Quantum Electronics

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Module No. # 03

Second Order Effects

Lecture No. # 10

Non- Linear Optics – Quasi Phase Matching

So, we continue with our discussion on quasi phase matching. So before I start, do you have any questions from the earlier lectures?

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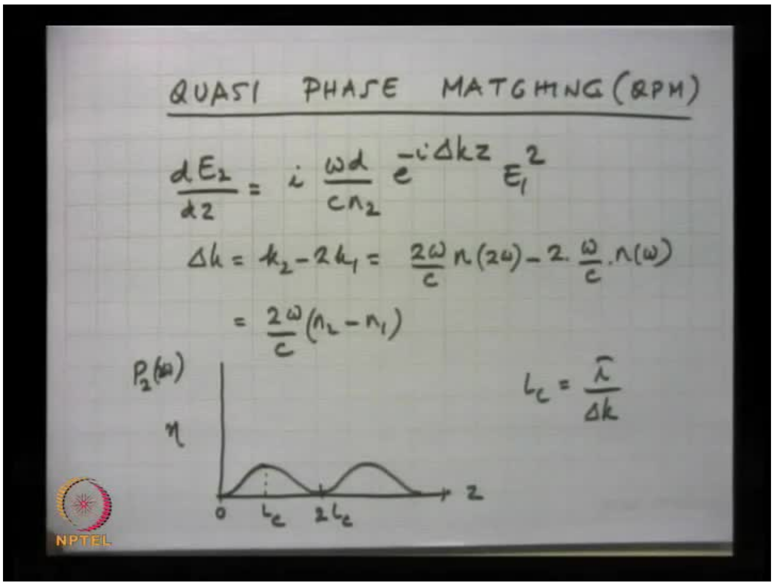

So, let us continue with our discussion on quasi phase matching also called Q P M. So, let us recall the electric field at the second harmonic, satisfies an equation given by i omega d by c n 2 exponential minus i delta k times z, where delta k is equal to k 2 minus 2 k 1, which is equal to 2 omega by c n at 2 omega minus twice omega by c n at omega, which we had also written as 2 omega by c n 2 minus n 1.

So, delta k is phase mismatch between the waves at omega and 2 omega, and depends on refractive indices of the medium at frequency omega and frequency 2 omega; n 1 is n of omega, the refractive index of the medium at frequency omega, and n 2 is n of 2 omega, the refractive index of the medium at frequency 2 omega. What we also saw is that because of this term, so there is an E 1 square, because of this term, the amplitude of the second harmonic field does not grow continuously, but oscillates.

So, let us recall; we have a curve looking like this; this is z and I can write here, either the power in the second harmonic or the efficiency it goes up and down periodically. This distance was called L c; this is 2 L c, L c is the coherent length of this interaction process and is given by L c is equal to pi by delta k.

So, what is actually happening is, as we discussed last time, the second harmonic electric field grows until some distance L c, at which point the phase difference between the non-linear polarization at frequency 2 omega and the electric field at 2 omega becomes pi, and beyond this point the non-linear polarization actually feeds energy into the second harmonic, which is out of phase with existing electric field resulting in a drop in the electric field of the second harmonic.

The electric field of the second harmonic becomes 0 at 2 L c and then again starts to grow. We also saw that, if you take typical values of refractive indices at frequency omega and 2 omega, because of dispersion the peak efficiency that you can get is extremely small.

So, it is extremely important to make sure that delta k is 0 or very close to 0; so that the efficiencies can be reasonably high. Now, if you can make delta k is equal to 0, we get what is called birefringence phase matching, which we have seen earlier. But I can actually use another technique to achieve, what is called as quasi phase matching. So, in this equation what we would like to do is to make d, a function of z and a periodic function of z.

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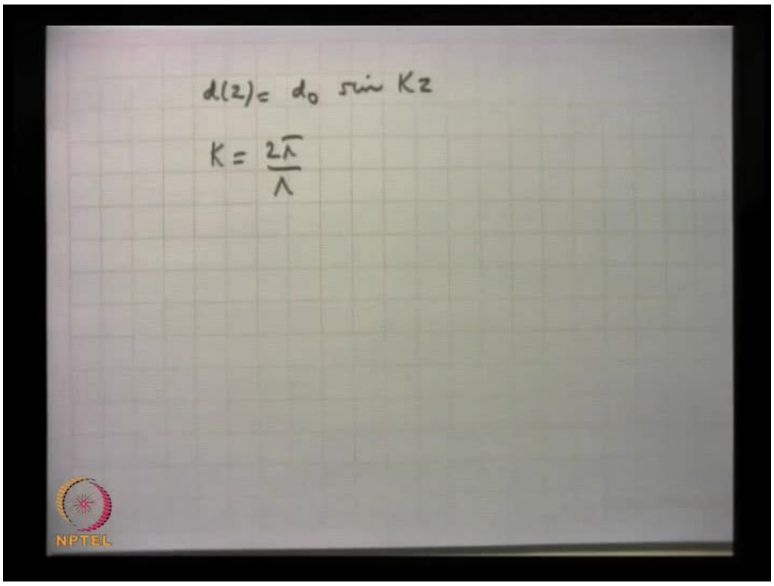

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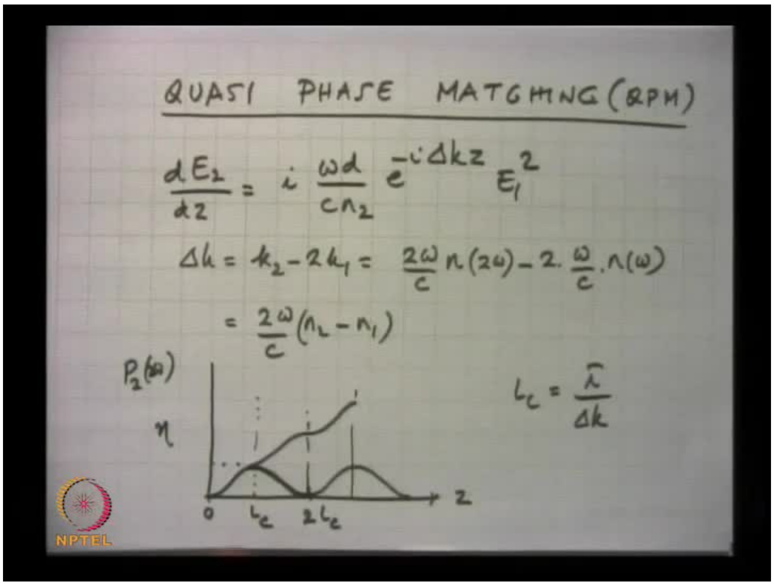

So, last time we saw that if I take d of z is equal to d 0 sin k z; let k is the spatial frequency of this variation and given by 2 pi by lambda, where capital lambda is the period of the variation of d, then what happens actually is that, at this point instead of the second harmonic decreasing in amplitude it grows; again, it continues to grow up to this point and then again, instead of dropping down, it continuous to grow.

So, there is a an increase of the efficiency of the second harmonic generation, because every time you have a phase difference of pi between the non-linear polarization and the electric field, you introduce a change of sin of d, which is essentially a change of phase of pi of the non-linear polarization term. So, this way you can actually overcome this decrease in efficiency and this technique is called quasi phase matching.

So, remember, what we had done was, we substituted this expression for d in this equation, and which you write sin in terms of two exponentials; what you find is, you have one exponential term, which is exponential I capital K minus delta k z, and you have a another exponential term, exponential minus I k plus delta k z.

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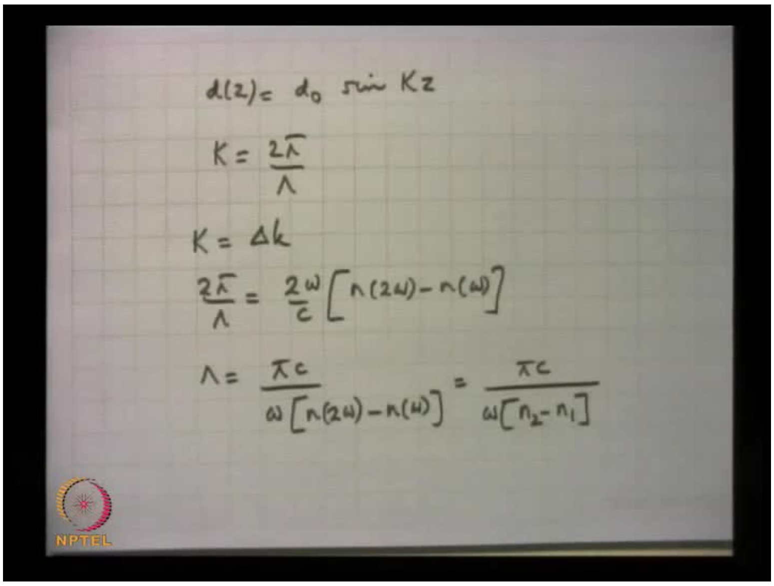

So, if I can make capital K is equal to delta k; one of these terms has no phase terms and that term, when I integrate this equation gives me an increasing electric field of the second harmonic. So, what is the spatial frequency required for phase matching or quasi phase matching k is equal to delta k. This implies that 2 pi by lambda is equal to delta k, which is 2 omega by c into n of 2 omega minus n of omega or lambda is equal to pi c by omega into n of 2 omega minus n of omega, which is equal to pi c by omega into n 2 minus n 1.

This period of quasi phase matching depends on the refractive indices at the frequency omega and 2 omega, and also of course the frequency omega itself. We had seen last time that if you take a typical wavelength of eight hundred nanometers, we can calculate what is the capital lambda required in the case of Lithium Niobate and we found it is of the order of three point three microns or so. So, it is a very small period required which means you need to exchange; you need to change the direction or the sign of the d coefficient every 3.3 microns or so.

Also note that this particular period depends on the frequency or the wavelength. So, if you take second harmonic generation of 800 nanometer wavelengths to generate 400 nanometers, I need to know the refractive index of the medium at 800 nanometers n 1; I need to know the refractive index of the medium at 400 nanometers, which is n 2 and then I can substitute into this equation and get the period.

Now, if you change this wavelength slightly say from 800 to I change it to 810 nanometers, then note that the frequency will change, the refractive indices will change and so the required capital lambda will also change. So, what this implies is that if you take a particular fundamental wavelength, calculate the corresponding grating period required for quasi phase matching and make a device with that particular period, that device will work perfectly at the chosen frequency at which you have designed the substrate.

