**Quantum Electronics**

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**Module No. # 03**

**Second Order Effects**

**Lecture No. # 12**

**Non - Linear Optics (Contd.)**

So, today, we will use this overhead projector like this, and come back to that lecture and see how it progresses. Do you have any questions?

There was a question raised last time, this is regarding the two solutions that we have obtained, for, when we tried to solve the problem for complete conversion, that means, essentially assuming, that E 2 also changes with that.

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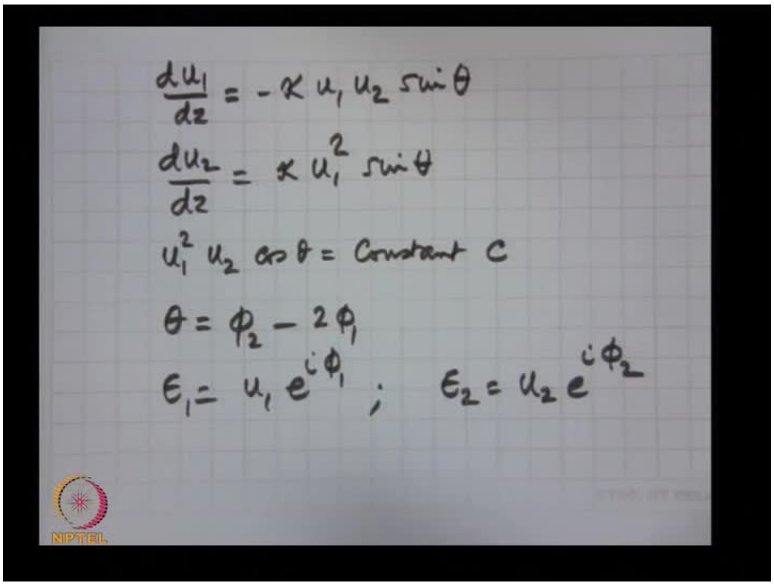


So, let me recall those equations; we have actually derived three equations. Let me write those equations; du 1 by dz is equal to minus kappa u 1 u 2 sin theta, du 2 by dz is equal to kappa u 1 square sin theta; and, we have also obtained u 1 square u 2 cos theta is equal to constant, some constant c. So, that means, as the waves progress, u 1 is the amplitude of the electric field at frequency omega, u 2 its amplitude electric field at 2 omega and cos theta was defined as phi 2 minus 2 phi 1; and so, E 1 was written as u 1 e to the power of i phi 1 and E 2 was written as u 2 exponential i phi 2.

So, there is no approximation in these equations; we are assuming that there is an interaction between omega and 2 omega only. Now, what we said was, for example, first situation is, when I have only the fundamental wave incident on the crystal, which means, that I have a situation where you have the crystal here and we have omega incident.

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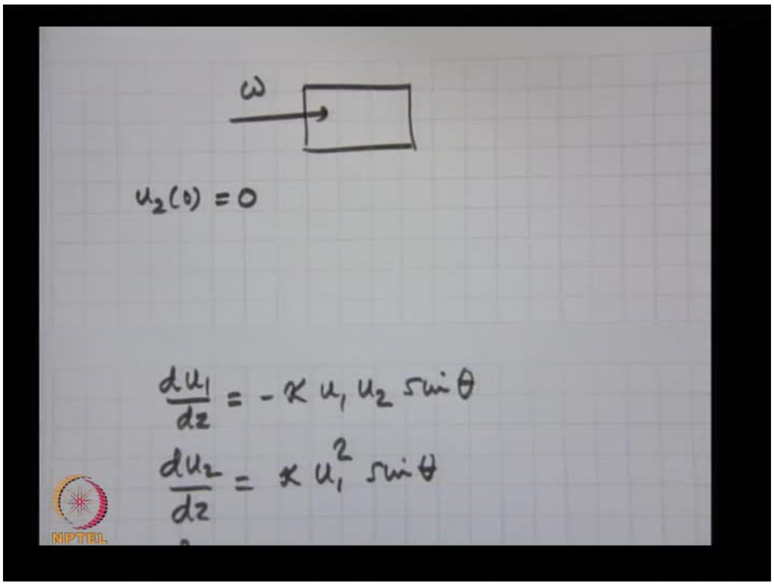


So, the condition you immediately get is u 2 of 0 is equal to 0; there is no second harmonic incident. So, this implies from this equation, this immediately implies, c is equal to 0; a constant is equal to 0.

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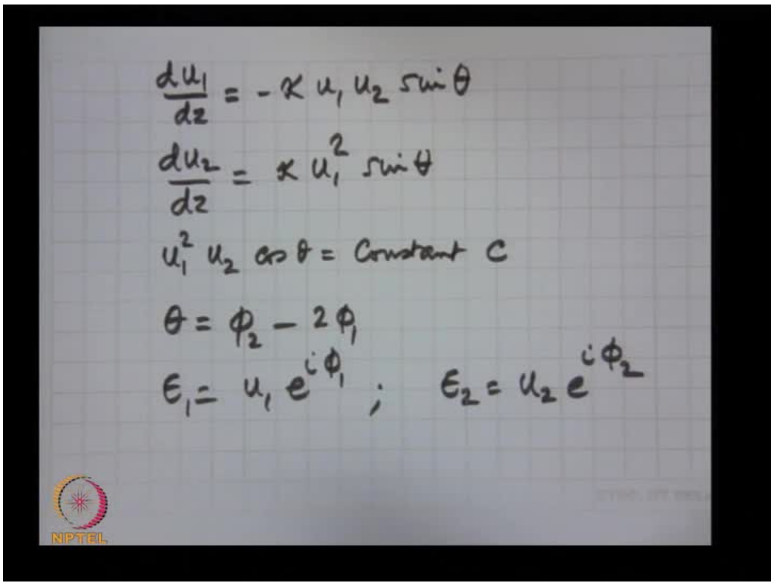


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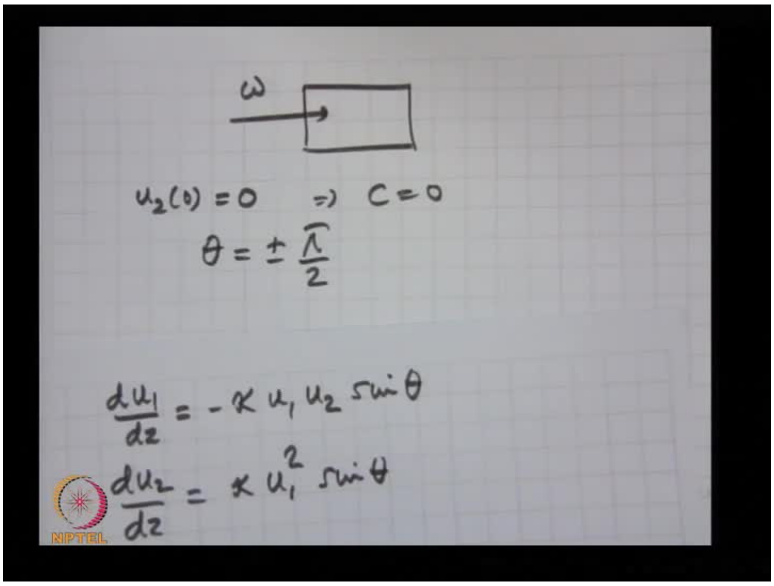


So, we said that this particular equation now implies, because u 1 and u 2 are not necessarily 0 as z changes, cos theta must remain 0; that means, theta must be plus or minus phi by 2; because c is a constant independent of z, constant of motion, so, as z changes, as the wave propagate, u 1 and u 2 change, but theta will remain such that c remains 0; theta will remain as plus or minus phi by 2.

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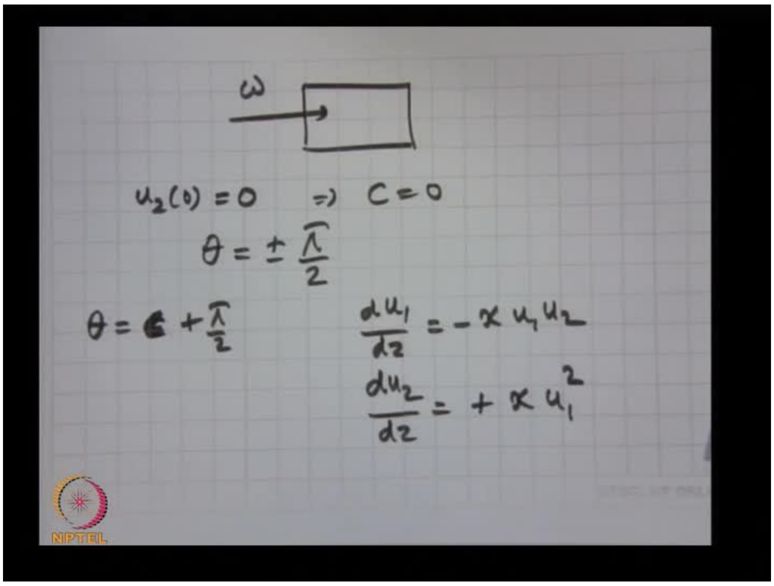


Now, let us look at which of the solution will work out. The first thing is, suppose I assume theta is equal to plus phi by 2, which is what we did in the class earlier; so, the theta is equal to plus phi by 2; then, you look at this equation, du 1 by dz is minus kappa u 1 u 2 and du 2 by dz is equal to plus kappa u 1 square.

