Plasma Physics

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Lecture No. # 09
Electromagnetic Wave Propagation
Inhomogeneous Plasma

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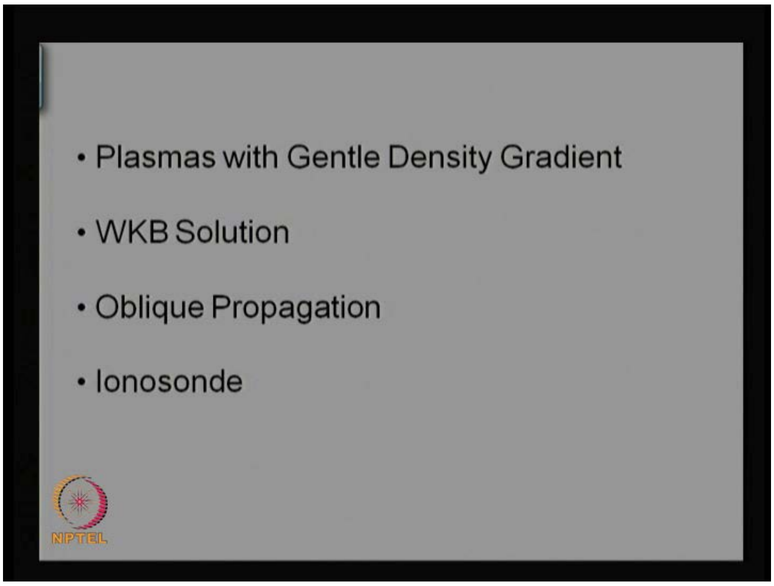

Today, I would like to discuss the propagation of electromagnetic waves in inhomogeneous plasmas. Well, I will discuss a few cases where plasmas with gentle density gradient are found, and then discuss the propagation of waves on the basis of WKB solution. And then I will consider the case of oblique propagation of an electromagnetic wave to a density gradient in a plasma, and finally I like to discuss the application of this analysis to a technique called ionosonde to determine the electron density profile in the earth ionosphere.

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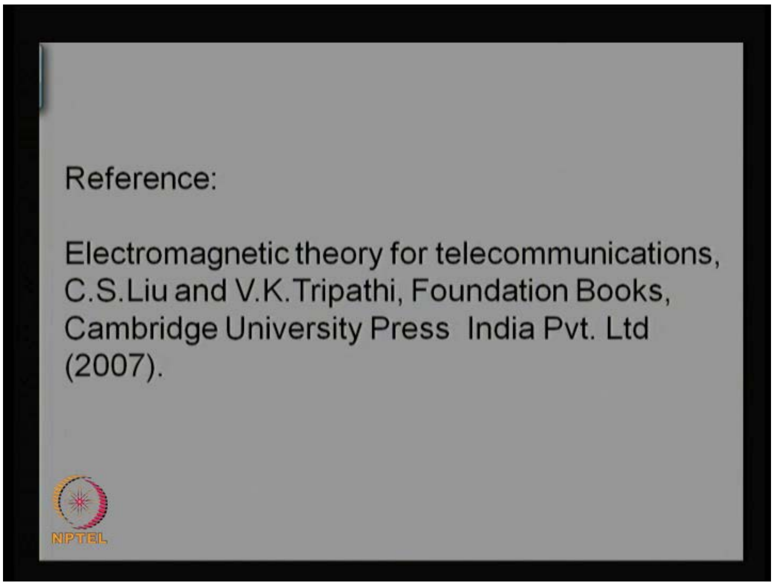

The reference for today's presentation is the book electromagnetic theory for telecommunications, that professor C S Liu, and myself wrote and published by Cambridge university press.

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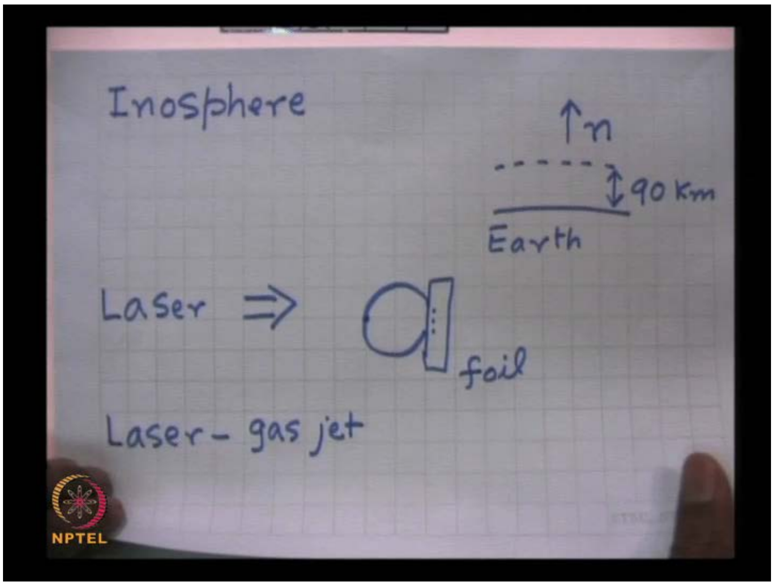

Well, let me cite a few example where realistic plasmas are of inhomogeneous density variation, for instance ionosphere look at the earth ionosphere earth ionosphere is the region of earth atmosphere which is ionized. So, if you take earth like this then the ionosphere starts at a height typically about 90 kilometers but, the density suddenly does not change from zero in the atmosphere to finite value in the ionosphere, it gradually builds up rather the density increases as you go up so electron density end increases with height its nearly zero at the boundary and as you moves up it increases.

So, if you want to launch an electromagnetic wave from the ground based transmitter, the wave will penetrate into the ionosphere but, the ionosphere is not of constant density but, of gradual density variation, so this is a typical example of inhomogeneous plasma. Currently there is lot of interest in laser interaction with targets for instance consider a thin metal foil, if you launch a laser beam on the metal foil and the metal foil and the laser intensity is large, the laser penetrates just a little in the material but, it can convert this into a plasma and plasma expands in time. So, after a little while of onset of the laser, if you examine the density variation of or the character of plasma that is formed outside this is called a plasma plume.

The plasma is created at this boundary here and this is expanding outside, so if you look at this plasma plume, the density here is the minimum and as you move into the plume the density increases, so this is another example of gradual density variation. Third example which is of current interest is, what we call as laser, this is the example of a foil; metal foil and then there is an example of laser gas jet interaction. what you do in laser gas jet interaction?