So, at 800 nanometers if you calculate the period and make a periodic domain periodic reversal of sign of d at the period corresponding to 800 nanometers, it will work perfectly at 800 nanometers. Now, if you change this input wavelength, the period that you have in your crystal does not correspond to the period required for the new wavelength. In this case, what will happen is the efficiency will drop down, because this capital K corresponding to what you have made in the crystal, does not correspond to what is required to quasi phase match the new frequency.

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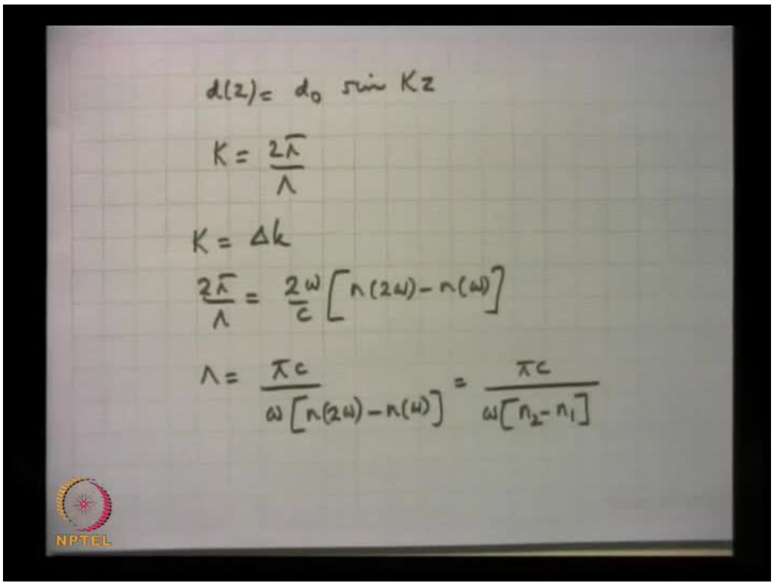

So, this is the frequency dependent period here; so for any particular input frequency, if you want to generate second harmonic, you need to calculate the corresponding quasi phase matching period and make a device with that period and of course, as you will notice that this will have a certain bandwidth, which means that if your frequency deviates from the chosen frequency at the input, then the efficiency will drop down. I will come back to this point a little later.

Now what happens is to create a sinusoidally varying d coefficient is not very easy, because remember, materials have a certain non-linear coefficient and how do I make it sinusoidally varying along the propagation direction?

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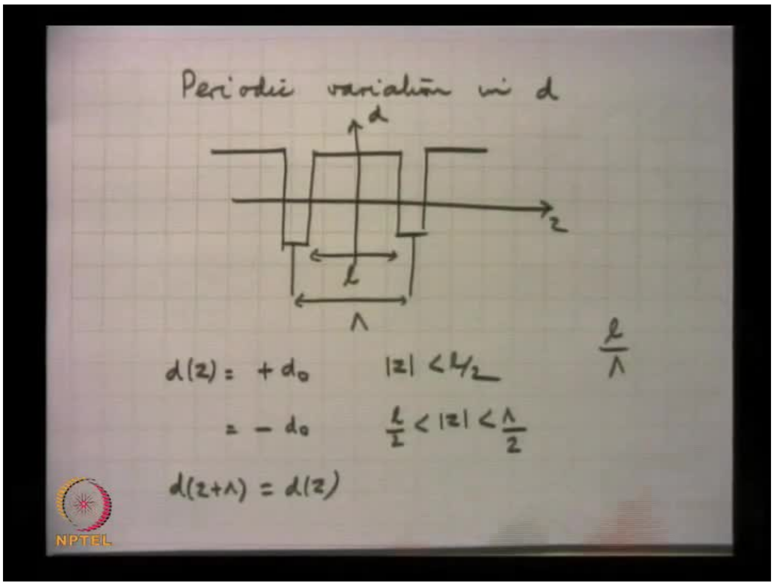

So, what is done? Now, instead of having a sinusoidally variation in d, I choose a periodic variation in d periodic variation in d, and actually I can have crystals with d positive and negative. Remember, d is the one of the elements of the non-linear tensor, the sign of the d coefficient depends on the orientation of the axis; so you can actually change the sign of d by changing the orientation to crystal axis.

So, it is possible to have d variation, which is either plus or minus, and so let me consider a situation where I have, if I plot d verses z, let me assume I have a variation like this. So, this is plus d and this is minus d; this is the period and this is length l. So, in one period, I have positive value of d over a length l, and over a length lambda minus l, I have a negative value of d. So, d of z is plus d 0 for mod z less than l by 2 and is equal to minus d 0 for l by 2 less than mod z less than lambda by 2.

That's the periodic function periodic function with a period capital lambda and this ratio l by lambda is called the duty cycle. So, overall length l, the non-linear coefficient is positive and overall length lambda minus l, the non-linear coefficient is negative and the function is periodic.

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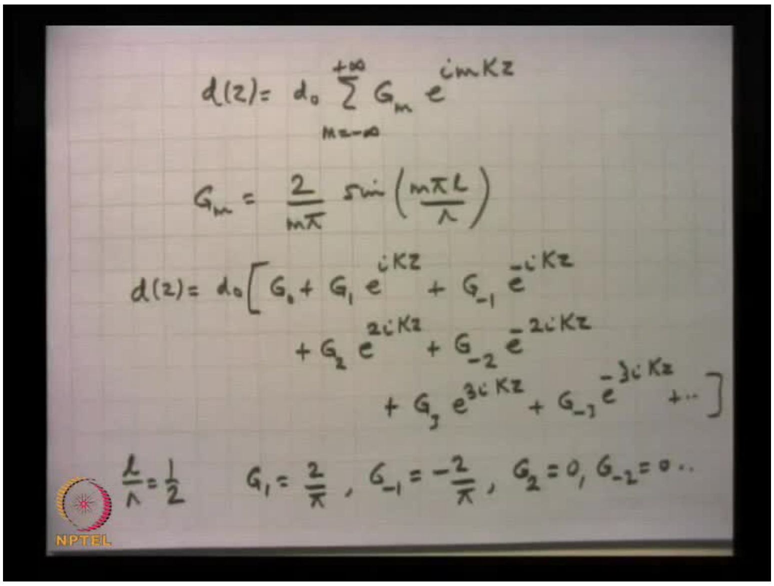

So, d of z plus lambda is equal to d of z. Now, because it is a periodic function, I can always make a Fourier series expansion. So, if I want to write a Fourier series expansion, I can write d of z as d zero sigma g m exponential i m k z.

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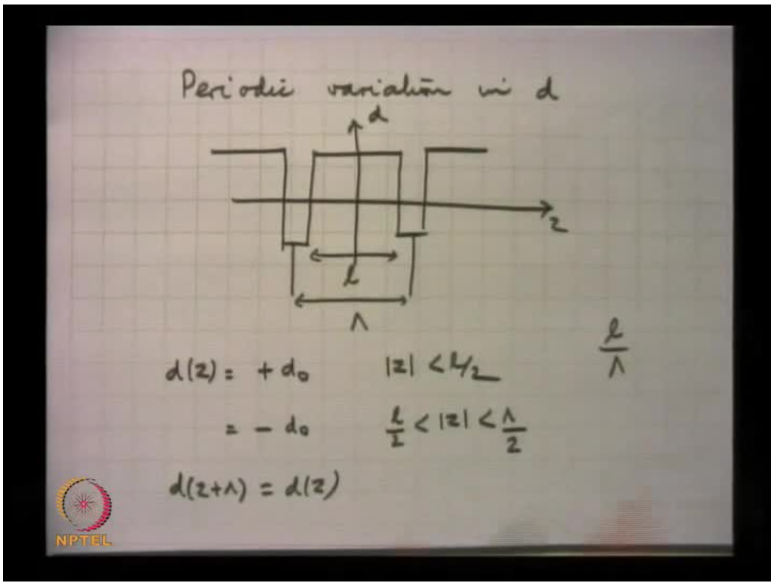

This is a periodic function with a fundamental period capital lambda and so fundamental spatial frequency 2 pi by capital lambda. So, when I make a Fourier series expansion of this d as a function of z, I leave it as a problem to you; that you can actually show that this is d 0 sigma g m exponential i m k z with m going from minus infinity to plus infinity.

This is an exponential Fourier series; instead of, writing sin and cosine Fourier series, I am using an exponential Fourier series and what I would like you to do is, take this periodic function and calculate the Fourier coefficient G m, and I would like you to show that G m is equal to 2 by m pi sin m pi l by lambda. These are the Fourier coefficients corresponding to various values of m. So, actually d of z can be written as d 0 times G 0 plus G 1 exponential i k z plus G minus 1 exponential minus i minus i k z plus G 2 exponential 2 i k z plus G minus 2 exponential minus 2 i k z plus G 3, and so on.

These are the various Fourier terms in the expansion and the various coefficients- Fourier coefficients are given by the value of G m which is written here. So and of course this depends on the duty cycle small l by capital lambda. So, if you take for example, a duty cycle of half that means over a length capital lambda by 2, I have a positive value of d and over a length lambda by 2; again, I have a negative value of d.

So, duty cycle of 50 percent, which means half the half the period of the non-linear coefficient is positive; the other half it is negative with the same magnitude. Then what you can see here is, from here G 1 is equal to 2 by pi, G minus 1 is minus 2 by pi, what is the value of G 2? Note, if l by lambda is half, G 2 will have a sign pi term that will be 0; so G 2 is 0; G minus 2 is 0. In fact, you can show that all even series - even terms in the expansion are 0, giving you only the odd terms in the expansion.

Similarly, G 3 will be 2 by 3 pi sin sin 3 pi 3 pi by 2 which is minus 2 by 3 pi, and so on. So, you can have you can find out all the expansion coefficients in Fourier series; so please do derive this equation here, find out what are the Fourier terms in the expansion.

So, instead of having a periodic sinusoidal variation in the non-linear coefficient, we have a periodic variation and that periodic variation can be written as a Fourier series and this series contains many exponential terms here.

