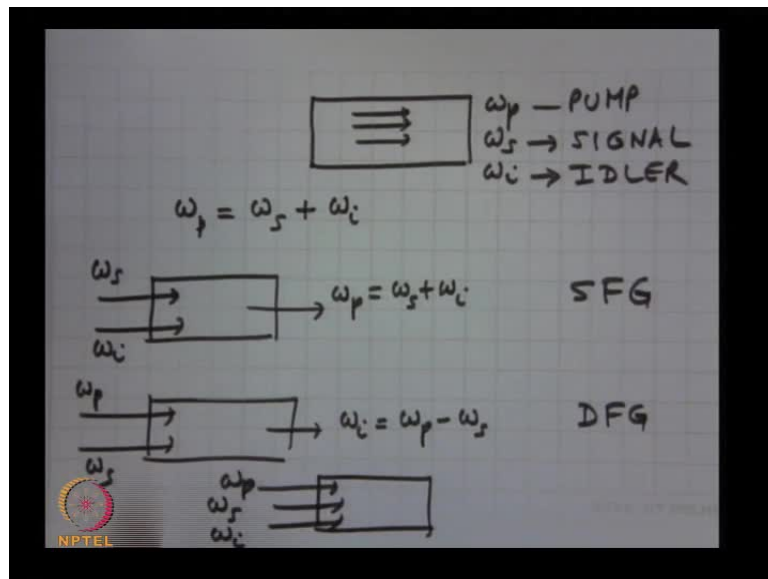


Quantum Electronics
Prof. K Thyagarajan
Department of Physics
Indian Institute of Technology, Delhi

Module No. # 03
Second Order Effects
Lecture No. # 13
Non Linear Optics (Contd.)

Today, look into the three wave interaction process, where we have three frequencies simultaneously present inside the media. So, second harmonic generation and the degenerate parametric down conversion will be the special cases of this.

(Refer Slide Time: 00:45)



So, the problem we are looking at is - you have a crystal in which there are three frequencies present simultaneously - ω_p , ω_s , and ω_i ; they are satisfying the condition ω_p is equal to ω_s plus ω_i .

So, depending on input conditions, I will have either of the various situations. So, for example, I can have an incidents of ω_s and ω_i to generate a new frequency

ω_p is equal to ω_s plus ω_i , this will correspond to some frequency generation, because the new frequency coming out is the sum of these two frequencies.

If you have an ω_p incident and an ω_s incident, I will generate an ω_i which is ω_p minus ω_s ; this is difference frequency generation. p stands for pump, s stand for signal, and i stands for idler; these are names which have been borrowed from microwave technology.

So, here is, in the situation, photons at frequency ω_p , one photon at frequency ω_s and one photon at frequency ω_i , combine to form one photon at frequency ω_p . In this process, photon and ω_p frequency interacts with ω_s frequency photon, and the ω_p photon splits into an ω_s photon and an ω_i photon. So, in the process generating an ω_i photon, and as you can see, the number of ω_s photons will increase.

So, ω_s will get amplified and in the process of generating the difference frequency. So, **this is the**, I will show you that this is a parametric amplifier, the signal will get amplified and in the process generate a new frequency ω_i . So, the term comes from the fact that because this is the input signal, which is usually weak and you have a strong pump coming in, you can amplify the signal ω_s and as a bonus you get a new frequency generated, which is ω_i , the idler.

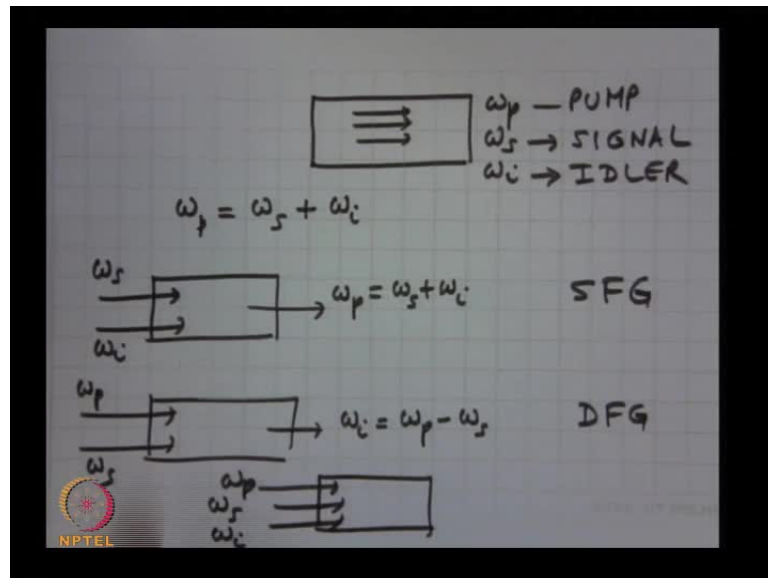
So, this is more for amplification or generation of a new frequency. If I use it as a generator of new frequency, I will amplify the signal, **so, I will**, but my interest is in ω_i . But if my interest is in generating in amplifying ω_s , I will amplify ω_s , but in the process generating ω_i , **which will be**, which I do not need to worry about that ω_s frequency will come out and I pick up the **(())** ω_s frequency.

Sir for connect frequency ω_p will split into ω_s and ω_i that is why ω_s will (()) Yes, because I cannot generate ω_i , I generate ω_i by splitting ω_p into ω_s and ω_i .

