Quantum Electronics

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Module No. # 03

Second Order Effects

Lecture No. # 14

Non-Linear Optics (Contd.)

Now, let me just briefly discuss the first question is essentially to look at (( )) biaxial medium and circularly polarized plane wave that has to propagate unchanged. This will happen provided the two eigenmodes have the same velocities. Circular polarization has nothing to do with the problem. Any polarization state will retain itself provided the two eigenmodes have the same speed. So, all I need to do is to calculate what is the angle with respect to z-axis, where the two eigenmodes have the same speed. So, I have done in the class the refractive index as seen by one polarization, which lies in the xz plane and the other polarization, which is parallel to the y-axis. So, all I need to do is to put the refractive index of the y-polarization state as the same as the one which lies in the plane of the vapor.

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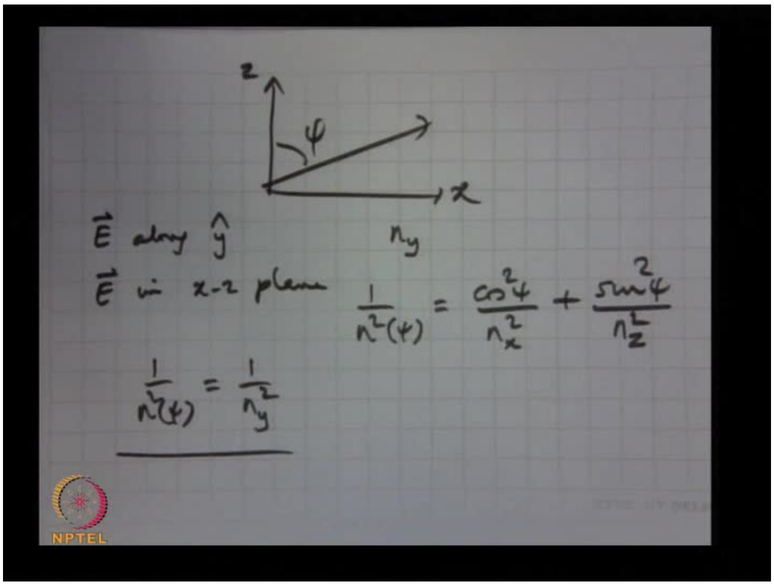

So, essentially, the problem is, I have x and z with some propagation direction. So, if you propagate in the xz plane, remember, I have done in the class, one polarization is E along y cap and the other one is E in xz plane. And, the two refractive indices we had calculated – one was n y; the other one was 1 by n square of psi is equal to cos square psi by n x square plus sine square psi by n z square. So, I had put this 1 by n square of psi is equal to 1 by n y square. And, solve the equation for psi. That will give you the direction of propagation in which the two eigenmodes have the same speed. And so, polarization state will not change as the wave propagates.

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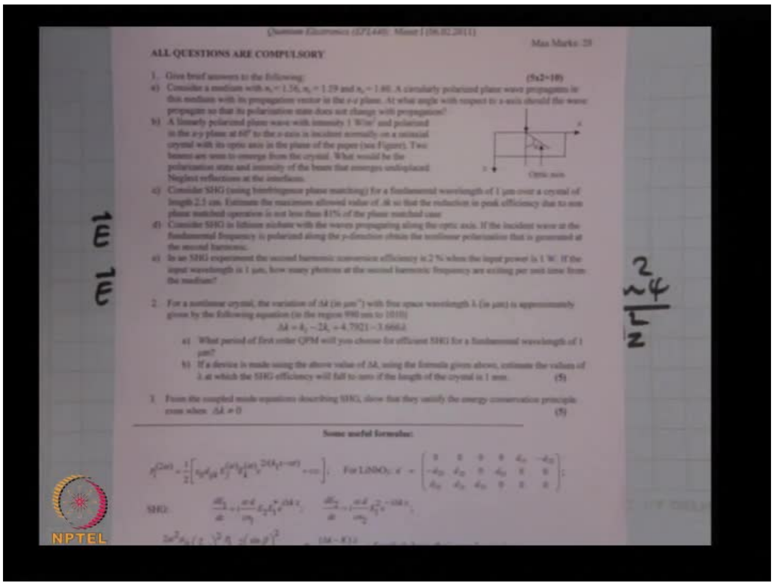

The second part was essentially… The second question is breaking up the input polarization state into one which is ordinary and the other is extraordinary. The ordinary one is perpendicular to the plane of the paper; extraordinary one is parallel to the plane of the paper. And, the ordinary one will go undeviated, and so, the electric field, which is incident like this has one component along the ordinary axis and the other on extraordinary axis. So, the ordinary one will propagate unchanged. And so, it is the y-polarization direction and the intensity of this is cos square of this angle, that is, 0.75 watts per square meter.

c problem was very simple. It is essentially using this formula for the sinc function. And, many people have made a mistake there. Actually, you have to calculate what is the efficiency with delta k is equal to 0; and then, calculate the value of delta k for which the efficiency will be not less than 81 percent of the delta k if it is equal to 0 value. And, some people have actually forgotten about the z square factor earlier in calculation of the efficiency. So, there is a problem there. The d part was similar to what we have been doing in the class; essentially, the polarization state of the omega wave is along the y-axis; the electric field has only y component. And, from the d tensor, you can actually calculate the polarization generated in the medium at second harmonic.

The e part is essentially to realize that the number of photons that you are generating is half the number of photon that you are losing. So, unfortunately, some people have used the wavelength of the fundamental in calculating the number of photons; not the wave in the second harmonic. So, please remember that you can calculate the power of the second harmonic, which is 2 percent of the input power, because the efficiency is given a 2 percent. And, that power divided by S cross of S cross into 2 omega, because 2 omega is the frequency of the output photon.

In the second problem, the second question, I had sort of numerically estimated at delta k function as a function of wavelength. And, the first part was simple. Actually, what I wanted in the second part, it was not very clear in the question. Actually, the question was if I made a device with the capital K value, which is calculated above… So, there was a mistake in the question; there was a sort of not mistake, ambiguity in the question. And so, I marked both answers to be right. So, what I wanted to calculate is if I assume a device with that capital K obtained in the first part, what is the bandwidth over which I will have significant efficiency of second harmonic generation?

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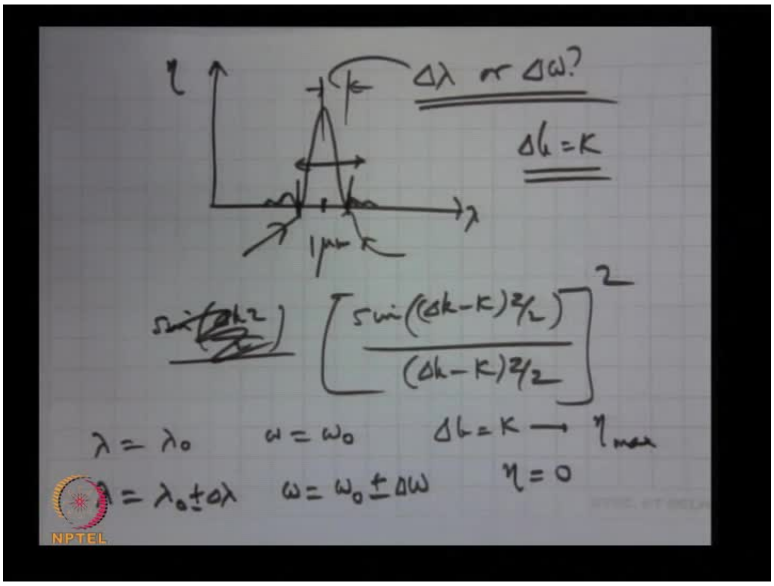

For example, if you would plot efficiency of the function of wavelength, you would have got something like this. This is 1 micron here, because this is the place, where delta k is equal to K. So, I wanted actually these numbers here; this value and this value. Just to give you an indication of how large is the bandwidth. So, please so such you who have not done this, go back and just do this. From this question, please calculate what this delta lambda is; the bandwidth over which the second harmonic efficiency has its peak.