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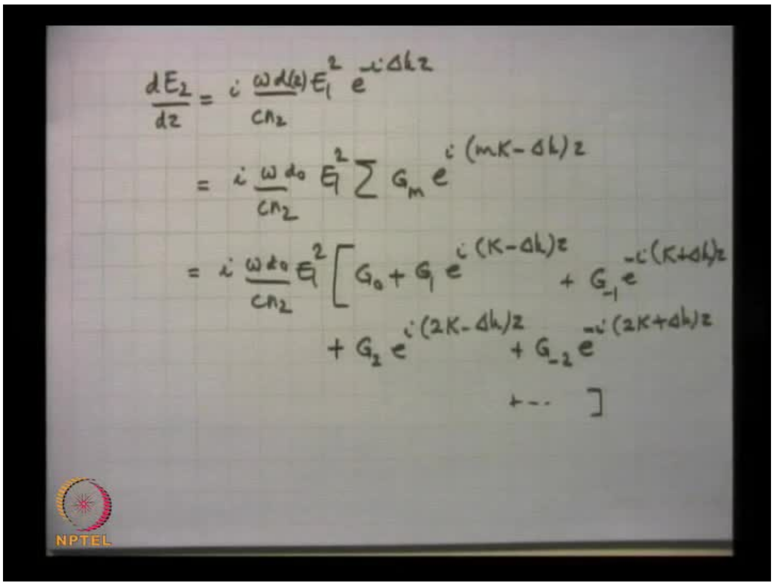

So, let see what happens to this expansion; so let me write again the equation for E 2, d E 2 by d z is i omega d by c n 2 E 1 square. Now, d i is you need to substitute here; so d is a function of z exponential minus i delta k z, which I write as i omega by c n 2 d 0 E 1 square sigma G m exponential i m K minus delta k z.

So, if I want to write the first few terms, so I have omega d 0 by c n 2 E 1 square into G 0 plus G 1 E to the power i K minus delta k z plus G minus 1 exponential minus i K plus delta k z plus G 2 e to the power i 2 K minus delta k z plus 1 G minus 2 e to the power minus i 2 K plus delta k z, and so on.

Now, I can integrate this equation sorry there is a exponential minus i delta k z here; G 0 there is no exponential i k z here, I still have an exponential minus i delta k z. So, I can actually integrating this equation assuming again like we did before, that the change in the energy of the fundamental frequency omega is negligibly small, which means the efficiency small, and that case I can assume E 1 square to be a constant and I can immediately integrate this equation you get the following expression.

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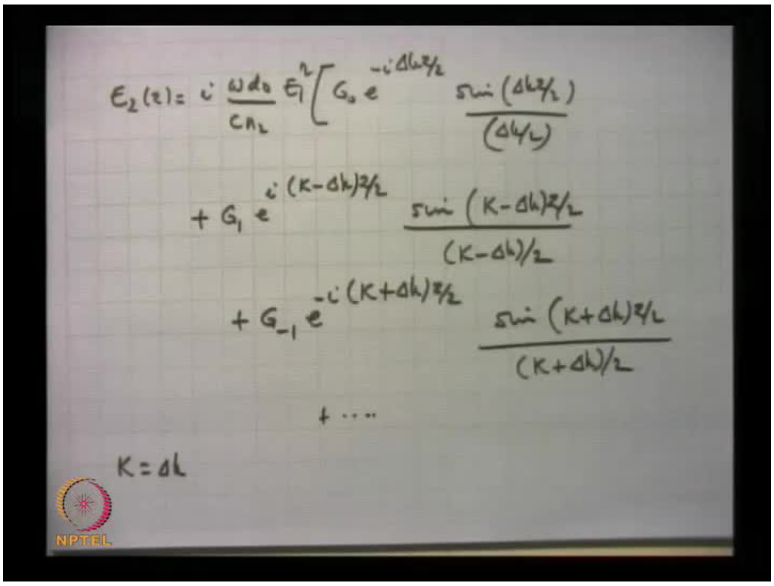

So, E 2 of z is equal to i omega d 0 by c n 2 E 1 square. So, let me now skip a few steps; I can actually integrate, and from 0 to z and what you will get is G 0 exponential minus i delta k z by 2 into sin delta k z by 2 by delta k by 2. I have integrated and simplified exactly like we did before plus I will have G 1 exponential i k minus delta k z by 2 sin K minus delta k z by 2 by k minus delta k by 2 plus G minus 1 minus i K plus delta k z by 2 into sin K plus delta k z by 2 by K plus delta k by 2, and so on.

So, I can actually first integrate this equation, because if I assume E 1 square is a constant, then the only term I need to integrate are these terms here and I get this sin delta k z by 2 by delta k by 2, and so on. Now, what can I do is, in quasi phase matching what I try to do is, I choose a value of capital K such that one of these sin terms one of these terms becomes just disappears from here; which means for example, if I choose capital K is equal to delta k, then this term will give me… what will be the value of this term? When capital K is equal to delta k, this will be z; this factor remember, here it is sin del k minus delta k z by 2 by k minus delta k by 2. So, if I multiply and divide by z this becomes a sync function and at 0 argument, the sync function is 1 and I get z.

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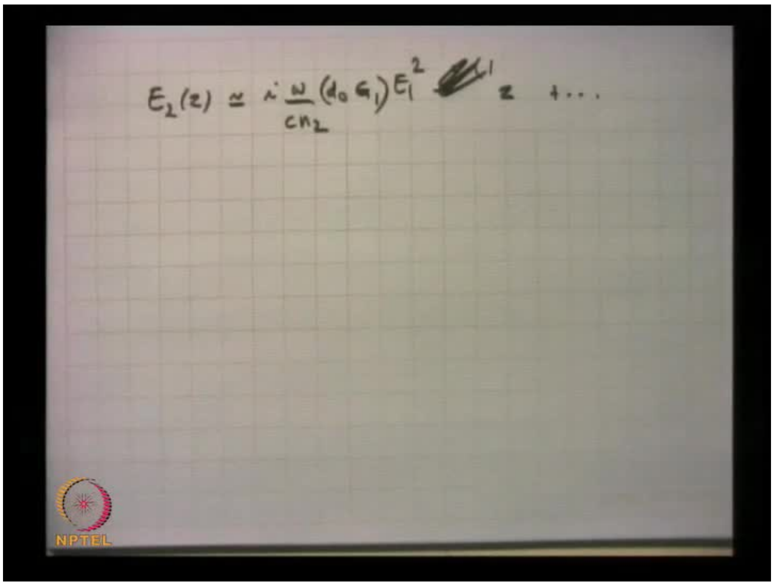

But all the other terms will still have a finite sin term setting here, and so I can neglect the contribution for of all the other terms and what I will be left with is E 2 of z will be approximately given by… If I can neglect all the other terms and if I choose capital K is equal to delta k, I will have i omega by c n 2 d naught G 1 E 1 square into exponential i.

So, k is equal to delta k; so this goes off and I have simply z plus the other terms, which are neglected. So, note here, that the expression that I get is very similar to the situation when delta k was 0. Except that, now, the non-linear coefficient, instead of being d 0 is now, d 0 times G 1, where G 1 is the Fourier amplitude of the exponential i delta k in the i k z term. The first Fourier expansion coefficient and it is proportional to length here; so as the length increases, as I showed you before, the second harmonic field will keep on increasing.

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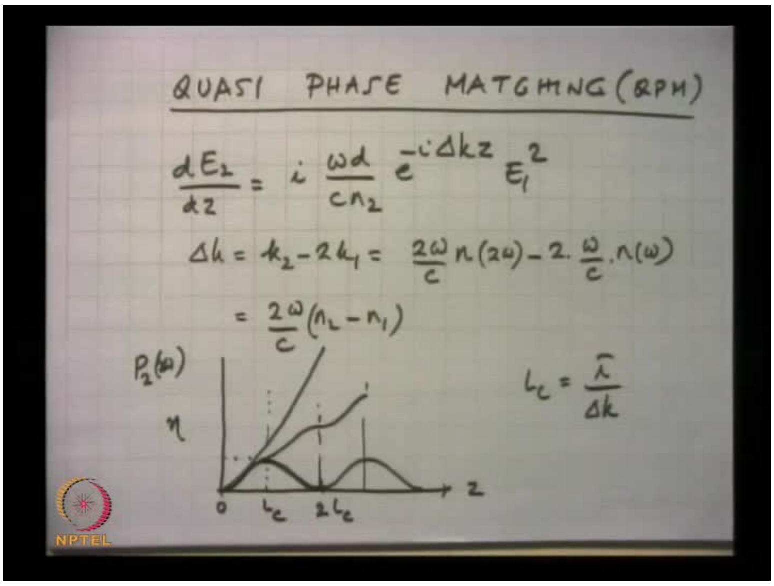

This is exactly what the thing shown here; it just keep on increasing, because of quasi phase matching. Remember, if there was perfect phase matching this should have gone like this.

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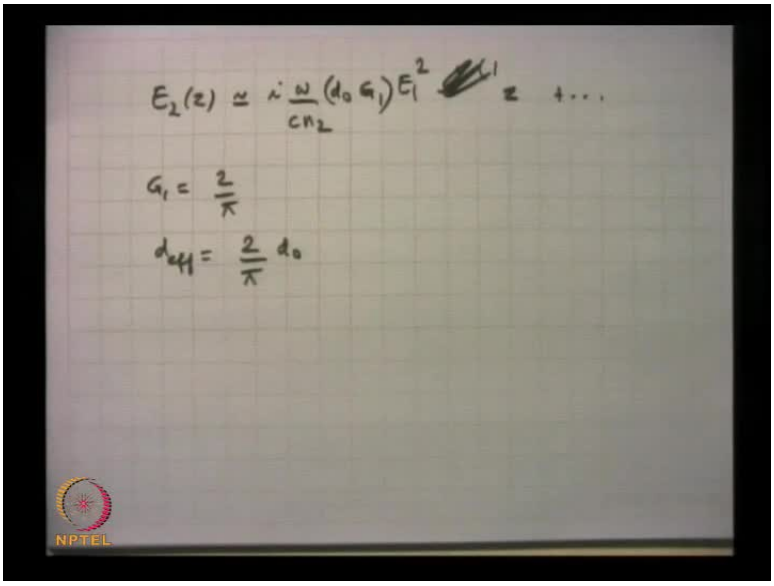

It would I mean continuously increasing, but it is increasing much slowly because the effective non-linear coefficient now becomes d 0 times G 1. So, if you choose the first this capital K is equal to delta k, G 1 is equal to 2 by pi; so the effective non-linear coefficients becomes 2 by pi d 0. And if I can neglect other terms what you can see that the second harmonic field now grows with z and I will have much higher efficiencies, than if I did not have phase matching or if I did not have quasi they phase matching.

