by 4  $z$ ; it is a standard way of solving ordinary differential equation.

Now, can you tell me, can it amplification, if  $\Delta k$  not equal to 0? What is a condition under which I will have amplification? This should be real.

Sir, it should be multiplied by simplification where  $2g$  square in  $E z$  also there is  $z$  here

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$$\frac{\Delta k^2}{4} < g^2$$

$$\kappa_s \kappa_i |E_p|^2 > \frac{\Delta k^2}{4}$$

$$\frac{\omega_s d}{c n_s} \cdot \frac{\omega_i d}{c n_i} \cdot \frac{2 c \mu_0}{n_p} \frac{P_p}{S} > \frac{\Delta k^2}{4}$$

$$P_p = \frac{n_p |E_p|^2 S}{2 c \mu_0}$$

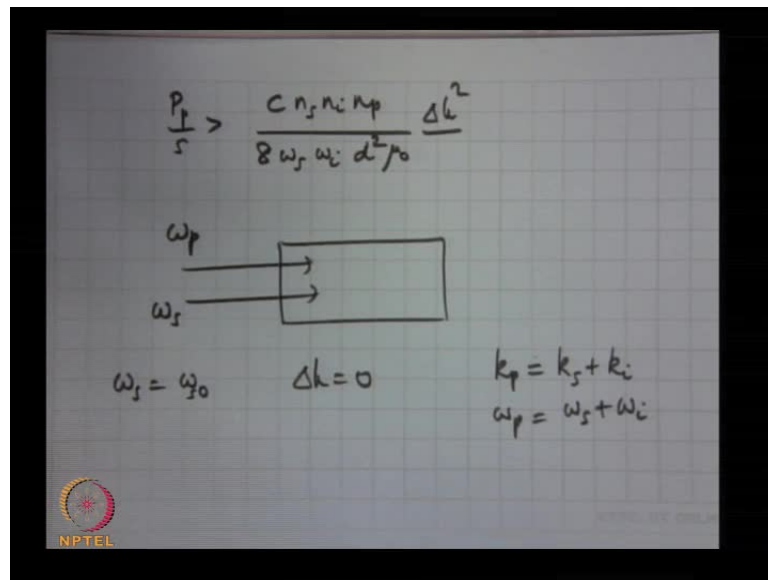
$$|E_p|^2 = \frac{2 c \mu_0 P_p}{n_p S}$$

So, this square root should be real, then I have an exponential increasing solution. If this square root is imaginary, then I have an only oscillatory solution. So, the condition for amplification if  $\Delta k$  not equal to 0 is,  $\Delta k$  square by 4 must be less than  $g$  square.

If  $\Delta k$  square by 4 is less than  $g$  square, then in this solution I have exponentially increasing and decreasing solutions. So, I can calculate from here, so  $g$  square is  $\kappa_s \kappa_i |E_p|^2$  must be greater than  $\Delta k$  square by 4. So,  $\kappa_s$  is  $\kappa_s$  let me substitute,  $\omega_s d$  by  $c n_s$   $\omega_i d$  by  $c n_i$ . Now, I substitute for  $|E_p|^2$  in terms of pump power; so, let me recall again, the pump power is  $n_p$  by  $2 c \mu_0$   $|E_p|^2$  into the area  $S$ ;  $|E_p|^2$  is equal to  $2 c \mu_0$  by  $n_p$  into  $P_p$  by  $S$ .

So, this into  $2 c \mu_0 n_p$  into  $P_p$  by  $s$  must be greater than  $\Delta k$  square by 4, because one of the  $c$  goes off. And so, I get a condition for this quantity this is the intensity of the pump -  $P_p$  is the pump power divided by area of cross section is the intensity of the pump.

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So, I get a condition  $P_p$  by  $s$  is must be greater than  $c n_s n_i n_p$  divided by  $\omega_s \omega_i d^2$  into  $\Delta k$  square by 8, so 8 either here; ya, mu naught.

So, if you have perfectly phase matched, then the signal will get amplified as soon as the pump is crystal. Let me assume in phase insensitive case, so I have add the crystal, I have putting pump inside and I have a signal coming in, so I switch off the pump, the signal does not get amplified. As soon as I switch on the pump, if I am perfectly phase matched, the signal at the output is more than signal at the input always.