So, ω_i will get generated and come out and every time an ω_i photon appears, an ω_s photon also has to appear, because of energy conservation. So, the number of ω_s photons will increase as the interaction proceeds. So, ω_s will get simply

amplified. I can actually come to a situation which we will discuss later that I have all three incident.

(Refer Slide Time: 00:49)



Here, the omega s will either can amplify or attenuated, depending on the phase at which these frequencies are being incident. In this case, omega s gets amplified irrespective of the phase of the signal; so, this is called phase insensitive amplifier. If I put all the three frequencies simultaneously, the omega s signal can get amplified or attenuated depending on the phase relationships between the wave omega p, omega s, and omega i.

So, I can use this process as a phase sensitive amplifier or a phase insensitive amplifier. The case we had considered earlier, where omega s was equal to omega i. This process with omega s equal to omega i is nothing but second harmonic generation, and this process with omega s equal to omega i is nothing but the parametric down conversation process we had studied.

So, the case we had considered is a degenerate case, where omega s and omega i become equal, but in general, omega s need not be equal to omega i. So, this is the most general case of three wave interaction where I have three frequencies interacting simultaneously within the crystal - one at frequency omega p, one at omega s, and one at omega i, and the three frequencies satisfy this condition that omega p is equal to omega s plus omega i.

(Refer Slide Time: 06:40)

$$P_{NL} = 2\epsilon_0 d E^2$$

$$E = E^{(\omega_p)} + E^{(\omega_s)} + E^{(\omega_i)}$$

$$E^{(\omega_p)} = \frac{1}{2} \left[E_p e^{i(k_p z - \omega_p t)} + c.c. \right]$$

$$E^{(\omega_s)} = \frac{1}{2} \left[E_s e^{i(k_s z - \omega_s t)} + c.c. \right]$$

$$E^{(\omega_i)} = \frac{1}{2} \left[E_i e^{i(k_i z - \omega_i t)} + c.c. \right]$$

$$k_p = \frac{\omega_p n_p}{c}, \quad k_s = \frac{\omega_s n_s}{c}, \quad k_i = \frac{\omega_i n_i}{c}$$

So, we use the same procedure as we did before. This interaction is completely non-linear; so, what we need to do is remember in second harmonic, what did we do? We started from this equation P non-linear is equal to 2 epsilon 0 d E square, where E is the total electric field within the crystal. A second harmonic case, we said E consists of waves at frequency omega, added to omega.

Now, in this case, the electric fields will consist of three parts: electrical field at omega p, electric field at omega s, and electrical field at omega I, all three frequencies will be present; so, E the total electrical field will actually consist of E at omega p plus E at omega s plus E at omega i.

So, for example, I will have E at omega p, I will write as half of E p exponential i k p z minus omega p t plus complex conjugate; k p is the propagation constant of the wave at frequency omega p, and E p is complex electric field of this omega p wave; this is assumed to plain wave propagating along the z direction, and as before, because of this non-linear interaction, E p will be a function of z.

Similarly, I will have for E at omega s half of E s exponential i k s z minus omega s t plus complex conjugate, and E at omega i will be equal to half of E i exponential i k i z minus omega i t plus complex conjugate. E p, E s, and E i are the electric field amplitudes of the fields of the waves at omega p, omega s, and omega i respectively.

So, k_p depends on the refractive index of the medium at frequency ω_p . So, k_p is ω_p by c into the refractive index at frequency ω_p , which I call n_p for example. So, k_p will be something like ω_p by c into n_p ; similarly, k_s will be ω_s by c into n_s , and k_i will be ω_i by c into n_i .

So, now, what do I do? How do I proceed? **I have to calculate**, I have to substitute in to the non-linear wave equation. Remember, we wrote the electric fields first, before that I need to calculate now, what is the non-linear polarization generated at these three frequencies? (Refer Slide Time: 10:03) So, I have to substitute the sum of this equation in to this equation and pick up terms, which will give me non-linear polarization at frequencies ω_p , ω_s and ω_i .

Now, can you tell me from here, what will be P_{NL} non-linear at ω_p ? So, I want to write P_{NL} non-linear at frequency ω_p , so, what I have to do is - to substitute the sum of these three and square it. So, what will be the term I will get at ω_p frequency?

(Refer Slide Time: 10:40)

The image shows three equations for non-linear polarization components P_{NL} at different frequencies, written on a grid background. Each equation is followed by '+ cc' (complex conjugate).

$$P_{NL}^{(\omega_p)} = \frac{1}{2} \epsilon_0 d \left[2 E_s E_i e^{i\{(k_s+k_i)z - \omega_p t\}} + cc \right]$$

$$P_{NL}^{(\omega_s)} = \frac{1}{2} \epsilon_0 d \left[2 E_p E_i^* e^{i\{(k_p-k_i)z - \omega_s t\}} + cc \right]$$

$$P_{NL}^{(\omega_i)} = \frac{1}{2} \epsilon_0 d \left[2 E_p E_s^* e^{i\{(k_p-k_s)z - \omega_i t\}} + cc \right]$$

An NPTEL logo is visible in the bottom left corner of the slide.

So, $2 \epsilon_0 d$ into E square will be half $\epsilon_0 d$, please note that this d is an effective non-linear coefficient, I am not writing d_{ijk} . For a given orientation of the crystal, for a given propagation direction, for given polarization states of these waves, **I can**, and for a given crystal, I can obtain this equation by substituting and calculating like we did for $k_d p$ and for lithium niobate, may be will two can example later, but d is an

effective non-linear coefficient which depends on the state of polarization of the waves at ω_p , ω_s , ω_i and also the non-linear tensor of the crystal.