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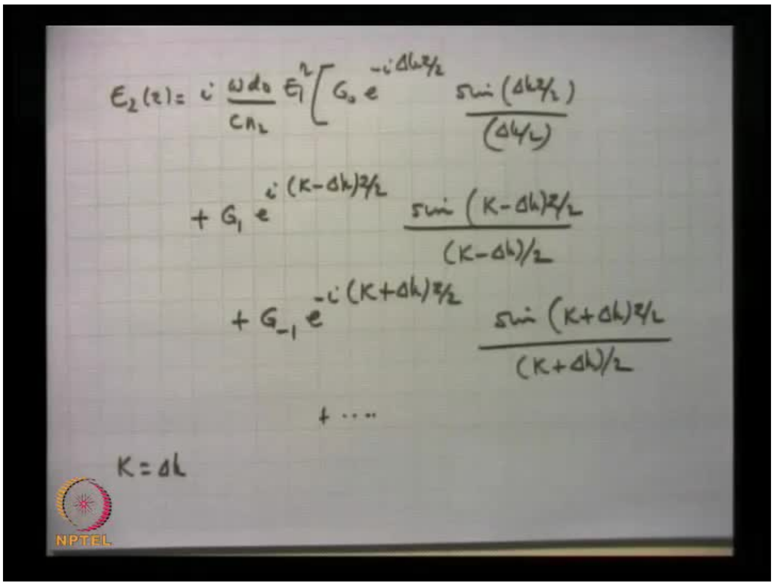

So, by choosing capital K is equal to delta k what I am essentially doing is, making one of these sin terms… give me z here, instead of an oscillatory solution and my second harmonic field grows rather than being oscillatory. Because I am choosing the first Fourier coefficient capital K is equal to delta k, this is called first order quasi phase matching.

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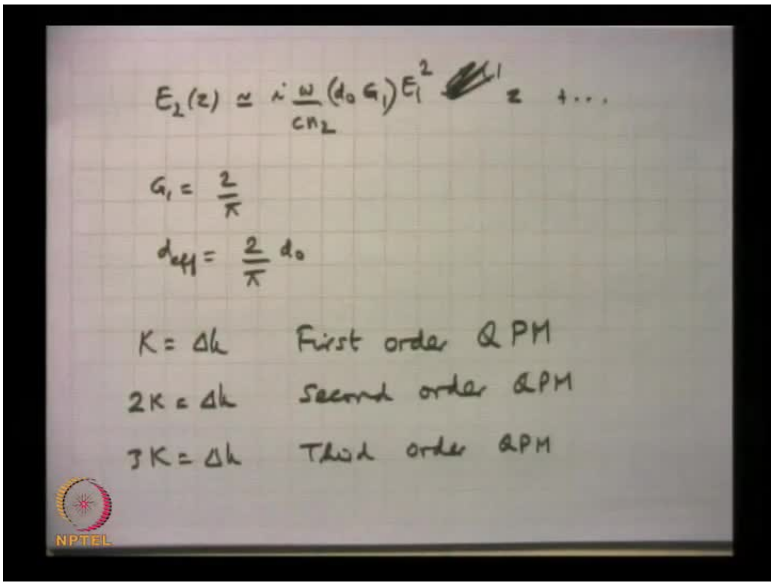

So, if I choose k is equal to delta k, this is first order Q P M; if I choose 2 k is equal to delta k, this is called second order Q P M, and if I choose 3 k is equal to delta k, it is called third order Q P M. So, what I have calculated is, for a first order Q P M, if you took if you took a second order Q P M, I will have d 0 times G 2 here; if I choose a third order Q P M, I will have d 0 times G 3 here.

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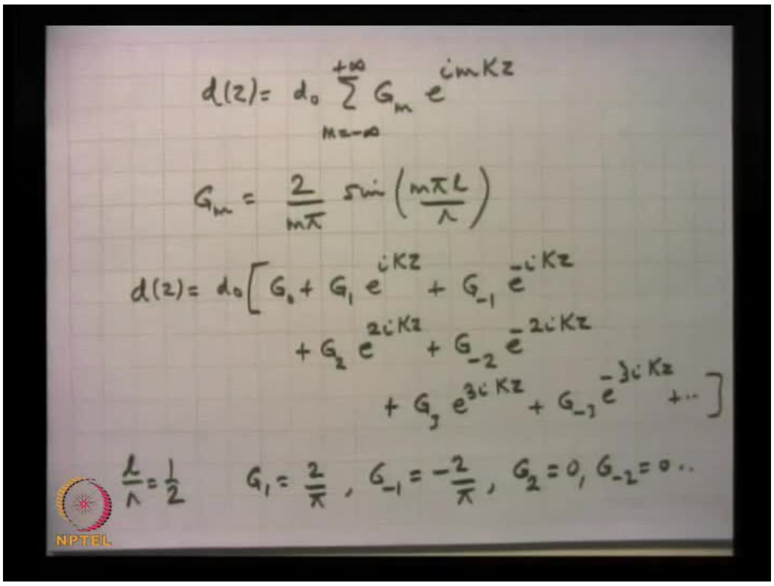

Now, please note that as you go to higher Fourier coefficients, the amplitude of the Fourier term keeps on dropping down. Because I have just calculated G m is 2 by m pi sin m pi l by lambda, and if you have a larger values of m, you have an increasing denominator here; so G 1 is 2 by pi, G 3 is 2 by 3 pi, G 5 is 2 by 5 pi, and so on.

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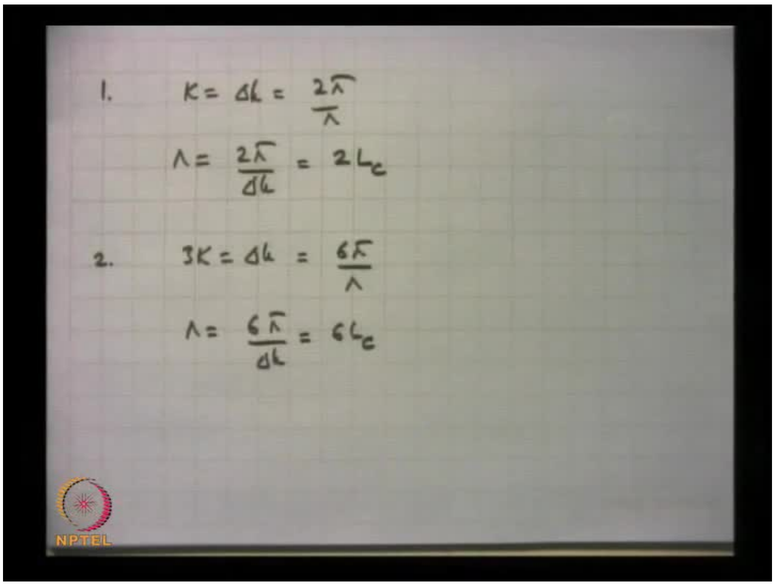

But what is advantage of going to higher order Q P M? So, for example, if I were to use first order Q P M, I need to choose k is equal to delta k, which is equal to k is equal to 2 pi by lambda; so the period required is 2 pi by delta k. And what is pi by delta k? pi by delta k is the coherence length; so this is twice L c that is visible from here; that I need to have the period to be twice L c.

I need to change sine of the d coefficient after half the coherence length here, and then I get quasi phase matching and so if the coherence length 3.3 microns, I need to have a periodicity of 6.6 microns. Every 3.3 microns, I need to reverse the direction or change the sign of d. If I were to use third order Q P M, then I need 3 k is equal to delta k, which is equal to 6 pi by lambda; so lambda is equal to 6 pi by delta k, which is equal to 6 times L c.

I gain in terms of the period required for quasi phase matching. Remember, to fabricate this is not so easy; so I need to worry about fabrication problems. So, if I need to reverse the sign of d coefficient every 3.3 microns, it is much more difficult than to do at 600 L c, which is about 19 microns or so. So, it is much easier to make third order quasi phase matching than first order quasi phase matching, but what is it what is the price I pay?

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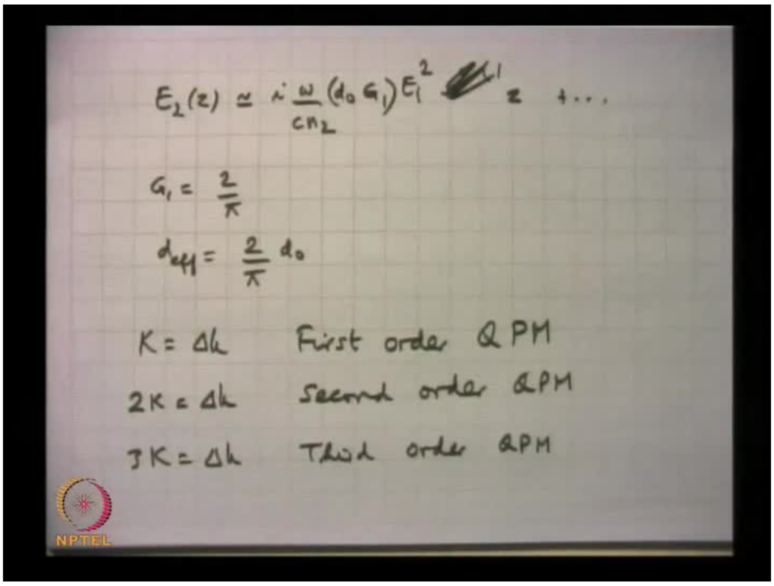

You see here, that the non-linear coefficient - effective non-linear coefficient, instead of being 2 by pi d 0 for first order Q P M becomes 2 by 3 pi d 0 for third order Q P M. And because the efficiency of second harmonic generation depends on the square of the non-linear coefficients, note that the second harmonic power is proportional to E 2 square, and E 2 square is proportional to d 0 G 1 whole square; it is proportional to G 1 square or G 3 square, G m square.

So, if I go to higher order quasi phase matching, the corresponding Fourier coefficients have smaller magnitudes and the effective non-linear coefficient will be smaller; and so the efficiency will decrease as you go to higher order Q P M.

So, already if you have perfectly phase matched, the effective non-linear coefficient will be d 0. If you have first order Q P M, the effective non-linear coefficient is 2 by pi d 0 and because the efficiency goes up as G 1 square, it will be 4 by pi square times the efficiency, if you had perfect phase matching. And pi square is of the order of 10; so 4 by pi square is about 0.4, that is, 40 percent. So, if you have perfect phase matching for certain efficiency, if you go to first order Q P M, your efficiency drops on by 40 percent.