But if I am not perfectly phase matched, then there is amplification only if the pump power incident is more than this quantity. So, this will defined to me, a range of wavelengths or frequencies over which this amplifier will amplify the signal. Please note that I can achieve  $\Delta k$  is equal to 0 at one particular signal frequency.

So, suppose I take at following experiment. So, here is my crystal, I put in  $\omega_p$  and I put a signal  $\omega_s$ ; so, at one particular frequency, say  $\omega_s$  is equal to  $\omega_0$ ,  $\Delta k$  is equal to 0, it is called as  $\omega_s$  is 0.

One particular signal frequency satisfies this condition, that  $k_p$  is equal to  $k_s$  plus  $k_i$ . Now, if I vary my signal frequency around this wavelength, around this frequency, as I change my  $\omega_s$  from  $\omega_{s0}$ , this condition is no more satisfied, because  $k_p$  remains constant, but  $k_s$  and  $k_i$  both will vary.

Please remember, these two frequencies  $\omega_p$  is also equal to  $\omega_s$  plus  $\omega_i$ . So, if I keep the pump frequency constant and change the signal frequency, the idler frequency will change, because idler frequency is  $\omega_p$  minus  $\omega_s$ . When  $\omega_s$  and  $\omega_i$  change,  $k_s$  and  $k_i$  will change, because the refractive indices are functions of frequency. So,  $k_s$  and  $k_i$  change and this condition may not be satisfied.

So, as soon as they change my signal frequency from  $\omega_{s0}$ ,  $\Delta k$  will become finite. And when  $\Delta k$  becomes finite, the gain will decrease, because look at this equation, the exponential factor is even if  $g$  is bigger than this number, this quantity is smaller than exponential  $gz$ , which we got for  $\Delta k$  is equal to 0.

So, there is a lesser gain, the gain will start to fall. And when  $g$  becomes equal to  $\Delta k$  by 2, that is the critical point; beyond which, the gain will, there would be no gain, it is a oscillatory function now. The gain drops and so we can define what is called as a bandwidth of the amplifier. What is the range of signal frequencies over which the crystal will be able to amplify? That will be determined by how  $\Delta k$  changes the frequency.

So, at  $\omega_{s0}$ , I have  $\Delta k$  is equal to 0, there is amplification. As I change my frequency from  $\omega_s$  from  $\omega_{s0}$ , as I increase or decrease the frequency,  $\Delta k$  becomes finite; and when  $\Delta k$  becomes finite, the gain drops. And as I change the, as I increase  $\Delta k$ , soon I will reach a point, where the pump power is not sufficient to overcome this  $\Delta k$  given by this equation. And the gain, there will be no gain, there will be no more gain and the signal output will drop down.

So, like every amplifier, this amplifier also has a certain bandwidth over which the signal will get amplified and that will be determined by how this  $\Delta k$  varies with the frequency.

So, this is same situation whether you are looking at quasi phase matching or phase matching; for quasi phase matching, delta k will also have a capital K in that equation. So, delta k for birefringence phase matching is  $k_p - k_s - k_i$ ; delta k for quasi phase matching is  $k_p - k_s - k_i - \text{capital K}$ . So, there is an additional the periodic spatial frequency factor coming from the nonlinearity added to the delta k.

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$$\frac{P_p}{s} > \frac{c n_s n_i n_p}{8 \omega_s \omega_i d^2 \gamma_0} \frac{\Delta k^2}{\omega_0}$$

$\omega_p$   
 $\omega_s$

$\omega_s = \omega_0$        $\Delta k = 0$        $k_p = k_s + k_i$   
 $\omega_p = \omega_s + \omega_i$

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So, this gives me a condition, under which I will be able to amplify the signal. And one can actually calculate what is the bandwidth of this amplifier. **Let me before,** now that this is an amplifier; in fact, what we will do next is to look at a situation, where I want to put this amplifier within a pair of mirrors.

