Plasma Physics

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Lecture No. # 08
Electromagnetic Wave Propagation in Plasma
(Contd.)

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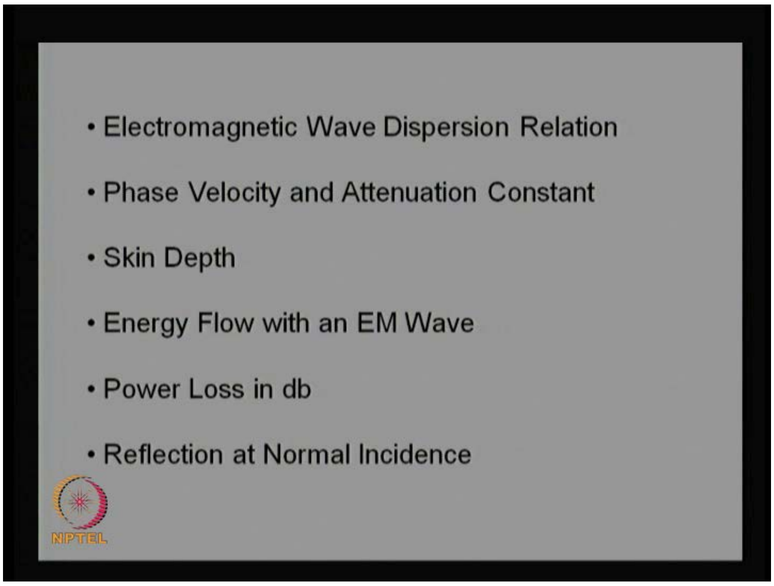

Friends, in this lecture I would like to continue our discussion on electromagnetic wave propagation in plasma. Here, we will discuss the electromagnetic wave dispersion relation, phase velocity and attenuation constant, deduce an expression for a skin depth in over dense plasmas. We also study the propagation of or the flow of energy with an electromagnetic wave, in observing media collision plasmas we will calculate the power loss in decibels. And if time permits, I will discuss the reflection of electromagnetic waves at normal incidence from plasmas.

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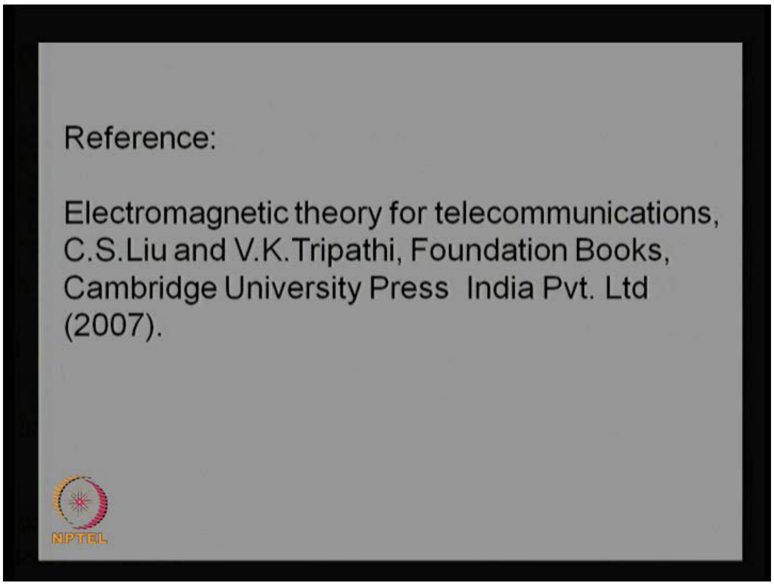

The reference for today's presentation is again the same book electromagnetic theory for telecommunications by professor CS Liu, and myself published by Cambridge university press.

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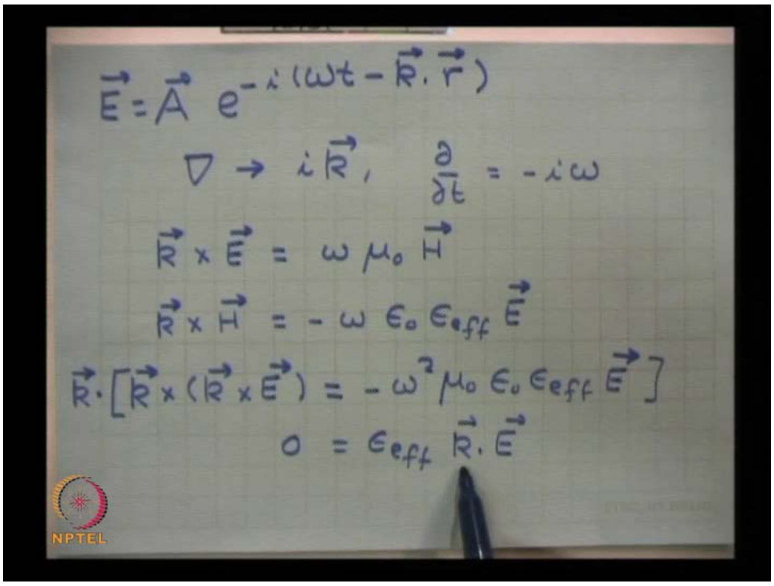

We will discussing the plane wave solution, like E is equal to A exponential minus i omega t minus k dot r under what condition this solution will satisfy Maxwell's equations. We consider third and forth Maxwell's equations and those equation on replacing del operator by i k and time derivative as minus i omega take the following form the third Maxwell's equation takes the form k cross E is equal to omega mu 0 into H and the forth equation is k cross H is equal to minus omega epsilon 0 epsilon effective into E they are coupled in E and H.