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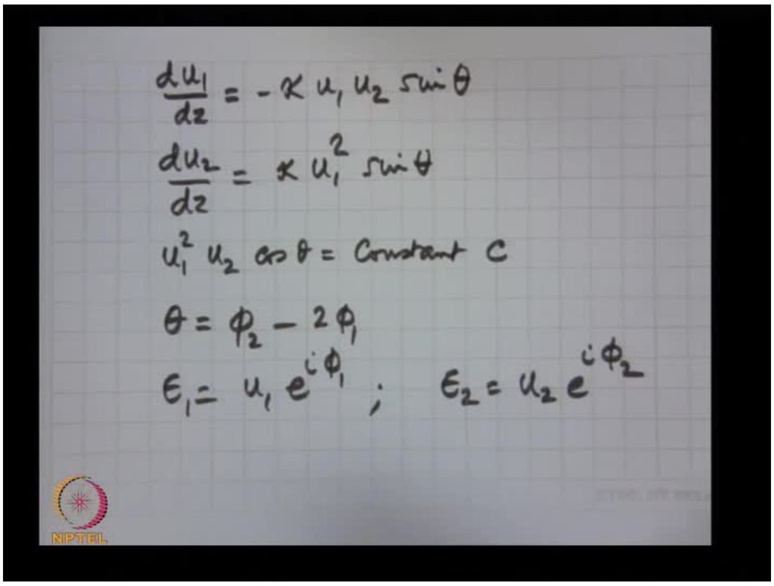


So, du 1 by dz is negative and du put by dz is positive; which means that, from z is equal to 0, if fundamental wave will go down in amplitude and second harmonic will increase; du 2 by dz positive, so u 2 will increase with z and u 1 will decrease with z. If I choose the other value of minus phi by 2, you will get du 1 by dz as plus kappa u 1 u 2, and du 2 by dz will become minus kappa u 1 square. Now, because you have not incident any second harmonic, this solution will not work.

So, theta automatically becomes plus phi by 2; all I am doing is, I am incident with the wave at omega frequency; the second harmonic gets generated within the crystal; the phase of the second harmonic will be, such that, it is phi 2 minus 2 phi 1 is equal to plus phi by 2 automatically. Because, a minus phi by 2 gives me a wrong solution; so, the minus phi by 2, I would have to decrease u 2, and u 2 is already 0. u 2 is the amplitude, the phase is contained in phi 2, so the second solutions theta is equal to minus phi by 2 is not an allowed solution in this problem; and, when the waves enter, when the omega frequency enters and propagates to the crystal, automatically, the second harmonic get generated, with a phase of, with the phase satisfying phi 2 minus 2 phi 1 0 plus phi by 2. You can ask any question.

Sir, one more doubt. Actually sir, it is at the boundaries. So, at the later stage, it also, theta is equal to phi by 2. Sir, I mean, added that later stage, there will be both of harmonic. So, how is it remembering that the earlier at the current position? So, why is it, it has to take care of what was at the volume? I mean, at the current position, there is both the waves, so why does not it work out with both?

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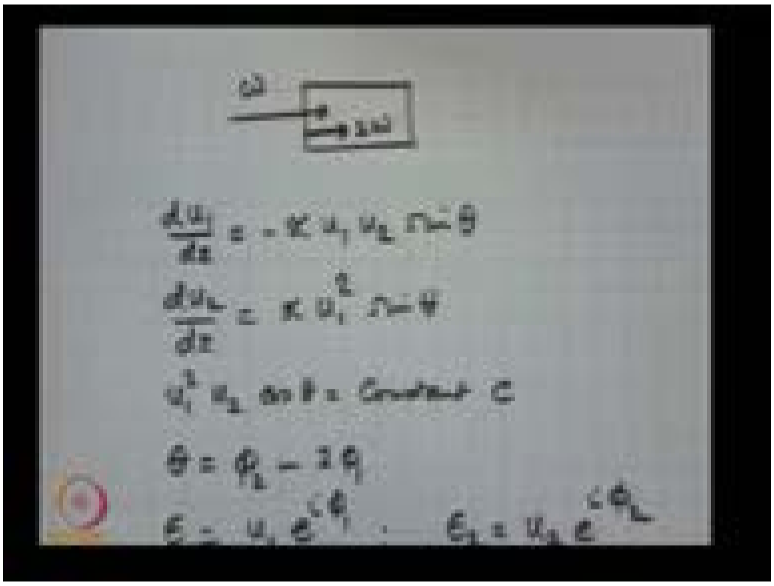

No. Theta is fixed to be plus phi by 2 because of my constant of motion; because my constant of motion says, u 1 square u 2 cos theta must be a constant. So, because I have started with u 2 is equal to 0, at z is equal to 0, c is 0; the moment c become 0 at z is equal to 0, it has to make maintain at c is equal to 0 always; because, this coefficient, u 1 square u 2 cos theta is independent of z; this what I showed you. So, because u 1 and u 2 are functions of z, the only way I can have this is a constant of motion is, theta becomes plus phi by 2 or minus phi by 2 automatically; and the minus phi by 2 solution is not allowed, so it becomes plus phi by 2; and it maintain plus phi by 2. The phase at which the second harmonic gets generated is automatically chosen by the interaction process, such that, phi 2 minus 2 phi 1 remains plus phi by 2 as the waves propagates.

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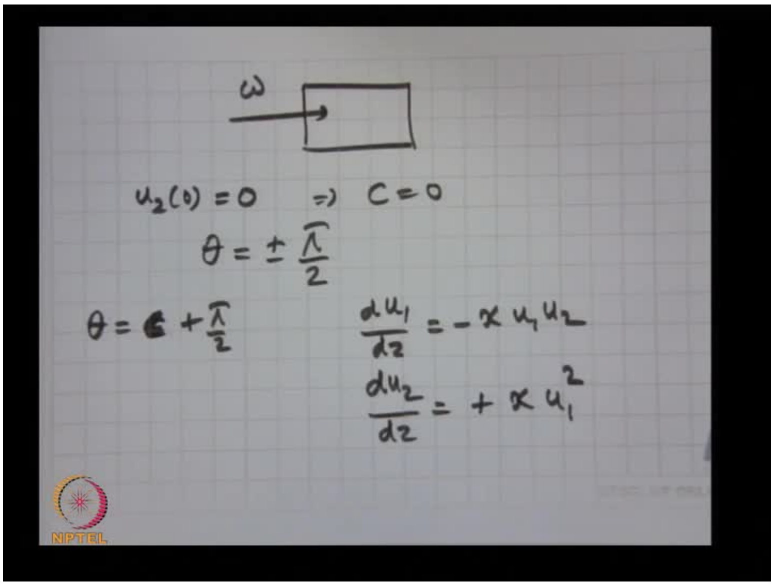


Now, because, as I showed you, if I solve these two equations simultaneously, I will never reach a situation when u 1 becomes 0; because, u 2 increases as tan hyperbolic function and the tan hyperbolic function will never become 1; it asymptotically tends to plus 1.

So, I will never reach a situation where all the fundamental gets converted into second harmonic, even if I have perfect phase-matched operation. So, as far as second harmonic generation from fundamental is concerned, the phase of the second harmonic automatically gets fixed at plus phi by 2; the generation is such that, it the second harmonic phi 2 becomes phi 2 minus 2 phi 1 is equal to plus phi by 2; it is just that it is fixed by the interaction process.

Here to be a single, it becomes constant as phi by 2

Yes

So, can we use this system as a memory system, I mean, which is remembering it phi by 2 (( ))

Yes. So, memory means, the phase difference between these two will always remain plus phi by 2, as the wave propagates; it does not change from plus phi by 2, always; if you start with 0 second harmonic with only the fundamental incident, theta is always plus phi by 2.

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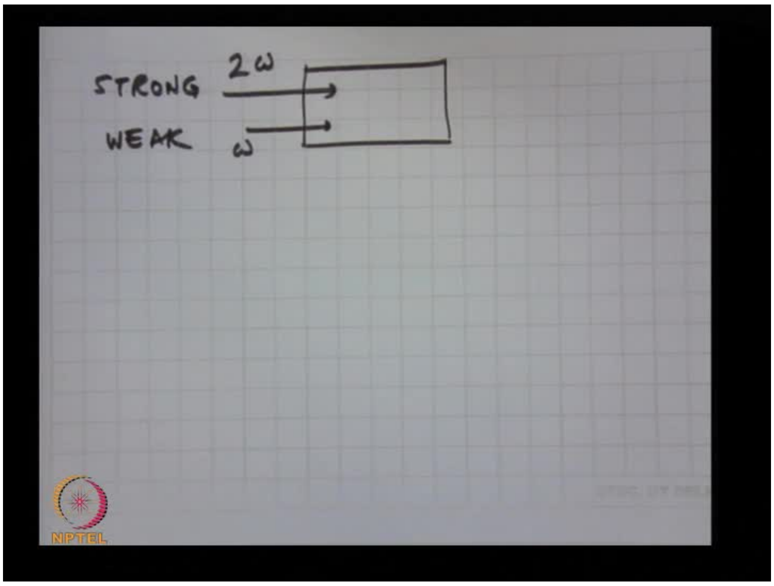


Now, if I look at the other situation, where I look at the amplification process; so here, I have not just omega incident, but I have a strong 2 omega incident and a weak omega incident. So, I have a strong 2 omega and a weak omega. Remember, the problem we started looking was, can I generate omega from 2 omega? And, I showed you, that in fact, if you look at these equations, you can show, that if your u 2 is finite and u 1 is 0 at z equal to 0, u 1 always remains 0, as I showed you from those first equations.