If you deviate in wavelength from this point much, this efficiency (Refer Slide Time: 06:38) will drop down to negligibly small value. So, sinc square function. And, what we have calculated is a sinc delta function – if I take was a phase matching, either sine delta k minus K into z by 2 by delta k minus K into z by 2 whole square. So, you can make delta k is equal to K at one particular wavelength, 1 micron. And then, if you change the wavelength, delta k will change according to the formula, which I have given. So, what will happen is, at some value of delta k, sine function will become 0 and you will get 0s of efficiency. And, this problem was just to give you an indication; in fact, it is an extension of a problem, which I had given in the class, which I still want you to do; analytical the expression for this delta lambda or delta omega.

What is the change in the wavelength from the central frequency or wavelengths at which the efficiency will drop to 0? I want you to calculate. So, it looks like if I give you a home work, you are not doing it. So, I would like you to submit it. Please submit this assignment next week. Please calculate what is the delta omega or delta lambda, so that the efficiency drops to 0. So, I have at lambda is equal to lambda 0 or omega is equal to omega 0, some particular frequency; I have delta k is equal to K. So, at lambda is equal to some wavelength plus minus delta lambda or omega is equal to omega naught plus minus delta omega, eta becomes 0.

If it is a constant divided by 2 n dash 2 omega minus n dash?

Sorry?

A constant divided by 2 n dash 2 omega minus n dash

What is n dash?

This dn by (( ))

Derivatives; so, why constant? There must be some

There was a value advantage

Yes, exactly; it depends on the difference in the dispersion values of… That means dn by d omega at the fundamental and dn by d omega at the second harmonic. That difference will determine what is the bandwidth of this interaction process; and, one can actually check that this bandwidth is usually very small, few nanometers; not more than that. So, that means the set wavelength of the laser has to be very precisely defined for a given cause phase matching interaction process. If your wavelength deviates by few nanometers, efficiency tops to almost 0. So, this is a problem just to give you some indication of what parameters are involved. So, if I want to increase the bandwidth of interaction, what I should do? The people who are interested in having a situation, where no matter if the wavelength of the laser deviates slightly by a few nanometers, efficiency should not drop significantly. So, what I should do? If I have an analytical expression, I would know what I need to do. To design a device, where even if the frequency of wavelength changes slightly, the efficiency does not drop significantly. So, that means, larger this delta omega is, (Refer Slide Time: 10:07) larger in one way.

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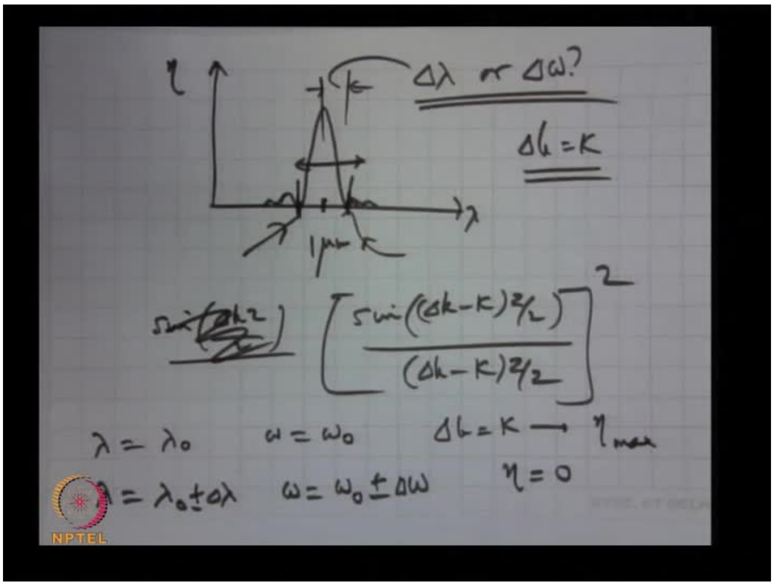

At what frequency? Please check carefully. Should the dn by d omega be less or should I relate dn by d omega at omega and 2 omega frequency or what? Please check this. So, I will… Please submit this next week sometime. So, this problem, actually, I did not want to give this in the question, because this is little more trickier.

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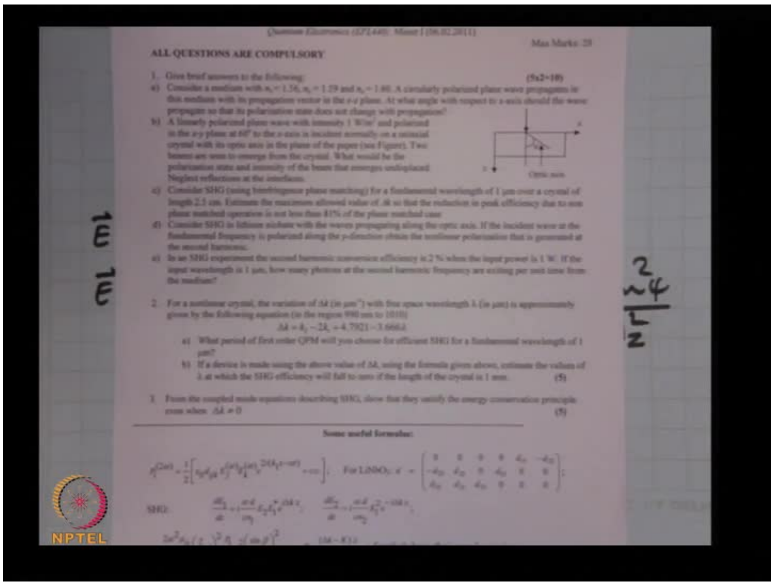

So, I actually simulated a delta k variation with wavelength for lithium niobate and just put it in the question paper. So, the second part was for this particular problem, because actually, remember, I have written that the region of wavelength for which this is valid is

990 nanometers to 1010 nanometers. If you do assuming delta k is equal to 0, this wavelength comes to 1.3 microns (Refer Slide Time: 11:03). So, this is not valid actually in that expression. This validity of this expression just goes off. But, anyway, I have still maintained that wherever there is little bit of ambiguity in this question.

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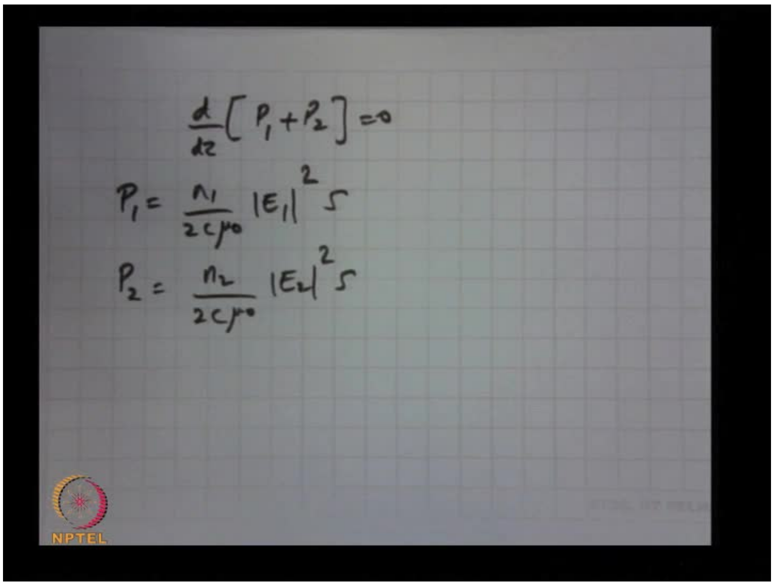

And, the last question, please do not forget the n refractive index sitting in the power expression. The power is related to n times mod e square. So, the fundamental, if the power in the fundamental, what you need to show is d by dz of P 1 plus P 2 is equal to 0. And, P 1 is n 1 by 2 c mu 0 mod E 1 square into the area. And similarly, P 2 is n 2 by 2 c mu 0 mod E 2 square into the area. So, using those two differential equations for E 1 and E 2, you can actually show E P 1, E by P 1 of P 2; and, this irrespective of delta k being 0 or not 0. If delta k was 0, then n 1 becomes equal to n 2 and it simple becomes d by d 0 mod E 1 square by mod E 2 square is equal to 0; otherwise, it is d by dz of n 1 mod E 1 square plus n 2 mod E 2 square, which is 0, because 1 by 2 c mu 0 S is common. So, if you substitute those differential equations, you will get this condition. So, please just do it for your own sake.