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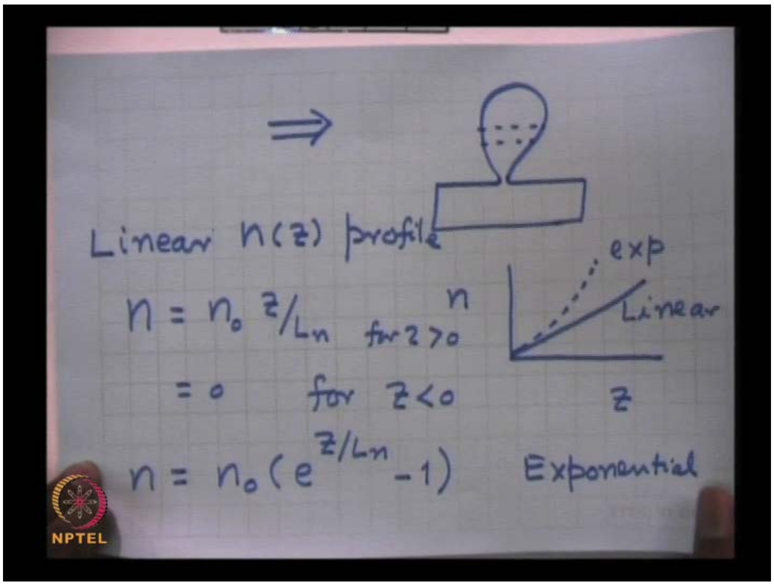

You have a container that has a gas at a high pressure and there is a small nozzle in here, through which the gas comes out, the gas forms sort of a plume here like this and you will shine a very high power laser into this gas then it forms a plasma here, in the region through which the laser propagates. If you look at the density in the plasma as you move from one end to the other end, the density varies at the edge here density variation is rather rapid but, then it becomes quite uniform then falls off. but its not that rapid it increases in several wave lengths. So, I can still treat the plasma boundary to be a diffuse boundary rather than a sharp boundary.

So this is another example, where density variation is important in many cases the density variation can be taken be linear, so we say that the density variation profile is linear n profile like density can vary like n is equal to n 0 z upon L n means If I plot a density on the y axis electron density and z is that distance from the plasma boundary this I have written for z bigger than 0 and this is 0 for z less than 0.

So, if you have this kind of profile then the density if you plot this as a function of z will be a straight line like this in some cases you may have exponential density profile, where density varies as some constant n 0 exponential of z upon L n some constant L n minus 1 in that case the density will vary little more rapidly like this. So this is called exponential density profile, this is called linear density profile, this is exponential both kind of profiles are observed in realistic laboratory plasmas and as well as in space plasmas.

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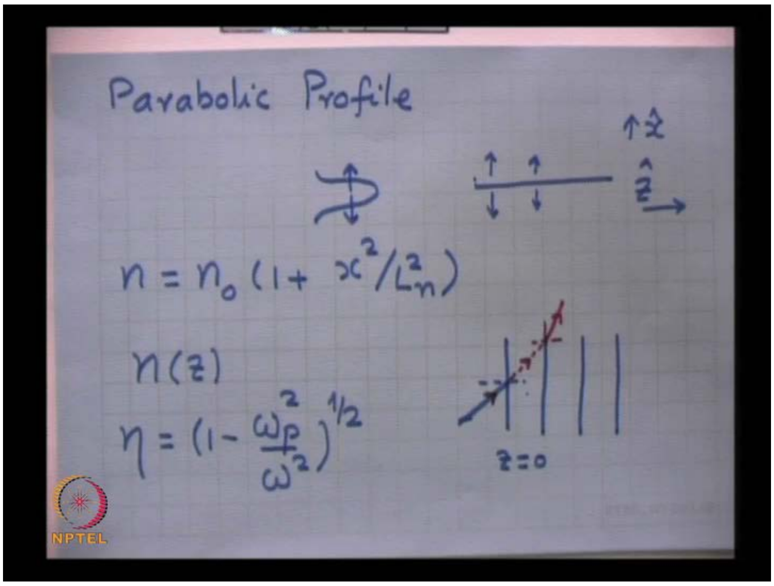

Another important density variation that occurs especially in laser plasma interaction is called a parabolic profile, what you have for instance you have gas jet target and (( )) laser here like this. This is the laser intensity variation, so when the laser travels through a gas jet it produces a plasma on the axis of the laser, whose density increases in this direction as well as in this direction this is say z axis and so I am considering a coordinate system where x axis is here and z axis is there.

So, in this situation density can vary with x and the profile usually that one considers say of this form density is minimum on the axis laser axis and it increases with x like 1 plus x square upon some constant L n square, n 0 is the density on the laser axis really what happens that when the laser propagates through a plasma its intensity being largest on the axis and less outside it exerts a radiation pressure force on the electrons in these directions and the electrons move out, sometimes they can carry the ions also along with them.

So there is a reduction in plasma density on the axis, so this is called a parabolic density profile. And this has been found to be very suitable for guiding the laser. So, this is a important problem in plasma physics to examine the propagation of laser in inhomogeneous plasma.

Before, I go into discussing the wave theory of wave propagation, let me tell you something regarding the qualitative aspects or physical aspects of what do you expect really in such situations. What you can consider is to be simple, consider a general density variation with z for instance, what you can do? You can think as if suppose this is my z equal to 0 plane from where the plasma begins this is free space here and the density is increasing for instance with z.

What you can do? Mathematically you can divide this plasma into many layers of increasing density but, in each layer the density is constant, you know the refractive index of a plasma is eta which is 1 minus omega p square upon omega square to the power half omega p depends on electron density.

So when density changes from place to place omega p also increases, as you z increases as a result eta decreases. So if you are launching a wave in free space from here, at some angle for instance then this ray as it goes from a rarer medium optically rarer medium to a denser medium sorry optically denser medium to optically rarer medium, because plasma is a rarer medium having refractive index less than 1.

So this will move away from the normal, I may draw this like this it will move towards this strikes the boundary between this layer and this layer again this will move further away from here so it may go like this. So the ray bends away from the normal as it goes from one medium to another medium this is what you physically expect, well one thing is very important in here and that is called the Snell's law.

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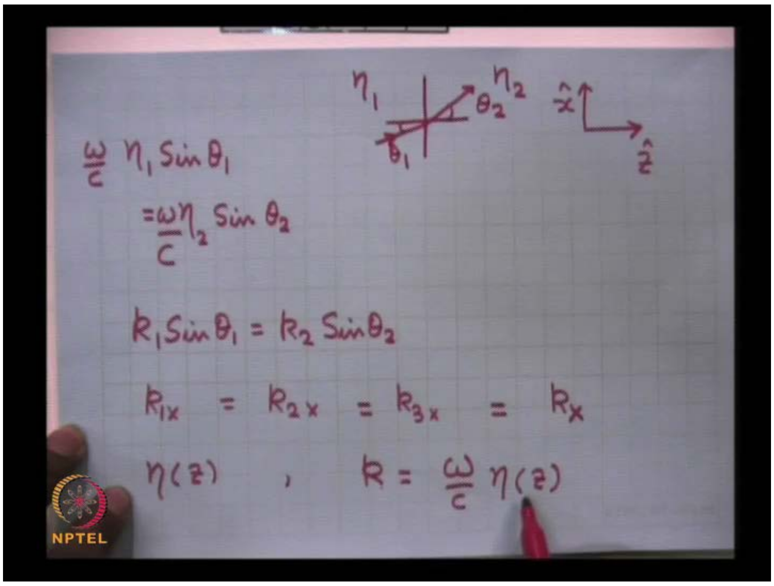

Which tells that if a ray goes from one medium at an angle theta 1 and then it gets out in second medium with angle of transmission equal to theta 2, if the refractive index of medium number 1 is eta 1 that of second medium is eta 2 then n 1 sin theta 1 is equal to eta 2 sin theta 2 this is Snell's law, to which if you multiply omega by c, where omega is the frequency of the light of the e m wave that you are launching.