If you go to third order Q P M it becomes another 1 by 9 times smaller, and that means another 10 times factor 9 times decrease in in efficiency. And so, of course, it is easier to make third order Q P M structures, but the price you pay is in terms of efficiency.

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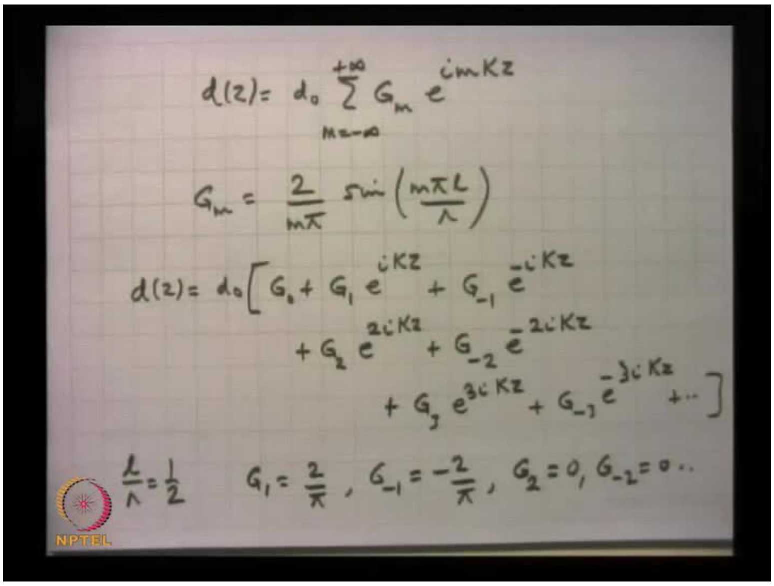

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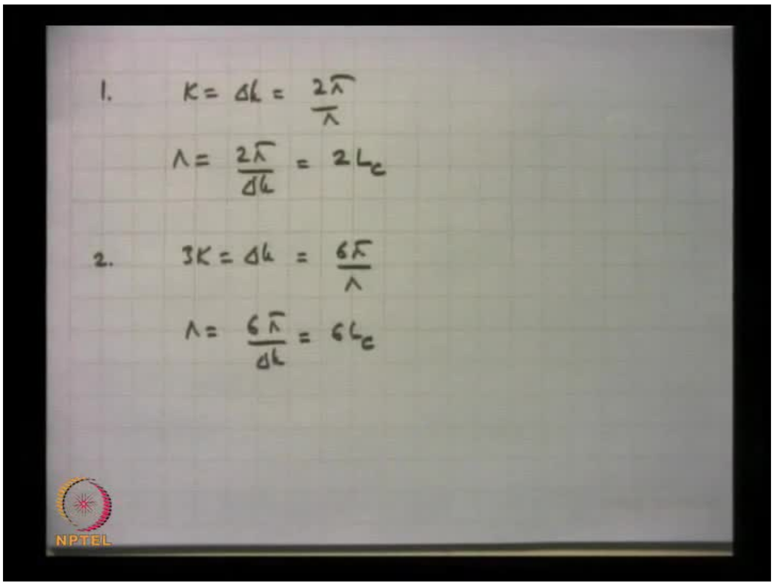

Now, I did not talk about G 2, because you note here, that if you choose a duty cycle of half; if l by lambda is half, then all even order coefficients are 0; so there is no G 2; so there is no G 2. So, in this expansion here, if your duty cycle is half, G 2 is 0; so there is no… You may be achieving second order quasi phase matching, but the effective non-linear coefficient is 0. See you will not generate second harmonic; if you choose a duty cycle of half.

So, I would not want to leave another problem to you; please find out, what is the best duty cycle that I must use, so that I am able to use the second order quasi phase matching term? So, what should be the value of… If I go back, look at this expansion here, what should be the value of l by lambda?

What should be the value of l by lambda here, so that I can have a finite value of G 2 and the maximum G2 that is possible? So, I leave this problem to you, and then correspondingly calculate what is the drop in efficiency, if I do second order of Q P M in comparison to the phase matched interaction process?

Now, so, for example, if I were to take the first order Q P M, then I can calculate; I had this expression here; let me write it again. So, this is the expression for E 2 of z; so from here, I can actually calculate the second harmonic power.

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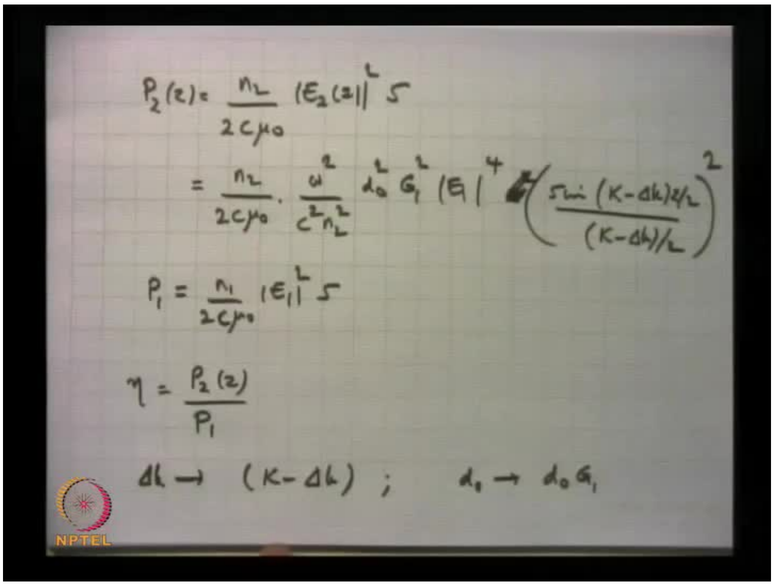

So, P 2 of z is equal to n 2 by 2 c mu 0 mod E 2 of z whole square into the area; so this is equal to n 2 by 2 c mu 0 omega square by c square n 2 square d 0 square G 1 square E 1 mod 4 z square. If I do not take exactly delta k is equal to capital K is equal to delta k, I will have essentially the term like, instead of, z square I will have sin K minus delta k z by 2 by K minus delta k by 2 whole square.

If k is equal to If capital K is equal to delta k exactly, then this becomes z square; otherwise, there is a sine term, which comes here. I can replace E 1 square in terms of

P1; remember, the fundamental power is n 1 by 2 c mu 0 mod E 1 square into area. So, I can substitute into this equation and calculate the corresponding efficiency of second harmonic generation P 2 of z by P 1. P 1 is assumed to be independent of z, because I am neglecting the amount of power that goes from the fundamental to second harmonic; I am neglecting the changes in E 1 as a function of z. Please remember that, I cannot generate second harmonic, unless I will unless I lose power from the fundamental.

So E 1, if E 2 changes, E 1 has to change, but I am neglecting the change in E 1 to be able to different able to integrate the equation; so that I get an expression for efficiency. So, you again substitute in terms of P 1 and calculate; so the only difference term perfect matching you will see is, instead of, sine delta k z by 2 by delta k by 2 whole square, I am getting sine K minus delta k z by 2 by K minus delta k z by 2.

So, delta k gets replaced by K minus delta k, and the non-linear coefficient gets replaced by d 0 times G 1 for first order Q P M. So, I can use this process, quasi phase matching to cancel the effects of delta k by an appropriate period of a reversal of the sign of the d coefficient. In birefringence phase matching, I needed to have a first medium, which has a birefringence; there must be an ordinary wave, there must be an extraordinary wave.

And the fundamental in second harmonic are to have orthogonal polarization states, but here please note, that I am not using any birefringence if there is a finite value of delta k between the fundamental and second harmonic. For example, if you have enter the same polarization, then it is medium is anisotropic and if they have the same polarization Lithium Niobate, let me assume, the omega frequency is extraordinary wave; the 2 omega frequency in an extraordinary wave; obviously, I cannot have birefringence phase matching, because the extraordinary refractive index at omega and the extraordinary refractive index at 2 omega are not equal.

So, delta k is not equal to 0, but if I can change the sign of capital as small d periodically with a period capital lambda having a spatial frequency capital K, then if which is equal to delta k, then I will have this for becoming z square and have a very good efficiency.

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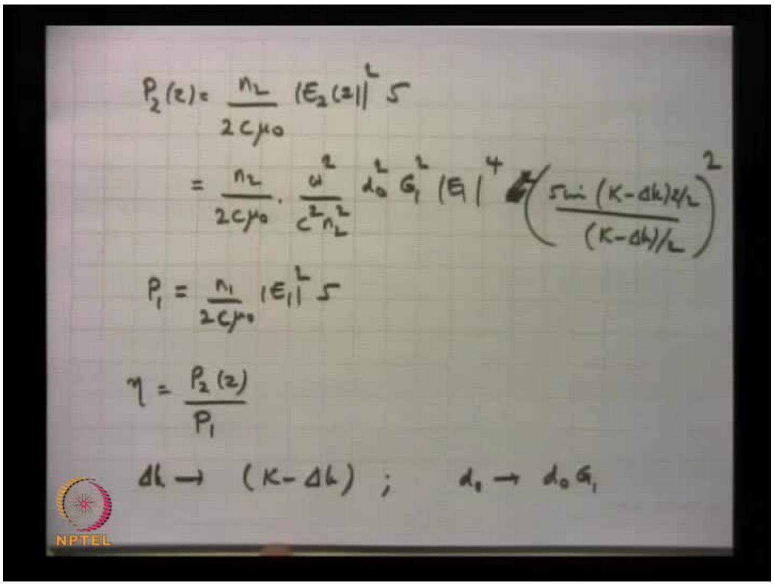

So that is the advantage of quasi phase matching; that I do not depend on the birefringence of the crystal and I can actually have a crystal, which is isotropic or in an isotropic medium, I can have interaction from omega to 2 omega for waves having the same polarization states and so on.

So that is the advantage of Q P M and this is now, a very standard technique for most third order non-linear processes. Now, I want to mention to you one thing, that suppose, I were to choose first order Q P M.