Why cannot we say that the energy density d by dp of the energy density will be the same, if I write that d by dp of the energy density is 0?

No, then, start the energy density; it is the amount of energy crossing a unit area per unit time or some area per unit time. That depends also on velocity, because energy density into velocity is intensity. So, the power that is crossing here should be the same, should be conserved. That will depend on energy density of course, and the velocity which is different at different frequencies, because the refractive index comes into picture. So, it is just not energy densities; it is the velocity also into picture, which is the intensity – amount of energy crossing per unit time either for unit area or in the total area. Any questions? Those such to whom I have to recheck the answer script, just give it back to me; I will have a look at it.

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Anything else? Just mark, just try this question.

Any questions?

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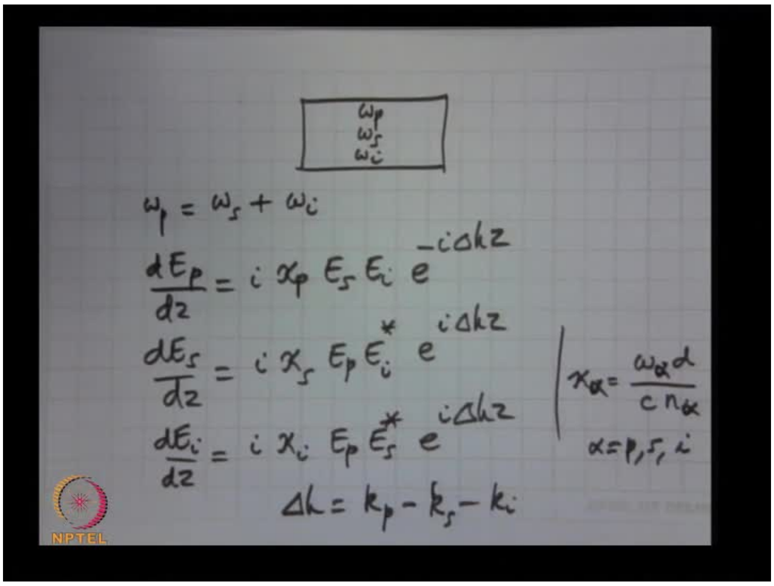

Let us continue our discussion on parametric amplification or different frequency generation. So, let me recall, what we have essentially is insert the medium; I have three frequencies simultaneously – omega p, omega s and omega i satisfying the condition omega p is equal to omega s plus omega i. So, as I said before, if the input corresponds to omega s and omega i simultaneously present at the input, then this is looking at some frequency generation. If I have input at omega p and omega i or omega p and omega s, it is to get amplification. So, all these processes are controlled by these three equations, which we have derived – dE p by dz is equal to i kappa p E s E i exponential minus i delta kz; dE s by dz is equal to i kappa s E p – what will I get? E i star exponential i delta kz; and, dE i by dz is equal to i kappa i E p E s star exponential i delta kz; but, delta k is k p minus k s minus k i. And, kappas or the coupling coefficients – so, kappa i is omega i d by c times n i; i corresponds to either the pump – p, s or i. Actually, I should some other index maybe. Kappa alpha is equal to… So, alpha can be pump, signal or idler.

Now, last time, what we did was, we started looking at the following problem; we started looking at difference frequency generation. What was the input condition we had assumed? We had assumed the input of pump and signal; p and s by the input. Now, before I relook at that problem, let me look at these equations and show you another condition that is being satisfied by these equations. Now, again, I will show this condition for delta k is equal to 0; but, please do it for delta k not equal to 0.

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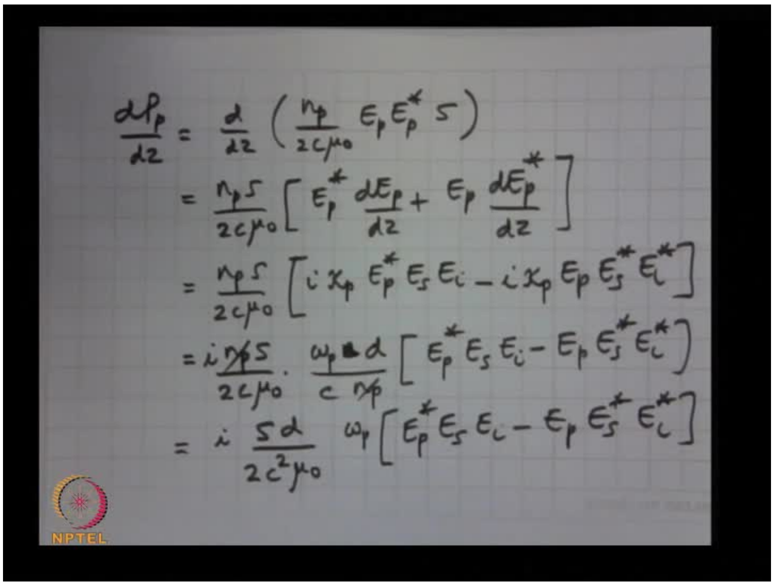

So, delta k is equal to 0. Let me calculate the following quantity – dP p by dz; P p is the pump power. So, this is d by dz of n p by 2 c mu 0 E p E p star into the area – intensity multiplied by area. So, this is n p s by 2 c mu 0 E p star dE p by dz plus E p dE p star by dz. So, I can substitute from here; I can substitute expression for dE p by dz from here (Refer Slide Time: 18:05) and dE p star by dz. So, I will get n p s by 2 c mu 0 i kappa p E p star E s E i. Please note that the second term is the complex conjugate of the first term. So, I will get minus i kappa p E p E s star E i star. The second term is the complex conjugate of first term. And, kappa p has only real quantities – omega alpha, omega, p, E, c and p – everything is real. So, I will have n p s by 2 c mu 0. I can take out kappa p – i times kappa p outside. So, I will have i here. Kappa p is omega p d by c times n p into E p star E s E i minus E p E s star E i star, which is equal to I – so, n p goes off – s times d by 2 c square mu 0 into omega p into E p star E s E i minus E p E s star E i star.

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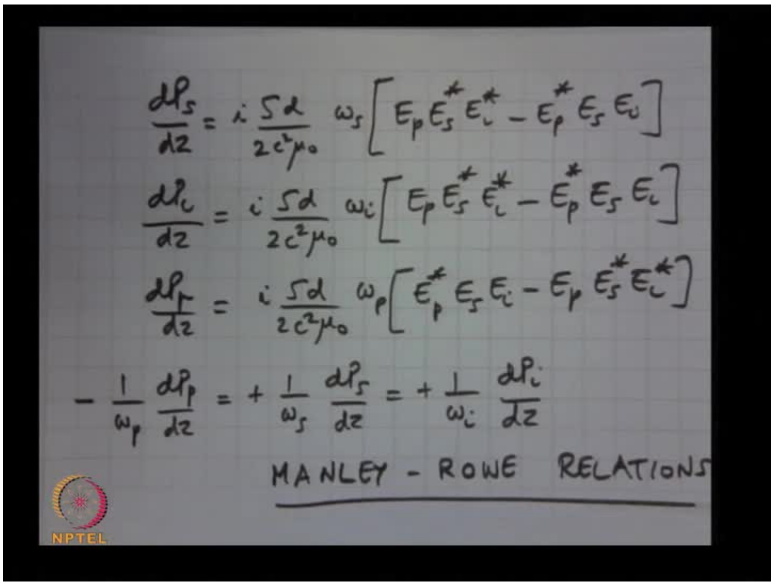

Now, I can do the same thing for dP s by dz and dP i by dz. So, I will leave this to you to please show dP s by dz will be i s d by 2 c square mu 0 into omega s into E p E s star E i star minus E p star E s E i. There if I derive equation… And similarly, the dP i by dz please show it is i s d by 2 c square mu 0 omega i E p E s star E i star minus E p star E s E i. Now, let me put these three equations together. So, I have the third equation here. Let me rewrite this third equation; dP p by dz is equal to i s d by 2 c square mu 0 omega p E p star E s E i minus E p E s star E i star. I have just rewritten the last equation for dP p by dz.

Now, what do these three equations tell me?