So, what I did was? I took k cross of the first equation and use the second one and I got this relation k cross k cross E is equal to minus omega square mu 0 epsilon 0 epsilon effective into E. First of all, if I take k dot of this equation then left hand side will be 0, because this is a vector perpendicular to k as well as k cross E. So, if I dot this equation with k, so multiply this equation by k dot left hand side is 0, so right hand side should also be 0; means epsilon effective into k dot E should be 0. Because, omega is a constant, epsilon 0 mu 0 are constant, so they cannot be 0 either this should be 0 or this should be 0.

So one simple consequence of this equation is that my solution given above will satisfy these equations only when either k dot E is 0 or epsilon effective is 0.

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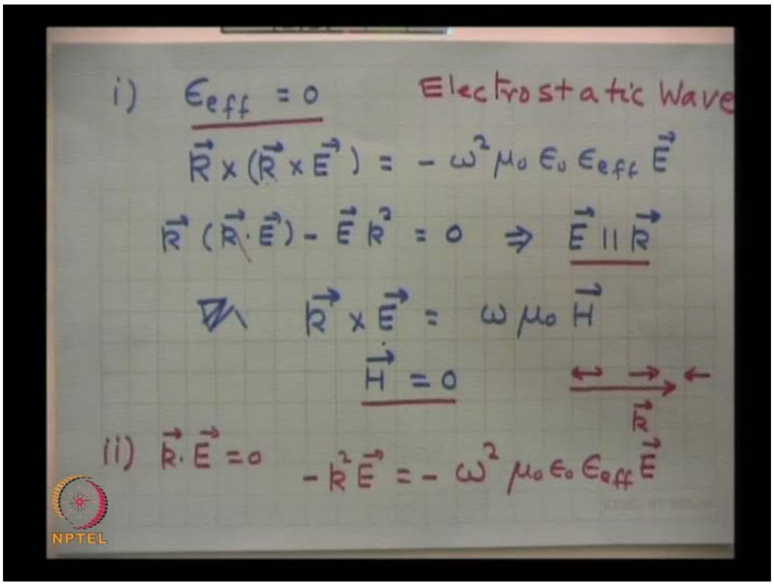

Let us see, one possibility is that epsilon effective you make 0, in view of this if I down my k cross k cross E equation again I can write down this k cross k cross E equation as let me write this, I think I here k cross E is equal to minus omega square mu 0 epsilon 0 into epsilon effective into E this was the basic equation, whose k dot I took and I got this condition that I can satisfy this equation and this is 0. When I put this equal to 0, what does this say this left hand side is identically equal to k, this k into k dot E minus E into k dot k which is k square and because I am choosing epsilon effective equal to 0 this should be 0.

This says that, this is a vector in the direction of E this is a vector in the direction of k these two can be equal, only when E is parallel to k so important consequence is that E is parallel to k and if you go to third Maxwell equation which I had written as curl of rather sorry not curls k cross E is equal to omega mu 0 H. So if E is parallel to k this term will be 0, so H will be 0 this is a very important consequence.

So whenever epsilon effective is 0, those waves will have electric field parallel to the direction of propagation and will have no magnetic field. So they cannot be called as electromagnetic wave. Because, there is no magnetic field they will be called as purely electric waves in plasma jargon they are called electrostatic waves. and if they are travelling in the direction of k, this is my direction of k vector then the electric field of these waves will oscillate in the same direction like this.

Now, what will happen in such a case they are called longitudinal waves and in some region the electric field will be in this direction at the same instant in some direction will be like this in some wave it will be like this. So, these electrons will be compressed somewhere they will be rarefied somewhere so this gives rise to (( )) charge oscillations or density oscillations. These waves are also known as density oscillations or propagating density perturbations they have given a name electrostatic wave.

This is a very important class of waves in plasma physics, lately these waves have been found to be very useful in electron acceleration and electron acceleration reaching the values of about 1 g e v have been achieved by these waves. So for particle acceleration these waves are very useful, well we shall discuss those utilities of these waves later on. second possibility to satisfy the k dot of this equation being 0 is that k dot E is 0. If, I choose k dot E equal to 0 then in this equation in this equation sorry if, I simplify this term will be simply equal to minus k square E because this is 0 this is the expansion of this term.

So if I choose k dot E equal to 0 this is 0 only this will survive, so put this in place of here and equate through this. because in this case epsilon effective is non zero. So, right hand side is not 0, so what you get here is minus k square E is equal to minus omega square mu 0 epsilon 0 into epsilon effective into E these waves have k given by this relation.

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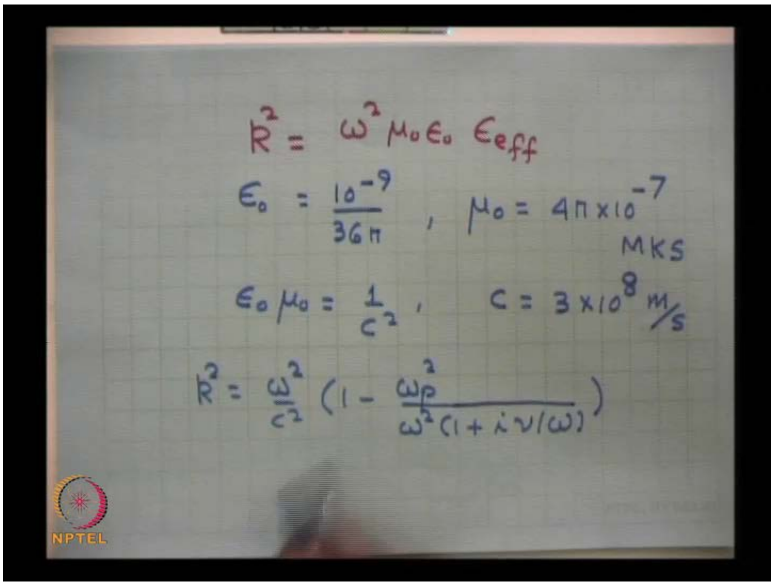