Now, can you tell me, if I substitute this, what is the term I will get at ω_p frequency? **With any additional factor multiplying**

(())

No, before that a multiplying factor I will have $a + b + c + d + e + f$ whole square; so, you are picking up a product of those two terms and there will be a factor of 2; so, you will have $2 E_s E_i \exp(i \dots)$, what will I get in the exponential?

(())

Not k_p

$k_s + k_i$ into z minus $\omega_p t$ plus complex conjugate, when you write all this and take a square, you will get twice $E_s E_i$, and actually, I would have got $\omega_s + \omega_i$ into t and $\omega_s + \omega_i$ is ω_p and plus its complex conjugate and nothing else.

Similarly, P non-linear at ω_s will be half $\epsilon_0 d^2$, what will I get? $E_p E_i \exp(i k_p z - i \omega_s t + \text{complex conjugate})$, please note, because ω_s is $\omega_p - \omega_i$, I will get $E_p E_i \exp(i k_p z - i \omega_s t + \text{complex conjugate})$ and I will get $k_p - k_i$.

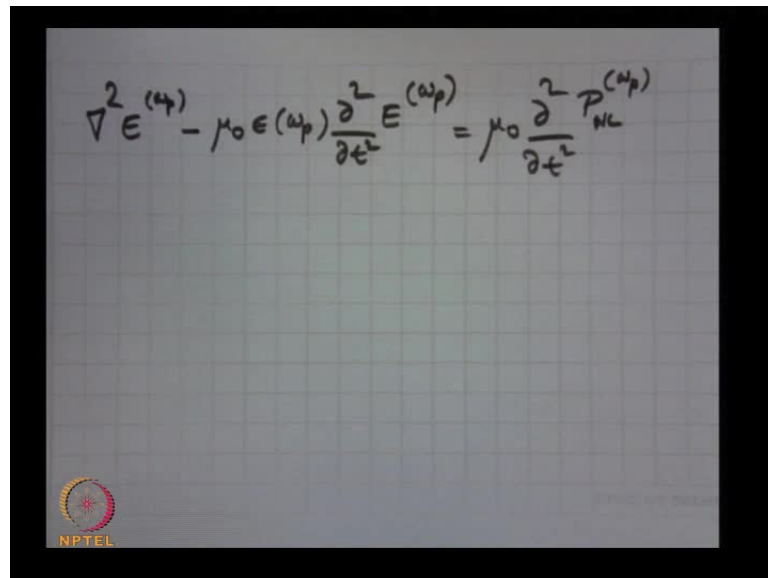
Similarly, P non-linear at ω_i will be equal to half $\epsilon_0 d^2 E_p E_s \exp(i k_p z - i \omega_s t + \text{complex conjugate})$.

We are using the same procedure as we did for second harmonic; this will contain more **polarization terms**, non-linear polarization terms and many other terms. We are not bothered about that because as we have seen already for efficient non-linear interactions, we need to consider only those terms which are close to phase matching.

So, we will assume that in this process, we somehow will manage, will find out what is the phase matching condition required for this process to take place and we are assuming

that it has been almost matched; so that I am only worried about three frequencies - omega p, omega s and omega i.

(Refer Slide Time: 14:49)



The image shows a handwritten equation on a grid background. The equation is:

$$\nabla^2 E^{(\omega_p)} - \mu_0 \epsilon(\omega_p) \frac{\partial^2 E^{(\omega_p)}}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}^{(\omega_p)}}{\partial t^2}$$

In the bottom left corner of the grid, there is a small circular logo with the text 'NPTEL' below it.

So, what did I do after this, for second harmonic? I substitute into the wave equation, remember, there was wave equation which we wrote down what was the wave equation? Suppose, I was looking at wave equation for omega p, I will have del square E of omega p minus mu 0 epsilon at omega p del square by del t square of E at omega p is equal to mu naught del square by del t square P non-linear at omega p.

For second harmonic, we had written del square E 2 omega minus mu 0 epsilon to omega del square by del t square E of 2 omega is equal to mu 0 naught del square by del t square P non-linear at 2 omega.

Each one of the frequencies must satisfy the wave equation. This is the source term, the non-linear polarization of the source term which is influencing the propagation of the corresponding frequency.

Similarly, I have an equation for omega s, where I just replace omega p by omega s, and another equation omega i, where I replace omega p by omega i.

(Refer Slide Time: 06:40)

$$P_{NL} = 2\epsilon_0 d E^2$$
$$E = E^{(\omega_p)} + E^{(\omega_s)} + E^{(\omega_i)}$$
$$E^{(\omega_p)} = \frac{1}{2} \left[E_p e^{i(k_p z - \omega_p t)} + cc \right]$$
$$E^{(\omega_s)} = \frac{1}{2} \left[E_s e^{i(k_s z - \omega_s t)} + cc \right]$$
$$E^{(\omega_i)} = \frac{1}{2} \left[E_i e^{i(k_i z - \omega_i t)} + cc \right]$$
$$k_p = \frac{\omega_p n_p}{c}, \quad k_s = \frac{\omega_s n_s}{c}, \quad k_i = \frac{\omega_i n_i}{c}$$

So, what I need to do is, now to substitute in this equation, substitute this expression for E of omega p and P non-linear at omega p on both sides and equate the terms coefficient of exponential minus i omega p t on both sides exactly the same procedure that I employed for second harmonic generation, and what did I neglect?