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First thing, I notice; so, I divide by omega p on this side; omega i – I bring it here; omega s I bring it here. So, what do I get? Please see, these are just the opposite. This is negative of this; this is negative of this. So, I get minus 1 by omega p dP p by dz is equal to plus 1 by omega s dP s by dz is equal to plus 1 by omega i dP i by dz. 1 by omega p dP p by dz – minus of that; we will just interchange these two terms; is equal to plus 1 by omega s dP s by dz is equal to plus 1 by omega i dP i by dz. These are called the Manley-Rowe Relations. So, how would I interpret these equations?

[Not audible] (Refer Slide Time: 23:10)

Yes. Please note that if I divide every term by (( ))

Rate of change of energy per unit energy of each wave sort of you can see…

Not the unit energy, per unit distance; rate of change in distance.

Rate of the energy per unit energy of the wave; I mean we divided with the initial energy of the wave and we say that

No. Where is the initial energy of the wave?

S cross omega means the initial energy of the…

That is energy of a photon. So, what is P p by s cross omega p? P p is the power – the amount of energy crossing per unit time – number of photons crossing per unit time.

[Not audible] (Refer Slide Time: 23:52)

Intensity is per unit area, but there is no intensity; I am talking about power. So, P p by s cross omega p is the number of photons at the pump frequency crossing per unit time that area; minus of that the rate of change of the number of photons at the pump frequency crossing is equal to plus of the rate of change of the signal photons is equal to plus of the rate change of the idler photon. What does it mean? It means that every time I lose one pump photon, if dP p by s cross omega p by dz is 1, if this is minus 1, this is plus 1 and this is plus 1. So, it means that every time the process, what is happening is a pump photon is splitting into a signal photon and an idler photon.

Please note, I have not brought photons at all into the picture till now. This is purely classical argument. This is purely classical analysis. I am just using Maxwell's equations.

But, if I interpret s cross omega p as a quantum, P p by s cross omega p is a number; it gives you the number of photons, because these photons per unit, which are crossing per unit time across the area. So, the rate of decrease of the number of pump photons is equal to the rate of increase of the signal photons is equal to the rate of increase of the idler photons.

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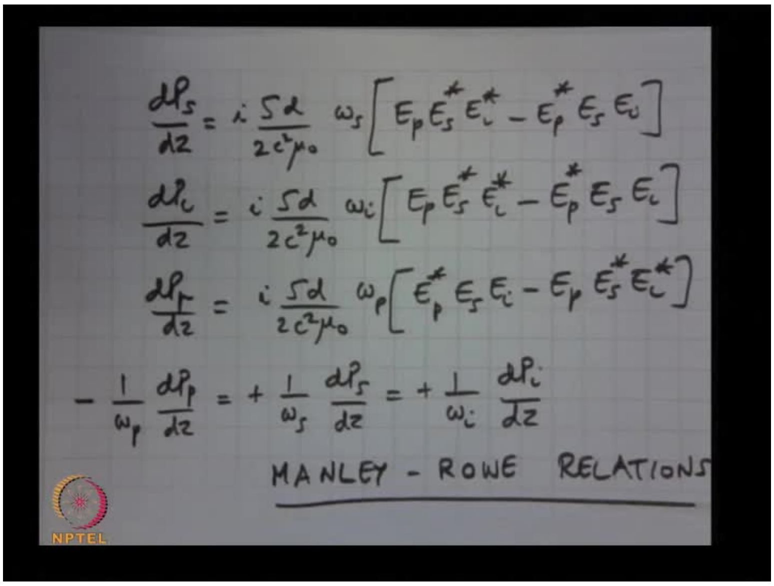

Now, I will leave it you to use this equation and you have to show that d by dz of P p plus P s plus P I, the sum of these three will simply be 0, because omega p is equal to omega s plus omega i. If I add these three equations, I will get this minus this (Refer Slide Time: 25:28) minus this. These three will be equal; I will get omega p minus omega s minus omega i, which is 0.

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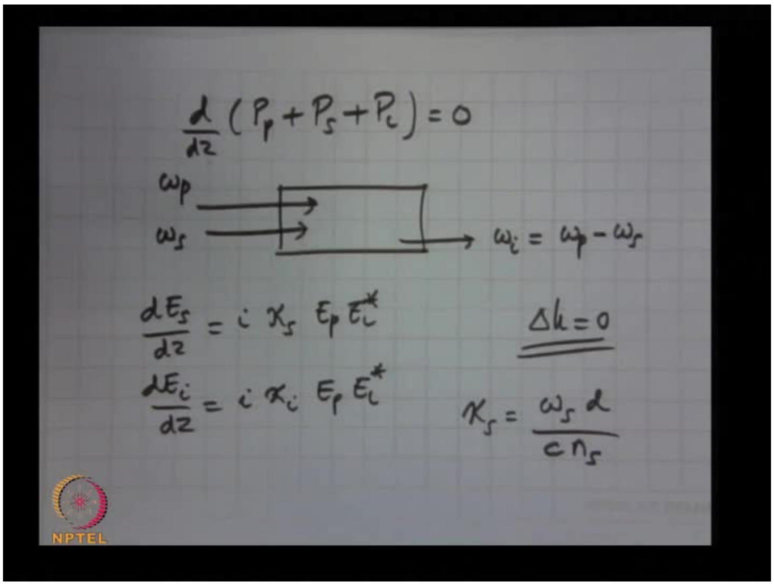

So, I will get the energy conservation condition d by dz of P p. What I am getting is d by dz of P p plus P s plus P i is equal to 0. So, this is a very important relationship, which tells me essentially that this process itself is as safe pump photon at frequency omega p are splitting to give you a signal photon and omega s frequency and an idler photon at frequency omega i. And, this is the condition satisfied by these three (Refer Slide Time: 26:09). This is a very important relationship. And, this is a quantum mechanical relationship also, because although the analysis is classical, it is showing that it is as if the number of photons, which I have lost in the (( )) Some frequency must be equal to the number of signal photons, are generated; and, the number of idler photons are generated. So, there is no conservation number. If I pump in 100 pump photons and convert them into signal and idler, I was having 100 signal photons and 100 idler photons. So, I come (( )) of 100 photons, but I come out with 200 photons, but each photon has a lower energy. There is no problem in energy conservation.

What we did last time was look at this equation. So, the problem we started looking was, I have omega p and omega s. And, we are looking at the generation of omega i, which is equal to omega p minus omega s (Refer Slide Time: 27:13). So, for this, we had to solve these two equations: d E s by dz is equal to i kappa s E p E i star and dE i by dz is equal to i kappa i E p E i star. This is assuming delta k is equal to 0. Let me emphasis again that even if delta k is not equal to 0, if I want to do quasi phase matching, I will get the same equations, except that in kappas, the d, which I am using will get replaced by an effective d value – 2 by pi times d if I am using first order quasi phase matching or 2 by 3 pi d if I am using third order quasi phase matching, etcetera, because I will pick up one of the Fourier coefficients of the periodic function; and, that will be responsible for the generation. So, kappa will just get replaced. So, kappa right now is omega s d by c times n s. And, d is the effective non-linear coefficient. If it is quasi phase matching and perfect quasi phase matching, these are again the same equations, except then I need to worry about d; d may not be the actual d coefficient of the crystal; it is the Fourier coefficient in the quasi phase matching case. So, these equations are for a situation where either perfect phase matching or perfect quasi phase matching with no exponential phase term sitting in these equations.

What I have shown you is the solutions that these two equations with this condition gave me the following two solutions.