So, I will write down k for these waves is equal omega k square is equal to omega square mu 0 epsilon 0 into epsilon effective. Let us understand the character of these waves, first of all mu 0 epsilon what? Epsilon 0 is the free space permittivity in m case units its value is 10 to the power 9 upon 36 pi, mu 0 is called free space magnetic permeability whose value is 4 pi into 10 to the power minus 7 in m case units both in M K S units. If, I put these values epsilon 0 mu 0 turns out to be 1 upon c square, where c is the velocity of (( )) in free space given by 3 into 10 to the power 8 meter per second.

So, I will put this in here and I will get k square is equal to omega square by c square into 1 minus for epsilon effective is omega p square upon omega square 1 plus i mu by omega this is the dispersion relation but, how about the character of these waves whether they are transverse or non transverse.

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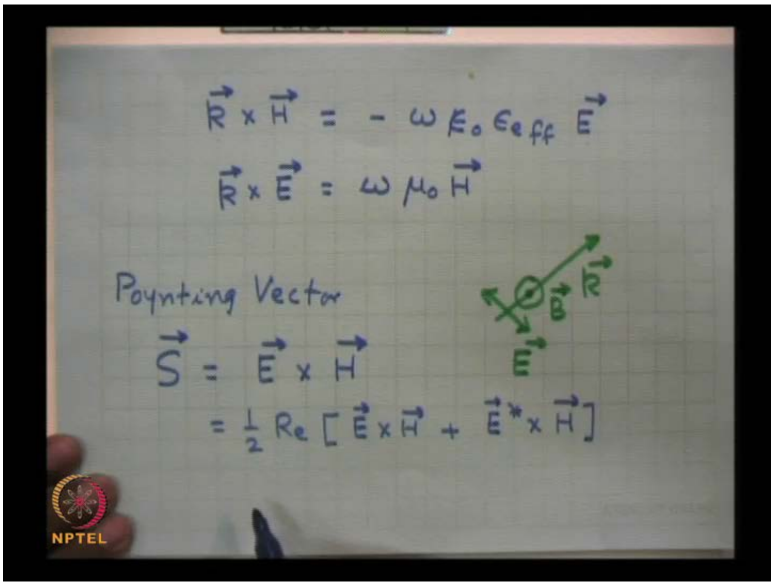

So, let us go step by step from forth Maxwell equation I had written, k cross H is equal to minus omega epsilon 0 epsilon effective into E this says that E is perpendicular to k and H both. Whereas, from third Maxwell equation k cross E was written is equal to omega mu 0 H; this says that H is perpendicular to k and E both.

So this equation tells E is perpendicular to k so suppose my wave is going in Z direction like this or some direction like this of k in the and the electric field has to be perpendicular. This will oscillate like this then magnetic field has to be perpendicular to both of these like this this is B field. So these are purely transverse waves, electric field is perpendicular to k vector, magnetic field also perpendicular to k vector and E and B are both perpendicular to each other.

Before, I proceed further I would like to recall a quantity called pointing vector. I will not drive this here but, I will borrow this result from general theorem that energy flow with a wave is denoted as turns out to be equal to E cross H. Here, real part of E has to be multiplied with the real part of H and as we have been doing earlier this implies that this is equal to half real part of E cross H plus E star cross H, so let me write down the values of E and H and see, what is the pointing vector?

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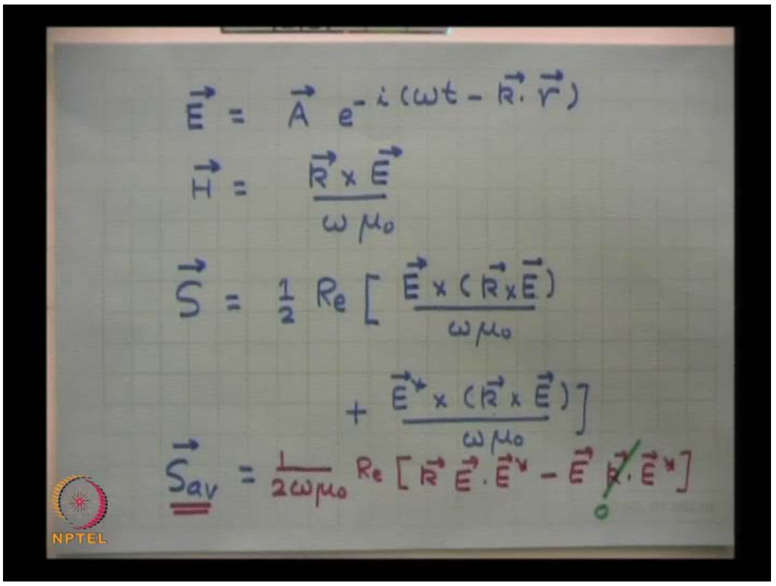

Pointing vector well, for this pointing vector I can write down E explicitly as A exponential minus i omega t minus k dot r and H, I just wrote down from third Maxwell equation is equal to k cross E upon omega mu 0. So what you will see here, in the pointing vector S is equal to half real part of first term is E cross H, so which is equal to E cross k cross E upon omega mu 0. the second term would be E star cross k cross E upon omega mu 0, you may note one thing in here E has an exponential factor as exponential minus i omega t minus k dot r this is also has.

