Quantum Electronics
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Module No. # 03
Second Order Effects
Lecture No. # 11
Non - Linear Optic

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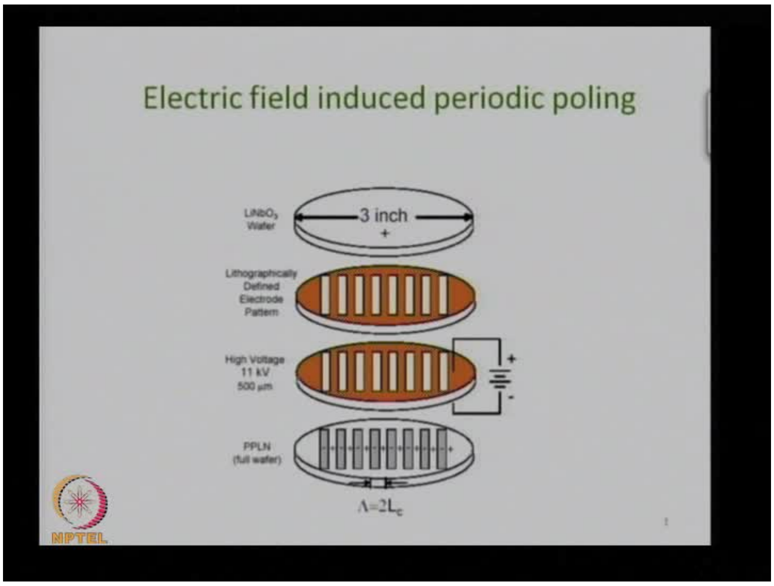

Before we continue with discussion of the mathematical formulation, I will show you some slides on this poling. How to generate periodically domain reverse structures in lithium niobate? So, as I was mentioning to you, lithium niobate is a ferroelectric material and the direction of polarization. There is a permanent polarization in the material. So, one takes a lithium niobate wafer, which is the plus is written there, that means the z axis is pointing up, its optic axis is pointing up, and then as I was telling you one lithographically defines an electrode pattern; and then applies a high voltage typically about as is written here, 11 kilovolts and 500 micrometers. So, that is 22 kilovolts per millimeter the breakdown strength is much higher.

So, what happens is where the electric fields are applied? Domain switch from vertically up to vertically down. So, you get a periodic poling of the crystal. And you can actually, have a more complex electrode structures to achieve this.

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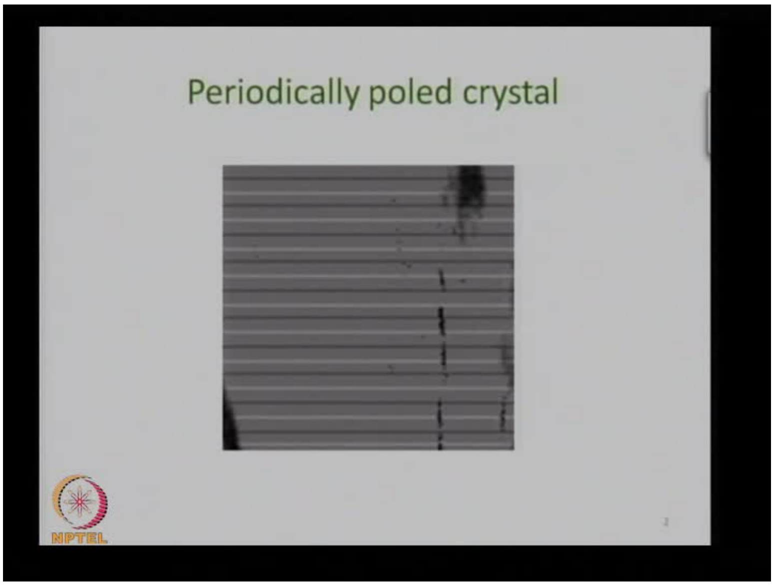

So, for example - let me show you, the top surface of the substrate after poling. And what happens, is you can actually show the poling, by etching the z plus and z minus surfaces get etched at different rates. So, this structure that you see is actually, the periodically poled substrate; with this in one of them and the other one is this, z minus z plus z minus z plus. Its periodic you can, you can, see it is very clearly periodic with a certain period. And the propagation direction is like this vertical perpendicular. So, these domains are like this and you are propagating perpendicular to the domains. So, you get a periodic poling which is possible in lithium niobate by periodically applying electric fields and so, this is now a standard technique, which is used to pole crystals of lithium niobate.

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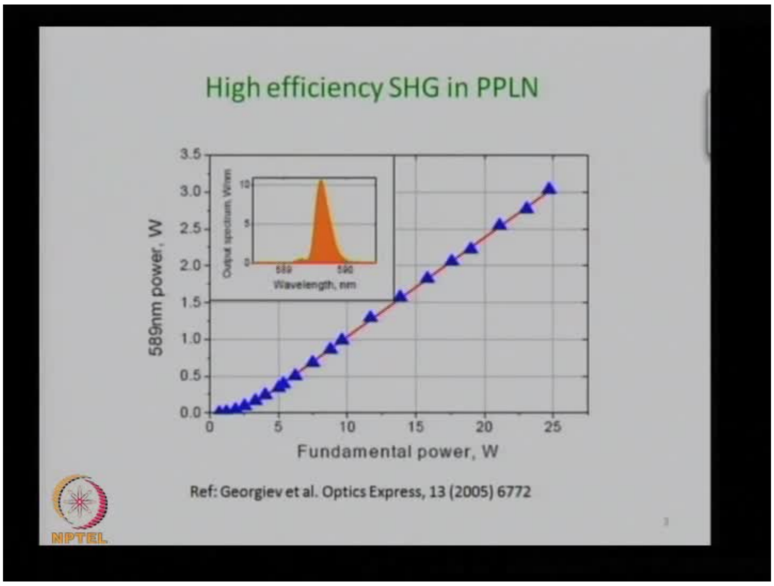

Let me show you some examples, of high efficiency generation in using periodically poled substrates; here is a report from a journal optics express, which is a journal published by the optical society of America and you can see here for about 25 watts of fundamental power, you can generate, something like 3 watts of second harmonic power. So, that is a good efficiency and as you can see that shows the change of the fundamental power versus the second harmonic power versus fundamental power. And this is just to show you that you can achieve very high efficiencies by using periodically poled crystals.

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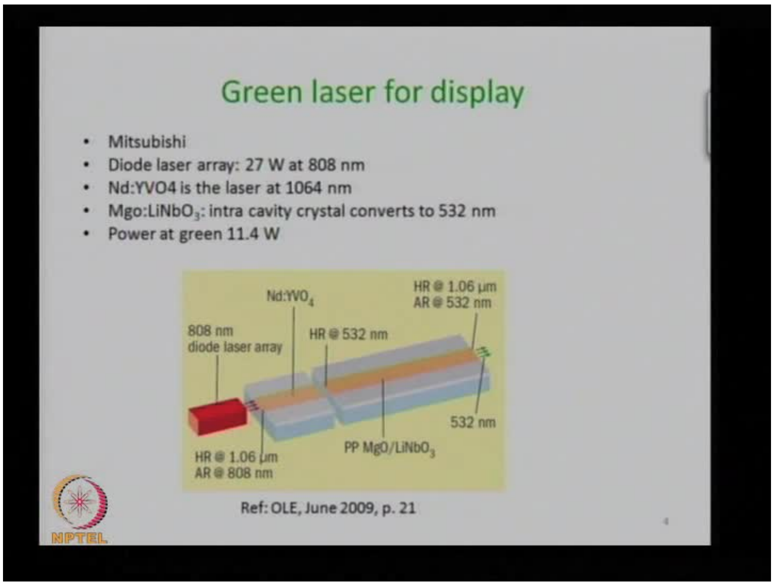

In fact, in a recent publication in 2009 June, they have actually shown, you can generate green light using this second harmonic generation. In fact, there are some laser pointers- green laser pointers they have actually a red laser, a second harmonic generator within the green laser pointer.

So, you can actually convert 808 nanometers to 404 nanometers actually it is 808 nanometers is used to pump a lutetium. Why we opt for crystal, that generate 1064 nanometer. The 1064 nanometer then is converted to 532 nanometers using a magnesium oxide doped lithium niobate crystal and you can see, the powers coming out at the green 11 watts. The argon laser, which is a huge laser in the laboratory that generates a few watts of power, so that is a huge power and these are now being seen for application for - I mean - laser displace, you have you need to have the three primary colors and this is one of the primary colors green laser.