So then omega by c you multiplied here and multiply omega by c here you know the product of omega by c into refractive index is called k. So this equation is equivalent to saying that k 1 sin theta 1 is equal to k 2 sin theta 2, what is k 1? I am considering a situation where density is changing in z and plasma is uniform in the x direction.

So what you are seeing here, that k 1 into sin theta 1 is the component of k vector of the wave along the interface or perpendicular to z axis. So I can call this is as k 1 x is equal to k 2 x. Similarly, if you apply the same Snell's law at the boundary between second layer and third layer this will be equal to k 3 x and so on means as an electromagnetic wave moves from 1 medium to another medium this k 1 x, k 2 x, k 3 x will remain same.

So, I can call them as a simply k x how about k z in any in any layer, so in any layer suppose the refractive index is eta z then total k is omega by c into eta z, so total k is decreasing as eta decreases with increasing density but, k x remains constant in order to have k vary k z must change.

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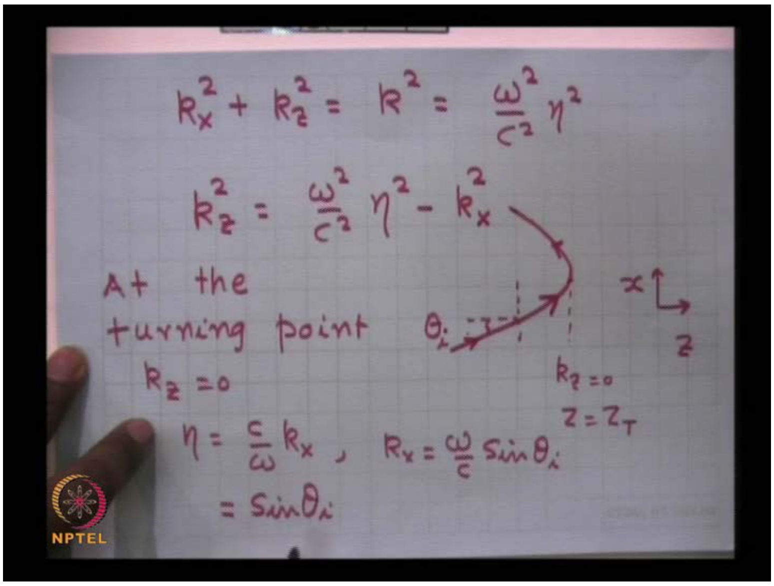

So what you really have, because k x square plus k z square is equal to k square which is equal to omega square by c square into refractive index square this equation tells me that k z square in any region, will be equal to omega square by c square eta square minus k x square this is a constant omega is a constant, eta is the decreasing function of z if the plasma density is increasing function of z so k z will decrease.

So, what you are having? If your medium begins at this point and if you are launching a ray here like this, the ray as it travels into a plasma of increasing density ray will bend like this and eventually will get out like this this is the direction of ray that we expect the issue is that at somewhere k z will become 0, whenever k x becomes equal to omega by c eta then k z becomes 0, so the point is called turning point where k z becomes 0, the wave vector travels in the x direction at this point. This remember this is my z direction this is my x direction. So at the turning point this is called turning point, I will call this z is equal to z t turning point.

K z is 0 which means eta becomes c upon omega into k x, if the wave is coming from free space this is free space at an angle of incidence is theta i then k x can be written as omega by c sin theta i because the magnitude of k vector in free space is omega by c, if the angle of incidence is theta i then the component of this k vector along x direction will be omega by c into sin theta i. So eta becomes is equal to sin theta i at the turning point.

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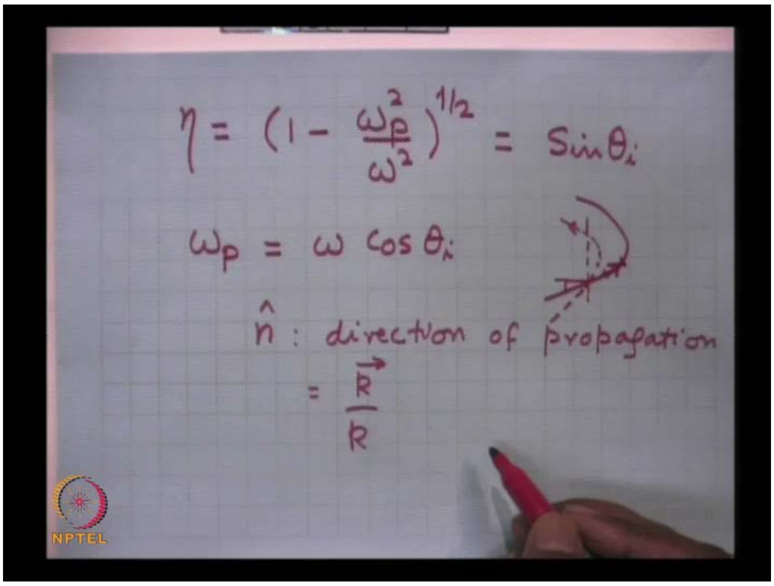

What is the consequence of this, what is eta? A refractive index of a plasma is eta is equal to 1 minus omega p square by omega square under root and I am saying put this is equal to sin theta i. If, you solve this equation it gives you at the turning point plasma frequency is equal to omega Cos theta i.

If, theta i is large then this quantity will be smaller, so at large angle of incidence this is my boundary between free space and plasma and the rate trajectory is like this. And If I had a higher angle of incidence like this then this ray will come back early, this is the first ray, this is the second ray, the first ray was coming at a small angle of incidence this is angle of incidence, the second ray was coming at a large angle of incidence so it travels only a little in the plasma it comes out.

So this is a important consequence of physical consideration of the or application of Snell's law in the propagation of waves in a inhomogeneous plasma. Well one thing I would like to mention in here, that if you want to determine the rate trajectory the equation of that trajectory of the ray can also be deduced on physical grounds what you except that a ray is travelling in a medium actually travels in the direction of propagation I will call the direction of propagation is n vector, this is the direction of propagation which is the same thing as k vector upon magnitude of k this is called the unit vector in the direction of wave propagation.