So, the product if you multiply them will be at 2 omega frequency time average of this function will be 0, this is the only term whose exponential time dependence will cancel the two factors because this will have minus i omega t and if you take complex conjugate it will give you minus i omega t plus i omega t so they will cancel.

So time average power vector will be always the term that will contribute as time average is equal to let me write down with a different ink because this is a important expression. This quantity is equal to 1 upon twice omega mu 0 and real part of this quantity, I will write down this as vector triple product as k vector here into E dot E star minus this E vector and k dot E star but, we have already seen k dot E is equal to 0. So this term vanishes, because they are transverse waves k is perpendicular to E, so this is the only term and you know E star is the real quantity.

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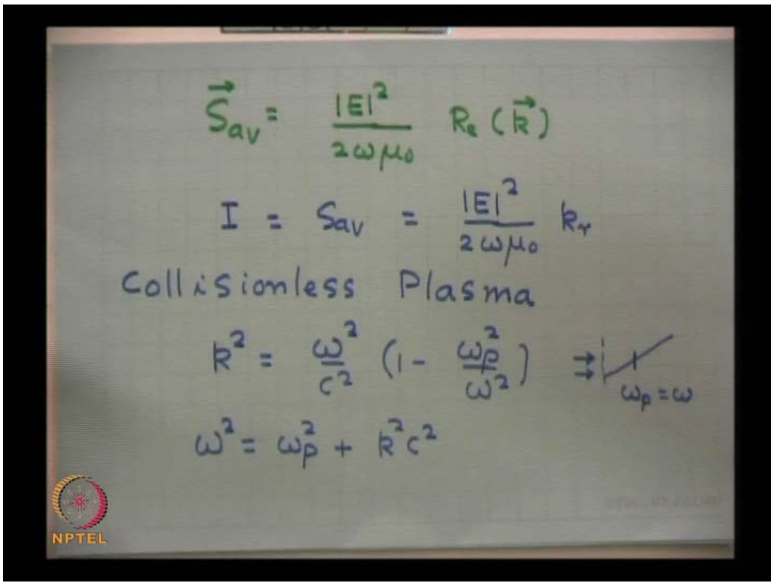

So this equation simply gives you time average pointing vector simply as average is equal to modulus of E square upon twice omega mu 0 into real part of k, if k is complex only real part of k has to be written there this is a very important result.

If, somehow we find that k is imaginary purely imaginary then there is no power flow, because time average pointing factor or energy flow is 0, the magnitude of this quantity is also called intensity of the wave or intensity of radiation.

So let me write down this intensity of radiation as average magnitude which is equal to modulus of E square upon twice omega mu 0 into k real part k r. Well, when we discuss different cases of wave propagation and different kind of plasmas we will make use of this expression.

So let me consider, the wave propagation as collision less plasma where collisions are small, what you get? k square is equal to omega square by c square 1 minus omega p square over omega square this can also be written as omega square is equal to omega p square plus k square c square. If, I want k to be real then k square should be positive.

So, if I want my wave to carry energy then omega must be bigger than omega p that is very important condition plasma does not allow the propagation of all sorts of waves it will allow propagation of only those waves whose frequency is bigger than omega p and. If omega somehow less than omega p then k will be purely imaginary and from that point on energy should be reflected back. Sometimes your plasmas suppose the plasma starts form here but, the density is low here and density increases in the plasma suppose density increases like this.

If you are launching an electromagnetic wave on a plasma, whose frequency is constant but, plasma frequency increases then it will go up to a region where omega p becomes equal to omega and beyond this omega p is bigger than omega the wave will not propagate it will come back this is a very important phenomenon called total reflection of electromagnetic wave from critical density plasmas or from critical layer which has been used in measuring the density profile of the ionosphere. I shall discuss this when I discuss the applications of these waves in a either today or in my next next talk. but let me mention something about this, that I can plot a dispersion relation omega versus k.

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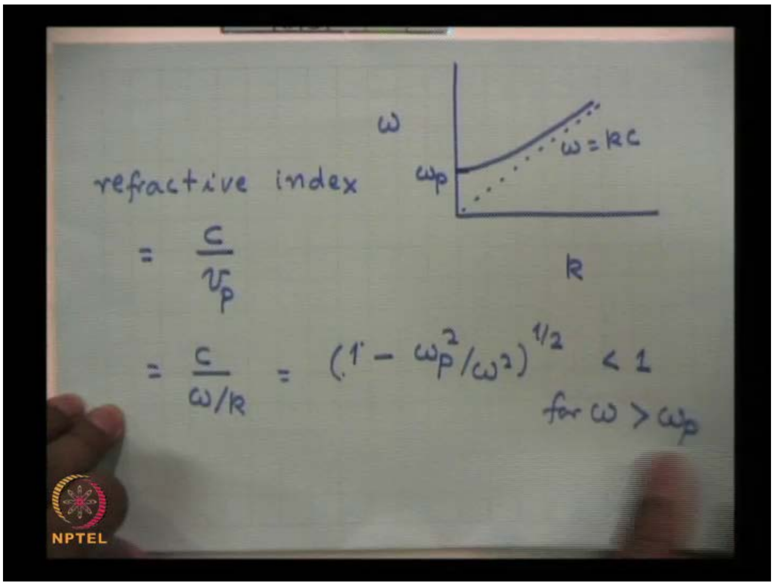

And this will be of this form omega normally is plotted on the y axis and k on the x axis it starts from omega p here, because waves of frequency less than omega do not propagate.