The second derivative of E p with respect to z, and I will use the condition that epsilon omega p and k p are related, k p square is equal to mu naught epsilon omega p into omega p square, the propagation constant and the permittivity are related through this equation.

So, if I use all this, this equation will simplify to the following equation. So, let me leave this substitution and simplification of this to you. Substitute the expression for E omega p from here, substitute the expression for P non-linear omega p from here, in to this wave equation and equate the terms the coefficient of exponential minus i omega p t on both sides.

(Refer Slide Time: 17:44)

$$\nabla^2 E^{(\omega_p)} - \mu_0 \epsilon^{(\omega_p)} \frac{\partial^2 E^{(\omega_p)}}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}^{(\omega_p)}}{\partial t^2}$$

$$\frac{dE_p}{dz} = i \frac{\omega_p d}{c n_p} E_s E_i e^{-i\Delta k z}$$

$$\frac{dE_s}{dz} = i \frac{\omega_s d}{c n_s} E_p E_i^* e^{i\Delta k z}$$

$$\frac{dE_i}{dz} = i \frac{\omega_i d}{c n_i} E_p E_s^* e^{i\Delta k z}$$

$$\Delta k = k_p - k_s - k_i$$

Neglect the secondary derivative E_p with respect to z and use the relationship between E epsilon omega p and k_p and you will land up with this equation - dE_p by dz is equal to $i \omega_p d$ by $c n_p E_s E_i e^{-i\Delta k z}$, where Δk is k_p minus k_s minus k_i .

It is very similar to the equation we had obtained earlier, dE^2 by dz is $i \omega d$ by $c n^2 E_s E_1^2$, etcetera, same equation, exactly a similar equation; except that, now, there is a particular frequency ω_p and n_p is the refractive index of the medium at the frequency ω_p , and Δk is k_p minus k_s minus k_i , there it was k^2 minus $2k_1$; so, if s is equal to i this simply becomes k^2 minus $2k_1$, exactly like the second harmonic.

So, this is the equation describing the change of E_p , the amplitude of the electric field at omega p frequency as the wave propagates and this depends on the non-linear coefficient d and the electric fields at signal and idler frequencies.

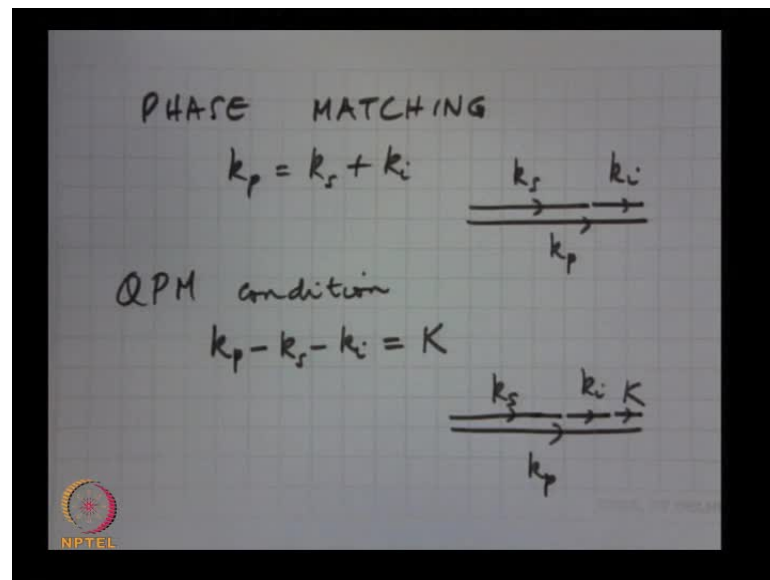
Similarly, let me give you the other two equations - dE_s by dz is equal to $i \omega_s d$ by $c n_s E_p E_i^* e^{i\Delta k z}$, and dE_i by dz is equal to $i \omega_i d$ by $c n_i E_p E_s^* e^{i\Delta k z}$. So, that is a separate definition of Δk .

So, you have three equations coupled equations, three coupled non-linear equations connecting the electric field amplitudes of signal, idler and pump.

The d is the **non-linear coefficient**, effective non-linear coefficient that is responsible for this interaction. Please note that if I do not choose the polarization states appropriately, the d element, the t tensor element, which is responsible for this may become 0. So, I have to be careful, but I am assuming that I am using a situation where the polarization states of signal, idler and pump are such that d coefficient is finite; there is a non-linear tensor element, there is a coupling between these three waves.

Now, as before, we will see that maximum interaction will take place for maximum efficiency of this interaction Δk must be 0, which is again the phase matching condition.

(Refer Slide Time: 21:04)



So, the phase matching condition, I will get here is k_p is equal to k_s plus k_i , which means that I must have in terms of photon picture - the momentum of the pump photon is the sum of the momentum of the signal and idler photons, that is generating or whatever it is.

In all the three process, whether it is sum frequency generation, difference frequency generation, parametric down conversion, whatever it is, I need to satisfy this condition for efficient interaction between these three waves.

Note that I can use the same quasi phase matching principle here, because in quasi phase matching, d becomes the function of z . So, what is the quasi phase matching condition? I

need to satisfy. So, if I had a d varying with z , like $\sin kz$, what should be the condition I will have to satisfy? $k_p - k_s - k_i$ must be equal to K , Δk must be equal to capital K . So, here, if I draw the vector diagram, I have a k_p , k_s , k_i . By convention, the signal frequency is supposed to be higher than the idler frequency.