Here, so far we have discussed the propagation of waves with velocity v p phase velocity we have not talked about group velocity, probably we will discuss it today or some other time the concept of group velocity. But, what really happens if you launch a wave into a plasma and the wave amplitude is limited in time it is not a continuous wave.

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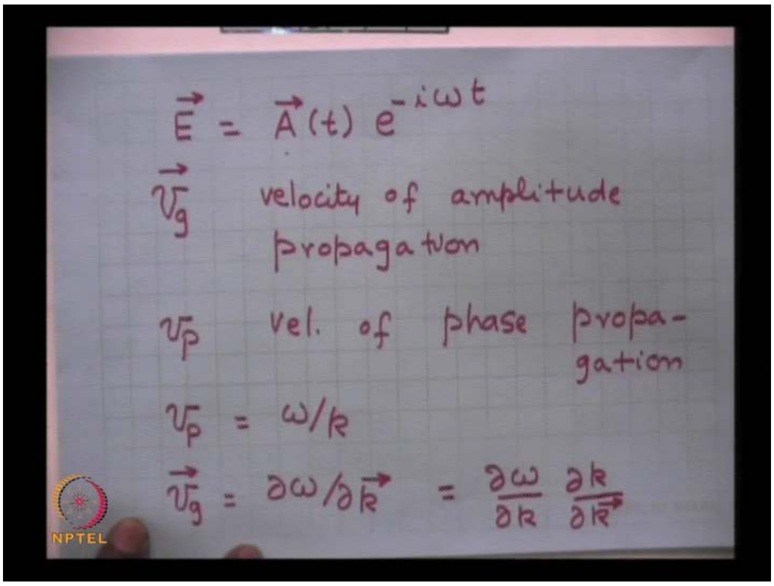

If you launch a pulse, then you can write down the wave field at the entry point as some function of time and exponential minus i omega t then later when this wave propagates through a plasma then the phase changes with z it is amplitude also changes with z, the velocity of amplitude propagation is called group velocity. So we denote this quantity called v g velocity of amplitude propagation, where as the velocity of phase propagation is called v p velocity of phase propagation.

There is a relationship between the 2 v p is denoted as omega by k and v g will be shown to be equal to delta omega by delta k it is a vector sign put there and vector sign there. v g has three components v g x, v g y, v g z delta omega by delta k x is called v g x, delta omega by delta k y is called v g y, delta omega by delta k z is called v g z. In a isotropic plasma omega does not depend on direction of k depends only on magnitude of k, so in that case this becomes is equal to delta omega by delta k is scalar into delta k vector delta k scale upon delta k vector.

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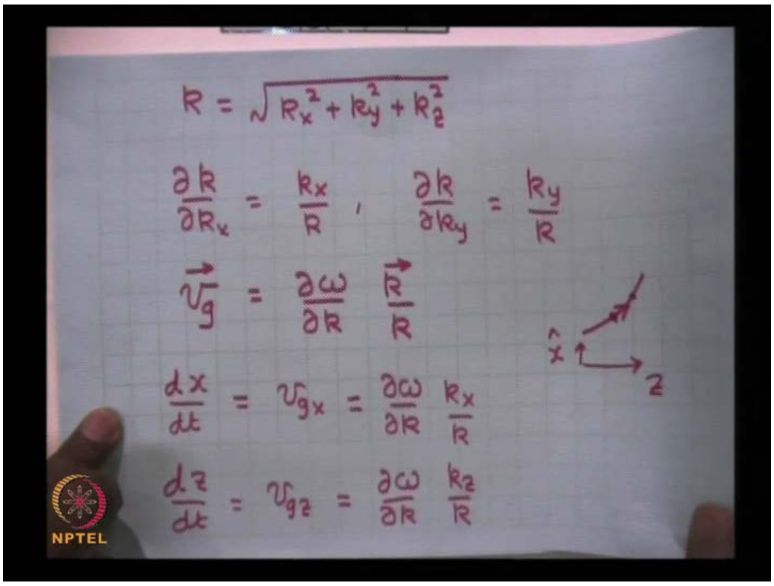

And if you differentiate k (( )) to k vector, what do you get? Because k is equal to under root of k x square plus k y square in general plus k z square, so if you obtain delta k upon delta k x, this will be equal to simply k x upon k. Similarly, delta k upon delta k y turns out to be equal to k y upon k, and similarly delta k by delta k z you can write. So, what happens that the group velocity can be written as delta omega by delta k magnitude into k vector upon k, which is n unit vector the direction of a propagation in a isotropic plasma.

Well, what is the consequence of this? If your ray is travelling like this this is the direction of your ray in a plasma, inhomogeneous plasma then suppose I am considering the wave propagation in x z plane this is my z direction and this is my x direction. So my wave is going in the x z plane, I am excepting that this velocity is in the x z plane also, v g has x component and z component also, what happens? I can write down the rate trajectory because when ray goes it the point moves from here to here.

So the distance travelled by the light by the electromagnetic wave in the x direction. I will call as d x by d t is equal to v g x the distance travelled by the ray in time d t will be d x this will be proportional to velocity put this is equal to delta omega by delta k into k x upon k take x component. And similarly, d z by d t is equal to v g z which is equal to delta omega by delta k into k z by k. If you divide these two equations you can write down d x by d z.

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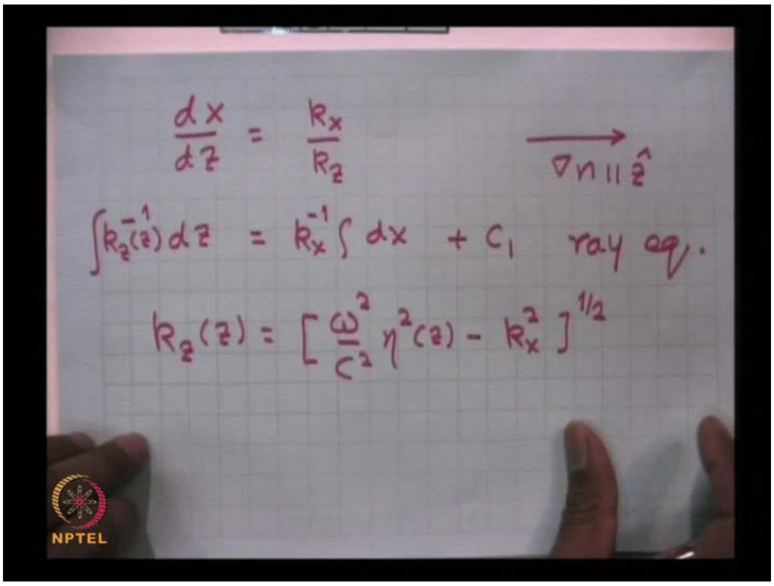

And the result would be d x upon d z is equal k x upon k z. please remember I just mentioned if the plasma has variation in density along z axis then k x is a constant only k z depends on z. So you can easily integrate this equation and you can write down d z into k z as a function of z that you know and integrate this is equal to k x is a constant into sorry I made a mistake this is inverse k z inverse into d z is equal to k x inverse into d x plus a constant of integration. k z let me write down explicitly k z as a function of z is equal to omega square by c square refractive index is square as a function of z minus k x square under the roo.