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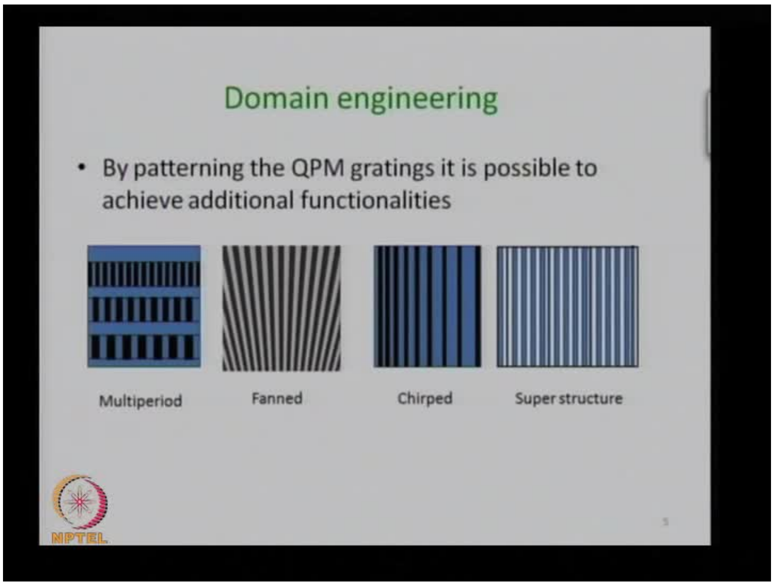

Let me, show you some interesting configurations, you can do by making crystals with multi periods as you have one period, then second period, third period. So, depending on which period you use, you can achieve phase matching for different frequencies, it is remembered that the phase matching period, depends on the frequency of the fundamental waves. So, you can actually make a crystal with multiple periods and use this multiple periods to achieve phase matching at different fundamental frequencies or you can actually, fan the grating like this. So, depending on the vertical position where you are coming. The period that will be encountered by the light will be different.

So, you can achieve tunable phase matching essentially. So, you can actually move the crystal and depending on where you are focusing the laser beam, you can have different phase matching condition satisfied; you can have a chirped, where the spatial period is changing with position. You can have super structure. So, there is a one period, which is the internal period and then there is another period, which is this is being repeated again and again the super period gratings. So, you can actually do a lot of interesting engineering to achieve different functions multifunctional, as you will see later on for difference frequency generation the periods are different. So, you can do second harmonic generation and then difference frequency etcetera, in a single crystal.

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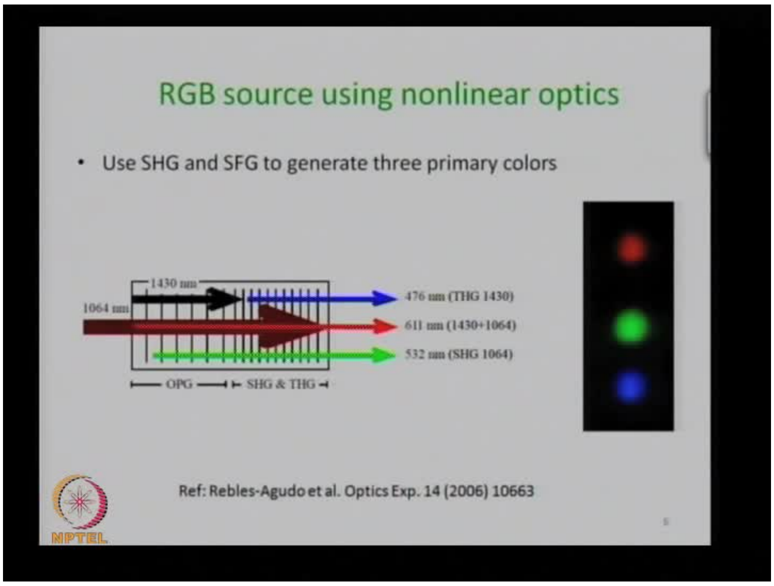

So here is an example of, how in a single crystal you launch 1064 nanometer light, use optical parametric generation to generate a new frequency. Then use second harmonic and third harmonic generation to generate multiple wavelengths simultaneously. This is a single crystal in which you generate multiple domains of different periods, each period is meant to achieve different nonlinear effects essentially. So, finally you can generate all the three primary colors. So, there is a lot of activity in using this nonlinear interaction to achieve new wavelengths, which are otherwise not possible, because conventional lasers have a fixed wavelengths which are emitted by the laser.

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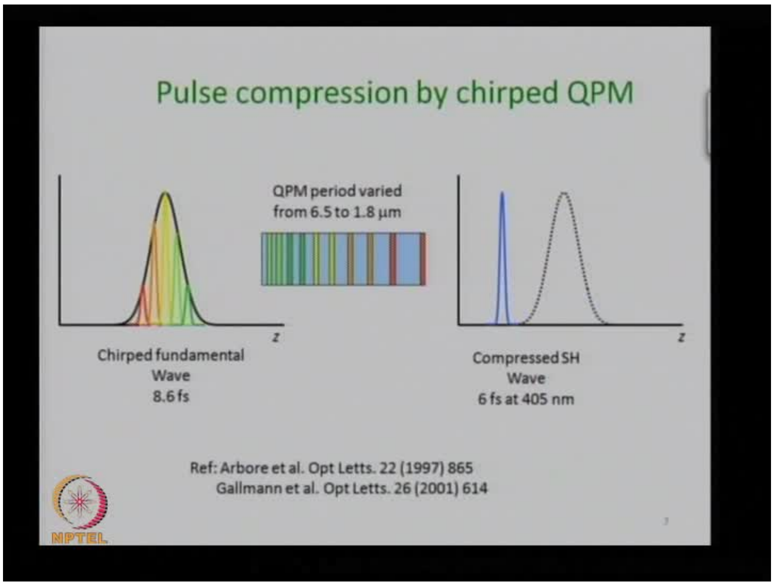

You can actually do more interesting things of compressing the pulse, where it generates a second harmonic and so here is a very interesting period paper in which you have a period varying from 6.5 to 1.8 microns. And that you can show is actually a fundamental waves, comes in with a duration of 8.6 femtoseconds; and it gets compressed into second harmonic to about 6 femtoseconds; see you can do all of change that is by essentially engineering the domains of the crystal, which is not possible if you have birefringence phase matching. So, quasi phase matching gives you a lot of flexibility in achieving multi-functionalities.

Sir, in the figure, that you showed that different colors were generated, three colors are generated. So, there what happens is that first the incident wave comes and its second harmonic successor gets generated? Then that second harmonic of that second harmonic is generated.

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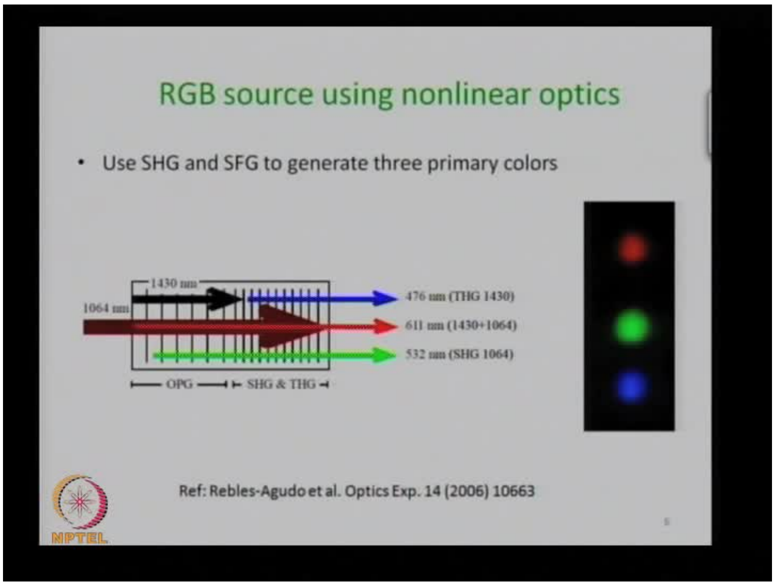

No, you are putting in 1064 nanometers; you are also putting at 1430 nanometers. So, it generates a difference frequency that is the optical parametric generation. So, we will discuss this difference frequency generation little later. Then there is the second harmonic of 1064 is 532 nanometers, which comes out. Then the 1430 and 1064 mix to generate a sum frequency generation at 611 nanometers and then, you have a 476 nanometers coming from the 1430 nanometer.

So, you actually multiple processes nonlinear process all depending on chi 2. Please remember, chi 2 process can be second harmonic sum frequency, difference frequency. So, the generated frequency is so powerful. Now, it is not very low power. Now, the efficiency is quite high that you can use the generated light to further undergo; further, nonlinear processes for which you require different periods and so the periods can be different at different position of crystal to compensate, for the phased mismatch for that interaction process.

Is it possible that the efficiency for second harmonic generation is that high, I mean if you use chirped gratings, these chirped grates, then is it possible that the efficiency for the first, second harmonic generation in so high; that further we can use a period that second harmonic of that second harmonic can be generated, I mean the efficiency can be…

no that is not chirped, I will have to generate two different gratings; one grating for fundamental second harmonic, another grating from second harmonic to the fourth harmonic of the fundamental .

2 omega to 4 omega

So we cannot do it in one?

No, because the period required are very different. One period is depends on n omega and n 2 omega, the another depends on n 2 omega n 4 omega and because of strong dispersion, as you go to the higher frequencies, the periods required may be very different, you cannot chirp it so much, there is no point in chirping, because chirping I need for a to extend in the bandwidth of the interaction process.

But otherwise, I know the period to convert from lambda 1 to lambda 2 omega 1 to 2 omega into 2 omega. Once I convert omega into 2 omega then I put another period to 2 omega to 4 omega.