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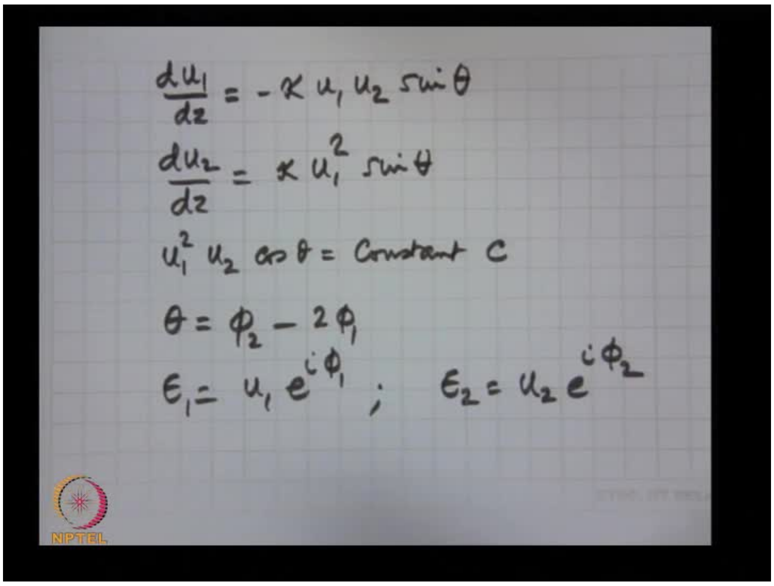


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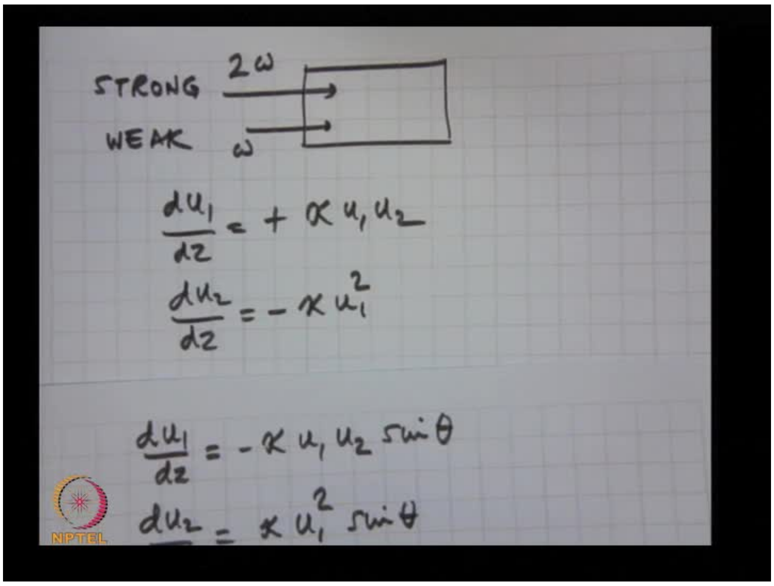


Now, because I am incident, both omega and 2 omega; as that is equal to 0, u 1 square u 2 cos theta is some value, depending on the value of u 1 and u 2 and theta and I choose. But if choose… So, now I have a choice of theta; if I choose theta up to be, such that, phi 2 minus 2 phi 1 is minus phi by 2, if I choose the condition that theta is minus phi by 2 then, I got an equation du 1 by dz is equal to plus kappa u 1 u 2 and du 2 by dz is equal to minus kappa u 1 square. In this case, the power in second harmonic will decrease, because du 2 by dz is negative and du 1 by dz is positive, so the fundamental frequency will can amplify.

Now, I have a choice of theta, because, at the input, I can choose theta is equal to minus phi by 2, c becomes 0, c is maintained at 0; and so, theta becomes minus phi by 2; so, sin theta becomes minus 1 and I get positive du 1 by dz and negative du 2 by dz. So, the omega frequency will get amplified if I choose this particular phase; if I choose the phase, theta is equals to plus phi by 2 then, I will have a power flow from omega and 2 omega.

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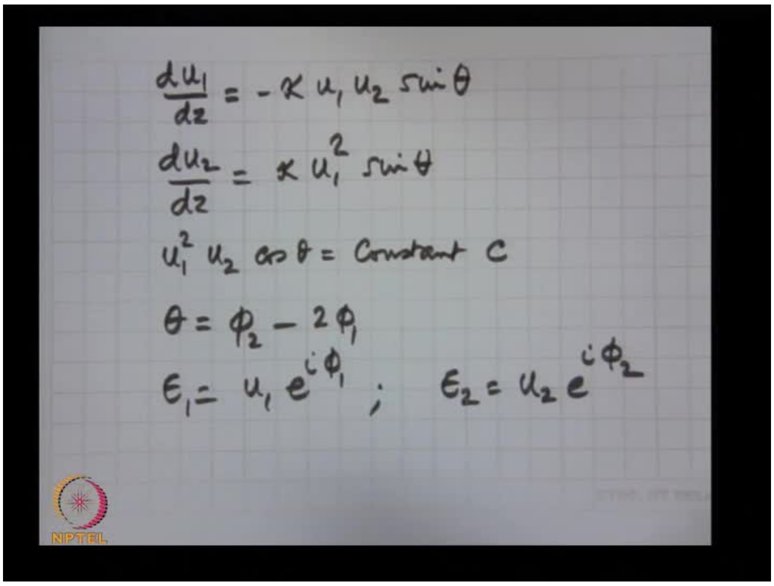


So, depending on the choice of my theta at the input, I can have an amplifier of omega or an attenuator omega; I can convert omega to 2 omega or 2 omega to omega depending on the value of theta, I chose at the input; and this is what I meant by saying, that this is a phase sensitive amplifier. Whether the omega frequency gets amplified or not, depends on the phase of omega, which is, if I fix a value of phi 2, as I change my phi 1, and whenever phi 2 minus 2 phi 1 is minus phi by 2, I would have amplification.

And whenever phi 2 minus 2 phi 1 is plus phi by 2, I will have attenuation of the omega frequency. So, this is what I showed you the other day; a slide showing that I can actually change the phase of phi 1 with respect to phi 2 and show experimentally, that whenever this quantity, it becomes minus phi by 2, I have an amplification; and whenever there is a plus phi by 2, I have an attenuation of the omega b.

So, that is a very interesting amplifier, it is a phase sensitive amplifier; and as i mentioned to you, we will later on discuss some quantum properties of this amplifier, because it has very interesting quantum features, in terms of noise of this amplifier vis a vis an amplifier, which is what is on population inversion principle, which is a typical amplifier that is used in lasers.

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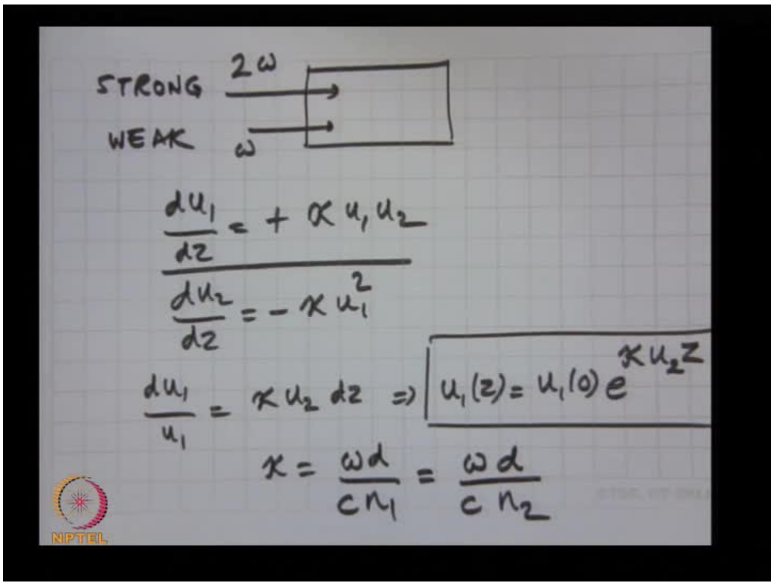


So, this is essentially, clear parametric amplification problem. So, let me try to look at an example and calculate what kind of amplification do I get, what is the kind of gain? So, if you take, for example, situation with theta as minus phi by 2, this is the equation I get for u 1 and u 2. If I assume no pump depletion, that means, if I assume 2 omega is very strong and the amount of power last from 2 omega is very small, I can assume u 2 to be a constant, integrate this equation.