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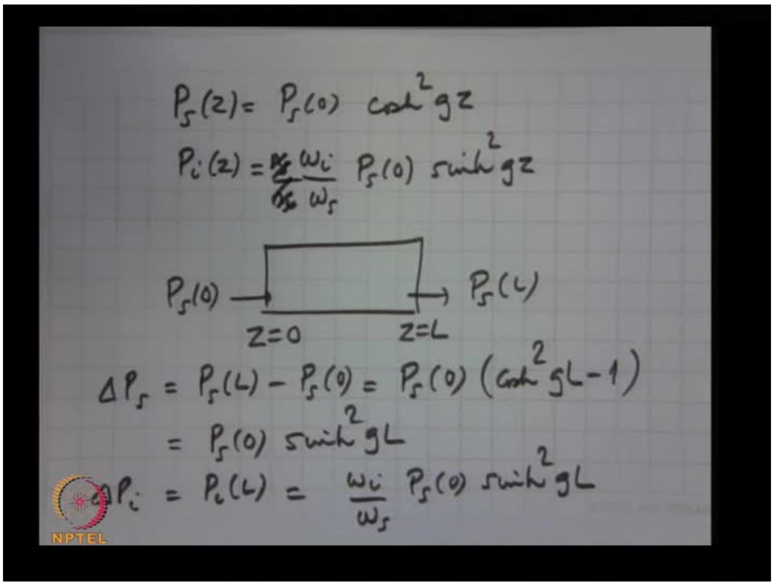

Can you now recall what was the solutions we got? P s of z is equal to P s of 0 into – what was the solution we had obtained last time? Cos hyperbolic square gz. And, P i of z was omega i by omega s into P s of 0 into sine hyperbolic square gz. So, what is the increase in the number of signal photons from z is equal to 0 to some value L? Suppose the interaction started at z is equal to 0, it comes to z is equal to L. So, here the input was P s of 0; here I am getting P s of L. So, delta P s – change in the signal power is P s of L minus P s 0, which is P s 0 into cos hyperbolic square gL minus 1, which is how much? Cos hyperbolic square gL minus 1 – sine hyperbolic square.

When we write P i of z

So, P I; that means n i times mod E i square

Yes. Pi 2 c mu 0 into the area of the b.

So, they should not be in n i

Sorry (Refer Slide Time: 31:25)

Electric field had n s by n i, kappa i by kappa s.

What was the expression we had obtained last time? Can you go back and look at the equation? Omega i by omega s P s 0 sin hyperbolic square gz. Yes, there is no n s by n i. In electric fields, we had this n i and n s also sitting, but not in the power. And, delta P i is simply P i of L, because P i of 0 is 0, which is equal to omega i by omega s P s 0 sine hyperbolic square gL.

Now, why is this extra factor of omega i by omega s sitting here?

Because they are not equal; otherwise, they will be equal where we had second harmonic parametric harmonic motions because omega and omega cancels out.

No, here, but mathematically, I have an extra omega i by omega s sitting in the expression for the change of power in the idler. So, it looks as if the power in the signal coming out; extra power in the signal is not equal to the power in the idler. Why?

Because the number of photons are same.

The number of photons, extra number of photons coming out at signal is delta P s by s cross omega s. The number of idler photons coming out is delta P i by s cross omega i. And, they are equal. So, delta P s by s cross omega s is equal to delta P i by s cross omega I, because it is the number of photons at the signal that have been added to the signal by the amplification process must be equal to the number of idler photons that have been generated in the down conversion process. And, because the idler frequency is usually smaller than the signal frequency, the power in the idler is smaller. It has the same number of photons as the additional number of photons in the signal, but because its photon energy is smaller, the power in this idler is smaller than the power in the signal, assuming omega s is bigger than omega i always. So, this factor is coming simply because of having same number of photons generated at the idler and signal by this process. Because every time I have added a photon in the signal, I have added a photon in the idler. And, the photon at the idler has a lower energy, and hence, the power coming out of the idler is smaller than the additional power being generated at the signal frequency. And also, notice that there is no dependence on phase.

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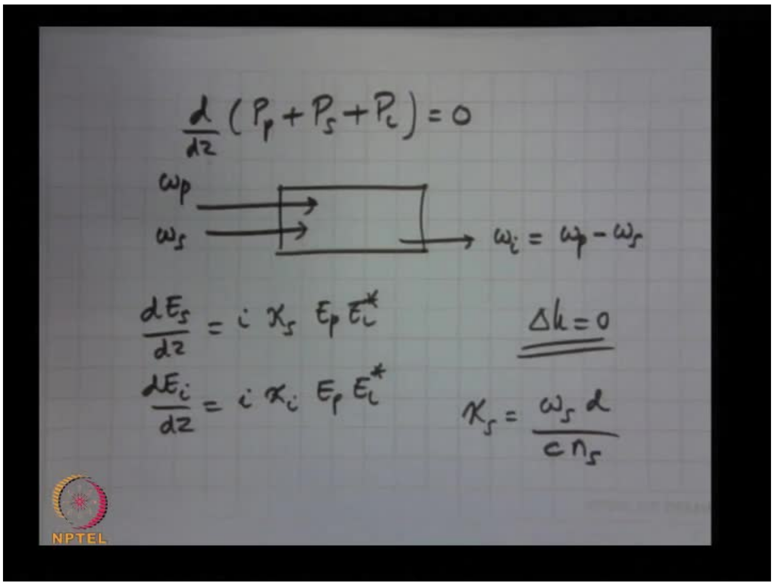

No matter what is the phase of this input signal here (Refer Slide Time: 34:33) with respect to pump, P s is always increasing with z.

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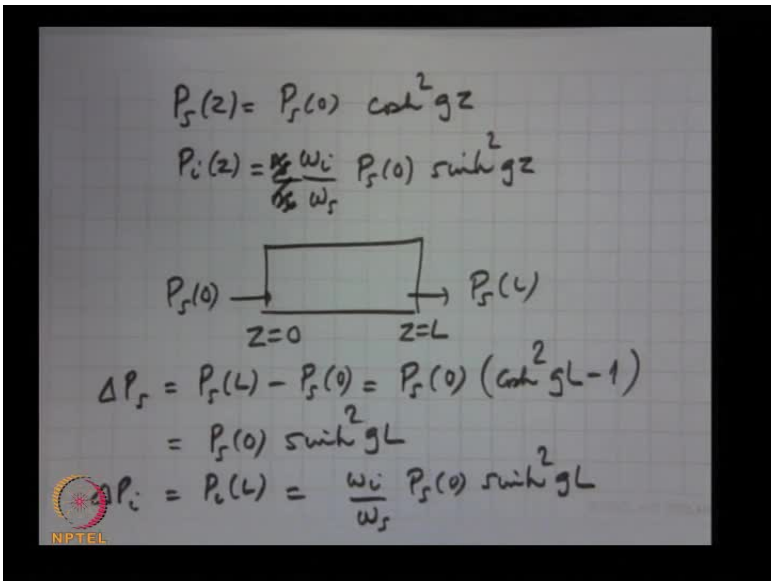

This is always increasing (Refer Slide Time: 34:39). Cos hyperbolic square gz is a an increasing function of z. So, this is a situation where it acts like a phase insensitive amplifier. This amplification process is insensitive to the phase of the input signals.

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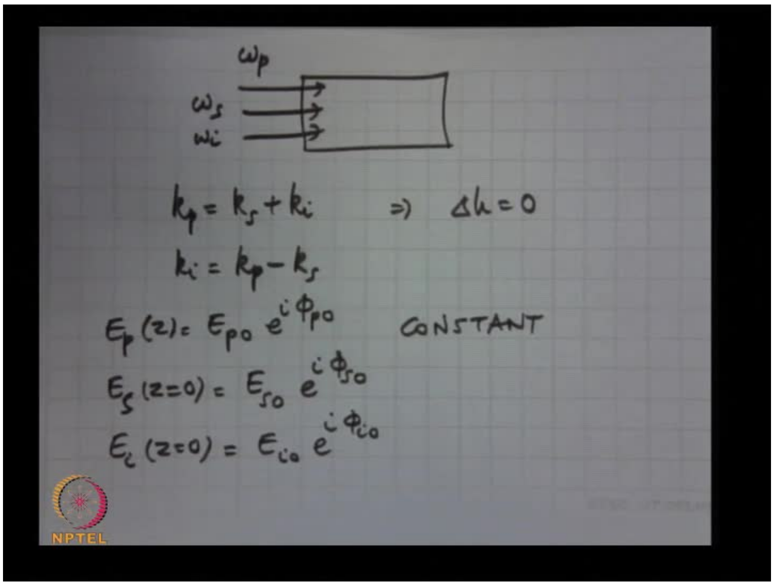

Now, let me go to the following situation, which is very interesting; where, I say that I have omega p, omega s and omega i – all three inputs simultaneously. I will show you that now, the signal will either get amplified or attenuated depending on the phase relationship between the idler signal and the fit pump. So, what is happening is, in this case, (Refer Slide Time: 35:36) because there is no third wave incident, the process is such that it always leads to generation of signal or idler photons from the pump photon irrespective of the phase of the signal. So, signal always gets amplified. No matter what signal phase is, these interactions between the signal and pump is such that you convert pump photons to signal and idler photons. So, the power is flowing from omega p to omega s and omega i continuously in this process.