Pump frequency is the highest frequency, because ω_p is $\omega_s + \omega_i$, the highest frequency among these three is ω_p , the pump frequency, then comes the signal frequency and then the idler frequency.

So, in the wave length space, idler is the longest wave length and pump is the shortest wave length. So, if I start with a wave length of 800 nanometers, so, 800 nanometer could be pump, 1300 nanometers could be signal, and I have something else which is around 1800 or 2 microns are something like as the idler. So, ω_p is equal to $\omega_s + \omega_i$, and conventionally, the signal frequency is higher than the idler frequency.

So, the signal wave length is shorter than the idler wave length. (Refer Slide Time: 23:49) **This will be**, in this quasi phase matching condition, this is k_p . I have $k_s + k_i$ plus K ; this is K , this is k_i , this is k_s , and it is for this reason that I am drawing the k_s vector to be longer than the k_i vector, because ω_s is higher than ω_i .

The refractive indices are not very different, they are different, but they are not very different. So, k_s is $\omega_s / c n_s$; k_i is $\omega_i / c n_i$, and because ω_s is bigger than ω_i , k_s is usually bigger than k_i .

So, the way I have drawn here, I have drawn the vector k_s to be bigger than k_i and k_p is the biggest vector. Here, this is the quasi phase matching where I am not perfectly satisfying the phase matching condition, but I am having a periodic variation in the non-linear tensor coefficient d , such that it compensates for this Δk that is appearing here.

So, the discussion that we had for QPM for second harmonic is exactly valid here, provided I choose a capital K which satisfies this condition, which is the quasi phase matching condition.

(Refer Slide Time: 17:50)

$$\nabla^2 E^{(\omega)} - \mu_0 \epsilon^{(\omega)} \frac{\partial^2 E^{(\omega)}}{\partial t^2} = \mu_0 \frac{\partial^2 P_{NL}^{(\omega)}}{\partial t^2}$$

$$\frac{dE_p}{dz} = i \frac{\omega_p d}{c n_p} E_s E_i e^{-i\Delta k z}$$

$$\frac{dE_s}{dz} = i \frac{\omega_s d}{c n_s} E_p E_i^* e^{i\Delta k z}$$

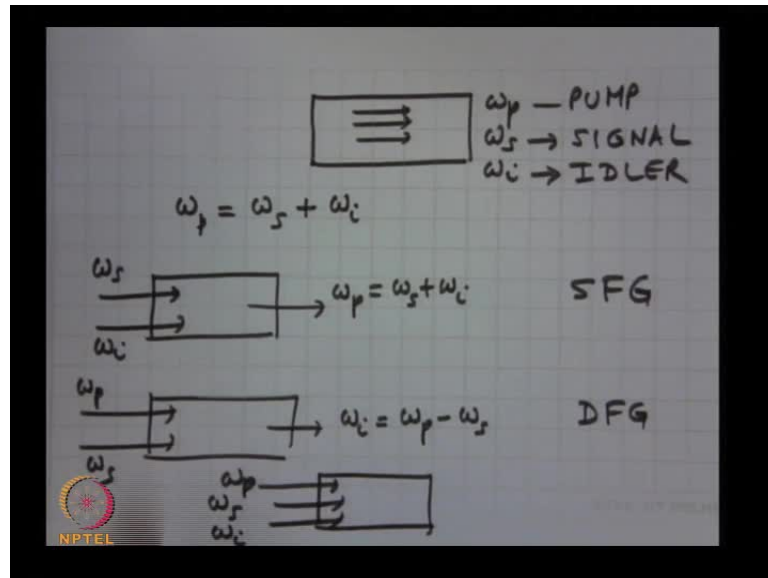
$$\frac{dE_i}{dz} = i \frac{\omega_i d}{c n_i} E_p E_s^* e^{i\Delta k z}$$

$$\Delta k = k_p - k_s - k_i$$

So, now, these three equations are the most general equations describing these three wave interaction process and we can use these three equations to study any one of the processes. Some frequency generation is difference frequency generation, and as before, I can show that if you have only the pump incident, you will not be able to generate signal and angular; this requires a spontaneous down conversation. So, if I take a crystal and shine only omega p, classically, just, omega p just propagates.

Quantum mechanically, I can show that an omega p incident can spontaneously generate an omega s and an omega i satisfying omega p is equal to omega s plus omega i.

(Refer Slide Time: 00:49)



Now, because this equation has infinite number of solutions; for a given ω_p , there are infinite combinations of ω_s , ω_i , which can sum to ω_p .

So, which pair will come out, that pair satisfying the phase matching condition. (Refer Slide Time: 26:10) Because I need to satisfy this condition, as well as the phase matching condition **if I need to satisfy**, if I need to generate efficiently which means, I need to satisfy ω_p is equal to ω_s plus ω_i and this condition or this condition.

There will be one pair of frequencies which will satisfy both these conditions; for given ω_p , k_p is fixed. For example, if I look at these two equations for a given ω_p , k_p is fixed and there is one ω_s , ω_i combination, which will satisfy both these equations that will be the one, which will be most sufficiently generated in this process.

Omega s omega i the first equation (()) we are getting omega p is equal to omega I, that equation is different

You have to be careful, because there is, what happens is - for second harmonic, we have only one wave incident. So, when ω_s becomes equal to ω_i , we did not consider there were two waves at ω which were incident; so, there is a factor of two which will be coming because of this problem. So, **I have to**, that is why I did second

harmonic from the first principle, so, I got an equation; I do the same thing here, but I cannot substitute ω_p is equal two ω and go there.