So this equation can be easily integrated and this is called the ray, the equation of the ray. So I think based on Snell’s law and physical considerations we have learnt that if we know the profile of refractive index variation with position, we can deduce the ray equation by integrating this equation. And obviously we have to know the angle of incidence because k x depend on depends on the angle of incidence of the ray. well with this in production let me go over to discuss the phenomenon of wave propagation in a inhomogeneous plasma.

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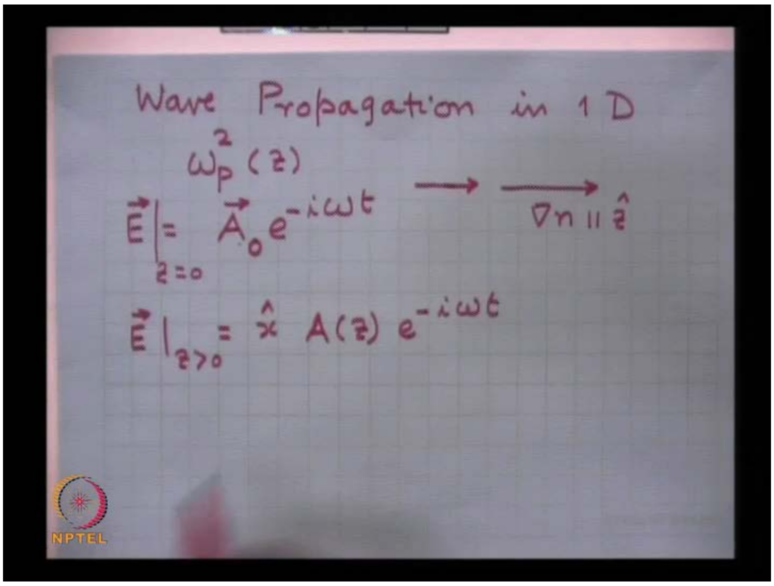

I will consider the propagation of wave in one dimension first. In one dimension, what you have? That I having a plasma where omega p square depends only on z and my wave is also going in the z direction, this is the direction where density is changing or omega p is changing, so this is the gradient of n parallel to z and my wave is also going in the same direction.

My electromagnetic wave I can write down, E is equal to some amplitude exponential minus i omega t at z equal to 0, say for instance I would like to find out how much the field looks at higher values of z, to be specific i because I have already learnt that when a wave travels in a plasma then the electromagnetic wave is transverse. So the A vector the amplitude has to be either in the y direction or x direction, so without any lose of generality I choose my x axis along the amplitude of the wave or A in the direction of x.

So, I will choose this E for z greater than 0 let me call the initial amplitude to be A 0 and let the amplitude afterwards becomes A which is a function of z, I do not know what is the x z dependence and time dependence I will take as i omega t. So first let me deduce the equation governing A and then under certainly approximations we will solve that equation.

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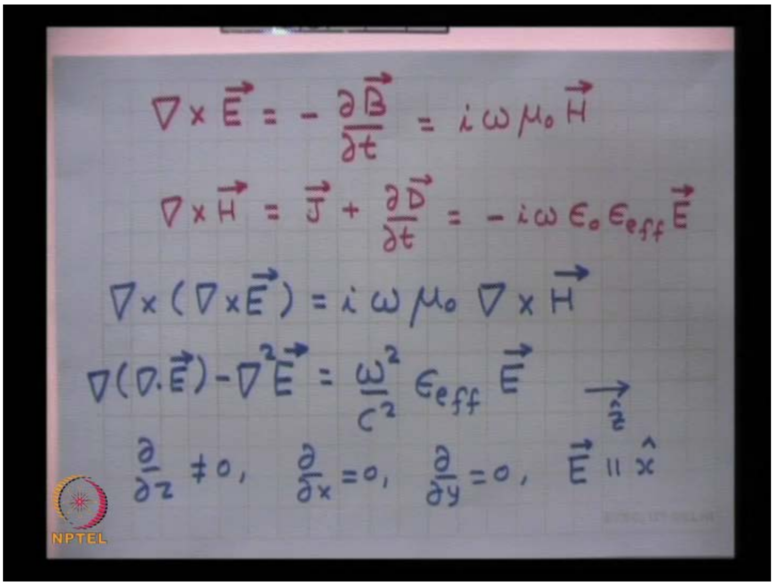

The relevant Maxwell's equations are curl of E is equal to minus delta B upon delta t this is the third Maxwell equation, because the time variation I have a specified as exponential minus i omega t, delta delta t i replace by minus i omega, so it becomes i omega mu 0 H. The fourth Maxwell equation is curl of H is equal to J plus delta D by delta t again replace delta delta t by minus i omega J by i omega as sigma E. So combine these two terms and you will get minus i omega epsilon 0 epsilon effective into E, this is how? The Maxwell's equations resemble a dielectric so this is the effective plasma permittivity.

Now, these two equations can be combined by taking curl of the first equation, so this becomes curl of curl of E right hand side becomes i omega mu 0 curl of H for curl of H, I use the second equation so the right hand side becomes omega square by c square epsilon effective into E and this I can break using vector identity into gradient divergence of E minus del square of E.

The issue is, what is the value of divergence of E? In my particular case because I am considering the wave propagation to be along z axis. So, I will choose delta delta z to be non zero but, I will choose delta delta x to be 0 and delta delta y to be 0, regarding the electric field by incident electric field is in the z x direction, so I want to choose E parallel to x axis.

So, If I take the x component of this equation you know that del operator means delta delta x is 0, so this term vanishes means in this particular case of wave propagation along the density gradient this term does not contribute at all. So, thus and del square becomes how much D to d z square because other derivatives are 0, so then this equation in this particular case becomes a differential equation in one variable.