So, you will send in omega and out will come 4 omega, you would not, you would not, care what is happening inside? So, it is possible in general to have multiple processes taking place simultaneously.

Point was that we can incident a light, which is at a given frequency – low.

At a lower frequency, which is on the lower end of the visible line? So, we can generate two omega from that. And then we get a net four omega, from that so we can reach from violet to red. From the in that process so using

No, this is the other way round red to violet.

Red to violet,

In only one input,

Sure, this is what I said, it is possible to go from omega to 4 omega, by going from omega into 2 omega and then 2 omega to 4 omega.

Essentially it comes from the outputs of the 2 omega also.

Surely, it is coming out here 532 is coming out here and 532, which is generated can also be mixed with other wavelengths at input to generate new wavelengths, for that I need to calculate, what is the QPM required? period required, and I need to make that period in microscope.

So, what we will now do is a continue our discussion, on the second harmonic generation that we were doing the other day, we were actually discussing the situation where the efficiency could be very high that is when I do not neglect the pump depletion.

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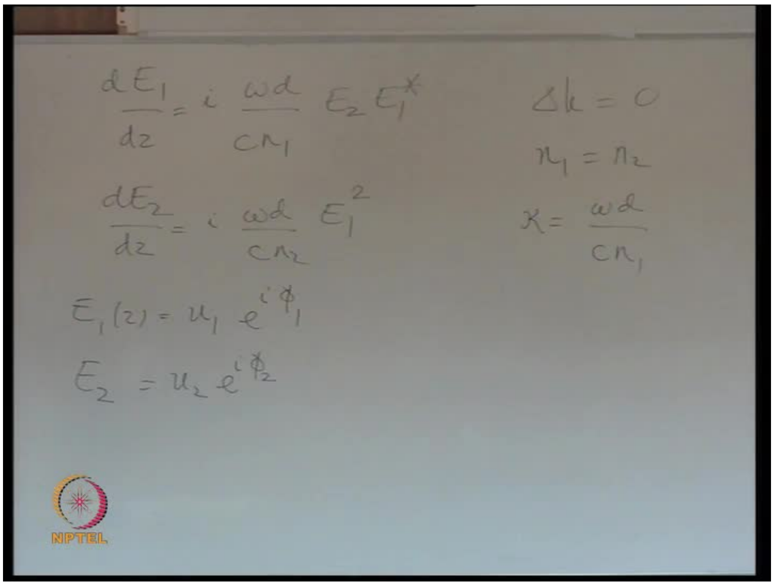

Remember, we started with these equations d E 1 by dz is equal to i omega d by c n 1 E 2 E 1 star and d E 2 by dz is equal to i omega d by c n 2 E 1 square. So, I am assuming delta k is equal to 0, I am assuming delta k is equal to 0.

Anyway let me lighten it, I forgot to switch.

Is that the maximum brightness can you see?

So, we had assumed delta k is equal to 0 phase dispatched is equal to 0 and let me call this coefficient as kappa. So, because delta k is equal to 0 n 1 is equal to n 2 and let me call this as kappa is equal to omega d by c n 1.

So, what we did was we substitute at E 1 of z is equal to u 1 exponential i phi 1 E 2 is equal to u 2 e to the power i phi 2 into these two equations, then where u 1 and e 2 u 2 and phi 1 and phi 2 are all real quantities. We can equate, the real imaginary parts on both sides and got the following four equations.

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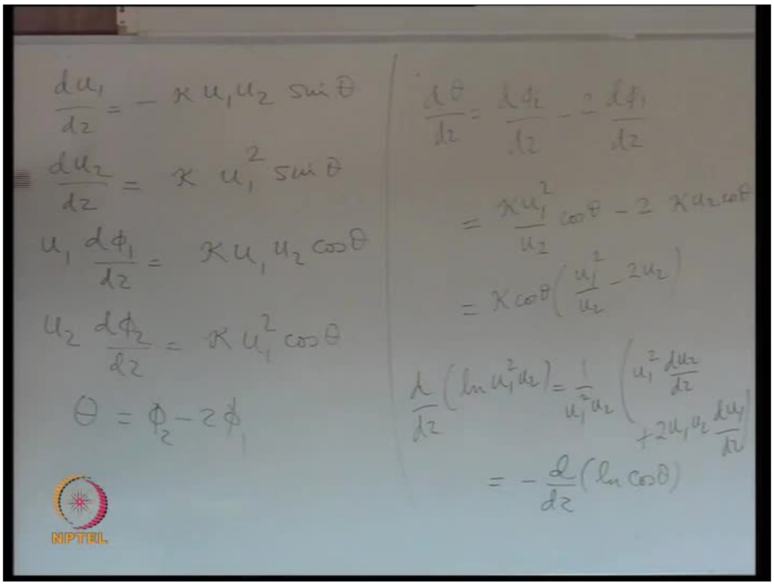

Let me rub these equations. So, what we got was du 1 by dz is equal to minus kappa u 1 u 2 sine theta du 2 by dz is equal to kappa u 1 square sine theta u 1 d phi 1 by dz is equal to kappa u 1 u 2 cos theta and u 2 d phi 2 by dz is equal to kappa u 1 square cos theta, where theta was defined as phi 2 minus 2 phi 1.

So, what I do is I substitute, the complex fields E 1 and E 2, which I write as u 1 times exponential i phi 1 and u 2 times exponential i phi 2 substitute into those equations and equate the real and imaginary parts on both sides; and I get instead of two complex equations I get four real equations now. What I will do is I can actually solve these equations, by the following procedure.

So, let me calculate, d theta by d E z this should be equal to d phi 2 by dz minus 2 d phi 1 by dz theta is phi 2 minus 2 phi 1. So, I substitute for d phi 2 by dz and d phi 1 by dz from here so, this is equal to kappa u 1 square by u 2 cos theta minus 2 kappa u 2 cos theta.

So, this is equal to kappa cos theta into u 1 square by u 2 minus 2 u 2. So, it is a mathematical trick that I am going to do, to just solve these equations. So, I get d theta by d z is so much. Now, let me calculate, so you can also calculate d by d z of log u 1 square u 2,

This can be calculated, this will be in terms of so… this is actually 1 by u 1 square u 2 into u 1 square du 2 by dz plus 2 u 1 u 2 du 1 by dz and I have expressions for du 1 by dz and du 2 by dz here which I can substitute here and this is simple exercise, you can show that this is actually happens to be equal to minus d by dz of log cos theta.

So, if you substitute for du 2 by dz and du 1 by dz here, you will see that the right hand side is essentially related to d theta d cos theta by dz. Due to this…

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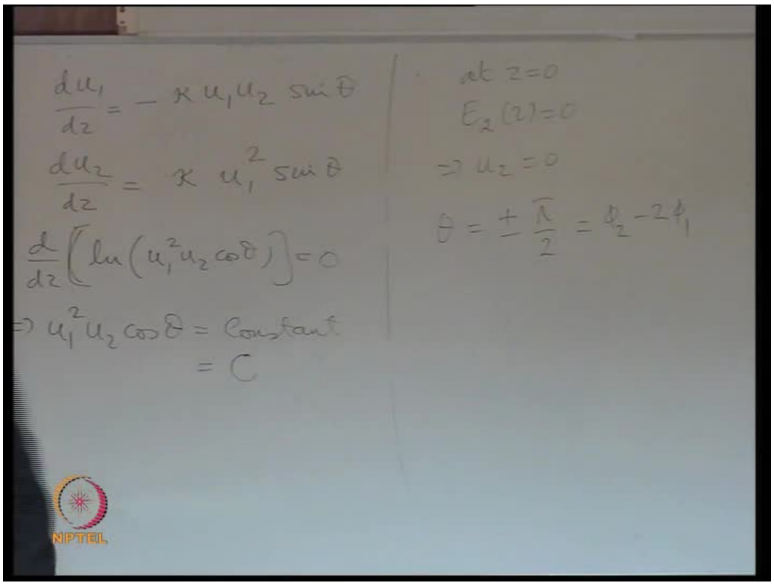

So, this is actually minus 1 by cos theta into minus sine theta d theta by dz. So, if you expand this you will get in terms of d theta by dz, you can use this equation and show that these two are equal, which implies essentially that d by dz of…. So, if I bring it to the left hand side I will get d by dz of log u 1 square u 2 plus log cos theta is equal to 0.

So, what does it imply log of u 1 square u 2 cos theta is equal to 0. So, this implies u 1 square u 2 cos theta is a constant of motion independent of z. As the waves propagate u 1 and u 2 and theta everything changes, but u 1 square u 2 cos theta remains constant.

It is just simple mathematical equation that I am using to get this condition. Now, you note that for the case of second harmonic generation, what is the initial condition, I have only the fundamental incident on the crystal. So, at z is equal to 0 E 2 of z is equal to 0 there is no second harmonic incident.