So, I will have du 1 by u 1 is equal to kappa u 2 dz implying, u 1 of z is equal to u 1 of 0 into exponential kappa u 2 z; you can integrate this equation and get the solution as u 1 of z is u 1 of 0 exponential kappa u 2 z. What is u 2? u 2 is the amplitude of electric field at the second harmonic, and the input which does not change in the approximation, we are looking at it. And kappa is the coupling coefficient which we had, omega d by c n 1 which is also equal to omega d by c n 2.

Because, we are assuming phase-matching under which we have written these two equations, delta k is equal to 0. So, what will be the power gain coefficient of the frequency omega? Gain coefficient of, in terms of power; this is a amplitude, remember; so, you have a gain coefficient if you have an exponential gamma z, gamma will be the gain coefficient.

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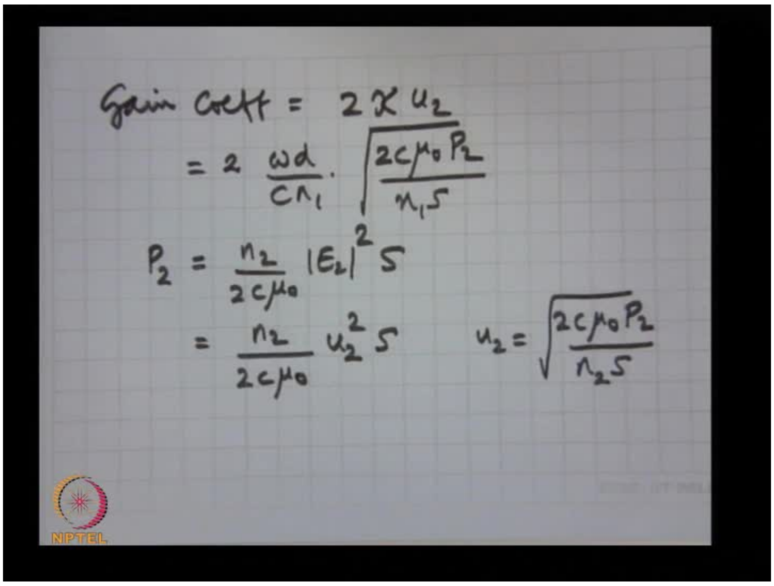


So, in terms of power, what will be the gain coefficient? 2 kappa times u 2, because this is the amplitude, the intensity will go as square of u 1, so it will be exponential to 2 kappa u 2 z; so, the gain coefficient is equal to 2 kappa u 2. So, let me substitute in this equation - 2 times omega d by c n 1. Now, for u 2, let me write this equation - P 2 is equal to n 2 by 2 c mu 0 mod E 2 square into area of cross section of the beam intensity multiplied by area cross section; this is equal to n 2 by 2 c mu 0 u 2 square into area.

So, this implies, u 2 is equal to square root of 2 c mu 0 P 2 by… So, the gain coefficient becomes this multiplied by this expression; so, u 2 which is square root of 2 c mu 0 P 2 by… this is n 1, because n 2 is equal to n 1. So, it depends on non-linear coefficient d, it depends on the power at the second harmonic; it is the presence of the second harmonic

2 omega wave which is actually leading to this amplification and converting from 2 omega to omega.

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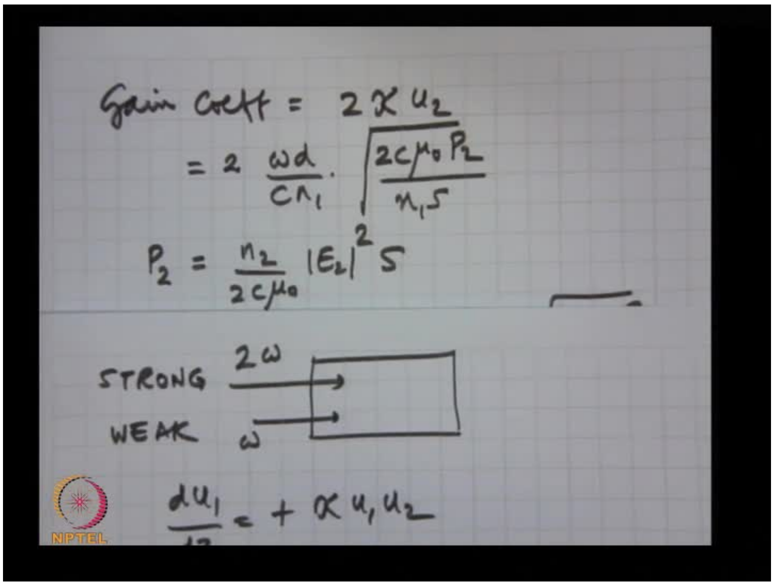


So, I mention this process, is that, what is happening here is, this 2 omega photon and the omega photons interact and this omega photon stimulates 2 omega photon to down-convert to 2 omega photon.

If I did not have this, if I have only 2 omega input, classically, I do not generate omega. But quantum-mechanically, this is an allowed process; 2 omega can be converted to omega, is an allowed process; just a reverse of omega can be converted 2 omega. So, that process is a spontaneous process and it is called spontaneous parametric down conversion. In the presence of omega, this omega line can induce this down conversion, which classically this question predicts; and this is something like a stimulated parametric down conversion process.

So, in the presence of this omega photon, this 2 omega photons are actually splitting and generating new omega photons; and it is exactly like the stimulated emission process, where there is complete phase coherence, means, the generated omega photons and incident omega photons; it is a phase coherent amplification, the spontaneous process is spontaneous, because there is nothing else to induce the emission process and so, that is a spontaneous parameter down conversion.

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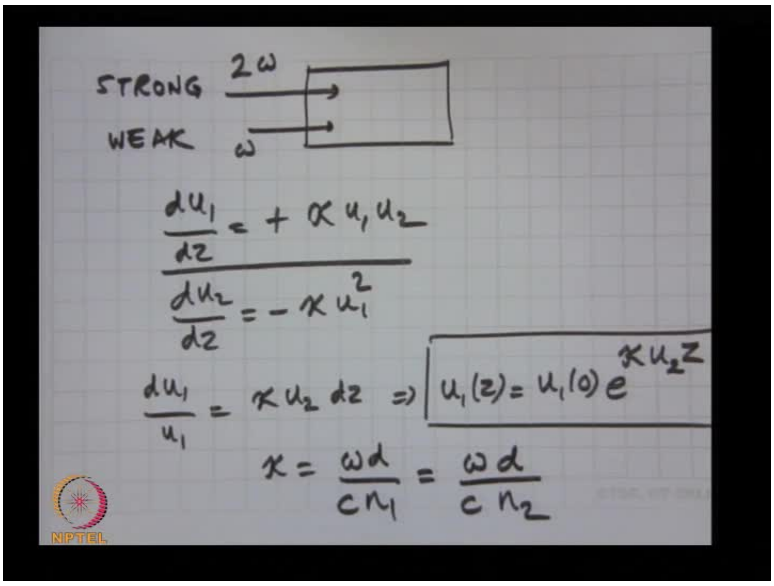


And here, it is a stimulated parameter down conversion process, where the omega photons interact with 2 omega photons inducing the 2 omega photons to split into 2 omega photons; and, such, that is the reason why the power in the fundamental wave is increasing with z; the electric field of the omega frequency is increasing exponentially with z.

Yeah, Mohit.

Sir, we have to induce this omega frequency.

When you are incident in the crystal, the presence of the omega photon along with the 2 omega photon, induces this down conversion process automatically. Let us continue in those equations.

But l i is already present there, or it has to the artificially.

I am incident from here.

We have to do it.

Yes, if I do not put this, then I have to have a spontaneous parameter down conversion process, which classical equation does not predict; if I input omega also along with 2 omega, then the classical equations predict an amplification process, then its consisted of quantum analysis and this process is essentially an induced process; it is like a stimulated emission process in a laser.

Sir, this process would not be continuous.

It will be continuous, because the omega is continuous to input.

Sir, but the (()) between the two thing, can we maintain that?

Yes, if I maintain, if I change it, obviously it will change.

It is not possible to maintain.

It is possible, because the technology is advancing; in fact, what people do to demonstrate is, I start from an omega, go to 2 omega, do not convert all the omega in to 2 omega; and then, I mix the 2 omega and omega and amplify the omega; and I change the phase between the omega and 2 omega waves from outside. so, I can actually show this amplification process; and to maintain the phase difference is not very simple; to maintain it at theta is equal to plus phi by 2 or minus phi by 2 constant.

Sir, the amplification keeps on being, then, as the wave propagates to other crystal.

Yeah.

So, let us say, from some part of the energy is converted from 2 omega to omega.

Yeah.

The light which rated at omega frequency; so, is it also in the same phase condition as…?

Yes, classically, the electric field simply increases in amplitude, so, there is… you cannot identify at the output, which is the wave which was incident, and which is the wave generated in the crystal.