Here (Refer Slide Time: 36:15) depending on the phase of these waves, I can either generate omega s and omega i from omega p or I can generate omega p from omega s and omega i, which is nothing but the generation. All three waves are present simultaneously; the phase matching condition for generating omega i from omega p omega s; what is the phase matching condition? k p is equal to k s plus k i. This implies delta k is equal to 0. This is the phase matching condition for either some frequency from omega s and omega i to omega p or different frequency from omega p and omega s to omega i, because that will be k i is equal to k p minus k s. This is the same equation, same condition. So, the condition for efficient interaction between these three frequencies such that omega s plus omega i is equal to omega p is that I must satisfy this condition.

Whether energy flows from omega p to omega s and amplifies omega s or energy flows from omega s and omega i to omega p and attenuates omega s, is determined by the phase of the signal. So, what I need to do is, again, we will assume no pump depletion. So E p of z is equal to some E p 0; let me put a phase here – this is exponential i phi p 0. This is assumed to be a constant E s of z at z is equal to 0 is E s 0 exponential i phi s 0. And, E i at z is equal to 0 is equal to E i0 exponential i phi i0. This is at z is equal to 0. Remember, E s and E i are the complex electric fields of the signal and idler frequencies. So, in the last class when we solved the problem, we assumed E i of z is equal to 0 is equal to 0. Now, I also assume that there is a finite idler present at the input and I refine the phases of the pump signal and idler at the input as phi p0 of phi s0 and phi i0. And, assume that the pump definition is negligible; the pump is powerful. So, I just neglect the changes in the pump electric field completely. So, I still need to solve only those two equations corresponding to E s and E i.

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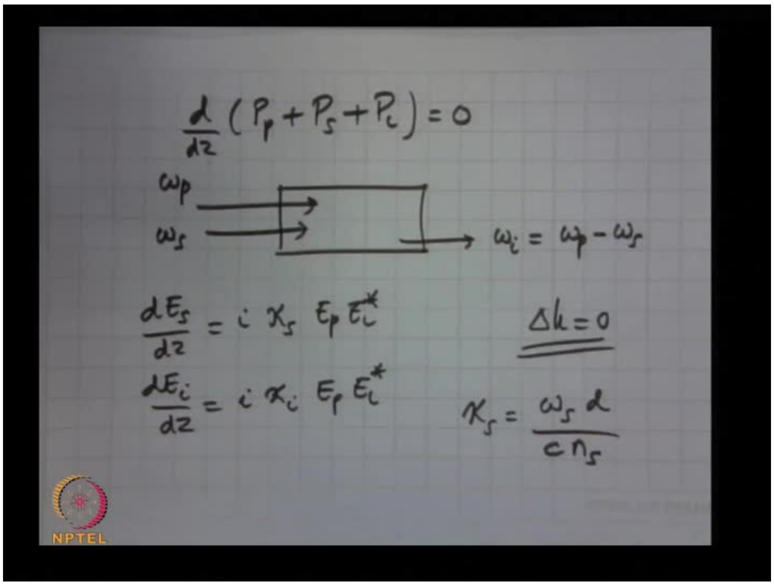

I need to solve again this pair of equations (Refer Slide Time: 39:09) with the boundary condition that this is satisfied (Refer Slide Time: 39:13).

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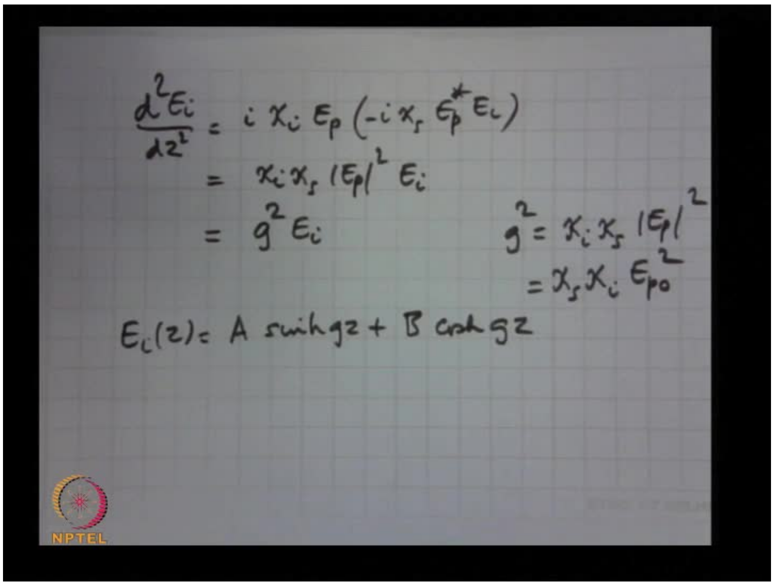

Now, remember, we had derived this equation last time; we differentiate this equation with respect to z. Assume E p is a constant and use the second equation. Actually, we had started with this equation (Refer Slide Time: 39:28). So, what we did was I differentiate the second equation.

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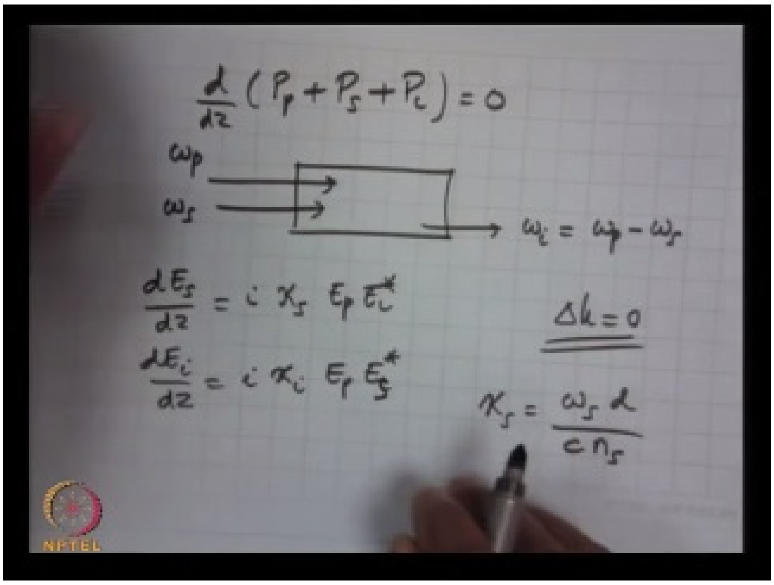

So, I get d square E i by dz square is equal to i kappa i E p – sorry, there is E s star; I am sorry, there is a mistake; here there is E s star (Refer Slide Time: 39:45).

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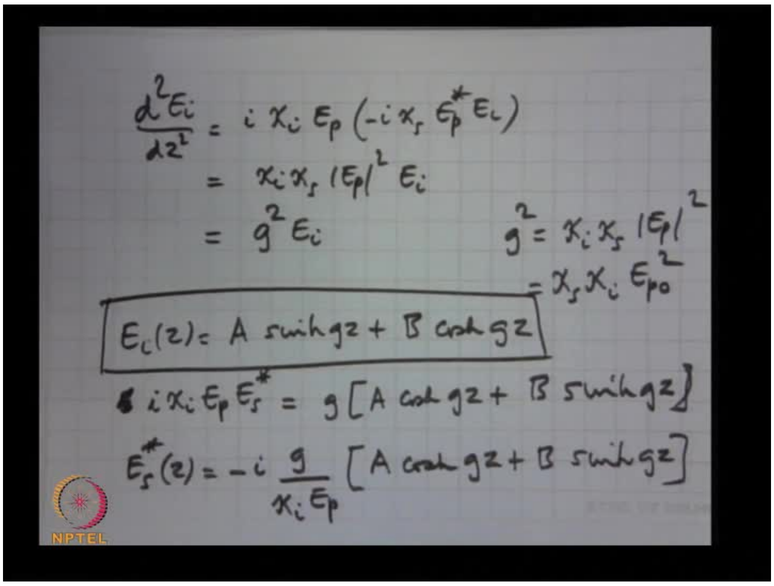

So, dE s star by dz is minus i kappa s E p star E i. So, this gives me kappa i kappa s mod E p square E i, which I called as g square E i. So, g square is kappa i kappa s mod E p square, which is actually kappa s kappa i E p 0 square. E p is assumed to be constant and E p is equal to (Refer Slide Time: 40:34) E p 0 exponential i phi p0. So, mod E p square is simply E p 0 square. So, the solutions of this equations are E i of z is equal to A sine hyperbolic gz plus B cos hyperbolic gz. And then, what did we do? We substituted this E i of z in this equation (Refer Slide Time: 41:01) and obtained in expression for E s star of z. So, what was the equation? Now, let me substitute here. So, i kappa i E p E s star is equal to d E i by dz; so, g times A cos hyperbolic gz plus B sine hyperbolic gz. So, E s star of z is minus i g by kappa i E p into A cos hyperbolic gz plus B sin hyperbolic gz; one solution for E i.