So, I have a crystal in which I launch wave at omega frequency. So, this implies u 2 is equal to 0 because E 2 is u 2 times exponential i phi 2 and u 2 is equal to 0 implies C must be 0 because at this is a constant of motion; so, at z is equal of 0 also it is valid. So, at z is equal to 0, I have u 1 square of 0 u 2 of 0 cos theta of 0 and because u 2 of 0 is 0 the constant is 0 and as it changes u 1 will change u 2 will change. So, how will I maintain this equation?

Theta has to be cos theta must be 0, which means theta must be plus minus pi by 2. So, theta will become and remember theta is phi 2 minus 2 phi 1.

Sir, we have taken that n 1 is equals to n 2.

Phase matched

We are solving this equation, only under phase matching, it is possible to solve in the original paper by Angstrom. I will give you the reference in 1962 physical review a. This equation is solved in more general terms, which in terms of elliptical integrals and so on. So, I am not going to do in details, but I just want to show you that it is possible to solve this equation, when you have phase matched situation, it is much easier. So, delta k is equal to 0; I get those two equations and then I am trying to solve this equation under this condition delta k is equal to 0.

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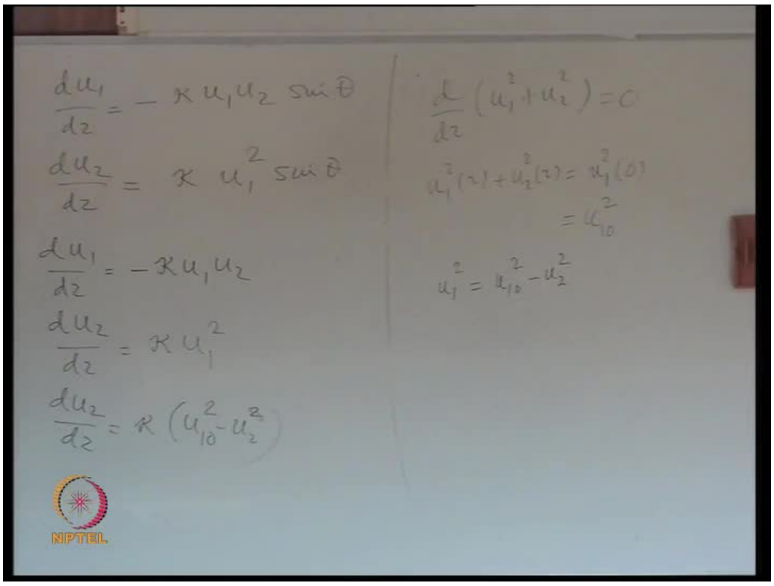

So, once theta becomes defined like this, these two equations will give me. Now, these two equations gets simplified to the following two equations. So, du 1 by dz. Let me, assume theta is equal to plus pi by 2, I leave the problem to you, to do for theta is equal to minus pi by 2 for theta is equal to plus pi by 2 minus kappa u 1 u 2 and du 2 by dz is equal to kappa u 1 square sine theta becomes 1. Now, again I want to leave for you to show these equations, satisfy the following condition d by dz u 1 square plus. What is this equation? Energy conservation, please remember this is under n 1 is equal to n 2 otherwise, you will have n 1 mod E 1 square by 2 c mu 0 plus n 2 by 2 c mu 0 mod E 2 square must be constant.

Because n 1 is equal n 2 is simply becomes E 1 square E 2 square or u 1 square is u 2 square. So, this implies u 1 square of z plus u 2 square of z must be equal to u 1 square of 0 because at z is equal to 0 there is no second harmonic incident, I am assuming that the crystal there is only fundamental wave incident in the crystal. So, let me call this u 1 0.

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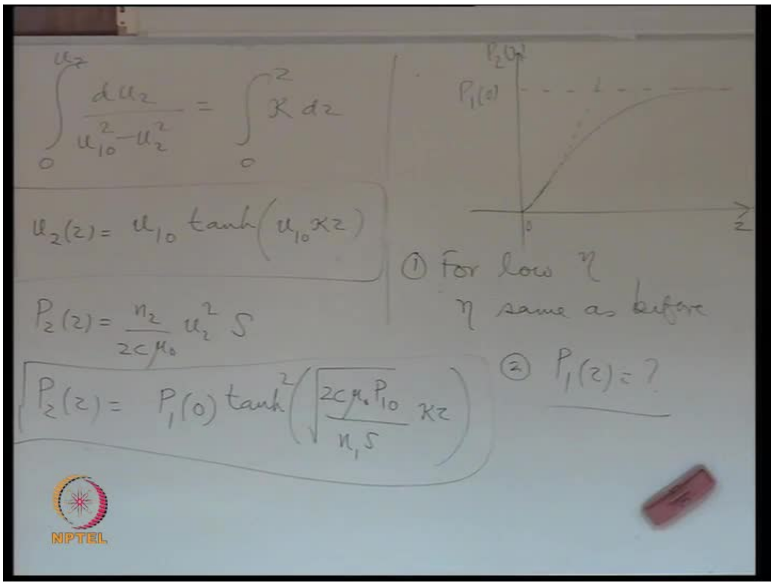

So, I need to solve this equation, so, u 1 square becomes u 10 square minus u 2 square and so, this equation simplifies to du 2 by dz kappa times u 10 square minus. So, I can actually integrate this equation, I will have integral du 2 by u 10 square minus u 2 square is equal to integral kappa d z from 0 to z from u 2 is 0 to from u 2 at z is equal to 0 u 2 is 0 at z the value of u 2 is so much, that is an integral function.

So, I will give you the solution, the solution comes out to be u 2 of z is equal to tan hyperbolic function, you can actually integrate this equation by transforming variables and show the solution of that equation that integral is in terms of tan hyperbolic inverse and so, you get this expression for u 2 of z from here, you can calculate P 2 of z is equal to n 2 by 2 c mu 0 u 2 square into area. n 2 by 2 c mu 0 mod E 2 square into area mod E 2 square is u 2 square into area. So, u 2 z square is given here and n 1 is equal to n 2 so this you can show again that this is P 1 of 0 tan hyperbolic square of you can this is actually square root of 2 c mu 0 P 1 0 by n 1 into area into I have replaced u 10, by the power in the fundamental and kappa is a known quantity omega d by c n 1.

So, the power in the second harmonic close with z like this, and if you all know time tan hyperbolic square function is a monotonically increasing function. So, from z is equal to… if you plot as a function of z it will go like this, the maximum power you can get is P 1 of 0 this is P 2 of z z versus z.

Now, I leave the following problem that if the efficiency is small then you can convert the tan hyperbolic function into…. So, if the efficiency is small that means P 2 z must be very very small that means, this argument must be small then you can except the tan hyperbolic function in terms of the argument. And I would like you to show that the efficiency comes out be exactly, what you have calculated before under the low pump depletion approximation.

That means, here the expression, which we calculated will go like this, it will be overlapping with this variation, until about a efficiency of a few may be 5 6 percent or 10 percent and then it will diverge up, because an increases quadratically with z the increase is got quadratic it is in the form tan hyperbolic function.

So, two things I want you to show, first thing is for low efficiencies P 2 of z for low efficiencies eta same as before and the second thing, I want you to calculate is P 1 of z. This is the power of the second harmonic, what is the corresponding power in the fundamental as z increases? Note that in this calculation, you can never convert all the power from the fundamental second harmonic, because that is a tan hyperbolic function it a synthetically approaches P 1 0, it is not a periodic exchange between omega and 2 omega.

So, omega you launch omega, you generate 2 omega continue to generate 2 omega and the 2 omega, power keeps on increasing. And if you have perfectly phase matched, it actually approaches the full efficiency of 100 percent at a essentially in finite lines.

Sir what is that dotted curve?

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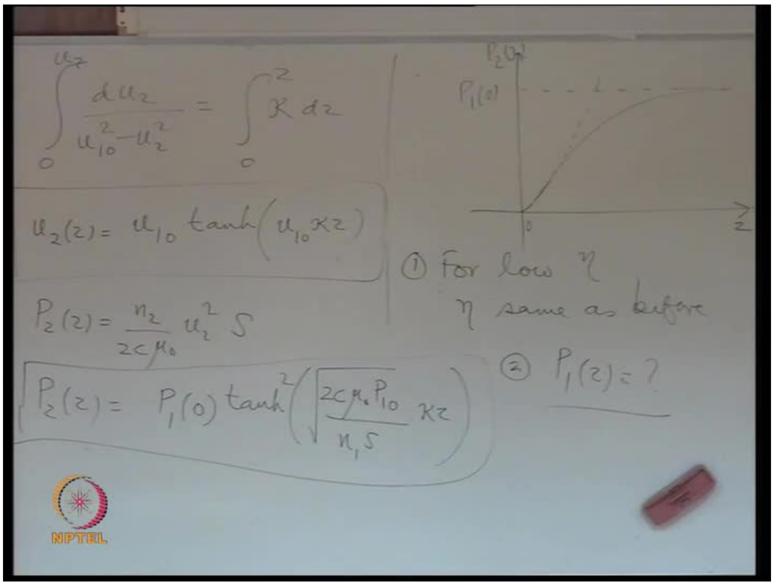

This dotted curve is for low efficiencies, what we had calculated sine square delta k z by 2 by delta k by 2 whole square in terms of that function. So, we had actually calculated earlier P 2 of z. Assuming, E 1 was a constant. So, on that approximation efficiency will be low and this is the low efficiency expression and this match for low efficiencies for higher efficiencies that equation cannot be used. And actual equation is this and the efficiency goes to hundred percent, as you increase the length of interaction.