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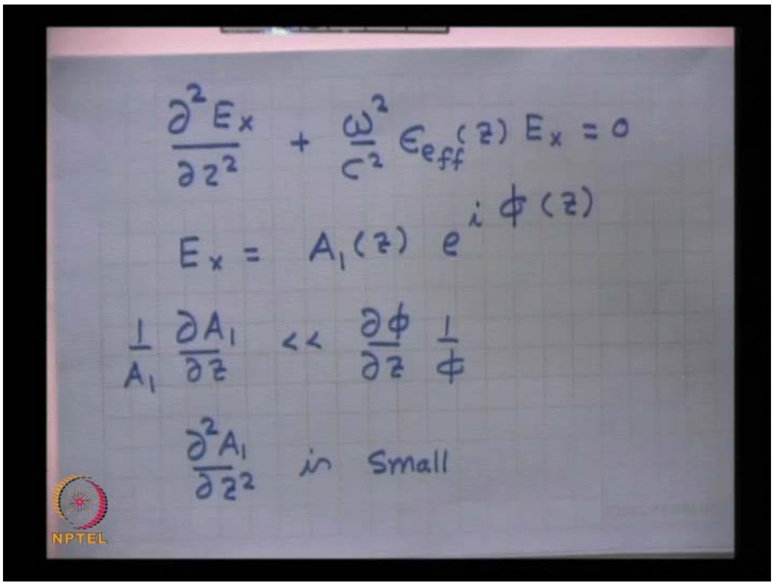

And it becomes d to E x d z square plus omega square by c square epsilon effective which is a function of z into E x is equal to 0.

There is no approximation so far but, if epsilon effective is general function of z, a general solution of this equation is very difficult, if this quantity varies very gradually with z then one can solve this equation in one approximation that I write down E x into a terms of two functions; one called amplitude function. So, let me write down this quantity say A1 some function of z into e to the power some i another quantity phi some function of z, where A1 is purely real and phi is purely real. I can call actually there is no need to put may be subscript one here but, does not matter.

So if this is the kind of dependence no approximation, I am just writing any complex quantity E x can be written as some amplitude which depends on z and some phase term but, from over a plane wave solution and a homogeneous medium we know amplitude is a constant this is a rapidly varying function of z as exponential of i k z.

So, here I am saying that when the plasma is inhomogeneous a still main z dependence comes through phi and v z dependence comes through the amplitude. So I am going to assume that delta A 1 by delta z is small is much less than delta phi by delta z, actually I should compare this with something so I will call this 1 upon A 1 and this as 1 upon phi. I will say that this is a much stronger z dependence than this 1.

So when I substitute this expression in the wave equation, I will say that first derivative I will certainly retain of A1 but, I will ignore the second order derivative of this so I am going to neglect d 2 A1 by d z square, this neglect of second order of derivative of A1 with respect to first order derivative of A1 or second order derivative of phi with respect to z is called W K B approximation. So first of all, we will employ this approximation and obtain the values of A1 and phi and then justify under what conditions this assumption is justified.

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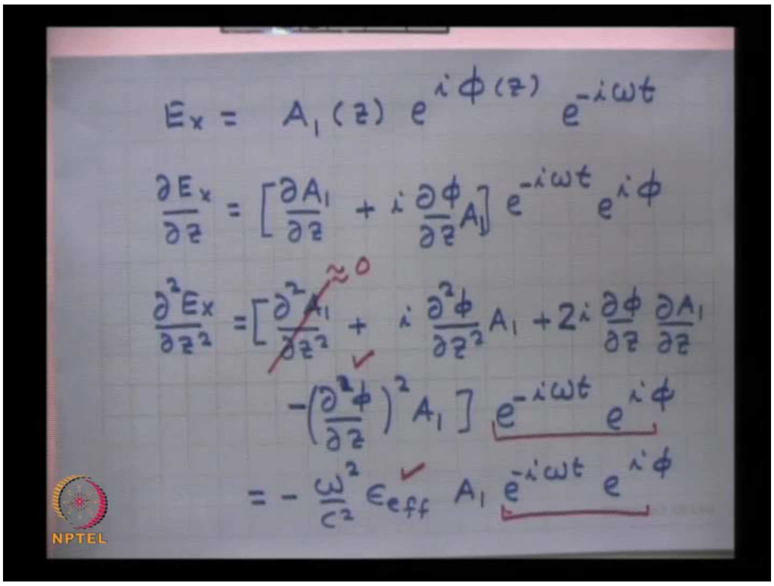

So, let me substitute this if E x, I am choosing is equal to a let me call A1 a function of z exponential of i phi then differentiate with respect to z obviously there is a time dependence is also there minus i omega t is already there. So delta E x by delta z will be how much this will be equal to delta A1 by delta z into this entire function plus i delta phi by delta z into exponential minus i omega t into exponential of i phi this is one derivative.

Second order derivative would be d 2 E x by delta z square is equal to d 2 A1 by d z square, which I am going to ignore in little while plus if you differentiate this you will get I, d 2 phi by d z square there is A1 here also, I forgot this write A1 here please write A1 there into A1 plus I will get i delta phi by delta z into delta A1 by delta z.

Then you start differentiating the exponential term, so you will get i delta phi by delta z into this terms so it becomes two times and then you multiply the differential coefficient of this with this term, so you will get minus d to phi d z sorry delta phi by delta z whole square into A1 multiplied by this exponential terms minus i omega t into exponential i phi. And this has to be put equal to in the wave equation minus omega square by c square epsilon effective into E x which is equal to this whole expression so A1 exponential of i minus i omega t exponential of i phi.

Now, please remember there are this is a common factor on the right hand side as well as on the left hand side this factor is common. So they will cancel out this from here through here and from here through here, they just cancel each other on both sides cancel them and equate the real part on the left with the real part on the right and imaginary part of these terms on the left should be zero, because there is no left imaginary part on the right after these exponential have been canceled.

So what do you get, on equating the real part means this this I am ignoring this is called W K B approximations we ignore. So real parts is simply this term equate this term to this term and imaginary part is this term plus this term is zero.

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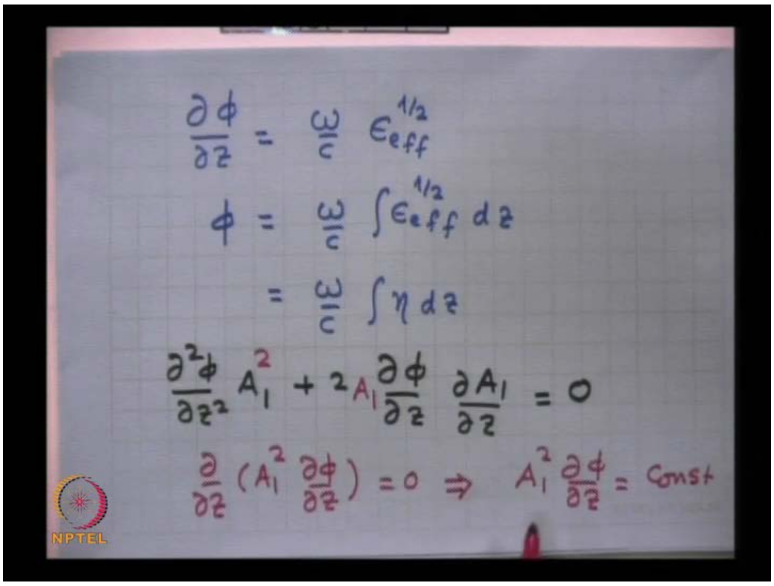

Let me write this this gives me delta phi by delta z is equal to omega by c epsilon effective to the power half. So phi can be easily obtained from this equation, so the phase of the wave a special part of the phase is omega by c under root of epsilon effective to the power half d z, because we had been calling this quantity as refractive index. I can also write this as omega by c refractive index into d z. If, the medium are homogeneous this is a constant you can take it out and d z simply becomes z integration, how about the this is by equating the real part on the left to the real part on the right.