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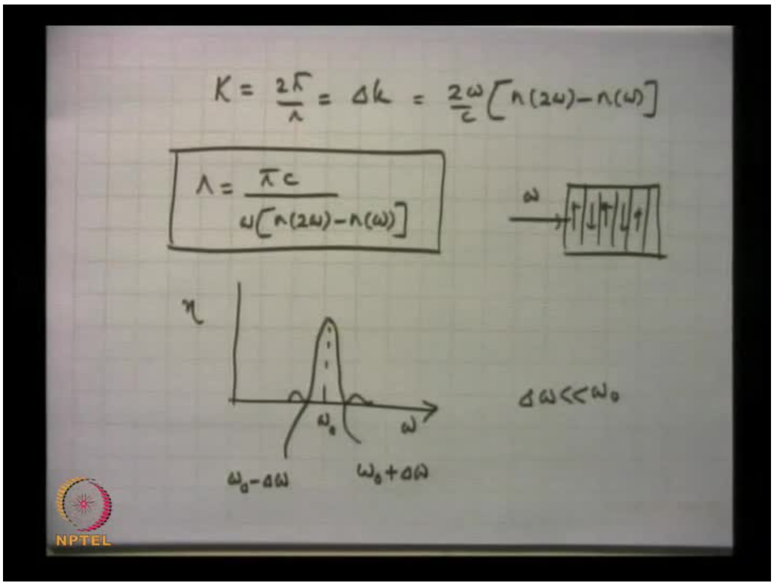

So, K is equal to 2 pi by lambda is equal to delta k; so this is equal to 2 omega by c n of 2 omega minus n of omega. So, the period required is pi c by omega into n of 2 omega minus n of omega. So, as I mentioned to you before, the spatial period required for quasi phase matching depends on the fundamental frequency.

So, if I take crystal and if I launch certain omega, the sign reversal of d positive negative, positive negative depends on the frequency. If I vary the input frequency slightly the required period is different from the period that exists in the medium and the efficiency will drop down.

So, in fact if you were to plot the efficiency as the function of frequency, what you will get is a curve like this; this is the frequency omega 0 for which you have chosen the capital lambda, and as certain frequency separated from here by delta omega, assuming delta omega much less than omega 0, deficiency will drop to 0.

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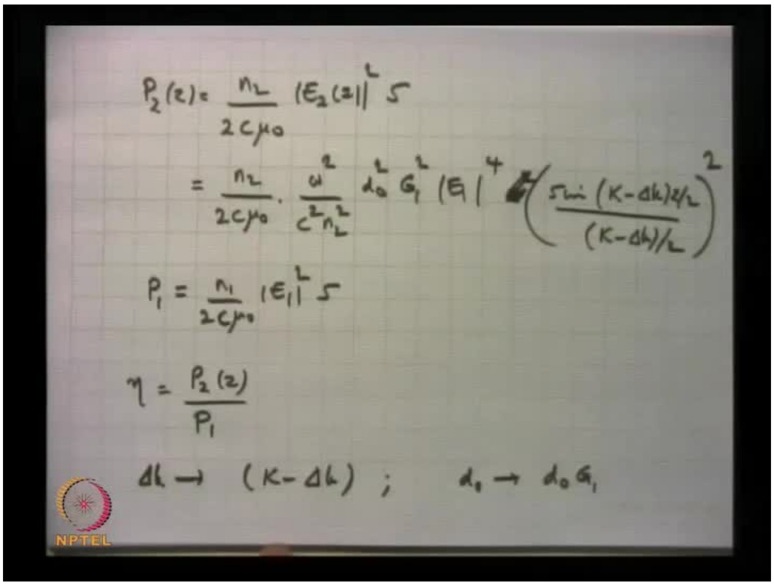

In fact, you can look at this equation and tell me what is the value of delta k at which deficiency will drop to 0? Deficiency will drop to 0, when the sine function becomes 0 that is K minus delta k into… If the crystal length is l, K minus delta k into l by 2 is equal to pi; so I can immediately calculate.

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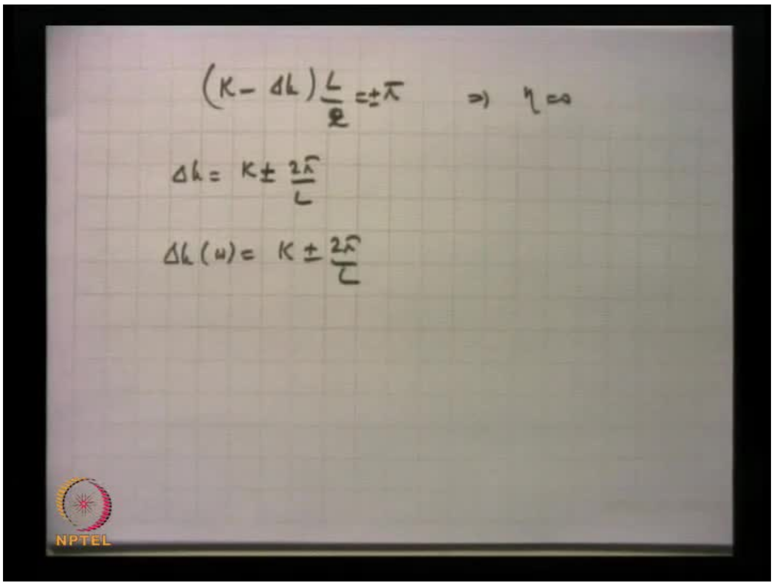

So, when K minus delta k into L by 2 is equal to pi deficiency becomes 0, and that means delta k is equal to K minus, actually plus minus pi; so delta k is equal to K plus or minus 2 pi by L.

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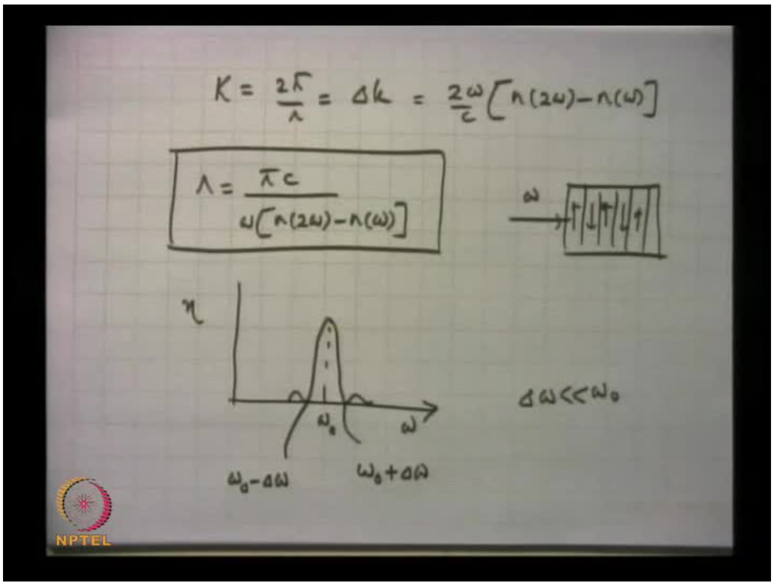

And this K, delta k is the function of omega; please note that delta k is a function of omega. So, in this curve what I have drawn here is, this is the frequency at which I have designed my quasi phase matching period for which capital K is equal to delta k.

As I move away from here, delta k changes and when delta k becomes k of plus minus 2 pi by L, the corresponding efficiency becomes 0. So, I can man-width of this interaction process, which is approximately delta omega over which the second harmonic, this quasi phase matching crystal will work efficiently.

So, if you were to deviate your input frequency by more than of the order delta omega, then the efficiencies of second harmonic generation will become very poor. The longer the crystal, the narrower is the bandwidth of second harmonic interaction process. So, if you take a long crystal with the domain reversed crystal, then the bandwidth is much narrower than if you have a shorter crystal.

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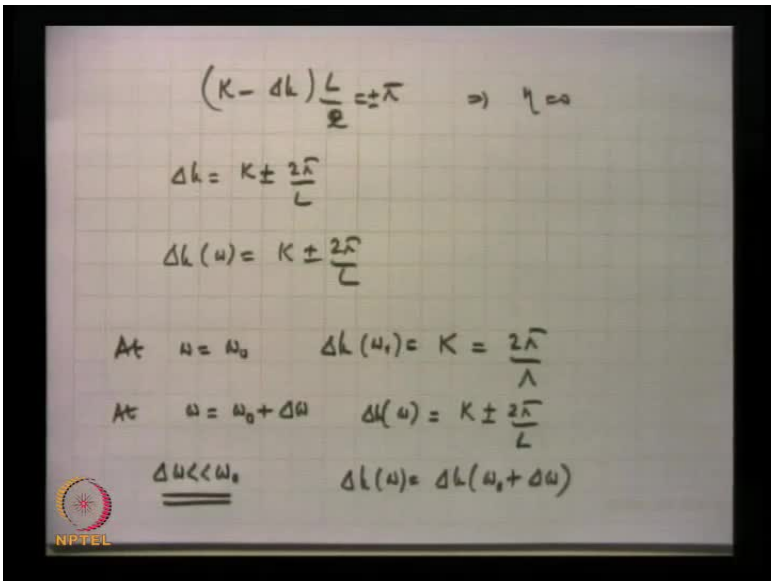

So, again, what I would like you to do is, please note that, let me give you a problem. The problem is at omega is equal to omega 0, delta k of omega 0 is equal to K; this is equal to 2 pi by lambda. At omega is equal to omega 0 plus delta omega delta k of omega is equal to K plus minus 2 pi by L plus or minus you find out.

So, if delta omega is much less than omega 0; from this expression here, you can write delta k of omega is equal to delta k of omega 0 plus delta omega. Make a Taylor series expansion, use this equation here and calculate delta omega, an expression for delta omega. What is the bandwidth of the interaction process in terms of the refractive indices or in terms of whatever it is?

So, I will leave it as a problem to you to calculate, what is the bandwidth of this interaction process how does in terms of frequency delta omega or delta lambda? So, this is important in a practical device, because I need to know, how precisely I need to control the frequency of the input laser, so that I keep having high efficiency second harmonic generation.

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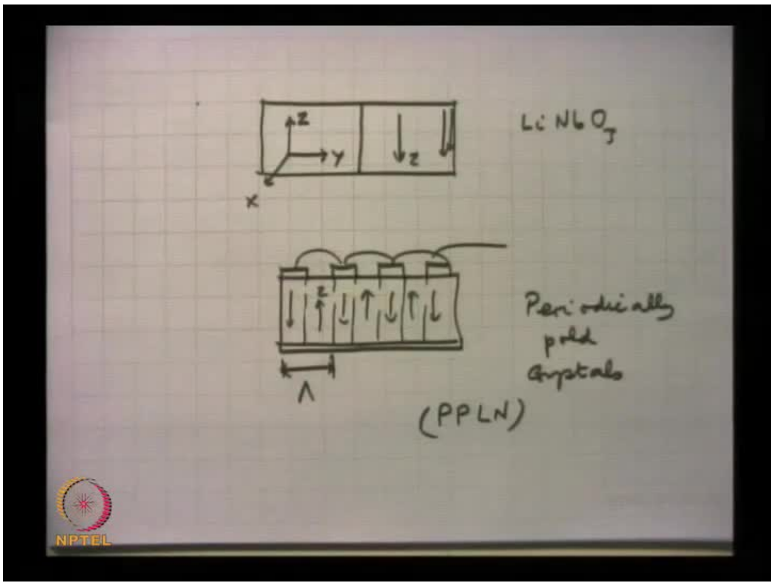

Now, obviously, the question arises, how do I change the sign of d? Now, there is a standard procedure now, which is employed. So, let me take an example of Lithium Niobate Linbo3, this crystal is a ferroelectric crystal; so it has a spontaneous polarization.