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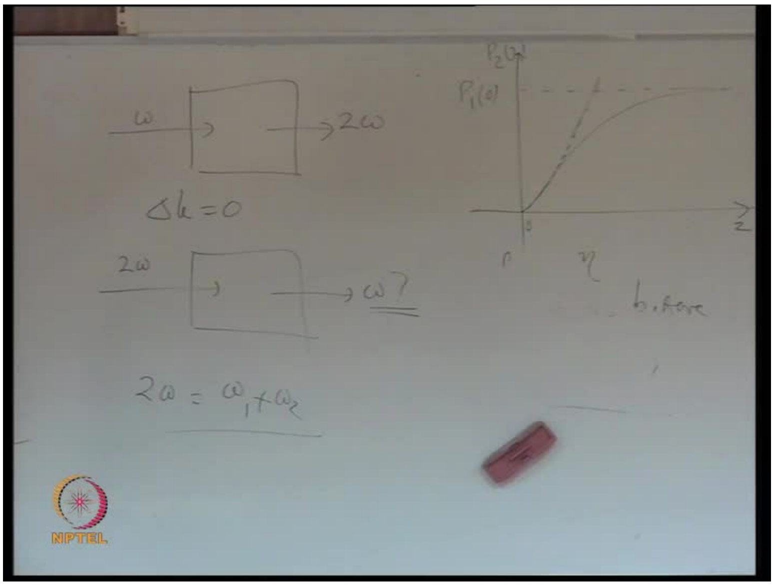

So, now let us look at another problem, and that is as I was mentioning to you before, generation of omega from 2 omega. So, what we have discussed is I have a crystal, I launch omega and I generate 2 omega, second harmonic generation. So, for this delta k is equal to 0. Now I take the same crystal and I launch 2 omega generate omega; the sub harmonic generation.

Remember, we have, we have, two equations d by dz and d E 2 by dz. we solve the d E 2 by dz equation under low, under low pump depletion approximation and got a variation of we got certain efficiency; and we got an efficiency for complete conversion, for where you do not neglect the depletion. Can I reverse the problem and I say I launch on 2 omega and I want to generate omega.

So, here what is happening is 2 photons are frequency omega, merged to form 1 photon at frequency 2 omega; here 1 photon at 2 omega is suppose to split into 2 photons that frequency omega. This is also called parametric down-conversion, you are decreasing the frequency, you are generating a lower frequency from a higher frequency, it is called parametric down-conversion.

And in fact, this 2 omega can be written as a sum of a infinite sets of two frequencies, 2 omega is omega plus omega 2 omega is omega 1 plus omega 2 some omega 1 plus omega 2 is also 2 omega.

So, in fact, there are infinite combinations of this equation, of course, you need to satisfy phase matching condition to generate the new frequencies. So, we will go to this little later, but first thing, I want to address the problem is if I do not if I launch only 2 omega can I generate omega.

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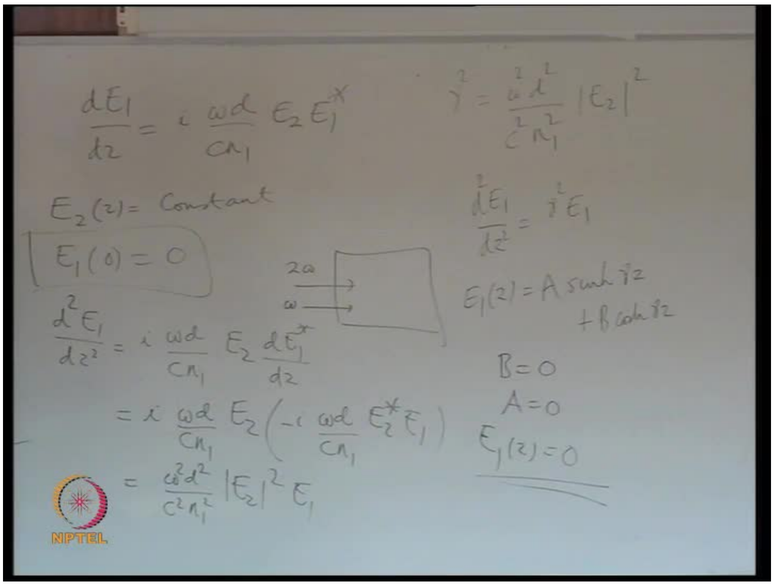

So, let me go back to this equation and let me assume no pump depletion approximation, which means I assume E 2 is a constant. So, the equation, which I need to solve now is just first equation I omega d by c n 1; with a condition that E 2 of z constant and E 1 of 0 that this I have at the input of a crystal, a second harmonic with a certain electric field amplitude E 2 and no incidence of any fundamental wave. There is no omega incident; there is only 2 omega incident.

In the first case, there was only omega incident. There was no 2 omega incident, I generated omega 2 omega from omega. Now, I am trying to generate, omega from 2 omega by launching only 2 omega.

Sir E 2 represents 2 omega and represents omega.

I continue to represent, the second harmonic by the electric field E 2 the fundamental by this electric field E 1. Now first thing you already notice here that, if E 1 of 0 is 0 d E 1 by dz at z is equal to is also 0; this equation because this is proportional to E 1 star, so, what you expect?

It will continue to says 0, let me calculate. So, let me differentiate this equation, d square E 1 by dz square is equal to I omega d by c n 1 E 2 d E 1 star by dz d E 1 star by d E dz. I substitute from here, so I omega d by c n 1 E 2 into minus I omega d by c n 1 E 2 star E 1, which is equal to omega square d square by c square n 1 square mod E 2 square into E 1.

Now, let me call this, gamma square its omega square d square by c square n 1 square into mod E 2 square. So, this equation is simply d square E 1 by dz square is equal to gamma square. What is the solution? E 1 of z is equal to or in terms of hyperbolic functions. So, what will it be A sine hyperbolic gamma z plus B cos hyperbolic gamma z.

Because, E 1 of 0 is 0. What should be 0?

B B

Because E 1 of 0 is 0 constant d must be 0, because E 1 of 0 is 0 plus B. so, B is 0 d E 1 by dz is also 0 at z is equal to 0 that means A is also 0.

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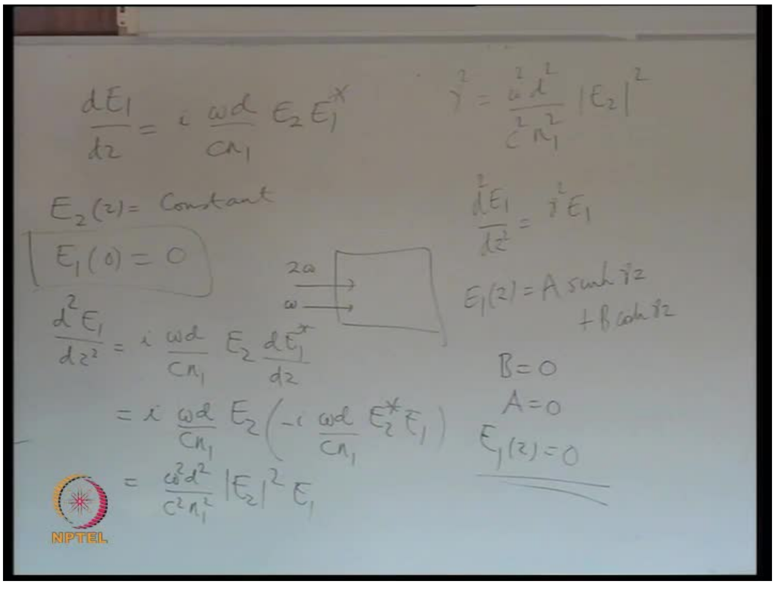

So, E 1 of z, so this gives me B is equal to 0, because E 1 of 0 is 0 and A is equal to 0, because d E 1 by dz is 0 at z is equal to 0. So, E 1 of z is equal to 0, no generation of sub harmonic.

Omega can get converted in 2 omega. Those equations predict, but those same equations tell me that I cannot convert 2 omega at omega, there is nonlinearity, there is nonlinear effects. The powers are large there are strong electric fields, but no generation of omega.

When I do an experiment, I find omega and the explanation is quantum mechanical. Note that according to quantum mechanics, I cannot have complete vacuum in terms of no electric fields and no magnetic fields. I cannot have a situation, where the electromagnetic energy is 0; there is always 0 point energy, 0 point fluctuations and it is so 0 point fluctuations, which will induced the second harmonic to down converter.

For example, if I do not put this condition and I as, if I assume a finite electric field to be incident at frequency omega, I can solve this equations, Because, then those both of them do not become 0 and I will get a solution to the problem.

Let me try to analyze, for example ,so let me assume, that instead of this, so the first thing is that without this noise, without this quantum noise, there is no sub harmonic generation, I cannot generate omega from 2 omega. I cannot generate. This is called, what is called as spontaneous parametric fluorescence or Spontaneous Parametric Down-Conversion SPDC?