When you equate the real imaginary parts, you will get imaginary parts give you d to phi by d z square of A plus twice delta phi by delta this is A1 rather actually this A1 everywhere. I made a mistake this is A1 into delta phi by delta z into delta A1 by delta z is equal 0, because there is no imaginary term on the right hand side these two equations can be combined into one, If I multiply both terms by a quantity called A1.

So, let me multiply this by A1, so If I multiply this by A1 this becomes A1 square and A1 i multiply here but, this 2 A1 into d A1 by d z becomes d 2 A square by d z and this can be written simply as delta delta z of A1 square delta phi by delta z is equal to 0, you can just check it, which means this is a constant term?

A1 square delta phi by delta z is equal to constant, let me put the value of delta phi by delta z from this expression here its omega by c into epsilon effective to the power half, which simply says that because omega by c is a constant.

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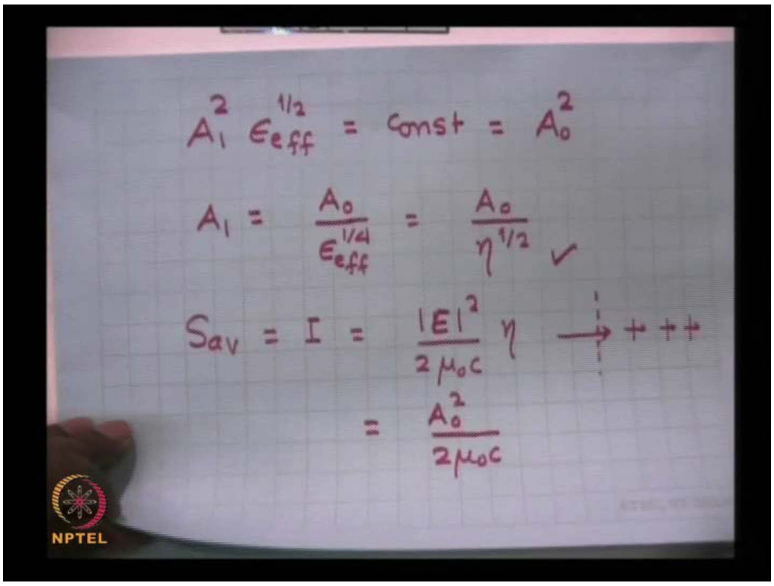

So this equation tells you that as the waves travels A1 square into epsilon effective to the power half is equal to constant. Initially, if the wave started from free space I will call this quantity initial amplitude of the was A 0 and epsilon effective was unity in free space.

So this is square A 0 square for A1 and this what you get? So what you are getting here the A1 amplitude of the wave in the medium is initial amplitude A 0 divided by epsilon effective to the power one-fourth or A 0 upon refractive index to the power 1 by 2.

This expression can be physically understood, if you recall that as the that in a plasma, the time average pointing vector which is called intensity of the wave was equal to modulus of E square upon twice mu 0 c into refractive index. So, what is happening here? As E if you put the value of E it becomes simply A1 square, so it becomes A 0 square upon eta will cancel out because 1 upon eta will be from here ,and eta will cancel upon twice mu 0 c.

So, what is happening? If you are launching a wave into a plasma whose density is zero here, and density increasing in the interior then what you are excepting that the wave amplitude after all total power of the wave if reflection is ignored passing through any unit area should remain constant. after all whatever electromagnetic energy is entering here must pass through here must pass through here must pass through here everywhere it must pass.

So, if there is no absorption as we are ignoring absorption here by taking eta to be real as the wave travels its energy crossing should remain same. So, if this has to remain constant but, eta is decreasing as the plasma density is increasing the refractive index decreases. So, E must increase that is why this amplitude is increasing so just by to converse the energy a denominator a refractive index under root comes in the denominator this ensure the energy conservation in the medium. So it is a important physical interpretation of this dependence.

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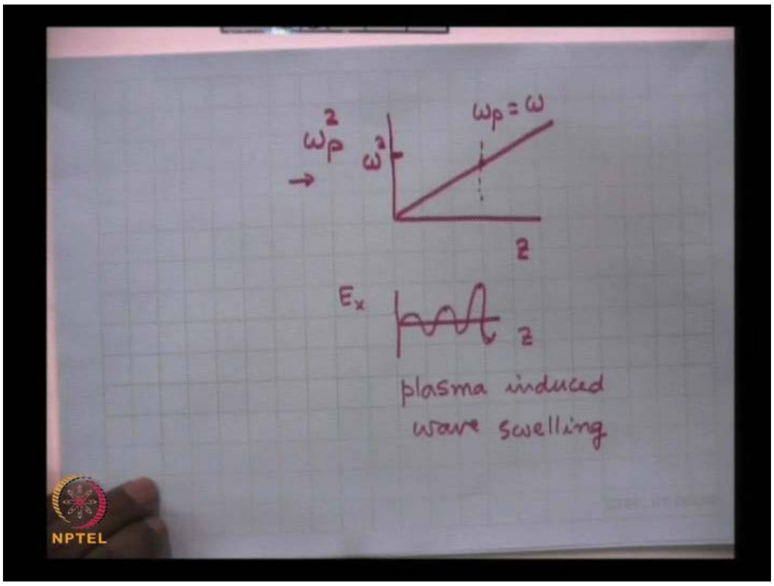

Well, it has a very important implication let me elaborate on this suppose I have a plasma whose density I am plotting like this, suppose the density is increasing like this.

So, If I plot omega p here and plot z here omega omega p square rather. because this is proportional to density so this is my density profile written in terms of plasma frequency square and suppose at some point omega p becomes is equal to omega, so this point is called omega square means at this point. So, if the wave is coming from here it will penetrate up to this distance at this point omega p becomes is equal to omega, up to this at this point your refractive index becomes zero beyond this refractive index is imaginary the wave does not travel.

So what is happening, If I plot the wave wave amplitude will be like this we have just seen, so the if the wave at a given instant of time if you see the oscillator electric field of the wave plot it will be like this.

Actually from W K B theory, when eta goes to zero your amplitude goes to infinity because eta to the power half comes in the denominator and there actually this theory fails, because the amplitude variation becomes quite rapid so d to a by d z square that we had ignored its neglect cannot be justified so theory does not hold but, if you do a little more careful calculation then the amplitude is not infinite. But, it is contained but never the less as the wave travels, its amplitude becomes larger and larger this is called plasma swelling of plasma induced swelling of the wave amplitude.