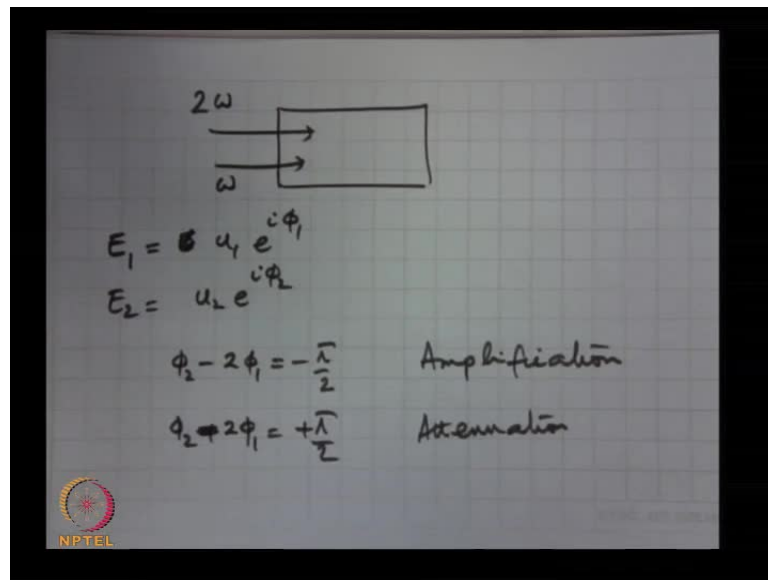
A pair of mirrors forms what is called as a resonant cavity; it is a resonator. So, when I put this amplifier within a pair of mirrors, I will convert this amplifier to an oscillator. In electronics, when you have an amplifier, if you feedback energy into the amplifier, you can make an oscillator.

So, similarly, I have an amplifier and if I feedback the energy generated by the amplifier into the system back again, I can convert the amplifier to an oscillator and that is what I have called as an optical parametric oscillator.

So, we will come to that little later, but right now what we have seen is, by these three wave interaction process between a pump signal and idler, I can actually amplify the

signal and I can operate the amplifier in either phase: sensitive fashion or phase insensitive fashion. Now, before we go into the oscillator, I want to discuss one interesting feature. Because this we will need, when we come back to the quantum picture of light and look at this same process in a quantum mechanical analysis picture.

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What I want to show you is, suppose I taken, let me go back to the old problem where I had a 2 omega-omega system; so, I have 2 omega coming in pump and omega. So, this is what is called as a degenerate optical parametric amplifier, because in this case omega s is equal to omega i, and omega p becomes 2 omega. The same situation as before, except that now, the signal and idler frequencies are the same.

So, I may have, for example I can have a 1 micron wave coming in from here and this is two microns - smaller frequency the higher wavelengths – so, I can have a degenerate parametric amplifier; the equations are simpler here. So, I would like to show you that... So, what we had seen before is, what was the condition for amplification; remember, we had written E 1 is equal to u 1 exponential i phi 1 and E 2 is equal to u 2 exponential i phi 2; and we had shown that, if phi 2 minus 2 phi 1 is equal to minus pi by 2, there is amplification just like here.

Look at the condition we got for amplification and attenuation, phi p minus 2 phi 0 2 phi 1 phi 2 minus 2 phi 1, so this is phi 2 phi 1 phi 1. So, phi 2 minus 2 phi 1 is minus pi by 2

in amplification and  $\phi_2 + 2\phi_1$  minus  $2\phi_1$  is equal to plus  $\pi/2$  is attenuation.

Now, what happens if my  $\phi_1$  does not satisfy either of these conditions? So, what I am going to show you is, that signal can be broken up into two components: one with satisfy this condition and the other satisfying this condition. This condition, that component of the signal which satisfies the first condition will get amplified; that component of the signal which satisfies the second condition gets de-amplified. What will be phase difference between the two signal components?

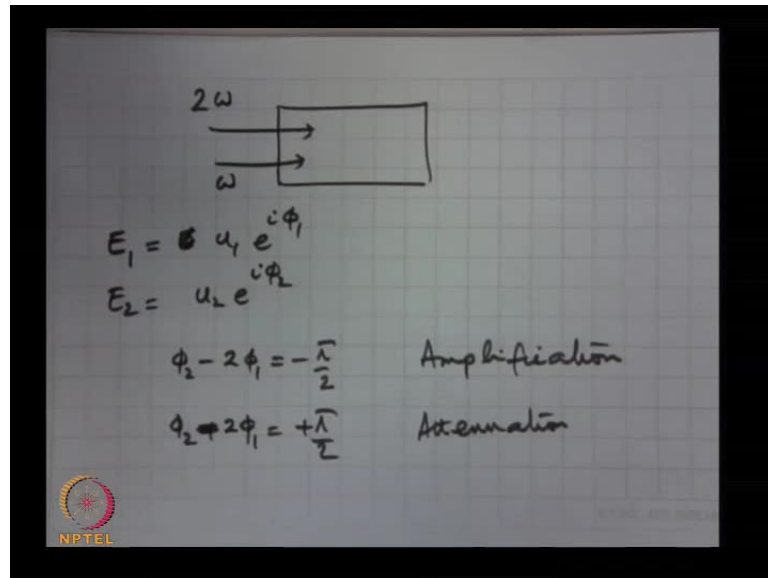
Because  $2\phi_1$ .