And for example, let me call this as z axis of the crystal; so x may be here, y may be here; so these are the three principle axis of a crystal x y z. It is a uniaxial crystal; so the optic axis is along the z axis and what happens in this crystal is, there is a spontaneous polarization, that is, take that is there along the z axis.

Now, what I can do is, if we apply very strong electric field in the reverse direction, suppose, I apply a strong electric field in the downward direction, I can actually reverse the direction of the z axis by flipping the atoms within the crystal. And what happens is for example, if I take this half of the crystal and if I apply very strong electric field in the downward direction, then I can change the orientation z axis from pointing up to pointing down.

It is just like in magnets; you have north pole, south pole and you can flip it out into make it south pole, north pole by applying a very strong magnetic field in the reverse direction. So, you can actually change the orientation of the spontaneous polarization direction by applying a very strong electric field in the opposite direction.

So, one of the techniques, it is called the electric field poling. So, what you do is, you apply; you put in periodic electrodes. So, this is a crystal having say z axis like with originally; so you apply; you connect all this and apply electric field in the downward direction; when you apply a strong enough electric field, what happens is the optic axis in this position reverses its direction.

So, here, there is a strong electric field in the downward direction, which changes the z axis from pointing up to pointing down. And similarly, here, again there is a strong electric field from pointing up to pointing down and this is the period of domain reversible.

So, this is called periodic poling, that means you are poling the crystal orientation periodically and so this is are called periodically poled crystals. And in fact, this is called periodically poled Lithium Niobate; if it is the Lithium Niobate also called in short form as (( )) that means these are Lithium Niobate crystal in which the direction of the spontaneous polarization is periodically reversing in the crystal. But please note, that the linear property of the medium is the same along the z direction, along this direction, but its optic axis reversing its orientation.

So, what is the effect of this direction change of optic axis? Again, please note, that the d tensor we have written d i j k; it is a tensor and it is written in the principle axis system. So, in the principle axis system these coefficients - d coefficients are all written in the principle axis system.

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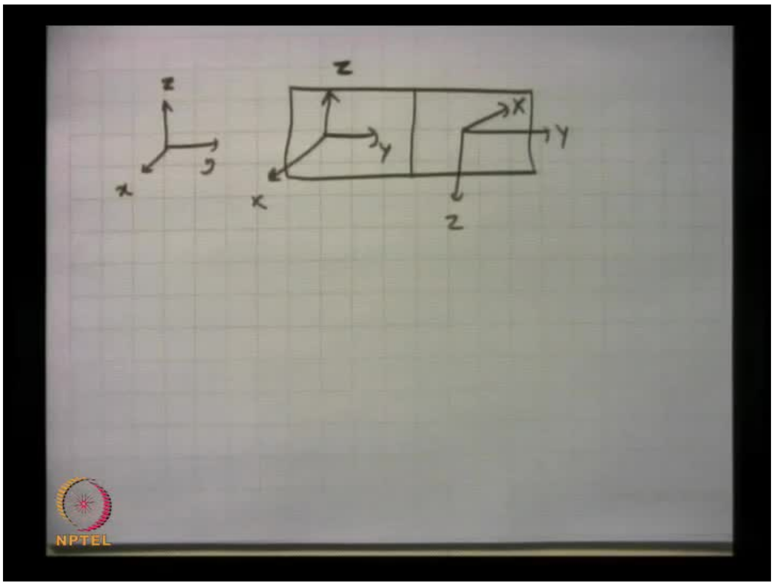

So, if I were to consider a crystal with 2 parts, for example, in one part let me assume the z axis was here; this is x and this is y, and in the other part, I had z like this, y like this and x like this; I have changed the direction of z, I have to keep the right handed coordinate system. So, in this coordinate system here, the d tensor, which I have written is valid in this coordinate system; the d tensor, which I have written is also valid; provided, I choose this coordinate axis. But please note, that the equation which I am writing are in a laboratory coordinate system. So, they could be like this for example, this could be x y z.

Let me call this as capital Z, capital X, Capital Y; this is the principle axis system; this is the laboratory coordinate system. So in this part of the crystal, the laboratory coordinate system and the crystal principle axis exactly match; so the d tensor written in the principle axis system is also the d tensor in the laboratory coordinate system. In the second part of the crystal, the laboratory coordinate system and the principle axis system do not coincide. The d tensor is written in the principle axis system; so I need to transform the d tensor from the principle axis system to the laboratory coordinate system. So, what do I? I use the transformation properties of tensors.

Note that, between these two, all I have done is to change x to minus x, z to minus z and keep y the same; so this system and this system are related through x to minus x and z to minus z and y to plus y.

So, I leave it as a problem to you that please calculate, when you rotate the principle axis system around the y axis by 180 degrees, which are the elements of the d tensors, which change sign; which are the elements of the d tensor, which do not change sign. That means I need to write the d tensor of this part of the crystal in the laboratory coordinate system and compare that with the d tensor of this half, and you will find you may find that some of the tensor elements do not change sign; some of them would have changed sign. So, those elements which have changed sign, I can use them for quasi phase matching; if the particular non-linear coefficient does not change sign obviously, I cannot use that for quasi phase matching.

Please go back and look at your earlier notes on tensor analysis; find out, how to transform tensors it is very simple; it is not a general angle, it is just x to minus x, z to minus z, y to plus y. And so, you can actually use this transformation properties of tensors to find out, which d i j k elements here are negative of here, and which d i j k elements do not change sign. And you will see for example, d 3 3 is the largest non-linear coefficient Lithium Niobate and that changes sign. So, you can actually use d 3 3 for quasi phase matching.

So, please go back and do. In case, you have a problem; we can discuss that in the class and analyze this little more carefully. Do you have any questions in quasi phase matching? So, let me let me just recollect what you have done in the equation for second harmonic. There is an exponential minus I delta k z, when I integrate over length, this exponential factor gives me an oscillatory solution and to remove this oscillatory solution, I must make delta k is equal to 0. If I choose delta k is equal to 0 the second harmonic power grows quadratically with length and I have very high efficiencies.

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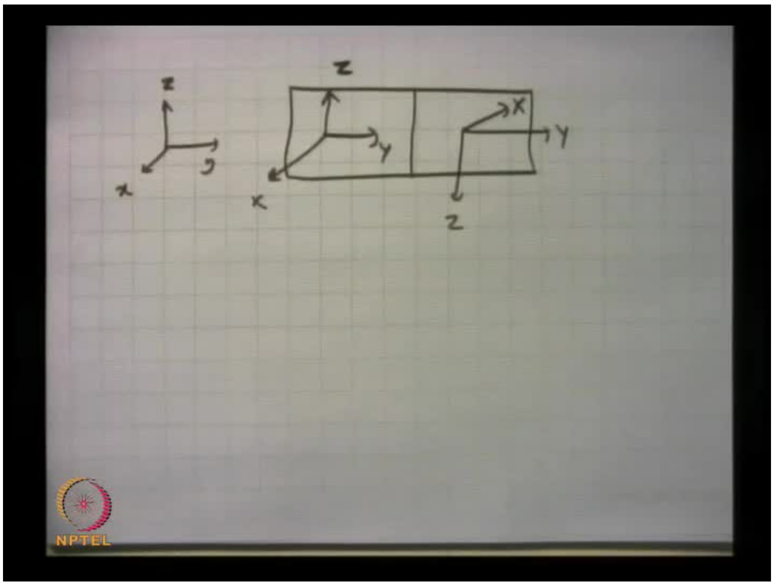

And the way to achieve this is called birefringence phase matching. Now, the problem with birefringence phase matching is, I need… the crystal has to be anisotropic, and the polarization states of the omega and 2 omega are orthogonal to each other, which means there are some d elements - some d tensor elements, which are responsible for this interaction and they may not be the largest.

So, to overcome this problem we have discussed quasi phase matching in which I do not take a crystal with a uniform d value, but I periodically reverse the sign of d, the non-linear coefficient. By doing that I can use this periodic variation in d to cancel the effects of exponential minus i delta k z, and I can achieve quasi phase matching. And then I do not have to have a birefringence crystal; I can use the same polarization state for omega 2 omega and achieve very good conversion efficiency.

So, the period required for this quasi phase matching depends on the frequency of which you want to do second harmonic generation, and it also depends on the refractive indices of the medium at omega and 2 omega.

You can do first order quasi phase matching; you can do third order quasi phase matching, and so on. And of course, you pay a price compared to perfect phase matching and the price is in terms of efficiencies. Because there is an effective non-linear coefficient now, which is less because of the Fourier expansion term and so you have a dropped efficiency. So, if you take a 50 percent duty cycle, I showed you that the drop in efficiency is about 40 percent. And so, but you can still have phase matching and may be you can use actually you can use higher non-linear coefficient of the d tensor to achieve increased efficiencies, and this is now, the standard technique, which is conventionally used for most of the chi two processes.

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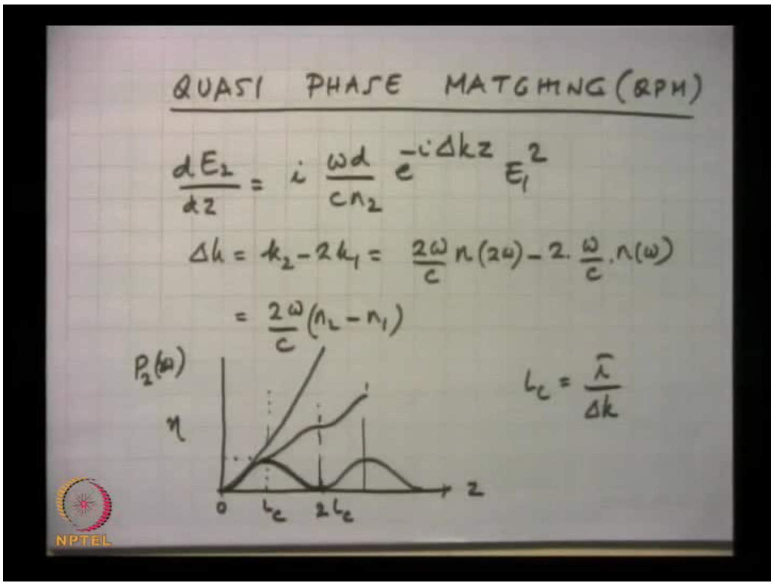

Now, if there are no questions, what we would like to next do is, to look at exact solutions of the problem. Remember, we have solved the equation for E 2 by assuming that the conversion from omega to 2 omega is of low efficiency, which means that I have assumed that E 1 as constant, when I integrated this equation either in perfect phase matching either with no phase quasi phase matching or with quasi phase matching.