So, k is not real for them and this will go like this and asymptotically this dispersion relation approaches a line called omega equal to k c line called light line. And we can define a quantity called reflective index, a reflective index is defined as velocity of light in the medium in free space to velocity of light in the medium but, we know v p is already omega by k. So this is c upon omega upon k or k c by omega and if I put the value of k it turns out to be equal to 1 minus omega p square upon omega square under root. So this quantity is less than 1, for omega bigger than omega p and its imaginary which has no meaning for omega less than omega p, so we should not talk in terms of.

Because space velocity actually does not have any meaning there. So, let me introduce a term called skin depth characterize the penetration of fields in a over dense plasma a plasma where omega p is bigger than omega is called over dense plasma and a plasma where omega p is less than omega is called under dense plasma.

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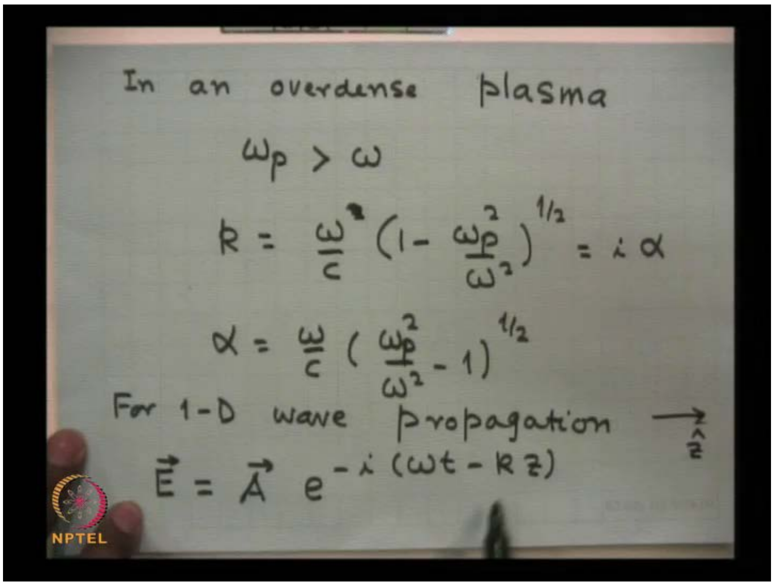

So, in an over dense plasma over dense where I am choosing omega p bigger than omega what is my k is equal to omega square sorry omega by c into 1 minus omega p square by omega square to the power half so k is purely imaginary. I can call this is equal to i times alpha, where alpha I will call as omega by c into omega p square by omega square minus 1 to the power half. If, I put k is equal to i alpha in my electric field expression, what do I get but, k is a vector quantity I am little worried so to appreciate the physical consequence of k being imaginary. I will consider the wave propagation one dimension for 1 D wave propagation.

What I am going to do? I will will write down say electric field is equal to A exponential minus i omega t minus k z, let my wave is be travelling in the z direction this is my k vector or the wave is going in this direction say z axis space variation is only with z. but k is imaginary, if I put k is equal to i alpha here.

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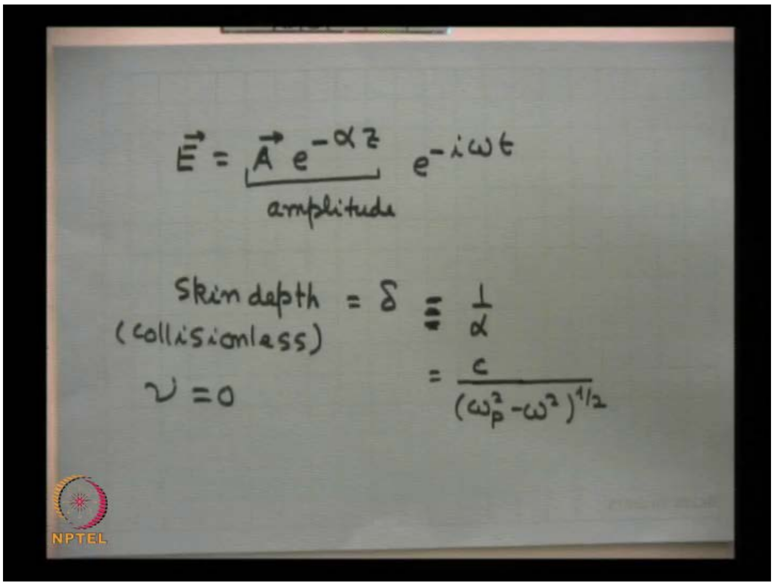

This equation can be written as E is equal to A e to the power minus alpha z exponential minus i omega t. So, in an over dense plasma, phase term does not have any z dependence? No z dependence, in phase and this entire thing is called amplitude then and amplitude will fall off with z exponentially. Alpha has the dimension of one upon length and I introduce a quantity called skin depth which is equal to delta and this is 1 upon alpha by definition. And if I put the value here of alpha from the previous expression it is c upon omega p square minus omega square under root. Well this is in the limit when there are no collisions, I consider collision less skin depth then the collision frequency was taken to be 0. However if nu is included what happens let us see.