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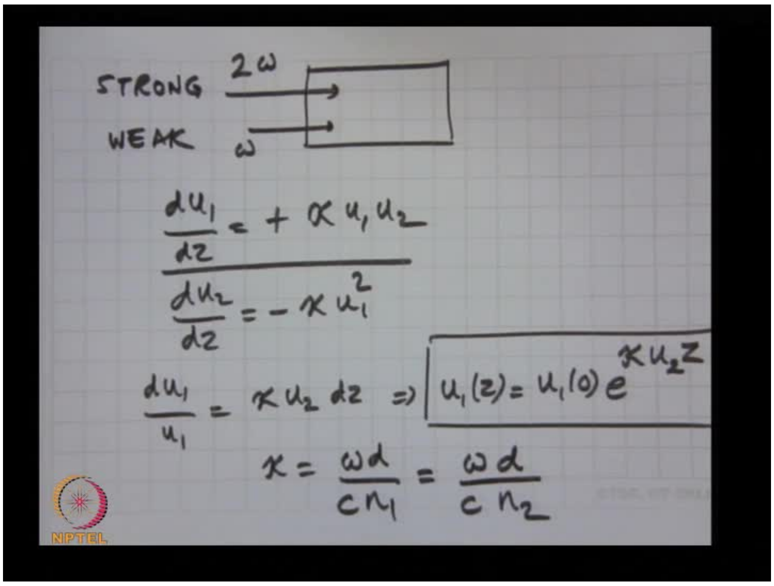


Right now, no photons, it is classical waves - right? So, the waves are incident, the wave is incident with certain amplitude, the wave comes out to the larger amplitude; of what portion of this was incident and what was generated, I have no by way of finding out; because, it is just increase in amplitude, just like a simulated emission process. In photon picture, it becomes little trickier, because I have to then, define, what is the phase of a photon? That is very tricky. So, right now, the photon picture I am just bringing in to show you, that actually, quantum mechanically, the omega is getting amplified because, the 2 omega photons was splitting and generating omega.

But experimentally, a gravity, we will, we do not need a pump or we do not need a omega frequency here; we could just stand at 2 omega and we would get a omega, because it is quantum-mechanically allowed.

Yes, you will get omega, which is spontaneous parametric fluorescence, but intensity will be very weak - with this process, has certain efficiency; we will calculate those numbers little later and efficiencies are very poor. The efficiency is… The emission probability is, as if you had 1 photon incident at omega. How much of photons will it generate? That will be what you will get in a spontaneous process; here, I am doing, what I am doing is, if I put even very little, 1 nano water of power at omega frequency, there are enough number of photons; huge number of photons are coming in and this process amplifies.

So, in an amplifier, it is interesting because, I can amplify any frequency omega; all I need is a crystal and a wave at a different frequency, at a higher frequency; as I told you, this is called degenerate case, where this is half the frequency, that means, 2 omega splits into 2 omega photons, but later on, just I have to finishing this - what we will look at is, when I have an incident photons splitting into 2 photons of lower frequencies, they need not be have the same frequencies.

And physically, when we performing the experiment…

Yeah.

So, both the beams have to be align on the same spot it…

In the same direction of propagation because, that is your direction where I am phase-matching.

No, direction is like, it can be spatially apart also.

No, they cannot be, they have to over-lapping; because, otherwise, if your 2 beams going like this, they are not interacting at all; I need to have an interaction between the 2 physically; so here, I am looking at plane waves, so, its approximation in the sense, that is a plane wave, it is a use wave; everywhere they are overlapping; I cannot have 2 plane waves which are not overlapping with their infinite extend.

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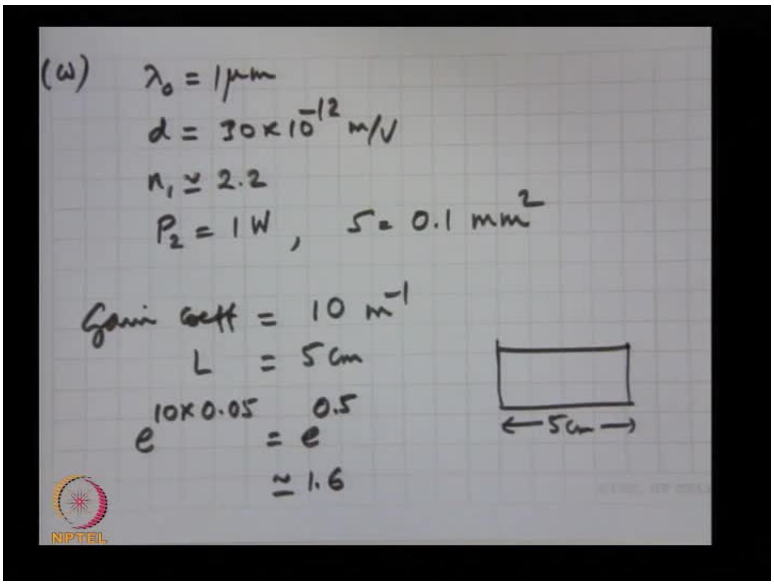


So, let me presume numbers and calculate, what kind of the gain coefficient we will get. So, the gain coefficient is given here; so, let me take a typical example. So, here is an example, where my incident lambda corresponds to 1 micrometer; this is the corresponding omega frequency; we take lithium niobate 30 tends to minus 12 meter per volt; we need a refractory index above 2.2, is extraordinary index of lithium niobate. Let me take a power of 1 Watt and let me take an area of 0.1 millimeter square. So, let me substitute these numbers into this equation. So, the gain coefficients will be…. So, you have in this equation, you have everything; you have omega from this wave length; you have the d; you have velocity of light in free space and 1 is known S is known; c, mu, 0 and P 2, everything is known. So, if you substitute here, what you get is, this is about, what units will it have?

One by length

One by length? It is exponential. kappa is… right? is the gain coefficient, so that is actually, is meter inverse; that is not a large gain coefficient because, suppose, I take a crystal of length 5 centimeters, so, by what factor will the power increase at the output?

E

Sorry. No, not E; here is the gain coefficient, length is 5 centimeter.

(( ))

E to the power 10 times 0.05, which is 10 e to the power 0.5, which is about 1.6. E is about 2.7; so 1.62 or something like that case. So, it is only 60 percent increase of power in a length of 5 centimeters. If you compare those with an amplifier with, simulated, I am sorry, population inversion, these are very very small.

A semiconductor laser has a very huge gain in thousands of meter inverse, because of population inversion. So, as an amplifier, this is not very interesting; but remember, if I put this amplifier within a laser cavity, within the pair of mirrors, I can convert the amplifiers as an oscillator. You have studying lasers in some course; so, I take this amplifier, put a pair of mirrors on either side of the amplifier, the stimulated emission that happens in this spontaneously generated light can reflect back and force inside the cavity; and as it propagates, it gets amplified and you can form a what is called as an optical parametric oscillator. This is an optical parametric amplifier where you have input; signal gets in getting amplified; there you have an oscillator in which you do not generate, you not put anything from outside except for the 2 omega beam.

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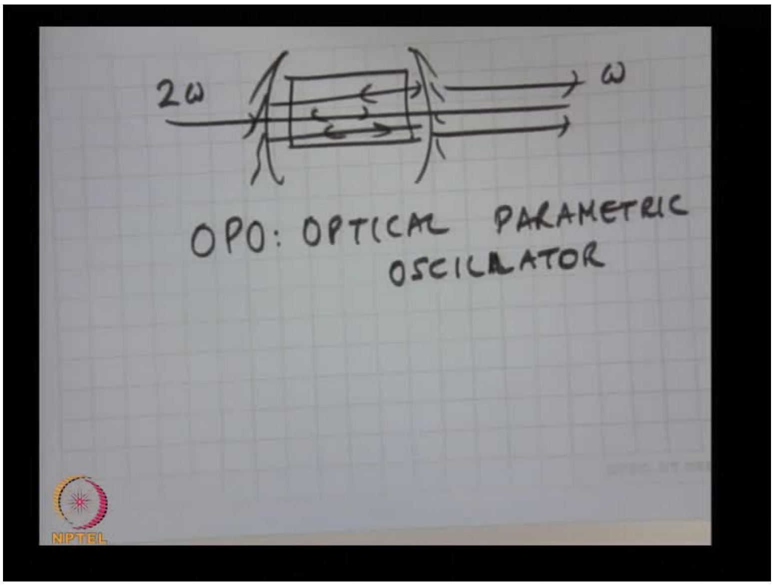


You need to put the 2 omega beam, so you can build an optical parametric oscillator, which we will discuss a little later by taking this amplifier and putting between a pair of mirrors. So, i have 2 omega coming in from here and omega get generated inside and I get omega coming out. The 2 omega passes through this mirrors, I could have situate… different situations are possible, so we will look at these; this is called the OPO Optical Parametric Oscillator. This is one of the very important tunable lasers available in the market; it is tunable because, I can actually, as I will show you later on, I can change the phase-matching condition and get the full frequencies coming out from here.

So, I will just to become a little more clearer as we go to interaction with 3 different waves, 3 different frequencies; here, we are only look at 2 different frequencies, omega and 2 omega. So, in general, I can have omega 1 omega 2 omega 3, 3 waves interacting; and, I will show you that this is a very interesting source of coherent radiation because, I can actually have a tunable output coming out. From here, so, I can start with a visible laser here, and come out with an infrared laser tuned over a very broad range of wave lengths.

So, all though the amplification is small, this interesting property of this amplifier is that, as I will show you later again, that, for an amplifier, a very important property of an amplifier is the noise added by the amplifier; every amplifier has noise, which means, if you have signal coming at 1 amplifier containing some noise, that amplifier will amplify the signal, will amplify the noise input, and we will also add its own noise. So, usually the signal to noise ratio which is a very important property at the output, is always worse than at the input; because you have a certain signal and noise, both of them get amplified with the same amount, and then you also have a noise generated by the amplifier itself.