And so that expression, which I get can be used only for low efficiencies, may be 5 percent, may be 8 percent, but what will happen if my efficiency becomes larger? I cannot use those solutions; so we need to look for solutions and what I will like to do is, to look at the solutions for a perfect phase match situation and get exact solutions to the coupled equation problem.

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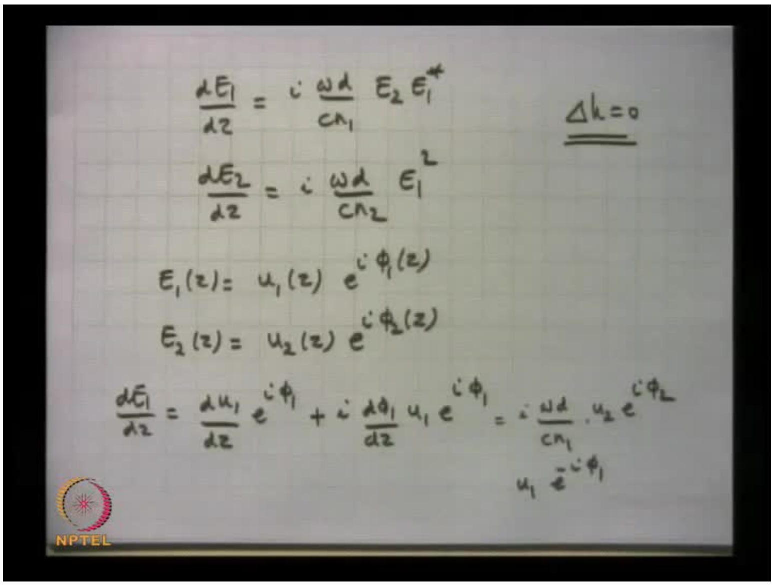

So, let be recall the 2 equation; so d E 1 by d z is equal to i omega d by c n 1 E 2 E 1 star and d E 2 by d z is equal to i omega d by c n 2 E 1 square. I am assuming delta k is equal to 0. What we did was we solved this equations assuming E 1 is a constant; actually including the phase mismatched term and we got a solution.

Now, I want to solve these 2 coupled equations and not to make this approximation of neglecting variation E 1. So, let me give you a few steps here and we will go to the complete solution in the next class. So, what I do is please note that E 1 and E 2 are the complex dielectric fields.

So, let me write E 1 of z is equal to u 1 of z exponential i phi 1 of z E 2 of z is equal to u 2 of z exponential i phi 2 of z; u 1, u 2, phi 1, phi 2 are all real terms. So that is the amplitude of the electric field; that is the phase of the electric field, the amplitude of the second harmonic, the phase of the second harmonic.

So, I want to substitute this into this equation. So, for example, let me calculate d E 1 by d z is equal to d u 1 by d z E to the power i phi 1 plus i d phi 1 by d z u 1 E to the power i phi 1, which is equal to i omega d by c n 1 E 2 E 1 star. So, u 2 E to the power i phi 2 into E 1 star, which is u 1 E to the power i phi 1; I am substituting this into this equation; so E 1 d E 1 by d z is equal to i omega d by c n 1 E 2 into E 1 star.

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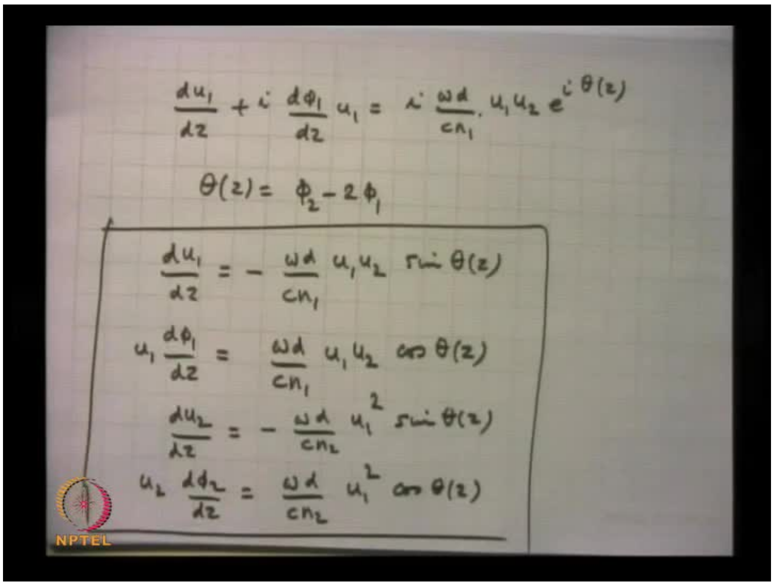

So, if I take exponential i phi 1 on the other side, what I will get is d u 1 by d z plus i d phi 1 by d z into u 1 is equal to i omega d by c n 1 u 1 u 2 exponential i theta of z; let be call this, but theta of z is phi 2 minus 2 phi 1.

Now, I can equate the real and imaginary parts on both sides and I can get 2 equations; so d u 1 by d z is the real part; so I have minus omega d by c n 1 u 1 u 2 sin theta of z and d phi 1 by d z is equal to into u 1 omega d by c n 1 u 1 u 2 cos theta of z.

So, what I have done is, I have written the electric field as a product of the amplitude term and a phase term here; substitute it into this equation and this one equation here, gets replaced by 2 equations here; similarly, I can substitute for E 2 and E 1 in the second equation and I get two more equation.

So, let me write those two equations; please check it yourself d u 2 by d z is equal to minus omega d by c n 2 u 1 square sin theta of z, and u 2 d phi 2 by d z is equal to omega d by c n 2 u 1 square cos theta of z.

Four equations instead of, 2 complex coupled equations, I have 4 coupled equations; now, all real equations here in terms of u 1, u 2, phi 1 and phi 2 and how they vary with z?

So, what we will do in the next class is to solve these 4 equations and get the solutions and I will show you that I can get the exact solution of these equations; these are under phase matching conditions only.

So, effectively what you will analyze is, here is my crystal delta k is equal to 0 and I have a frequency omega incident here with amplitude u 1 and with a phase phi 1, what is it that is get, what is the u 2 here that is getting generated? So, this is the frequency 2 omega that comes out. So, we will be able to obtain a solution to the second harmonic exact solutions under phase matching conditions.

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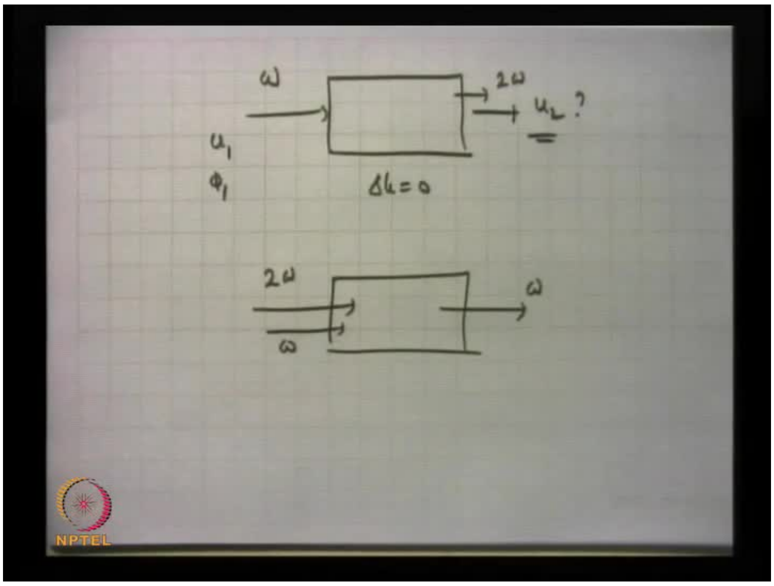

And later on I am going to use the same set of equations for the reverse problem of generating omega from 2 omega, actually this problem, where instead of going from omega to 2 omega, I go from 2 omega to omega.

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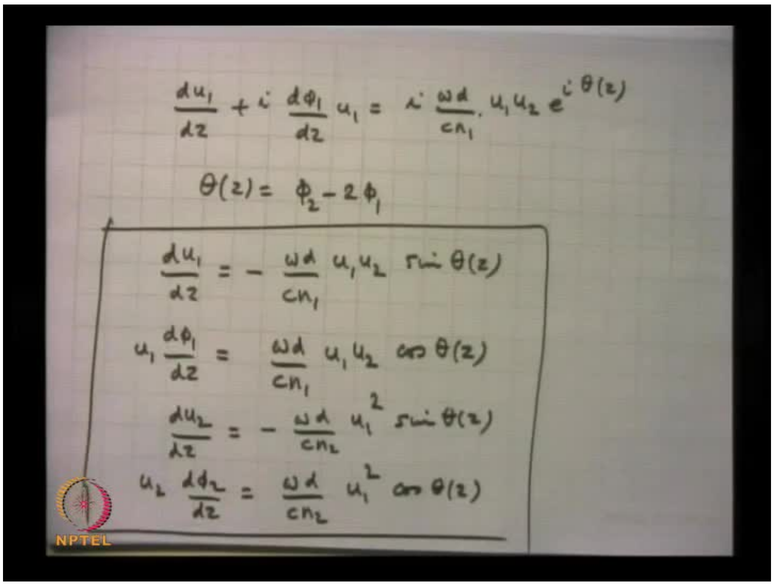

This is called the parametric amplification and I will use the same set of equations, and these equations are very interesting, because they give me a complete analytical solution to the problem of second harmonic generation under phase matched operation condition.

Do you have any questions? So, what we will do next class is to solve these equations and get complete solutions.

Thank you very much.