Because, then remember here, there are two waves incident; I did not consider two ω incident and one or two ω have been generated; so, there is no direct transfer for here to there.

(Refer Slide Time: 28:08)

DFG

$\Delta k = 0$

$$\frac{dE_s}{dz} = i\kappa_s E_p E_i^*$$

$$\frac{dE_i}{dz} = i\kappa_i E_p E_s^*$$

$$\frac{d^2 E_i}{dz^2} = i\kappa_i E_p (-i\kappa_s E_p^* E_i)$$

$$= \kappa_s \kappa_i |E_p|^2 E_i$$

$$\kappa_s = \frac{\omega_s d}{c n_s}$$

$$\kappa_i = \frac{\omega_i d}{c n_i}$$

NPTEL

So, now, the first example let us look at difference frequency generation. So, we just look at this today and understand what is happening in the difference frequency generation.

So, what is difference frequency generation? I have the non-linear crystal, I launch an ω_p wave and an ω_s wave, and my objective is to generate a wave at the difference frequency ω_i , which is ω_p minus ω_s and I will show in this process, ω_s will get automatically amplified.

So, I can look at this problem as if I want to amplify ω_s or to generate ω_s , it is a same problem. So, how will I solve these three equations? So, what is the approximation I will make (()) depletion which means, I will assume E_p is almost a constant, so, I do not have to worry about this equation. I have to only solve these two equations simultaneously, assuming E_p is a constant.

So, and, so, let me first assume Δk is equal to 0 to get some easy solutions. So, I will have two equations $\frac{dE_s}{dz}$ is equal to i . Now, let me write this as κ_s into $E_p E_i^*$ and $\frac{dE_i}{dz}$ is equal to $i \kappa_i E_p E_s^*$. So, I am solving the equations for the case Δk is equal to 0, because I know already that the maximum generation of difference frequency will take place, if I satisfy the phase matching condition.

So, in these two equations, I am going to assume E_p is a constant. So, let me, for example, because I am looking at generation of E_i or the electric field at frequency ω_i , let me differentiate the second equation. So, I will get $\frac{d^2 E_i}{dz^2}$ is equal to $i \kappa_i$; so, κ_s is $\frac{\omega_s}{c n_s}$, and similarly, κ_i is $\frac{\omega_i}{c n_i}$.

So, $i \kappa_i E_p$ into $\frac{dE_s^*}{dz}$, which is the complex conjugate of this one, which is minus $i \kappa_s E_p^* E_i$; (Refer Slide Time: 30:56) so, this is equal to $\kappa_s \kappa_i \text{mod } E_p^2$ into E_i .

I am assuming E_p is a constant - is a high power pump coming in, so, I am assuming E_p is a constant. (Refer Slide Time: 31:30) So, let me call this as, g^2 is this coefficient $\kappa_s \kappa_i \text{mod } E_p^2$, this is g^2 .

What is the solution of this equation?

So, E_i of z sin hyperbolic (()) hyperbolic

(Refer Slide Time: 31:51)

$$E_i(z) = A \sinh(gz) + B \cosh(gz)$$

$$i \kappa_i E_p E_i^*(z) = g(A \cosh(gz) + B \sinh(gz))$$

$$E_i^*(z) = \frac{-i g}{\kappa_i E_p} (A \cosh(gz) + B \sinh(gz))$$

$$g^2 = \kappa_s \kappa_i |E_p|^2 = \kappa_s \kappa_i E_p^2$$

$$E_i^*(z) = -i \frac{\sqrt{\kappa_s \kappa_i}}{\kappa_i} (A \cosh(gz) + B \sinh(gz))$$

$$= -i \sqrt{\frac{\kappa_s}{\kappa_i}} (A \cosh(gz) + B \sinh(gz))$$

So, E_i of z is equal to $A \sin \text{hyperbolic } gz$ plus $B \cos \text{hyperbolic } gz$, how do I get the solution? For E_s , I have another equation, but they are not independent remember.

We can subtract the power

No, I want, not in terms of power, in electric fields. So, what do I do? I have a solution for E_i .

(()) the linear energy will be proportional in both of them

But, you see, I want electric fields, not powers. Powers will be (()) square, I want the electric fields.

I can again differentiate the first equation, substitute from the second equation, but I left two more constants c and d ; I do not have to do that, I can substitute my solution here and get equation for E_s star. Actually, I can do c and d , and then I have to substitute back in to this equation and make sure that they are satisfying this. So, instead of that, I just substitute E_i of z in the second equation and get the following equation. So, $i \kappa E_p E_s$ star of z is equal to $d E_i$ by dz , which is g times $A \cos \text{hyperbolic } gz$ plus B times $\sin \text{hyperbolic } gz$.

So, E_s star of z is equal to $\text{minus } i g \text{ by } \kappa E_p A \cos \text{hyperbolic } gz$ plus $B \sin \text{hyperbolic } gz$. I remember, we had written g^2 is equal to $\kappa s \kappa i \text{ mod } E_p$ square.

Now, let me assume that I defined the phase of the pump as 0 , that is E_p is the real quantity; E_p remains constant and I take the phase of the pump as the reference phase, I relate all phases to that pump phase.