Because, this is  $2\phi_1$ , remember. The  $\phi_1$ , the phase difference between these two components will be  $\pi/2$ , because this is  $2\phi_1$ . So, if I fix  $\phi_2$ , the  $\phi_1$  satisfying this equation and  $\phi_1$  satisfying this equation, the difference is  $\pi/2$ . So, if one varies the  $\cos \omega t$ , the other one vary a  $\sin \omega t$ ; so, these are called quadrature component of the signal.

If I give any signal  $\sin \omega t + \phi$ , I can write it as  $\sin \omega t \cos \phi + \cos \omega t \sin \phi$ ; so, there is a  $\sin \phi$  component varying a  $\cos \omega t$  and a  $\cos \phi$  varying a  $\sin \omega t$ . So, the  $\sin \omega t$  and  $\cos \omega t$  terms, which are called quadrature components, because there is phase difference of  $\pi/2$  between them.

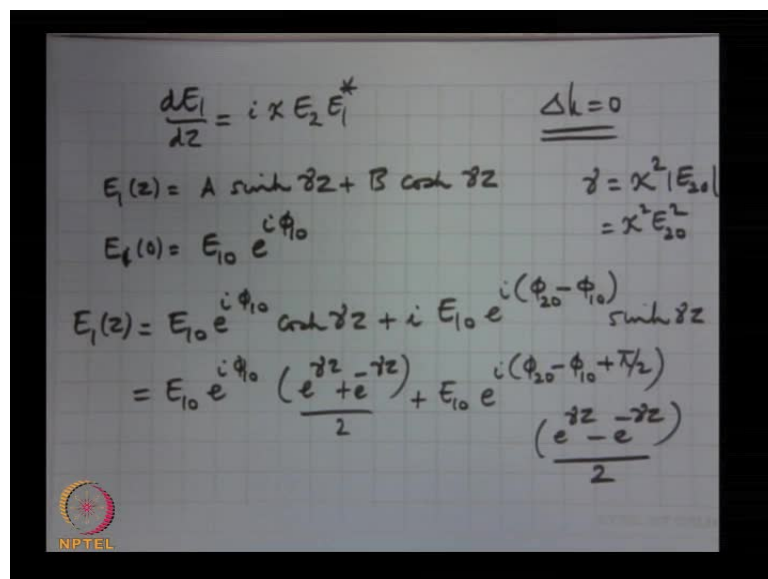
So, what I want to show you explicitly is, one quadrature will get amplified, the other quadrature gets de amplified; this happens only in a phase sensitive parametric amplifier. If you do not have a phase, if you would have a phase insensitive amplifier, both quadrature's, there is no quadrature's there, every gets amplified. And because of this phase sensitive nature, these amplifiers have much lower noise figure, which makes, they add much less noise to the signal than a phase insensitive amplifier.

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So, I will come back to this later again, when we look at some quantum noise that it generated exemplifier. But let me now try to show you, that I can write any signal which is coming in as a sum of two components: one component getting amplified and other getting de-amplified; it is a little bit of algebra. It is nothing difficult but, let me write down the solutions that we had got before.

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So, what we need to do is to solve this equation again. We go back to the case where we had looking at 2 omega-omega case, **E 2 E 1 star**. So, we had obtained this following