So, this is a very important property of all amplifiers; the change in the signal noise ratio by the amplifier; and as I will show you later, that this amplifier can operate in a region where it does not add any noise to the signal. So, it can act like a noise free amplifier, and that is very interesting for many applications, where, in many times, your signals are very weak and if you want to amplify without adding noise, that is very interesting; any weak signal, if you try to amplify, that weak signal could be comparable to the noise levels and you will be worst of by amplifying than without amplifying. So, this is a very important property of this amplifier; other than that, these amplifiers are very important for making an oscillator.

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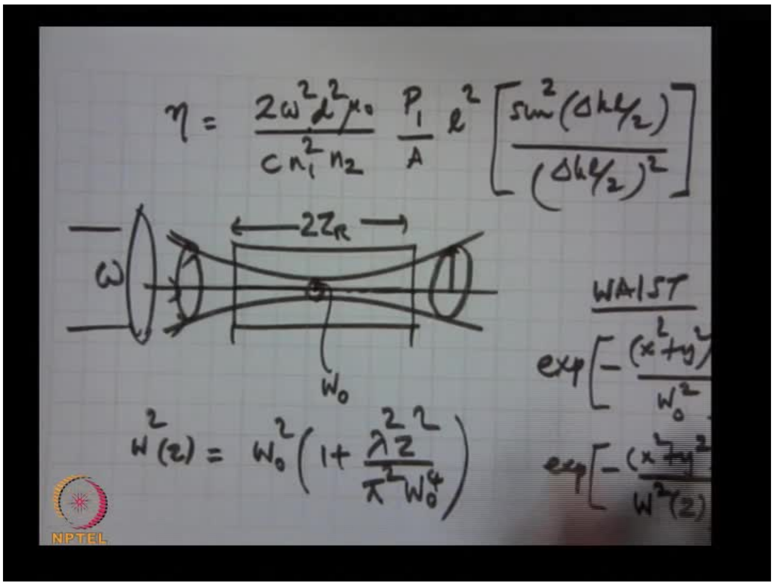


Before we move on to further discussion on C wave interaction, what I would like to do is, briefly present one interesting aspect, which we have a brief heuristic derivation or something which is very important. Remember, if I go back and look at the expression for efficiency for second harmonic, when we neglected depletion of the second harmonic; if you remember, we had obtained this equation eta is equal to 2 omega square d square mu 0 by c n 1 square n 2 P 1 by A l square into sin square delta k l by 2 by delta k l by 2.

So, what I have done is, I multiplied and divided by l square, keep this as a sync function, and there is an l square here, so, there is an efficiency dependent on l square; and this is assumed, this is derived assuming, that the omega wave and the 2 omega wave are plane waves; it also shows me that I can increase the efficiency by increasing P 1 by a, which means, for the same power of the fundamental, I can increase the efficiency by focusing, then we make omega frequency; I can reduce the cross sectional area of the omega frequency. So, let me try to approximately use this equation and try to estimate - Should I too much focus or should I less focus, what should I do?

So, for this, let me assume that I have a Gaussian beam, usually the laser beams have a Gaussian transfer distribution, so, I have a Gaussian beam which I try to focus into the crystal, this is that crystal. The omega beam comes in, there is a lens here, which focuses the omega beam into the crystal. So, this is the waist size of the Gaussian beam W 0.

Remember, I had given this formula right in the beginning; the 1 by width of the Gaussian beam changes with z according to this equation.

Actually, this line which I have drawing is the distance from the axis where the amplitude reduces by a factor of E. So, the Gaussian beam has a variation like this, exponential minus x square plus y square by W 0 square; I must write… This is at the input at z is equal to 0, at the waist; and at any other value, it goes as exponential minus x square plus y square by W square of z. So, this is a Gaussian beam which is getting focused and expanding from the crystal. So, what I need to do is… So, remember here, that if I try to…, in my hand touches, that will (( ))

So, this is the change of this size of the beam, area of a cross section; this W is the radius over which the most of the Gaussian beam's amplitude is present. So, beam coming like this, you focus, it goes down and goes up like this; and this W of z defines this distance. So, the cross sectional area is so much here; the cross sectional area is so much here; a cross section area is so much here. The beam starts with the large cross sectional area, becomes focus, and comes out.

Now, for Gaussian beams, I can define a distance where W changes by a factor root 2. What is that distance? From this formula, z becomes equal to, when this quantity becomes 1, then W increases by a factor of root 2. So, this is called the Raleigh range, so, this distance is equal to pi W 0 square by lambda. When you chose a distance is equal to pi W 0 square by lambda, then W of z at that point is root 2 times W 0.

So, I can approximately say, that the beam maintains its cross section over distance of the order of Z R; beyond Z R, these part size of the beam becomes more than root 2 times W 0. Now, in a focus beam like this, the intensity is varying as a function of z; in a plane wave, the intensive of the fundamental does not vary with z because, it is not getting focused; here, the beam is getting focused, so, the intensity starts from some value here, maximum at this point, and then starts to decrease again here; so, I can approximately define a distance, 2 time Z R, that means, from this value to this value, this is the distance over which the beam will maintain its approximate cross section; it is not exactly W 0 all the way, but it is W 0 here, it is root 2 times W 0 here, root 2 times w 0 here. So, I can approximately say, that if I use a crystal of length 2 Z R and W 0 is the spot size of the at the waist of the Gaussian beam, then I can achieve an efficiency assuming that the power, the intensity remains almost constant across this length.

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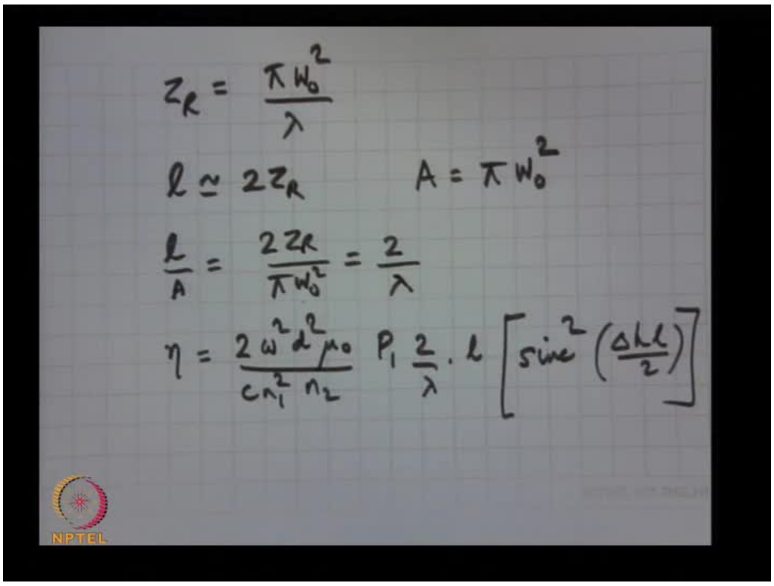


So, unlike a plane wave where I can chose arbitrarily A and l; here, for a chosen value of A, the area of cross section, l gets fixed; because, if I chose a certain W 0 value, the Z R value gets fixed because of W 0 and the wave length. If I chose a smaller W 0, that means, I focus more, z, I will reduce; if I focus less, I can have a longer length of interaction.

So, the length of interaction and the area of cross section of the beam, then get couple to each other; they are not independent quantities now; so, I can use this equation approximate and approximate that, l is 2 times Z R; the length of interaction can be assume to be 2 times Z R and the area of cross section A is equal to phi W 0 square. So, this quantity l by A which appears in this equation, becomes equal to… l by A, so, l is 2 Z R by A, is phi W 0 square, which is equal to 2 times 2 by lambda.

Because this equation here, Z R is phi W 0 square by lambda, so, l by A is 2 divided by the wave length of incident wave; this is assuming that the length over which I am interacting is approximately twice the Rayleigh range, where the spot size goes from root 2 times W 0, starts from root 2 times W 0, becomes W 0 here, and becomes root 2 times W 0. So, if you now use these equations for l by A, what you see immediately is, eta becomes equal to 2 omega square d square mu 0 by c n 1 square n 2 P 1 l by A is 2 by lambda into l into this sync function, sync square of delta k l by…

So, what you find is, when your phase matched with plane waves, efficiency increases quadratically with length of interaction; if you double the length of interaction, you will have 4 times the intensity, 4 times the efficiency. But, in actual practice, when you try to focus the beam, the efficiency does not increase the l square, this increases l; because, you need to worry about the fact, that if you try to decrease the area of cross section, the length of interaction will also reduce automatically.

Your crystal may be very long, but beyond a certain length, the intensity are too low for efficient conversion. So, this is a short of an approximate derivation or approximate estimation of what will happen to the efficiency, if I were to use a focused beam rather than a plane wave. So, in a plane wave, the efficiency goes quadratically with length for phase matched interaction; for a focused beam, the efficiency actually goes down; it only increases as a function of length or, proportion of length. So, if you double the length with the focused beam, keeping in mind, that this focusing and the length are related to each other, you will have only double efficiency, not four times the efficiency.