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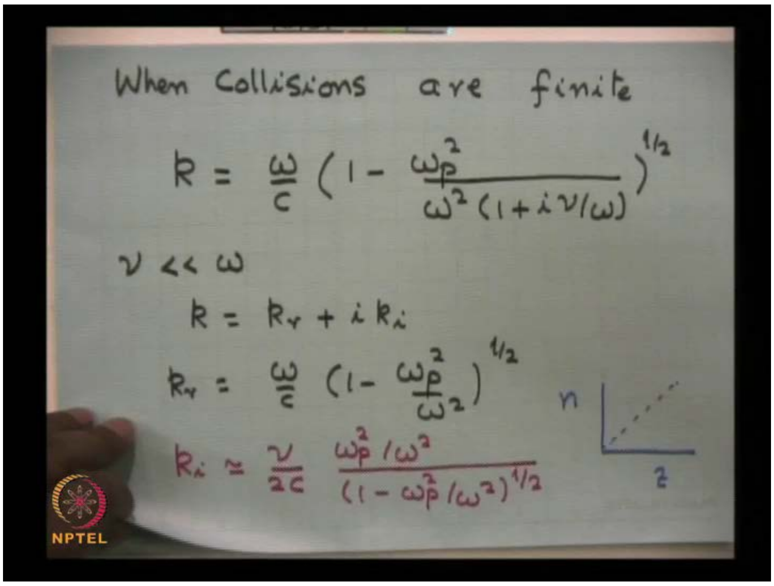

So when collisions are finite, what do you expect? your k is equal omega by c, 1 minus omega p square upon omega square 1 plus i nu by omega to the power half for propagation of waves of frequency omega p, in a plasma nu is always less than omega p indeed much smaller than omega p in all plasmas, nu is always less than omega p and we have learned that if we want the wave to travel in a plasma with very little attenuation omega should be bigger than omega p.

So in the limit where nu is much less than omega you can simplify this expression. I can write down k is equal to k real plus i time k imaginary imaginary part comes because of this nu term.

Simplify this you get k real nearly equal to omega by c into 1 minus omega p square by omega square to the power half and imaginary part of k is k i of the order of if you simplify this expression it becomes nu upon 2 c into omega p square over omega square multiply divided by k r, well it is not k r this is 1 minus omega p square by omega square to the power half this sort of expression you will get. You may check here, that nu if you increase a plasma with high collisionality will give you higher value of k imaginary and imaginary part of k always gives rise to damping of the wave it is also called damping coefficient or attenuation constant and in the vicinity of omega equal to omega p this term is most dominant.

So, when wave goes from a low density plasma into high density plasma two things happen collision frequency increases. Because, this depends on plasma density and because of this omega being closer to omega p. Now, denominator becomes smaller and there is enhancement because of that also, this k i is very strongly increasing function with plasma density. So, if you are launching a wave into a inhomogeneous plasma whose density suppose a plasma has a density variations like this density 0 here. Suppose a plasma density is increasing like this you are launching a wave, what will happen? Initially collision frequency will be very tiny I am talking of strongly ionized plasma where collisions frequency depends on electron density or ion density.

So, as the density increases nu will increase omega p will increase this vector will decrease and there is a considerable enhancement in k i. So, the wave gets very very strongly damped as it travels deeper and deeper in to a denser plasma so there is a message contained in this expression.

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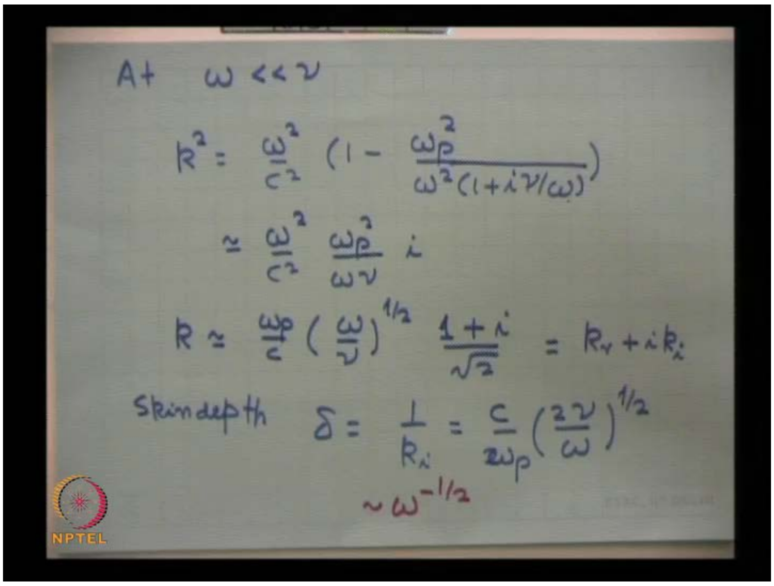

Another thing that you may note here, that if I am talking of very low frequency response at omega much less then nu, what will happen? These are the kind of waves that are of interest in the E L F range, because the frequency of E L F wave in less than collision frequency. So what will happen, in this case k square is equal to omega square by c square 1 minus omega p square over omega square into 1 plus i nu upon omega, what I have to do? I am choosing nu much bigger than omega, I can ignore this 1 and this becomes of the order of omega square by c square into omega p square upon omega nu into i. I have ignored one as compared to this term because this term becomes very large when omega is very small.

So this is the typical thing and k i becomes of the order of take the under root it becomes omega p by c into omega upon nu to the power half and this I will give you 1 plus i upon root 2 this is the under root of i (( )) i.

So k has a real part and imaginary part of equal magnitude. I can write this is equal to k real plus i times k imaginary of the same values. And skin depth in this case is defined as 1 upon k i imaginary part of k that will be equal to reverse of this, so c upon omega p into twice nu upon omega to the power half. The important thing is the dependence on this skin depth on frequency this skin depth decreases with frequency.

So this goes as omega to the minus half, please note at omega much bigger than nu we found that the skin depth was like c upon omega p kind of thing. whereas this is like this here.