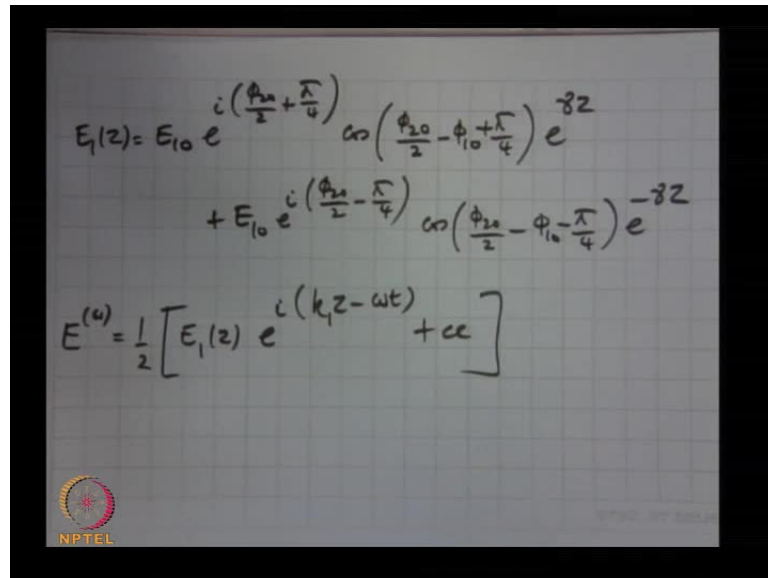
solution,  $E_1 z$  is equal to  $A \sinh \gamma z$  plus  $B \cosh \gamma z$ ; so  $\gamma$  was equal to  $\sqrt{\kappa^2 - \omega^2}$ , which is equal to  $\sqrt{\kappa^2 - \omega^2}$ . In this case,  $E_2$  is the pump,  $2\omega$  wave - electric field of the  $2\omega$  wave,  $E_1$  is the electric field of the  $\omega$  wave; then, we substituted  $E_1$  of 0 is equal, let me substitute,  $E_1 = E_0 \exp(i\phi_1)$ .

So, this also tells me what should be  $dE_1/dz$ , if  $z$  is equal to 0. So, I can find out both the constants  $A$  and  $B$ . Knowing  $E_1$  of 0, I know the constant  $B$ ; and knowing  $dE_1/dz$ , if  $z$  is equal to 0, I also know the constant  $A$  and we had obtained the solution actually. So, let me rewrite the solution  $E_1 = E_0 \exp(i\phi_1) \cosh \gamma z$  plus  $i$  times  $E_0 \exp(i\phi_2) \sinh \gamma z$ .

This contains the phase nature - phase sensitive nature, because if I take out  $\exp(i\phi_1)$  common, you can see that if  $\phi_2 - \phi_1$ , and this I take care of themselves, then you have  $\cosh$  plus  $\sinh$  and you have an exponential increasing solution. So, this solution we had obtained earlier, where  $\phi_1$  is the phase of the  $\omega$  wave at the input,  $\phi_2$  is the phase of this  $2\omega$  wave - it is a degenerate parametric amplifier case.

Now, I will tell you the simplifications I do and I will finally give you the final results. All I need to do is, a right cross hyperbolic function as the sum of exponential; so, what I do is this, I write as  $E_1 = E_0 \exp(i\phi_1) [e^{\gamma z} + e^{-\gamma z}]$  plus  $i$  times  $E_0 \exp(i\phi_2) [e^{\gamma z} - e^{-\gamma z}]$ , now let me write this as,  $E_1 = E_0 \exp(i\phi_1) \cosh \gamma z$  plus  $i$  times  $E_0 \exp(i\phi_2) \sinh \gamma z$ . I write the  $\cosh$  function in terms of exponentials,  $\sinh$  in terms of exponentials and then I collect the terms containing  $e^{\gamma z}$  and the terms containing  $e^{-\gamma z}$ .

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$$E_1(z) = E_{10} e^{i\left(\frac{\phi_{20}}{2} + \frac{\pi}{4}\right)} \cos\left(\frac{\phi_{20}}{2} - \phi_{10} + \frac{\pi}{4}\right) e^{\gamma z} + E_{10} e^{i\left(\frac{\phi_{20}}{2} - \frac{\pi}{4}\right)} \cos\left(\frac{\phi_{20}}{2} - \phi_{10} - \frac{\pi}{4}\right) e^{-\gamma z}$$
$$E^{(\omega)} = \frac{1}{2} \left[ E_1(z) e^{i(k_1 z - \omega t)} + cc \right]$$

So, let me give you the final result, which you can actually... so, you just have simplify this and the final results comes out to be, let me write down here. It is a bit of algebra and you will get,  $E_{10} e^{i\left(\frac{\phi_{20}}{2} + \frac{\pi}{4}\right)} \cos\left(\frac{\phi_{20}}{2} - \phi_{10} + \frac{\pi}{4}\right) e^{\gamma z} + E_{10} e^{i\left(\frac{\phi_{20}}{2} - \frac{\pi}{4}\right)} \cos\left(\frac{\phi_{20}}{2} - \phi_{10} - \frac{\pi}{4}\right) e^{-\gamma z}$ .