So, this is always to be considered when focusing beams because, the efficiency of non-linear interactions is proportional intensity of the incident wave, not the power, intensity of the incident wave; and to increase the intensity, you may try to focus, and if you try to focus, you are restricting the length of the interaction process. So, I thought, this may be, an interesting aspect which we have not derived rigorously, but we have just estimated from the plane wave expression, we have sort of used the approximation and tried to estimate, what will happen if I have a Gaussian beam instead of the plane wave.

So, do you have any questions? Tell now, what will we do is, after this, we will move in to a 3 wave interaction process; I will just introduce this today and in the tomorrow's class, before we go on to made minus. Yes, any questions? I have left the problem of calculating the bandwidth of interaction. Can any of you try it out? Yes, there is a problem in a tutorial sheet; on the sheet also. Number? But, to be able to obtain an equation for the bandwidth of interaction process,…

Is lambda wave or 2 waves?

No, you have to worry about dispersion d n by d omega.

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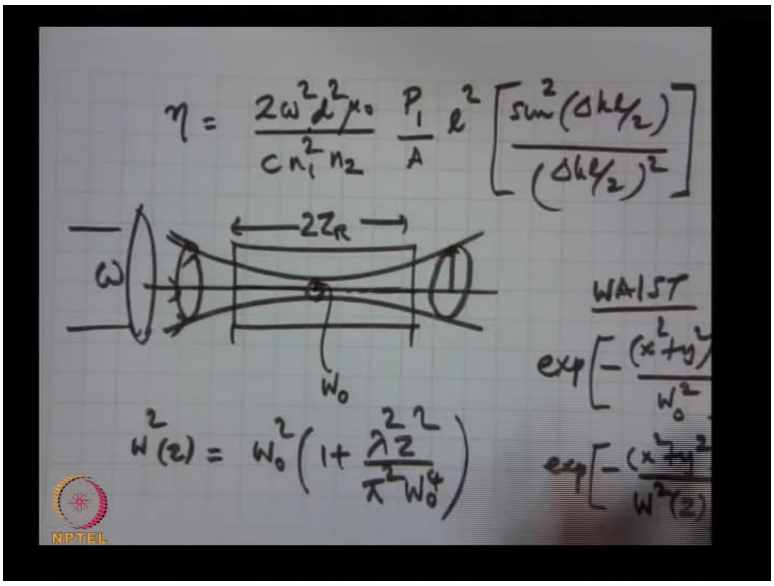


Please work it out again, otherwise, we will discuss in the class; the problem is that, if I look at this interaction process, if I look at this efficiency - at delta k is equal to 0, I have maximum efficiency, and as delta k changes, the efficiency drops down.

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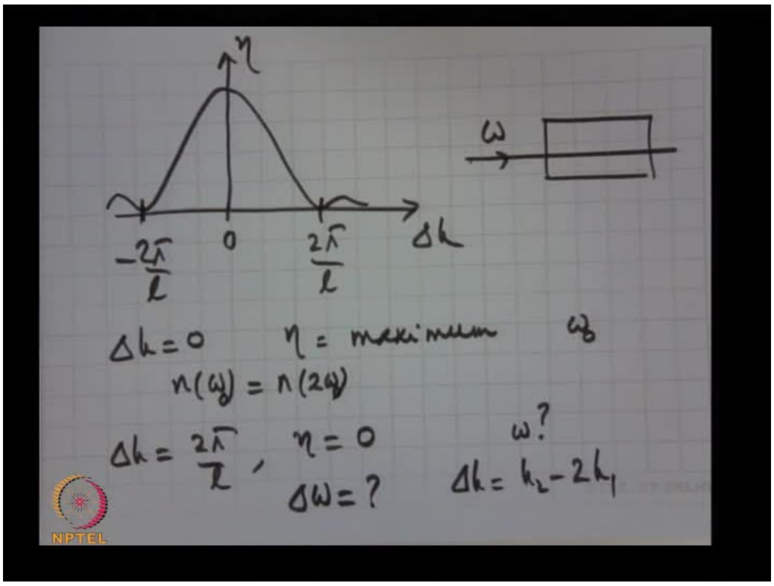


Remember, we have plotted this figure as a function of delta k, so, if I plot eta versus delta k, I will get a sync function like this. So, what is the value of this? What is the value of delta k when the efficiency goes to 0? delta k l by 2 is phi, this is 2 phi by l and this is minus 2 phi by l So, which means, at delta k is equal to 0, eta is maximum; and this corresponds to what? 1 particular omega value, some omega value, for which n of omega is equal to n of 2 omega.

Now, if delta k becomes is equal to 2 pi by l, the efficiency becomes 0. So, to what omega does this correspond? That is the question. Which means, I have this crystal; I have omega here, tunable laser. So, if I chose an omega, let me call this omega naught; if I chose an omega is equal to omega naught for which n of omega naught is n of 2 omega naught, I have delta k is equal to 0 and I have maximum efficiency.

Now, I change my frequency from omega naught, slightly away from omega naught; as I start to change my omega from omega naught, n of omega and n of 2 omega will vary and this condition will no more be satisfied; and when this condition is not satisfied, delta k becomes finite; and when delta k value becomes equal to 2 phi by l, for a given length of interaction, and for that frequency, the efficiency will become 0.

So, the question is, what is a change in frequency required, from the center of frequency, for the efficiency to drop to 0? That means, how precisely should I adjust this omega frequency for maximum efficiency? This is actually contained in the last problem here; also, with actually taking into numbers, because I have given this l m a equations, defining the refractive indices of the crystal as a function of wavelength, so, but what would be interesting is, obtain an analytical expression for delta omega; what in the delta omega for which delta k is equal to 2 pi by l? Because delta k is k 2 minus 2 k 1, so, this is k of 2 omega minus 2 of k omega. So, what is the shift from omega naught that I need to have, for the efficiency drop to 0, that means, what is the change in omega for which delta k, a corresponding delta k becomes 2 pi by l?

And that will me the bandwidth and this is very important because, when I tried performing an experiment, I would need to know - to what precision, I need to maintain omega? See, if omega changes by this value, corresponding to delta omega, the efficiency drops to almost 0. So, that is, the bandwidth of interaction and that I would like to do, sort of, calculate and obtain an expression for delta omega; and let me tell, it will contain the derivatives; because, you have make some Taylor expansion of n omega around omega naught.

If we assume delta omega to be very small, we can sustain with only one term.

Yes, but you need to get a delta omega value.

If we assume that delta omega is very small, as compared to omega.

Yes, we will assume; yes, I agree.

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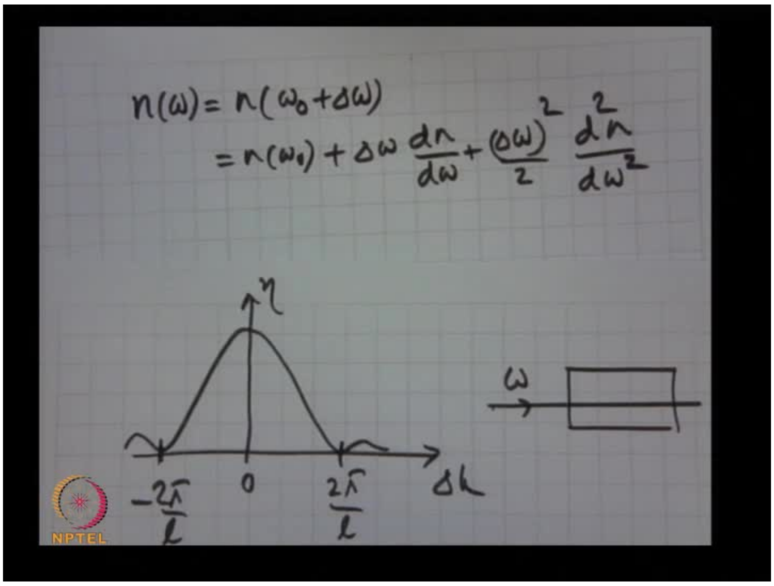


n of omega will be equal to n of omega naught plus delta omega; so, this is n of omega naught plus delta omega d n by d omega… plus you keep the next term and find out whether I need to keep it, I will check; similarly, I need to do for the second harmonic and the second frequency, and then,

Then, when we take the difference from the terms, we will…

Yes, so, I leave it to you; this is what I leave the problem to you, solve it out. Any other question?