So, I assume E_p is real quantity, so, this simply becomes $\kappa s \kappa i$ into E_p square. So, E_s star of z is $\text{minus } i$, so, this g is now square root of $\kappa s \kappa i$, E_p cancels off and I get κi into $A \cos \text{hyperbolic } gz$ plus $B \sin \text{hyperbolic } gz$, which is $\text{minus } i$ square root of κs by κi $A \cos \text{hyperbolic } gz$ plus $B \sin \text{hyperbolic } gz$ plus $B \sin \text{hyperbolic } gz$.

()

No, phase is also assumed to be 0 real, not decaying means, amplitude of E_p remains constant, but I am also assuming the phase, I am relating all phases to that I could have substituted, but then I will get an extra phase sitting here that is all. (Refer Slide Time: 36:03) If I had written E_p as E_p times exponential i by p , I will get here, there will be an exponential minus i by p because of this E_p here and there is $\text{mod } E_p$ square in g , but E_p in here, so, I will have an exponential. Some phase factor will be sitting, it does not matter, but we will come to it a little later when we have all three waves incident and I look at a phase sensitive amplification process.

Now, what is κ_s by κ_i ? Actually, I can substitute κ_s κ_i and I get this equation, so, this is E_s^* of z minus i . Now, κ_s by κ_i is how much?

()

There is also ω .

(Refer Slide Time: 36:54)

$$E_s^*(z) = -i \sqrt{\frac{\omega_s n_i}{\omega_i n_s}} (A \cosh gz + B \sinh gz)$$

at $z=0$, $E_i(0) = 0$, $E_r(0) = E_{s0}$

$$B=0 \quad \kappa \quad E_{s0}^* = -i \sqrt{\frac{\omega_s n_i}{\omega_i n_s}} A$$
$$A = i \sqrt{\frac{\omega_i n_s}{\omega_s n_i}} E_{s0}^*$$

So, I will have square root of ω_s and i by ω_i n_s into A \cos hyperbolic gz plus B \sin hyperbolic gz ; so, these are the two solutions, E_i of z and E_s^* of z . Now, how do I find out the values of A and B ? Apply boundary conditions.

So, at z is equal to 0; so, normally, what I will have is – here, this is my problem. So, z is equal to 0, which I call this plane E_i is 0. So, at z is equal to 0, E_i of 0 is equal to 0, and E_s of 0 is sum E_s 0, there is some signal incident at the input and no idler, just a signal.

So, what are the values of the constants I get? (Refer Slide Time: 38:12)

The first one tells me B is 0, because this is 0 at z is equal to 0 that means, B is equal to 0, and E_s 0 must be equal to minus i square root of $\omega_i n_s$ by $\omega_s n_i$ into A .

(Refer Slide Time: 38:37) Because if I substitute here, B is anyway 0; so I get A is into this is equal to E_s star, so, E_s star of 0. So, A will be equal to i times square root of $\omega_i n_s$ by $\omega_s n_i$ into E_s 0 star.

(Refer Slide Time: 39:14)

$$E_i(z) = i \sqrt{\frac{\omega_i n_s}{\omega_s n_i}} E_{s0}^* \sinh gz$$

$$E_s^*(z) = E_{s0}^* \cosh gz$$

So, here, at solutions, finally solutions which I get from these two equations **are** So, if I substitute the values of A and B in this E_i of z , so, E_i of z is equal to B 0 s A sin hyperbolic gz , so, i times square root of $\omega_i n_s$ by $\omega_s n_i$ E_s 0 star sin hyperbolic gz , and E_s star of z is equal to E_s 0 star minus i into this will be E_s 0, so, E_s 0 star cos hyperbolic. (Refer Slide Time: 40:00) A times this minus is actually E_s 0 star; so, I get A E_s star of z is E_s 0 star cos hyperbolic gz . So, at z is equal to 0, this is 0, and this is E_s 0 star.

So, first thing you notice is that, at z is equal to 0, there is no idler, but as z increases, the idler amplitude keeps on increasing. Sin hyperbolic function, how does it behave? As extent as the argument times infinity, it keeps on increasing; it is monotonically increasing function.

So (()) idler will keep on increasing. Of course, as before, I cannot use these equations when the efficiencies become very large. So, there is an increasing of the idler power, but the signal power is also increasing; cos hyperbolic gz is also an increasing function of z . (Refer Slide Time: 40:55)

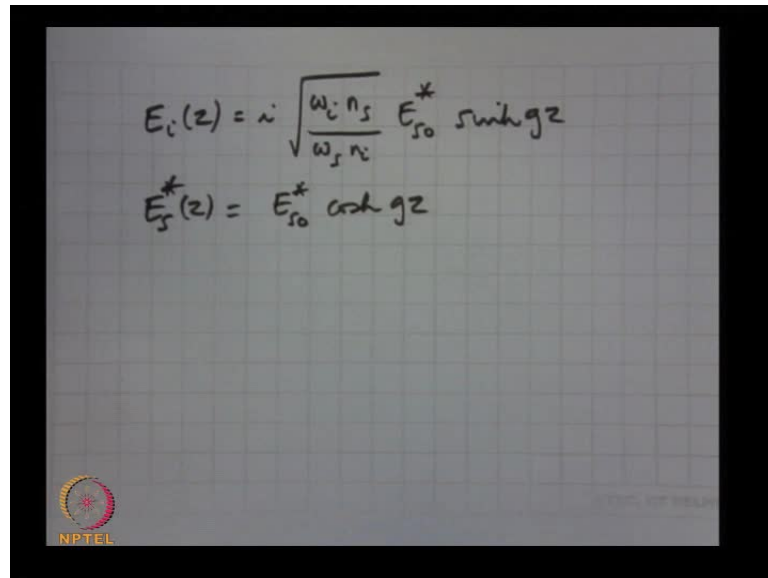
So, this process generates an idler, but also amplifies the signal and the amplification here is independent of the phase of the E_s the stage appearing here, but $\text{mod } E_s$ is square is always cos hyperbolic square gz .