It just algebraic manipulations between the terms; and what you get is one term which goes as exponential  $\gamma z$  and the other term which goes as exponential  $-\gamma z$ . If you choose  $\frac{\phi_{20}}{2} - \phi_{10} = 0$ , if you choose this to be 0, then this becomes... if you choose this to be 0, what happens the second cos?

That will be zero.

**They do not 0**, because there is a minus  $\frac{\pi}{4}$  here. So, there is one particular phase  $\phi_{10}$ , for which this is 0, the signal gets amplified; and if you choose this to be  $\frac{\pi}{2}$ , then the signal gets attenuated.

So, these are essentially this is the same equation, I have now molded in a slightly different algebraic form. And you can see that, this the phase difference between these two terms is  $\frac{\pi}{2}$ , is an amplitude term here, there is a phase difference of  $\frac{\pi}{2}$ . So, these are the two quadrature components of the signal: one of them gets amplified and

the other quadrature gets de amplified. So, if you wait long enough, this component will just disappear and this will keep on amplifying itself.

So, unlike a conventional amplifier, the phase sensitive amplifier will amplify or de-amplify depending on the phase of the signal and this will also happen for the noise that enters de amplifier. The noise that enters in one particular phase will get amplified, the noise that enters in another quadrature phase will get de amplified.

So, the amplifier is actually decreasing the noise, is de amplifying the noise, because the noise which comes in its random; one of the components is in this phase, the other component is in  $\pi$  by 2 order phase. And the amplifier actually manipulates the noise in such a fashion, that one of them gets amplified, the other quadrature phase gets de amplified.

So, I will come back to this issue again later, when we talk of little bit of the amplifier noise characteristic. But this is a very interesting features of phase sensitive parametric amplifiers, that this amplifier amplifies one quadrature and it de amplifies the other quadrature; so, do you have any questions?

Sir, this equation is resituate with the case, when  $\phi = 0$  **does not** does not satisfy that any use two conditions explicitly.

This is general, this is a very general solution; if  $\phi = 0$  satisfies, this is equal to 0, then I get amplification; if  $\phi = \pi$  satisfy, this is equal to 0, I get de amplification which are the two conditions which we have got. Otherwise, this is the way the signal will develop itself; instead of writing in sin hyperbolic, cos hyperbolic form, I am just writing an exponential forms.

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$$E_1(z) = E_{10} e^{i\left(\frac{\phi_{20}}{2} + \frac{\pi}{4}\right)} \omega\left(\frac{\phi_{20}}{2} - \phi_{10} + \frac{\pi}{4}\right) e^{\gamma z} + E_{10} e^{i\left(\frac{\phi_{20}}{2} - \frac{\pi}{4}\right)} \omega\left(\frac{\phi_{20}}{2} - \phi_{10} - \frac{\pi}{4}\right) e^{-\gamma z}$$

$$E^{(\omega)} = \frac{1}{2} \left[ E_1(z) e^{i(k_1 z - \omega t)} + cc \right]$$

So, to be apparent, that one component is getting amplified. But what is important to note is that, there is a phase difference of  $\pi$  by 2 between these two components. Please note that this is a complex amplitude, the total electric field at  $\omega$  is half  $E_1$  of  $z$  exponential  $i k_1 z$  minus  $\omega t$  plus complex conjugate; this is only the complex amplitude of the electric field.

So, the total electric field at the set of the fundamental is, half of this  $E_1$  of  $z$  exponential  $i k_1 z$  minus  $\omega t$  plus complex conjugate; this just not have the time dependent term. So, I must substitute here and you will automatically find that, this term and this term are  $\pi$  by 2 out of phase. So, I leave it to you, why do not you substitute this into this equation and write it finally in terms of a sum of two real terms: one which goes up exponentially and the other one which goes down exponentially.

So, this term, this plus this will substitute here, plus its complex conjugate; so, you can actually collect all the exponential  $\gamma z$  terms, collect all the exponential minus  $\gamma z$  terms, and you will see that the two components are  $\pi$  by 2 out of phase.