Plasma induced wave swelling, I am plotting field at a given instant of time as a function of z wave swelling this is a important consequence of W K B theory that wave amplitude becomes stronger and stronger as the wave approaches the critical layer where plasma frequency equals the wave frequency.

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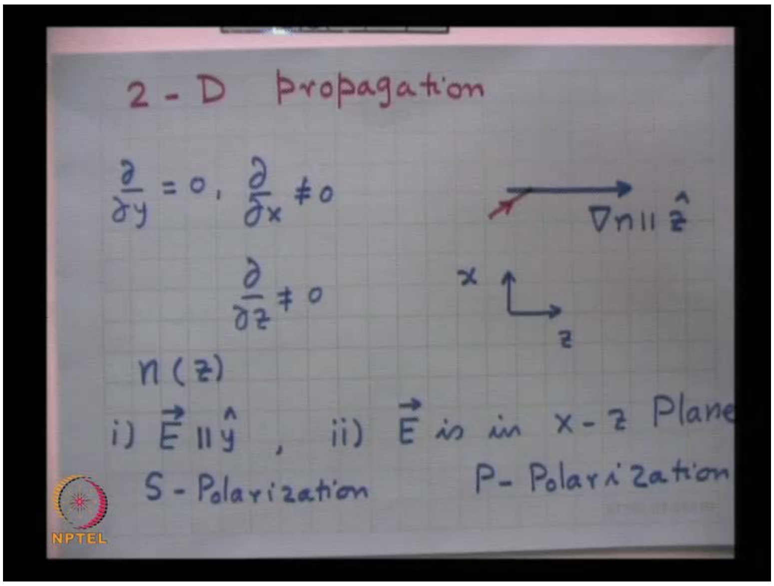

Now, a few remarks regarding the oblique propagation for that is called a 2 D propagation means, the wave is not traveling in the direction of density gradient but, at an angle to density gradient.

So, I have a plasma where density gradient is like this around z axis for instance but, my wave is coming at an angle I am launching a wave like this. So, I would like to find out as this wave travels into the medium, how does it its field vary with z. Well, let me specify the x axis also, I will choose this as the x axis and this is my z axis and I will choose my fields to vary in x and z but, being constant in y this is called two dimensional propagation.

So, I am choosing delta delta y to be 0 but, delta delta x is non zero and delta delta z is also non zero but, my properties of a medium the density of the plasma is a function of z alone. So from Snell's law we except that the wave vector of the wave in the x direction will not change with distance it will remain the same as it was in the beginning this one thing that we can recall from there but, we should be able to deduced the same thing from the Maxwell's equations and we shall do that.

Another thing that I would like to mention here there are two possibilities that this wave may be polarized this wave may be polarized perpendicular to the x z plane means there are two possibilities; that the electric field of the wave is parallel to y axis and second possibility is the electric field is in the x z plane is in the x z plane there are two possibilities let us consider either of the two. For the sake of simplicity, I will consider the wave propagation with y axis this polarization is called S polarization and the other one is called P polarization plane of incidence polarization.

So, the issue is if my wave is travelling in with electric field in the y direction like this and the medium properties do not change with y, you will just see that the wave will maintain its y polarization whereas, if the electric field of the wave is in this direction and the wave travels something very different happens what happens lets physically see.

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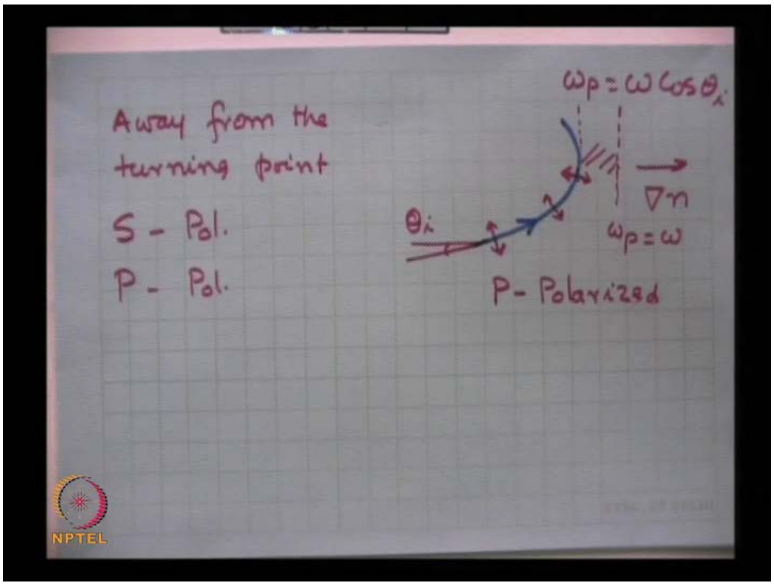

When the wave travels in a inhomogeneous medium and it curves like this and if the electric field is polarized in the plane of incidence like this then as the ray bends this electric field also bends, because it has to remain perpendicular to the direction of propagation its bends like this and if this point this becomes like this, so this is case of P polarized light. Now, there is a gradual rotation of the electric field of the wave as it travels in the medium and as you know from physical considerations that this is a layer called turning point, where omega p is equal to omega Cos theta I, where this wave was having an angle of incidence equal to theta i.

This is not equal to omega P equal omega. But, what happens at the turning point, the electric field of the wave is parallel to density gradient because this is the direction of density gradient. So when the electrons oscillate parallel to density gradient they always give rise to density oscillations.

Normally, we know that if the electromagnetic wave travels in a plasma it does not give rise to density oscillations but, because of the background gradient and density density gradient in background plasma density and when the electric field is parallel to the density gradient then it gives rise to density oscillations. We have learnt that a plasma has a natural frequency of oscillation is equal to omega P, so if this wave can tunnel from here to a region somewhere here there will be a region called omega P equal to omega critical layer.

So, if the gap between these two regions, these two layers is not large, then this wave can penetrate from here to here, and can resonantly excite a plasma wave there. So, there is a very special thing that happens here; therefore, P polarize light as the wave rotates or changes its orientation, the electric field also rotates and it may give rise to large density oscillations or the conversion of the wave into plasma wave can occur. Well certainly this is not within the limits of W K B theory, but this phenomena we shall discuss sometime during this course in some detail. So, this is a basic difference between the S polarized light, and P polarized light; the polarized light is polarized perpendicular to the ray like this, and its orientation does not change, it does not give rise to any density oscillations.

However, in the region away from the turning point like in this region etcetera, the W K B theory for P polarize light, and S polarize light is the same. So, we shall essentially consider or rather limit our discussion to region away from the critical layer or away from the turning point, away from the turning point S polarization, and P polarization have similar character. And the mathematical analysis will be taken up later to discuss the propagation of waves at oblique angle, in a inhomogeneous plasma. I think today, I stop at this stage. Thank you very much.