So, this signal will get amplified irrespective of the phase of the signal with respect to pump or idler whatever it is, independent of this input condition this signal will always get amplified provided, I satisfy the phase matching condition.

So, this is an example of phase insensitive amplifier, where the amplification takes place irrespective of the phase of the signal. (Refer Slide Time: 41:42) Why is this coming factor? Please remember that every time there is a signal generated, there will be an idler generated.

So, the number of photons that are coming out at the idler frequency must be equal to the increase in the number of signal photons coming out of the crystal, because I cannot generate a million idler photons without having also generated a million signal photons.

(Refer Slide Time: 39:14)


$$E_i(z) = \kappa \sqrt{\frac{\omega_i n_s}{\omega_s n_i}} E_{s0}^* \sinh g z$$
$$E_s^*(z) = E_{s0}^* \cosh g z$$

So, the increase in the number of signal photons must be equal to the generated idler photons, because at the input there was no idler, because every time an idler gets generated, a signal will be generated. So, we will discuss this, we will discuss these solutions a little more detail in the next class. (Refer Slide Time: 42:43)

So, what I will show you is that this is simply the process that the pump photon is splitting into an idler and signal photon, which is generating an idler photon, and a signal photon.

But please remember, here - these are Maxwell's equations, we have not quantized the radiation at all and these conclusions are arising just from the classical equations. I will show you that the number of idler photons coming out is exactly equal to the increase in the number of signal photons, and we will derive an equation called the **(C)** relation which tells you this relationship between the number of photons coming out at the signal idler. Also remember that if I have generated a million idler photons, how many pump photons should I have lost? **(C)** Same in one million.

So, if I had generated a million idler photons, I should have lost a million pump photons and I would have also generated a million signal photons; I am not conserving the number of photons, because I am splitting the photons.

So, I have a million photons coming in, I convert all of them idler and I also generate a million signal photons; there will be two million photons coming out, but each photons has a lower frequency there, a lower energy; so, energy conservation is always maintained. Do you have any question? Otherwise, you will have a quiz.

(()) does it come because of the boundary condition Right now, we have assumed that the input at only the signal and the pump. Later on, I will look at case where the input contains signal, idler and the pump. Because in the last case also when (()) In the 2 omega case

You see, in that case, I cannot have 1 omega coming in and 1 2 omega coming in and not the other omega coming in, because here I can, because I have three frequencies, I can have omega p and omega s and no omega at the input.

There if I put omega, then I have both the inputs, it is as if putting all three waves simultaneously. So, there was no situation of phase insensitive case in the 2 omega omega case. Here I can have either omega p, omega s input or omega p, omega s and omega i input.

(()) phase sensitive All of them will have to be phase sensitive And for theta equal to (()) This is always phase sensitive; so, the amplification not depends on the phase of the signal of omega with respect two omega. Here, because there is only omega s and omega i incident, the phases of the various waves automatically get fixed up to amplify the signal and to generate the idler.

What we require (()) ultimately they are still having a phase matching No, that is phase matching; phase match - that means, delta k is equal to 0, what I will show you later on is that if I input this signal, idler, and pump simultaneously, the signal will get amplified only under certain condition of phase relationship between the signal and idler and pump. If I change the phases of the signal, I will get de amplification, that means, I can either have omega p generating omega s omega i and amplifying omega s or omega s and omega i mix together to generate omega p, which will attenuate the signal.

(Refer Slide Time: 46:14)

$$E_i(z) = i \sqrt{\frac{\omega_i n_s}{\omega_s n_i}} E_{s0}^* \sinh g z$$
$$E_s^*(z) = E_{s0}^* \cosh g z$$

The image shows a handwritten slide with a grid background. At the bottom left, there is a circular logo with a star and the text 'NPTEL' below it.

So, depending on the phase relationships, I will either amplify the signal or attenuate the signal. So, the energy is getting converted some omega p to omega s or omega s to omega p, and that will be phase sensitive.

So, that will be phase sensitive nature of an amplifier; here, there is no phase sensitivity, because the idler automatically picks up its phase, so that the signal always gets amplified and that is the difference between phase insensitive and phase sensitive.

Phase sensitive we will look at little later, but right now, we will discuss a little bit more on these solutions and get some important conclusions from these solutions. **Anything else.?**

(Refer Slide Time: 47:13)

Q#3

Consider SHG in LiNbO_3
1 μm wave: Ordinary wave
0.5 μm wave: Extraordinary wave
both propagating along γ -direction
Obtain QPM period

λ	n_o	n_e
1 μm	2.236	2.160
0.5 μm	2.341	2.249

The image shows a handwritten slide with a grid background. At the bottom left, there is a circular logo with a star and the text 'NPTEL' below it.

So, we have a quiz now. Fine, the last quiz before the (()) This is quiz number 3. So, the problem is - we are considering second harmonic generation in lithium niobate, the 1 micro meter wave is an ordinary wave and the 0.5 micro meter wave is extraordinary wave and both of them are propagating along the y direction; this y is the principle axis, y direction. What is the quasi phase matching period required for this process - first order quasi matching period?