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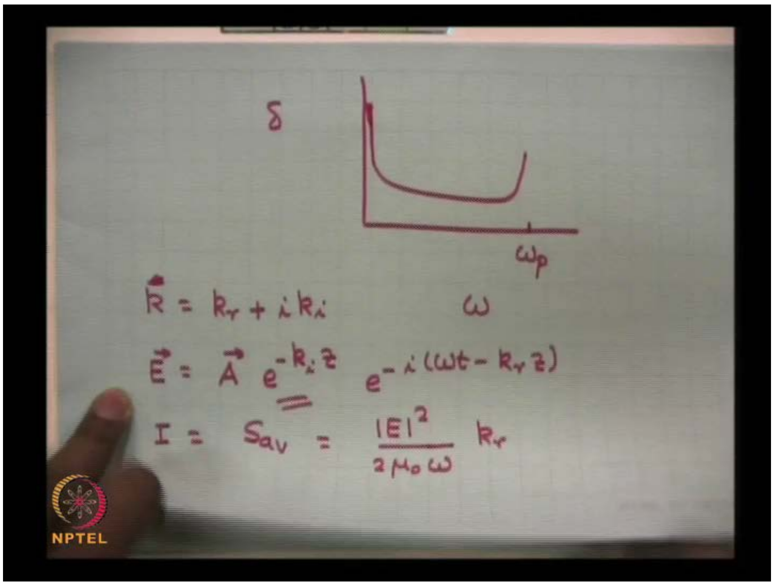

So, if I plot a graph of skin depth versus frequency. Delta versus omega at very low omega this will be large and as omega approaches omega p again this will be large. this is actually a logarithmic plots kind of thing, so this is skin depth would be quite large at very low frequencies like if you are sending a wave of radio frequency like 10 megahertz in a solid or in a high density plasma, in that case the skin depth would be quite large in several centimeters. Whereas, the same metal at microwave frequencies will have a very tiny skin depth may be of the order of a few microns. So, skin depth is a very strong function of frequency. I think, I would like to you know mention a quantity because when we talk of absorption of waves via collisions we always talk in terms of power loss in decibels.

So let me introduce this quantity here, whenever k is complex, k magnitude wise is equal to k real plus i times k i your electric field can be written as E is equal to a amplitude exponential minus k i z, for wave propagation on z axis exponential minus i omega t minus k r into z this sort of expression you get. And as I just told you that the pointing vector or intensity of radiation which is time average pointing vector is equal to modulus of E square divided by twice mu 0 omega into k real is the expression I have given to you.

So, if I put the values please note here when I take the modulus square only A square will not come exponential factor was also real and square of this will also come. So, what you get? your intensity will depend on z. Now, because of this vector this vector.

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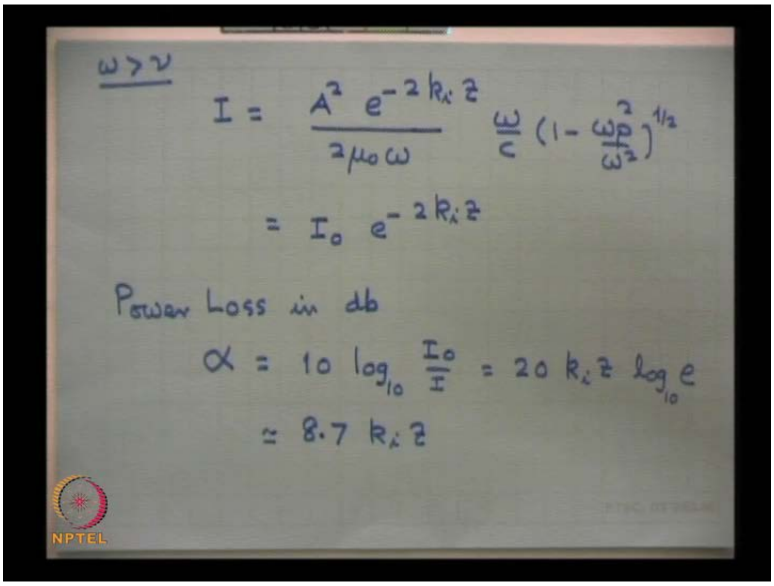

And the result would be that I can write down my intensity as this will be A square exponential minus twice k i into z upon twice mu 0 omega multiplied by k r, which is omega by c into 1 minus omega p square by omega square to the power half; this is a typical expression. In a weakly collision plasma, where I have assumed omega bigger than nu.

Well, all these vectors are constant in a plasma of uniform density only this is z dependence. So, I can write this is equal to some constant I 0 exponential minus twice k i z means the amplitude intensity of radiation decreases with z. Power loss in decibel is defined like this, as alpha which is equal to 10 log to the base 10 of i 0 upon i. And if you evaluate this quantity like this it becomes 20 into k i into z into log to the base 10 of log of E. Log of E is to the base 10 is 1 upon 2.3, so if you divide this by 2.3 this will be something like 8 point something 8.7 or something, 8.7 i guess into k i into z.

So in many experiments you measure the power loss in decibels over a certain distance z and hence k i can be calculated.

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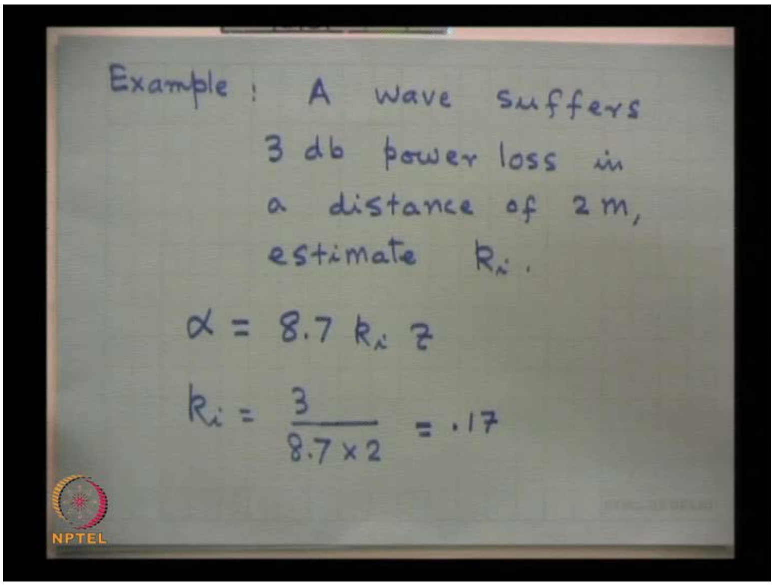