It is a spontaneous process in which the 2 omega photons spontaneously split into 2 omega photons, it is parametric down-conversion. It is a down-conversion process, because it goes from a higher frequency to lower frequency and spontaneously it has to happen.

So, SPDC this Spontaneous Parametric Down-cCnversion has a completely quantum mechanical origin and this situation is called degenerate parametric fluorescence, because the two frequencies which are coming out are the same.

At a split a 2 omega photon into two different frequency photons, whose sum of frequency is equal to 2 omega. So, that will be non-degenerate parametric fluorescence. This is called degenerate parametric fluorescence, degenerates means the output 2 photons have the same frequency.

The equations, we have derived is only for degenerate, because we have only consider two frequencies omega and 2 omega in the process, after we finish second harmonic generation and this discussion, we will going to more general case, where I can have three frequencies interacting with each other and that will be used for understanding more things about parametric processes.

Now, let me try to show you mathematically some very interesting features of this equations.

Sir, I seen that because use this same equations to that it is possible to generate omega 2

No, we have to go then we go to quantum mechanical description, then only I can show you but these equations can be used, when E 10 is not equal to 0, that means in addition to 2 omega, if I put a small omega here.

Sir, but this is a similar to the condition, when omega 1 got the when the electric field at omega got depleted and there was no (( ))

Exactly, so, it is in between condition, I launch omega in the case after some distance, I have both omega and 2 omega.

So, but according to it which the energy only gets pumped into E 2 and not and from E once this no.

You are right, but I will show you that depending on the phase difference between these two waves energy, can move from omega into 2 omega or from 2 omega to omega.

This is in the first case that we did where (( ))

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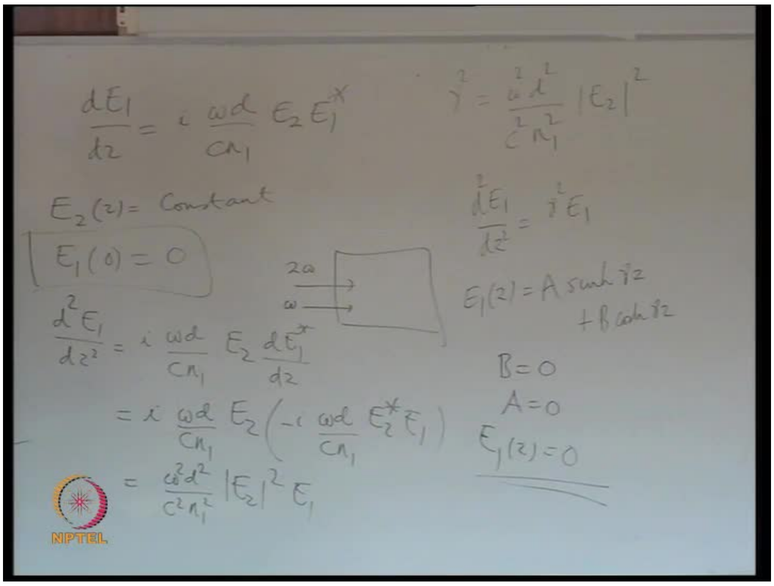

Because that says different number appear here, I will show you depending on the phase difference between these two waves, I can either go from omega to 2 omega to 2 omega 2 omega.

You said that quantum mechanically electromagnetic energy cannot be 0, you said that.

You cannot have a situation where electric and magnetic fields are 0.

If they are finite, so, how can we know that there must be some component at frequency omega presents that some harmonics are generated.

Because they are presented in all frequencies. In free space, all frequencies are allowed and so there is fluctuations corresponding to all frequencies, if you take a cavity a cavity only allows certain frequencies of oscillation and then noise is present at those frequencies; the other frequencies cannot exist in where inside the cavity.

I mean in theory assuming that these frequencies are very well defined and individual frequencies. So, this situation is where I have a strong 2 omega beam and a weak omega beam got again but this is very weak but not 0.

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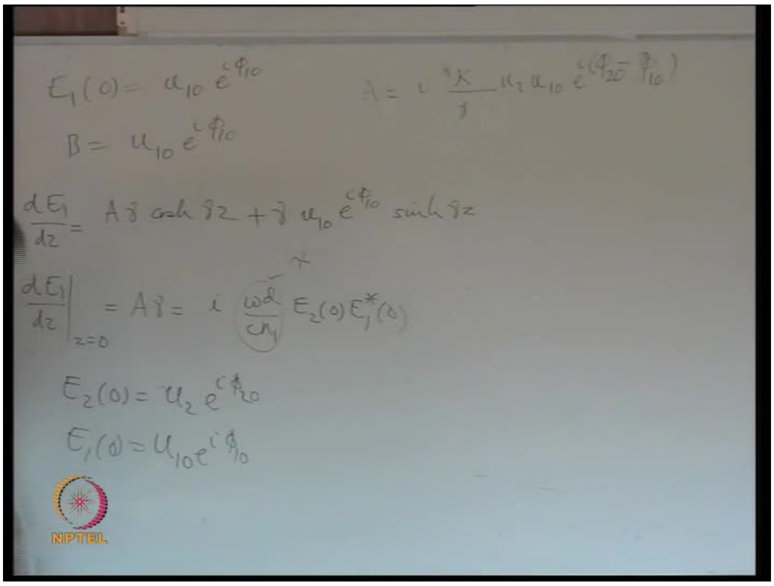

So, let me solve this equation. So, what is a expression, I will get now, so let me call this, this is not 0 .so, I call this let me write here again, E 1 of 0 is E 10. So, I get B is equal to E 10 from here.

So, d E 1 by dz is equal to A gamma cos hyperbolic gamma z plus gamma E 10 sine hyperbolic gamma z so d E 1 by dz at z is equal to 0 is equal to A gamma is equal to i omega d by c n 2 c n 1 into E 2 of 0 E 1 star of 0.

And substituting the expression for d E 1 by dz that is I omega d by c n 1 E 2 E 1 star. So, let me assume that E 2 of 0 is a u 2 E to the power i phi 2 and E 1 of 0 is u 1 phi 20 and exponential i phi 10.

There is a sine or sine hyperbolic

Sine hyperbolic, I am sorry, sine hyperbolic. So, I get an expression for A. A is equal to i omega omega d by c n 1. Remember, I have call this kappa so kappa by gamma u 1 u 1 u 2 this is a let me symbol.

So, let me call this, This is E 10 actually because E 1 of 0 is E 10. This is complex please just correct this. This is E 10 u 10 e to the power i phi 10 amplitude and phase so this is B is equal to u 10 e to the power i phi 10.

And this is actually gamma, anyway be second harmonic is constant, so there is no z dependence on the E 2. So, this is A is equal to i kappa u 2 u 10 exponential i phi 20 minus phi 10 minus phi 10, because you have an E 10 star E 10 star is u 10 exponential minus i phi 10 and so that is the expression for A.

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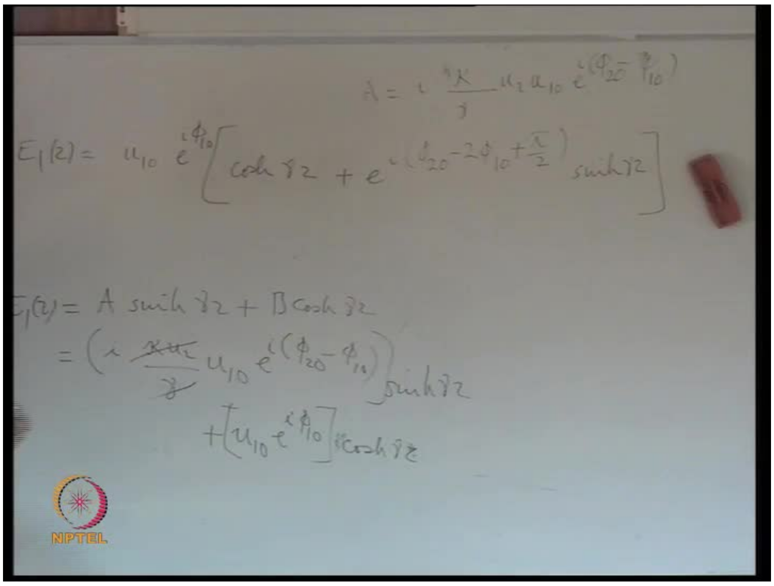

So, let me substitute into the solutions and I get a complete solution. As, so let me write this first, one first so what is what was E 1 of z A sine hyperbolic gamma z plus B cos hyperbolic gamma z. so, A is I kappa u 2 by gamma u 10 exponential phi phi 20 minus phi 10 sine hyperbolic gamma z plus u 10 e to the power i phi 10 cos hyperbolic and substituting the values of A and B.