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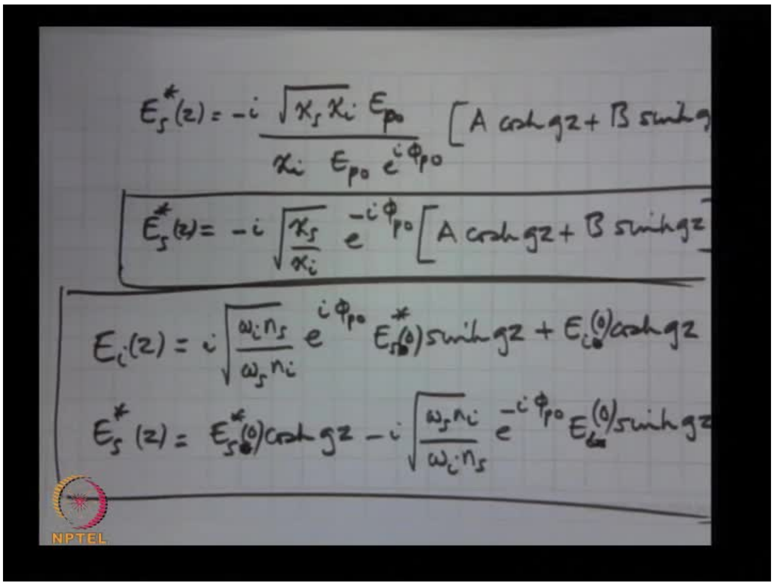

Now, I have an expression for g here. So, let me use that equation here. So, what will I get? I will get E s star of z equals minus i square root of kappa s kappa i E p 0 by kappa I – E p is E p0 exponential i phi p0 into A cos hyperbolic gz plus B sine hyperbolic gz, which is actually minus i under root kappa s by kappa i exponential minus i phi p0 A cos hyperbolic gz plus B sine hyperbolic gz. In the last class, actually, I had assumed here E p was real, but I have assumed a certain phase of the pump here. So, if you use linear boundary conditions, you will get the same solutions. Yes, questions?

In the earlier case, there was no initial E i

Yes

But, at the rate of process, E i was getting generated.

But, its phase was such that the cost was getting reduced. So, can we say that if this particular device acts as a source of coherent phase.

It is a source of coherent phase. The idler, which is coming out is a coherent source, is a coherent wave. The frequency that wave, which I am generating at the idler frequency is a coherent wave; it has at one particular frequency. Actually, there is a finite bandwidth; I will come to this problem later. But, it is generating electromagnetic wave at a particular frequency.

Apart from this, that the phase difference between the two waves that we get at the output – that also becomes constant with time and space.

No, not with constant based phase; if you go back and look at the earlier equation, theta was maintained at phi by 2. For the second harmonic, 2-omega omega process for example, it does not say that phi 1 and phi 2 are constants. For example, when I started with the condition that I had theta is equal to 0 at the input, theta is phi by 2 at the input; plus phi by 2 or minus phi by 2. It remains plus phi by 2 and minus phi by 2. It does not mean that phi 1 and phi 2 are constants.

No sir, I am saying that…

Phase difference may be remaining (( )) but I have to solve the equation. So, find out whether the phase difference between signal and idler remains constant. But, please note that these are two different frequencies. In principle, at any value of z if you look at, the phase difference will remain constant with time, because there is no time variation. We are assuming a situation, where the pump is a constant pump wave coming in. It (( )) generated a signal and idler. They will have a certain phase difference at the output and that phase difference will remain constant with time; there is no change with time, but the frequencies are different. If I try to put them on a detector, it will beat at the frequency omega s minus omega i.

No, sir, one more thing sir; suppose we use (( )) temperature experiments, we use sources. So, can I use such type of source in case of interferences?

Yes, but these two frequencies are different.

Yes sir; although these two are different, but with respect to two phases.

Sure, you will come to more intricate experiment later. When I interfere what is called as two photons interference experiment, we will discuss little later. But, these two are in that sense coherent. The photons coming out in signal and idler are actually from the same from photon. And, they have polarizations, which are very interesting. And, as I will show you later on, there are correlations beyond what we can imagine drastically.

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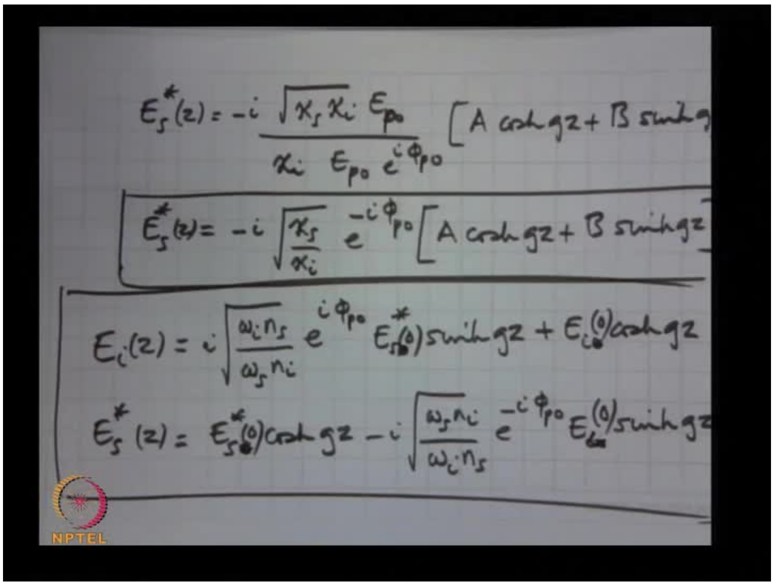

But, this is a purely classical equation (Refer Slide Time: 46:15) that I am solving and getting solution. And, there is a phase generated of the idler and signal in the first process, where there was no idler present. The phase of the idler gets automatically defined by the process and the signal and idler keep on increasing an amplitude. In this case, because I have chosen a phase difference between these three waves at the input, it is possible that energy flows from omega p to omega s omega i or the reverse process. So, what I need to do is to now… I have two equations – here is one equation for E s star; here is an equation for E i of z. There are two constants A and B; I have to apply this boundary condition E s at z is equal to 0 – this (Refer Slide Time: 47:03) E i at z is equal to zero, so much.

Now, I will leave this substituting and calculating in coefficients to you and let me give you the final equations for E i of z and E 2 – E s star of z. Here are the equations. Please go back and just obtain these equations; that means just obtain the constants. So, I have i square root of omega i n s by omega s n i exponential i phi p0 E s 0 star sine hyperbolic gz plus E i 0 cos hyperbolic gz; and, E s star of z is equal to E s 0 star cos hyperbolic gz minus i times square root of omega s n i by omega i n s exponential minus i phi p 0 E i 0 sine hyperbolic gz. Please use the boundary conditions. For example, here you see at z is equal to 0, E i of 0 is equal to E i 0.

Now, let me be little careful; one second. Let me write this as E s of 0; please correct this (Refer Slide Time: 48:45). This is because I have defined and E s 0 and E i 0 differently. Let me just call this; that is, the signal electric field complex conjugate signal electric field at z is equal to 0; E i of z equal to 0; E s star of z is equal to 0; E i of z is equal to 0. So, these are complex. They have the paste on inside. So, I think we will stop here now. What we will do in the next class is using these equations, (Refer Slide Time: 49:13) I will show you that… As you can see here, you are getting a sine hyperbolic term and cos hyperbolic term.