So, any input signal can be broken up into two components: one which gets amplified and the other component which is  $\pi$  by 2 out of phase, with that with respect to that getting de amplified. So, this concept we will use later, when we discuss little more on noise of amplifier.

Anything else?

Sir, the noise in this first part, I mean first component it gets amplified, the other gets de amplified; so, how can we say that its de amplifying the noise?

No, this one quadrature amplifier. See, for example,

But we should be concerned is, with what the output, the noise the output has it is, it has the same noise, because I mean we break the noise into two parts and one is decaying another is amplifying.

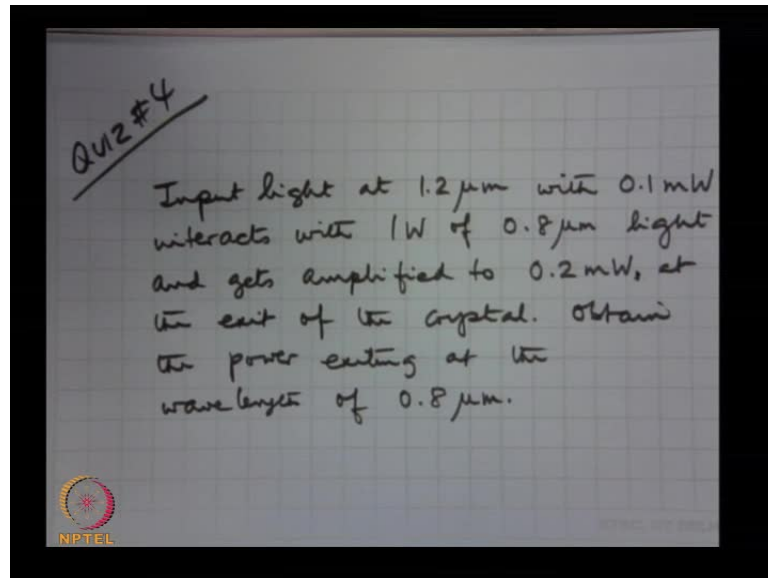
If you do the same thing for phase insensitive case, both components getting amplified; so, the noise already I can see that, the noise output here must be lower than the noise output in other case, because one of them is at least is de amplified by the amplified; not amplified. It is not even kept constant, it is de amplified. Now I will show you mathematically later on that the...

In theory, this amplifier is a noise free amplifier; it does not add any noise to the signal as it amplifies. Normally, there is **no** all amplifiers, standard amplifiers, phase insensitive amplifiers will always add noise **to the** while it amplifies. So, you put on the signal with a certain amplitude and some signal and noise, the signal gets amplified, noise gets amplified, and I prefer add this own noise. So, the signal to noise ratio at the input and signal to noise ratio at the output are not the same. In fact, this signal to noise ratio at the output is worse than signal to noise ratio at the input.

So, that is the price you pay for amplification, you add noise. But this amplifier I will show you later, can amplify the signal without adding any noise. And people would demonstrate experimentally that the noise figure of these amplifiers is much lower. In fact, there is a quantum limit - its 3 d b; that means, high gain amplifiers will worsen the figure the noise ratio by at least the factor of 2.

If your input signal at the noise ratio is 100, that signal is 100 times the noise and the output the best you can get is, the signal is 50 times the noise. Both are increased: the signal is increased, noise has increased, but the ratio as decreased by a factor of 2; that is the best you can get by quantum mechanics for high gain amplifier, but this one, the input signal to noise ratio and the output signal to noise ratio are the same.

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So, you amplify the signal, you amplify the noise, the amplifier does not add any noise to the signal. So, it is a very interesting feature of this and this is being used perceived for various applications ((no audio 48:27 to 49:58))

So, the problem is we have an input light at  $1.2$  micron wavelength carried  $0.1$  milliwatts of power entrance the crystal and interacts with a  $1$  watt power of a  $0.8$  micron wavelength light.

The  $1.2$  micron wavelength gets amplified to  $0.2$  mill watts, so it goes from  $0.1$  milliwatt or  $0.2$  milliwatts and as it comes out; now, what is the power exceeding at  $0.8$  microns at the output?

Sir,  $100$  percent efficiency. No, I am giving all of the numbers, so I think we will stop here.