So, I can write this as E 1 of z, Let me take this common now, remember, what was gamma square gamma square was omega square d square by c square n 1 square mod E 2 square. This is equal to kappa square into u 2 square and you find gamma square as omega square d square by c square n 1 square into mod E 2 square omega B by c n 1 is kappa and mod E 2 square is u 2 square so kappa times u 2 at is equal to gamma. So, these cancelled out and I write this as u 10 exponential i phi 10 into cos hyperbolic gamma z plus exponential I have i phi 20 minus 2 phi 10 plus pi by 2 the i factor multiplying here I have written as exponential i pi by 2 and then I have sine hyperbolic gamma z.

What I have done is I have used the initial conditions at z is equal to 0 to calculate the two constants A and B substituted into the solution and explained the solution like this now you see E 1 z is not 0.

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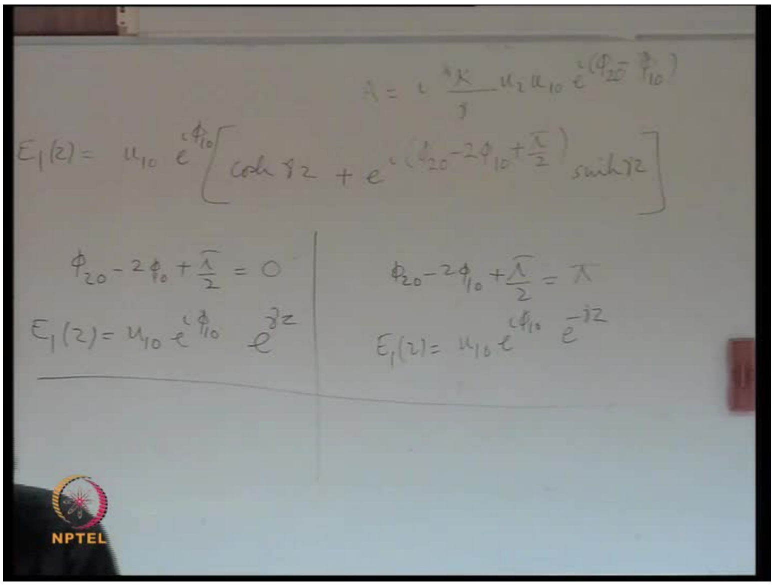

Now, I can have two situations suppose this coefficient – suppose - phi 20 minus 2 phi 02 phi 10 plus pi by 2 was equal to 0. The initial phase of the fundamental initial phase of the second harmonic. Remember phi, what is phi? 20 E 2 0 was u 2 0 exponential, I phi 20, this is the phase of the second harmonic 2 omega wave at the input this is the wave of the fundamental wave at the input the phase of fundamental wave.

Suppose I could satisfy this condition, what will happen E 1 of z? What happens to the sum? This becomes exponential, becomes 1 sum of cos hyperbolic and sine hyperbolic.

Becomes exponential gamma z, becomes exponential gamma z cos hyperbolic x plus sine hyperbolic x is exponential x cos hyperbolic x minus sine hyperbolic x is exponential minus x.

If I had this is one case, so, if I had phi 20 minus 2 phi 10 plus pi by 2 is equal to pi or pi then E 1 z will be u 10 e to the power i phi 10.

If I choose this exponential factor to this phase to be minus pi by 2. So, this is 0. So, this happens with 0 that means phi 20 minus 2 phi 10 is minus pi by 2, then the fundamental wave grows exponentially. This assuming, no pump depletion that means I am solving the equation assuming E 2 is a constant.

So, there is an exponential growth, but you cannot extend this to large values of that so, the omega frequency grows exponentially, if you choose this phase of phi 20 minus 2 phi 10 is plus pi by 2, if this difference is plus pi by 2 then the fundamental decrease and this decay, which means that the energy is transferring from omega to 2 omega here. And here it is transferring from 2 omega to omega, what is this amplification, u 10 was the input field at the omega frequency as z increases the omega frequency keeps on increasing in amplitude. This is an optical amplifier. This is called an optical parametric amplifier it uses the nonlinear property to amplify the signal and interesting thing is this amplifier depends on the phase of the input signal.

If your input signal phase phi 10 satisfy this condition, the signal will get amplified if the input signal phase, satisfy this condition it will get deamplified. This is not like laser amplifiers, if you take a laser amplifier irrespective of what you do to the input phase it is always amplified it uses population inversion. This uses a different principle completely it uses a nonlinear effect and this is called a parametric amplifier and this is a phase sensitive parametric amplifier.

Later on, I will show you the parametric amplifier can operate in phase sensitive fashion of phase insensitive fashion and you can use this phase sensitive nature of this amplification process to very interesting features in generating light with noise properties which are better than, what you can achieve with lasers.

You can use this process to what is called as squeezed vacuum to generates squeezed vacuum has certain fluctuations, you can reduce the fluctuations below, what standard vacuum has by using this kind of a process and that is called squeezed vacuum. And squeezing is a very important technique that is used today to reduce the noise of signals and this is used in many applications including in gravitational wave observatory. For example, there is a light orb interferometer which is which is being built to detect gravitational waves.

So, you need waves, which whose noise levels are very low and the…. So, this squeezed light which comes out from this kind of a process can be used to for applications where you want very low noise signals, we cannot have the quiz now.

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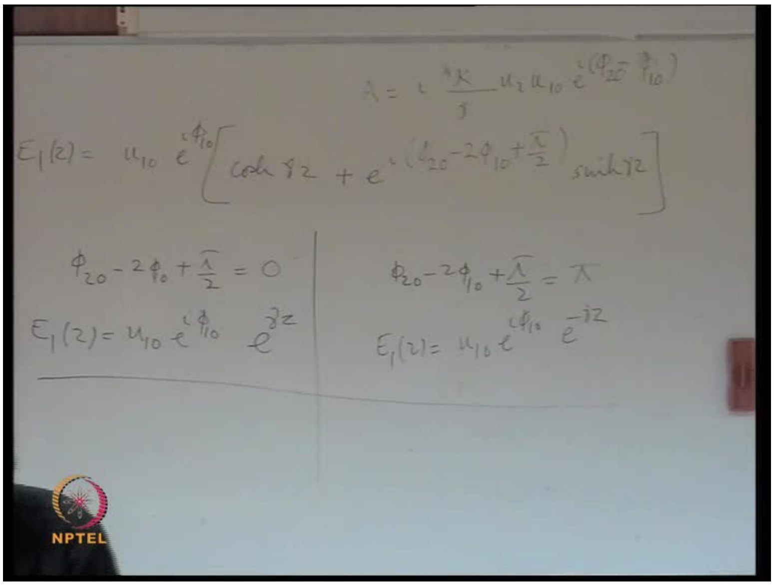

So, there is there is a very interesting feature here, so let me show you some slides before I stop. So. will have the quiz may be on thursday itself.

So, this is a very interesting feature of an amplifier, which does not use population inversion but which uses nonlinear effects and this the amplifier is phase sensitive. So, we will come back with this phase sensitive amplifiers a little later but let me finish the lecture by showing you some slides.

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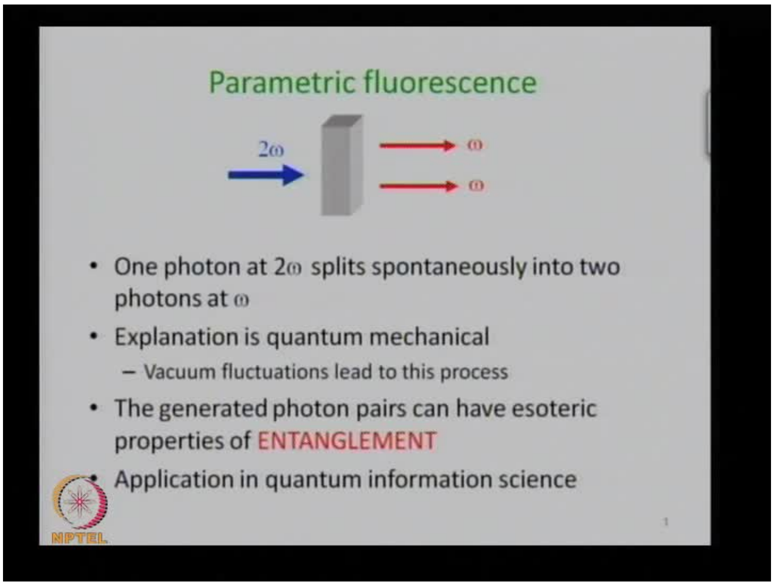

So, this is what is called parametric fluorescence. This is a degenerate parametric fluorescence where photons at 2 omega splits into 2 photons at omega and these 2 photons, which are coming out, actually are generated from the same photon 2 omega and they can possess certain peculiar features of what are called as entanglement properties of entanglement? So, there are correlations between these 2 photons, which are beyond what classical physics predicts and this process is very much used in in quantum information science.