If you make sure that you have sum of the cosine hyperbolic and sine hyperbolic, it will lead to an increase in amplitude, because cos hyperbolic gz plus sine hyperbolic gz is exponential gz. So, it all depends on the phase here, which is coming from here, the pump and the idler phase from here. So, what I will show you in the next class is depending on the phase at which the idler and signal and the pump are incident, E f and E i will increase with z or decrease with z.

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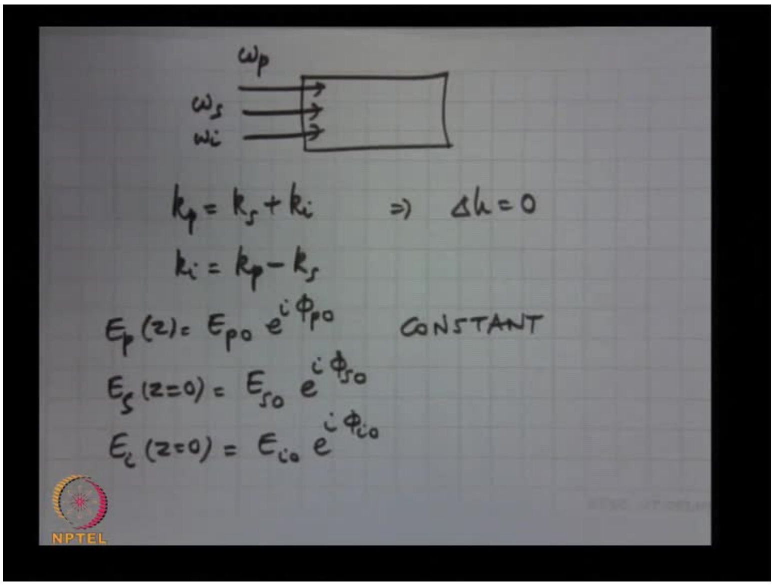

So, this signal which is input into the amplifier will either get amplified or attenuated depending on the phase of the idler. So, I can go from amplification to attenuation depending on the phase, which I chose of the idler frequency. And, I will show you some experimental plots, which people have obtained showing a very interesting behavior of this amplifier.

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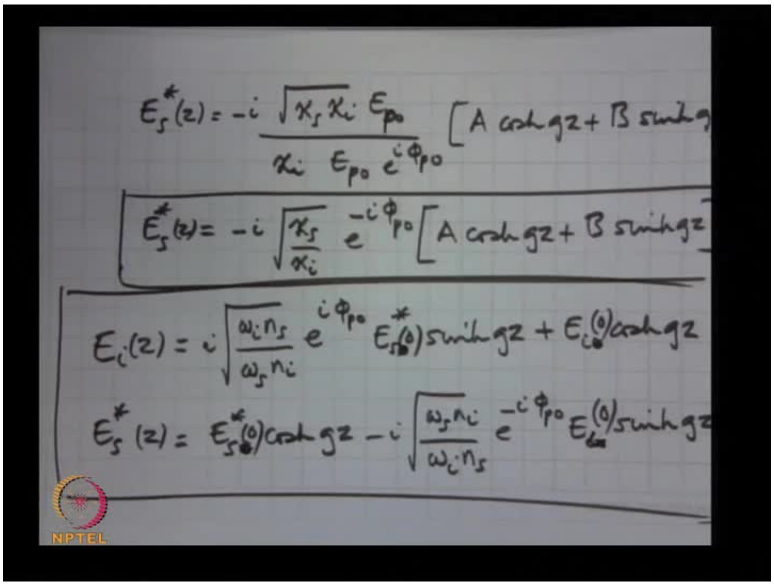

Also, note that in the phase insensitive case, the amplitude was increasing as (Refer Slide Time: 50:26) cos hyperbolic square gz. If I make sure that this leads to a sum of the cosine and a sine hyperbolic, it will go as exponential to gz, the power, which is faster. Cos hyperbolic square or exponential to x; cos hyperbolic square x and exponential 2 x; exponential is much faster. So, actually, in the phase sensitive amplifier, you can actually amplify for the same length for the amplifier; you can achieve much more amplification than in the phase insensitive phase. But, I need to maintain the phase relationship as a constant phase relationship between these waves. If the phase fluctuates, the signal will fluctuate from sometimes get amplified and sometimes get attenuated. So, that is an interesting problem.

And, in fact, in the next class, what I will show you is if I come with a signal with an arbitrary phase, I can break the signal into one component, which gets amplified, the other component which gets deamplified. You see you can break any signal; you have a sine omega d plus phi; it is sine omega d cos phi plus cos omega d sine phi. So, I can write as one sine omega d term and one cos omega d term. These are called quadrature components; one is varying as a sine, the other is varying as a cosine. So, I can actually break up any signal into two components: one at a certain phase; and, the other one – phi by 2 order phase with respect to the signal.

And, what will happen is, if this one is (Refer Slide Time: 52:06) getting amplified, this one will get attenuated, because there is a phi by 2 phase difference. So, I will show this explicitly next class for the omega to omega process. That means when you make omega s and omega i equal here, I get back into the degenerate parametric amplifier process, where omega p was 2 omega and omega s is equal to omega i. So, this three-wave interaction is more general. The second harmonic generation and omega 2 to omega and 2 omega to omega is a special case of this three-wave process, (Refer Slide Time: 52:40) where the idler and signal photons are the same frequency. Any questions?

Initially, we need two different frequencies; we will also control the phase difference between this; how is this generated in actually…

I can actually see at the lower frequencies like microwaves and radio waves; it is not very difficult to control phases of signals. But, the problem will arise at optical frequencies. So, what is now done is you generate these waves from the same original source wave and make sure that there is a constant phase relationship between these waves. If you do that, then you can achieve this kind of things; otherwise, you need to have mechanism by which you can keep control in the phase. Is the phase fluctuation is very slow? You can have feedback mechanism to control the phase; otherwise, it becomes difficult.

Sir, how is it that if we are using a laser in all the (( )) how many able to control this phase? [Not audible] (Refer Slide Time: 53:38)

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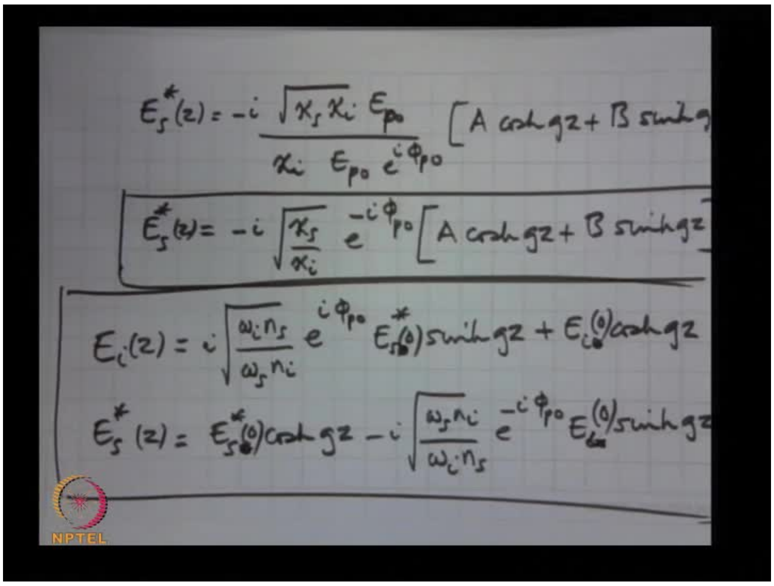

No, backup is a strong wave, which is coming from a laser pump source. Then, I have a signal and an idler frequency, which are phase-matched with the pump frequency. The experiments, which I will tell you, are actually generating all the waves from the same original source. So, they are all actually having a constant phase relationship. And then, they show that there is a phase-sensitive nature of the amplification process. But, later on, when we come into little bit of quantum mechanics, I will show you some interesting features of this process, because of the process in which omega to omega interaction is taking place. There this phase sensitive nature or two signals; one in a particular phase and one at a quadrature phase difference get amplified and deamplified – comes in very handy in sort of modifying the noise at vacuum level. This will become clearer when we come to little later. Anything else?

Thank you.