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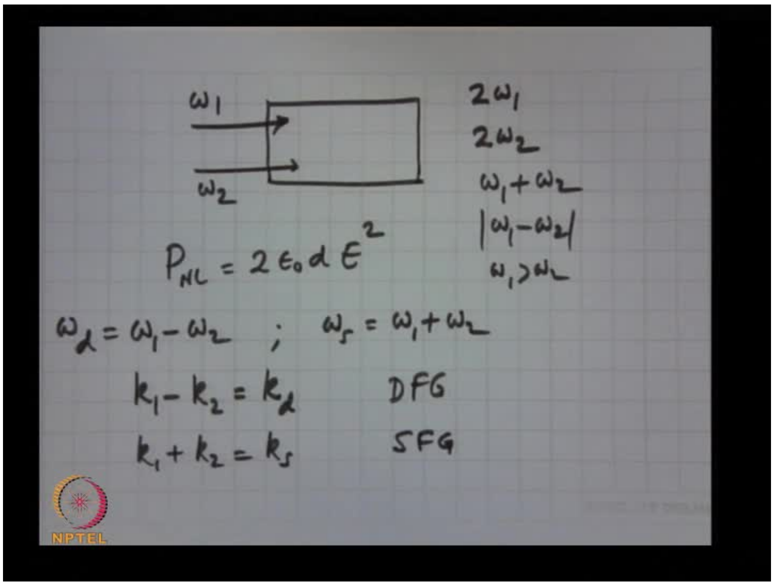


Let me just briefly introduce this other process, and we will do the mathematics in the next class. So, the problem is the following, that now I have omega 1 and omega 2 incident; two wave, waves at 2 frequencies are incident on the crystal; remember, non-linear polarization is 2 epsilon 0 d E square.

Now, I will get back to the scalar equation, because as I showed you, I can always obtain a scalar equation which I can which is obtained from knowing the polarization states of the waves and the crystal d tensor etcetera. So, I do not have to actually solve the equations with the complete tensor notation, I have sort of approximated by scalar equation. Now, can you tell me, what frequencies can come out from this interaction process? So, what can exist at the output? 2 omega, 1 can come out 2 omega, 2 can come out second harmonic omega 1, second harmonic omega 2; and then plus omega 2 and mod omega 1 – right? That is the only possible output. Now, what will you get from n, what frequencies will come out? Will all of them come out, only some of them come out, or what will determine which comes out?

Phase-matching condition

Phase-matching condition. So, for example, to generate omega 1 minus omega 2, what is the phase-matching condition I need to generate, I need to satisfy? So, let me assume that this is omega 1 is bigger than omega 2; let me assume omega 1 is bigger than omega 2, so, this is omega 1 minus omega 2; so, to generate omega 1 minus omega 2 which I call as the difference frequency, what is the phase-matching I need to satisfy?

k 1 minus k 2 is equal to… the difference in propagation constants, because you will see when you substitute here, and you get for the non-linear polarization velocity, remember, I had 1 quiz; the velocity of a non-linear polarization becomes equal to the velocity of the electromagnetic wave at omega d when you satisfy this condition.

So, this is our difference frequency generation; if you chose k 1 plus k 2 is equal to k s, where k s is the propagation constant corresponding to this frequency, then you will generate some frequency generation; and of course, if you satisfy the corresponding second harmonic generation conditions for omega 1 and omega 2, you will get the second harmonic omega 1 or second harmonic omega 2. Now, at normal practice, it is not possible to satisfy all these conditions simultaneously, for the same set of frequency at the input. So, you will generate one of these processes or none of these processes.

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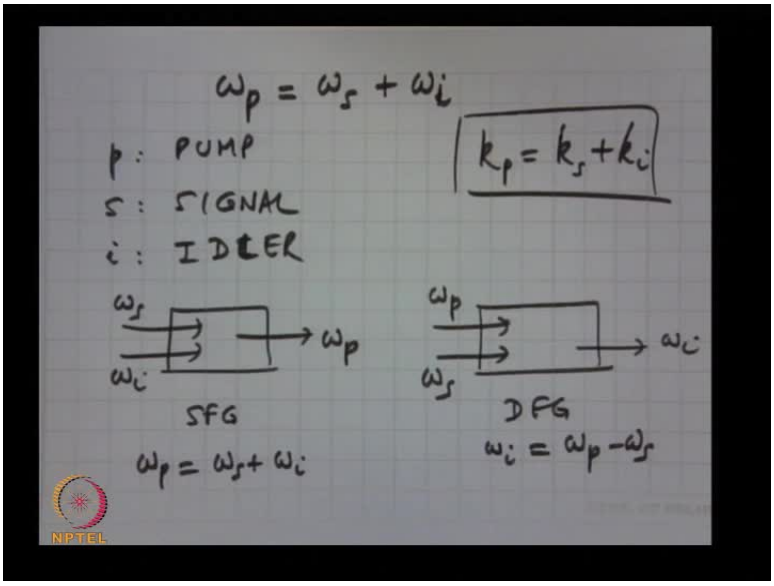


You may not satisfy any other wave matching condition, your omega 1 omega 2 incident, I mean, outcomes omega 1 and omega 2; nothing else. So, what we will study is, this process in which I have two waves incident, and generating a third wave; now, to be more specific, we will call this by the following names.

So, I will assume that the three frequencies are satisfying this condition. So, p stands for pump, s stands for signal and i stands for idler; instead of taking omega 1 minus omega 2 omega 1 plus omega 2, whatever it is, I will have three frequencies; they are defined, such that, omega s plus omega i is equal to omega p. So, if I want to look at some frequency generation, I will assume the incidence of omega s and omega I, to generate omega p.

This is some frequency; if I want to look at difference frequency generation, I will have omega p and omega s or omega i, generating omega i; and this is difference frequency. So, omega i here is equal to omega p minus omega s; and omega p here, is the same equation - omega s plus omega i. This is just to make a consistent, otherwise, what will happen is, if I give you omega 1 and omega 2, if I generate difference frequency, I must call if omega d; if I generate some frequency, I must call it omega s etcetera. So, just to avoid this confusion, the three frequencies are, which you will consider, are always related through this equation - omega p is equal to omega s plus omega i; if I want to look at some frequency, the input waves will be called omega s omega i.

If I want to look at difference frequency, I will call them omega p omega s, so, if omega s becomes equal to omega i, I come back to the old process, second harmonic generation and sub-harmonic generation. Because, if these two become equal, omega P becomes twice omega s and that is corresponded to second harmonic, this will then correspond to second harmonic generation, and this will correspond to the parametric down conversion.

In general, omega s and omega i are different. All I need to them, is to satisfy this sum must be equal to omega P; and all these processes will require the following phase-matching condition k p is equal to k s plus k I, that means, the propagation constant at the pump frequency must be equal to the propagation constant of signal frequency plus propagation constant at the idler frequency. I can actually generalize this to Quasi-phase-matching, which I will come to get little later. So, what we will like to do now, next time, is to derive equations relating the electric field amplitudes at omega p, omega s and omega i.

Till now, we have two equations corresponding to omega and 2 omega. Now, we have three equations, one corresponds to omega p, one omega s and one for omega I, so, depending on the process which I want to study, I will pick up those two equations, assume the third one to be under platted.

For example, here I was assume usually, that omega p, that is, which is called pump, is a very strong electric field; and this is a weak electric field; and this generates omega i. Both of them are strong, so omega P is very strong, so the electric field at omega P will be assumed, would be constant in solving those equations. If I look at this equation, I will assume one of this, omega s will be very strong and omega i will be very weak, and I will assume the electric field at omega s does not vary with z; I neglect that equation. So, I can use those equations to solve any of these problems.

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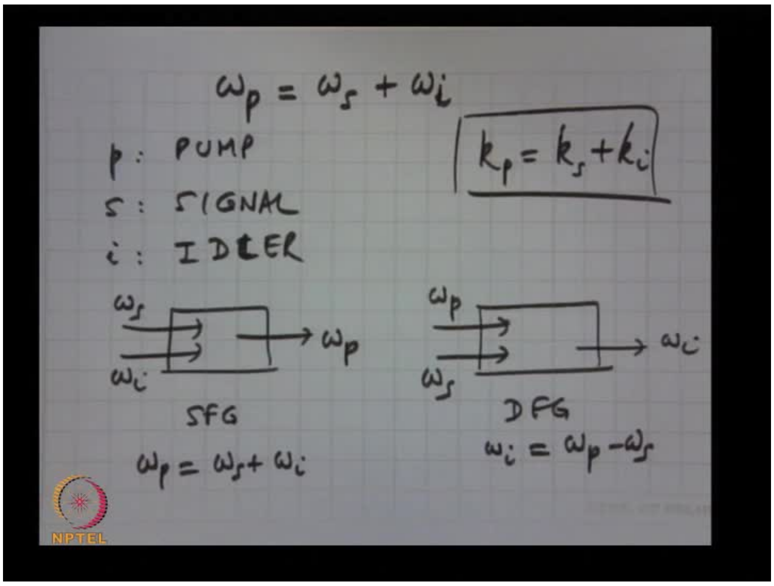


So, all the non-linear processes that you can generate with these three waves; this is called 3 wave interactions. The 3 waves are interacting to form all the non-linear processes; this process used for example, to convert light at infrared frequencies visible. This frequency is higher density frequencies, so, suppose I have a signal coming at 2 micron wavelength, I can mix the 2 micron wavelength with say, 600 nanometer wavelength to convert this to a new frequency, which is very close to 600 nanometers.

So, 2 micron I may not have good detectors; I have good detectors at the visible wavelength, so I can have a very interesting detector by using some frequency generation; I think, the frequency of the signal from 2 microns to 1 micron, half a micron and I have good detectors; silicon detectors are very good, very efficient in the visible region; this is used to generate infrared from visible, and this is what will be used for parametric amplifier and parametric oscillator.

In fact, I will show you here, these three equations that omega s signal gets amplified in this process. So, we will derive these three equations and solve these three equations in the next class. So, do you have any questions?

Thank you.