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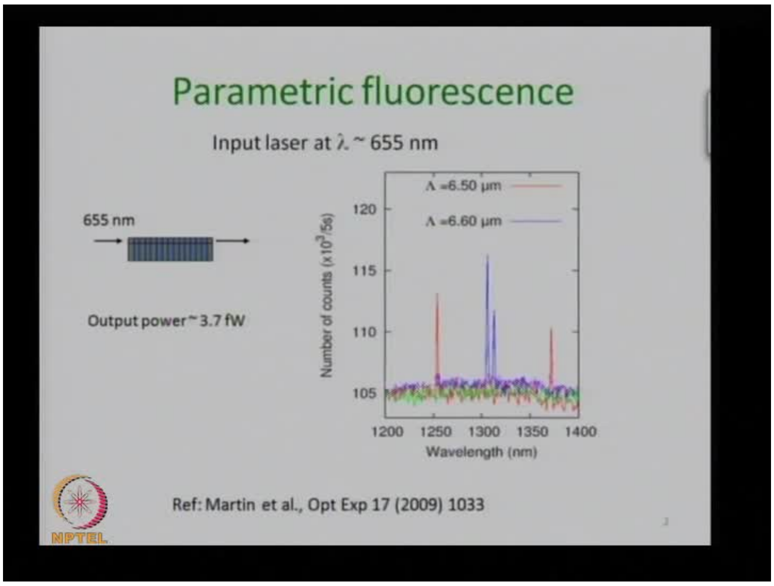

So, here is a typical example of light being pumped at 655 nanometers and out comes light at longer wavelengths, which is down-conversion and depending on the quasi phase matching period. The pairs of light wavelengths coming out are such that the omega 1 plus omega 2 is equal to the omega of the instant light. 655 nanometers photons are splitting into 2 photons of lower frequencies or higher wavelengths and the sum of these two frequency is equal to 655. The sum of these two is also 655 but in this case, these are coming out because you satisfy the phase matching condition for that pair of frequencies.

So, if you change the period of this quasi phase matching, you can generate different pairs of photons, which are coming out please store the power levels control watts very very low generation powers. These are this is called this is called fluorescence parametric fluorescence, you shine light at this wavelength and out comes light at new wavelengths, which are longer wavelengths than the input light.

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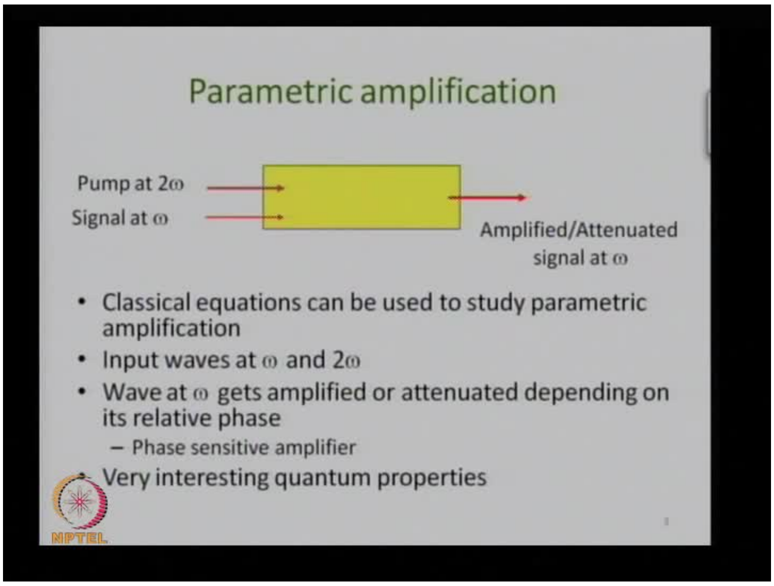

What we discussed today is a parametric amplifier, you can amplify input signal and this is phase sensitive. So, it has very interesting quantum properties, which we will come back when we discuss quantum mechanics at that time I will bring back this problem and we look at some quantum properties of this.

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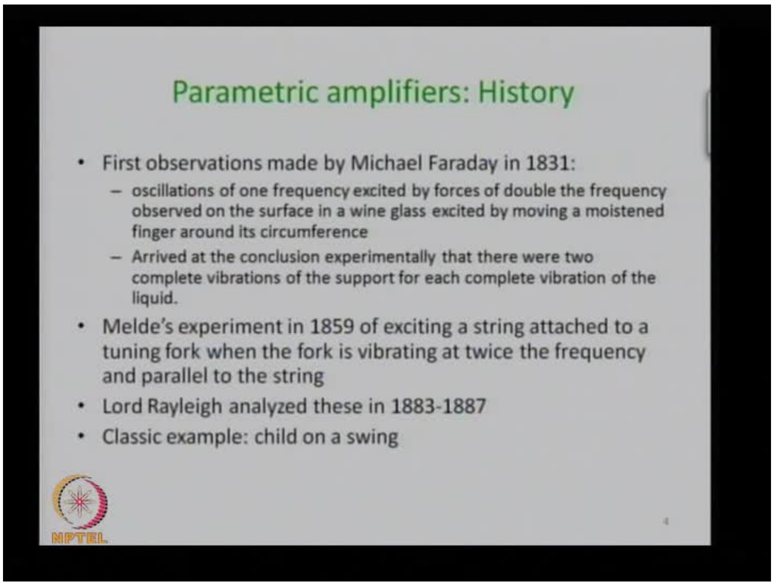

Incidentally, parametric amplifiers were investigated about Michael Faraday in 1831: so you take a wine, if you rub your finger on a wine glass, the wine started to vibrate and he found out that there were two complete vibrations of the support for one vibration of the liquid 2 omega omega.

Many of you may not have done this experiment - Melde's experiment where you can excite a string into oscillation by forcing it at the twice the frequency and not really as analyzed this, and let me show you classical example of a child on a swing.

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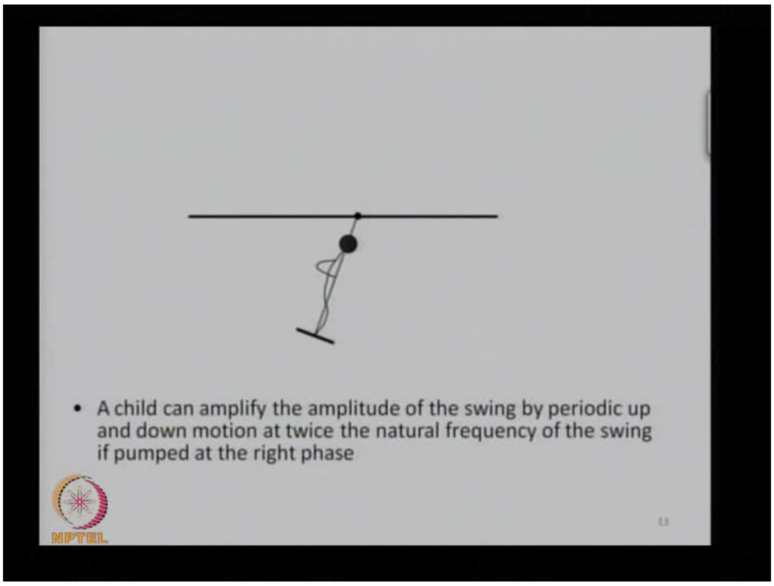

So, child sits then stands up, again sits back and stands up, two frequencies of the child sitting and standing for one period of the swing. The child is pumping the swing at 2 omega and the swing is oscillating at omega frequency.

And so, also note that the child has to pump the swing at the right phase, if it gets lot of phase, the swing will come down to rest and also nobody needs to push. The child can generate some noise in the swing and then start to amplify that then, the moment it starts to swing a little bit you sit and stand at the right time, the right phase and you start to amplify the signal. So, you can actually generate from noise amplify that noise and generate the oscillations.

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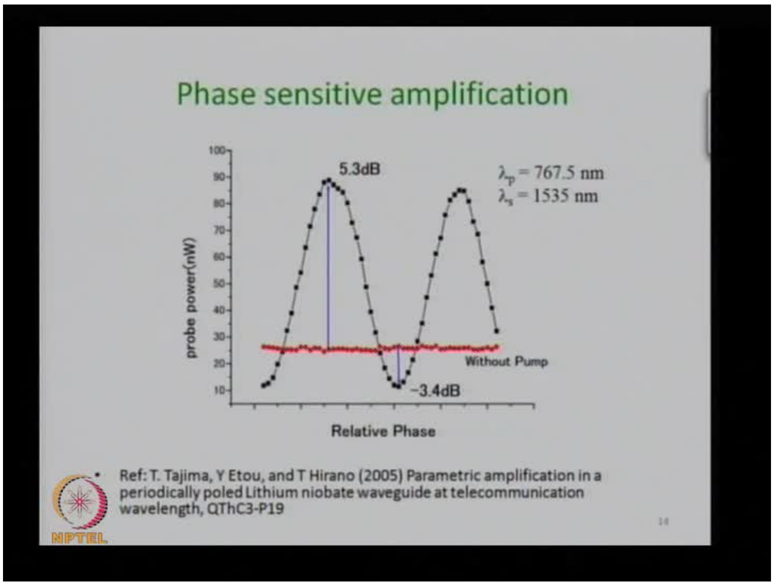

And here is some experimental results showing, how the amplification depends on the relative phase, which is what this equation predict and you can these are some experimental results showing, phase sensitive amplifiers and so, these are very interesting. So, there is a lot of work going on in phase sensitive amplifiers and as I said one of the most interesting application of this is in quantum noise reduction or generating non-classical states of light and so on.

Thank you.