I will give you an example I will give you an example A wave suffers say 3 d b power loss in a distance of say 2 meter, estimate k i. what should I do? It is very trivial alpha is 3 here, so expression was alpha is equal to 8.7 multiplied by k i into z is 2, so k i turns out to be equal to alpha is equal to 3 upon 8.7 multiplied by z which is 2, one can evaluate this quantity which is I think is equal to point this is I think 0.17 or so just just a very rough I have written and one can estimate so it is a simple calculation. The advantage here is, that once you measure the power loss in decibels you can calculate k i and k i if you put the expression for k i from there you can deduce the collision frequency.

So absorption measurement gives you information about the collision frequency, if you know plasma density that you can measure by some other method. And hence this is a important diagnostic technique also. I must also mention that when we were discussing the conductivity of a collisional plasma, rf conductivity of a collisional plasma we found that heating of the medium takes place only when collisions are there. and here we have noted that damping of the wave takes place, when k i is finite or collisions frequency is finite. So collisions are responsible for both damping of the wave and heating of the plasma, after all where the energy goes when the electromagnetic wave travels in a plasma and if it is attenuated it is attenuated by the electrons, which oscillate in the presence of the electric field of the wave and which undergo oscillations.

But when they collide with scatterers like ions or neutral atoms then the their velocity has a component in phase with the electric field and that gives rise to net power dissipation. So, electrons are heated and obviously when electrons collide with ions or neutral atoms they impart part of their energy to those (( )) and they can also be heated but, the temperature of electrons in such plasmas is always bigger than the ion temperature or neutral temperature.

So, I think these are some basic characteristics of electromagnetic wave propagation in plasmas. I would like to finally, discuss the implication of complex k on wave reflection, because we have just learnt that if wave is launched on to a over dense plasma then propagation constant is purely imaginary and hence time average pointing vector is 0. The question arises is there some electric field still in the over dense plasma or not, so to address this problem I would like to discuss the problem of wave reflection in a from a plasma boundary.

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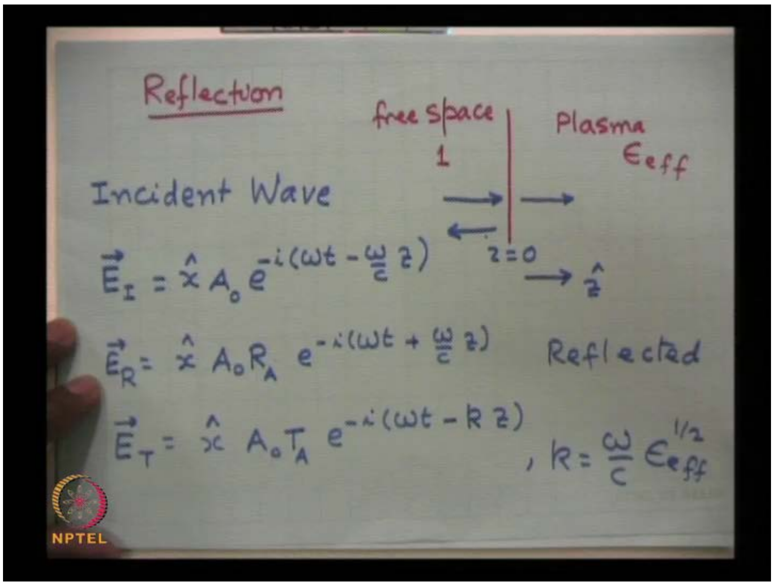

Consider, a boundary between a plasma here and free space there, plasma is characterized by a single quantity called epsilon effective and free space has this permittivity equal to unity it is a free space. What we do? We are launching an electromagnetic wave here, partly the wave will be transmitted in the plasma partly it will be reflected.

So there are three waves a incident wave a reflected wave and a transmitted wave and let me consider this direction as z direction and the interface as z equal to 0. So, the incident wave if I write E incident is equal let me take the this wave is polarized perpendicular to the direction of propagation, so either E x is finite or E y is finite or both can be finite. but without any loss of generality I will choose this to be x polarized and that the amplitude of this incident wave is A 0 and this is a wave travelling the positive z direction.

So, I will write down this is equal to minus i omega t minus free space, where this is effective permittivity is one, I can write down the propagation constant as omega by c and similarly, reflected wave will also have A 0, suppose the amplitude is reduced by a factor R A subscript means amplitude reflection coefficient.

The frequency would be same but, the direction of propagation is reversed. So, I will write down plus omega by c z this is a reflected wave and the transmitter would be E T again should have polarization to satisfy the boundary conditions, let the amplitude will be A 0 into T transmission amplitude of T A exponential minus i omega t, the wave will have a different k vector (( )) this k z. Where, k is omega by c epsilon effective to the power half.

So, now I know the fields I must apply the continuity conditions well there are two unknowns here, R A and T A. I require two boundary conditions; one boundary condition is that the tangential component of electric field must vanish here must be equal on both sides should be continuous and second tangential component of magnetic field should also be continuous.

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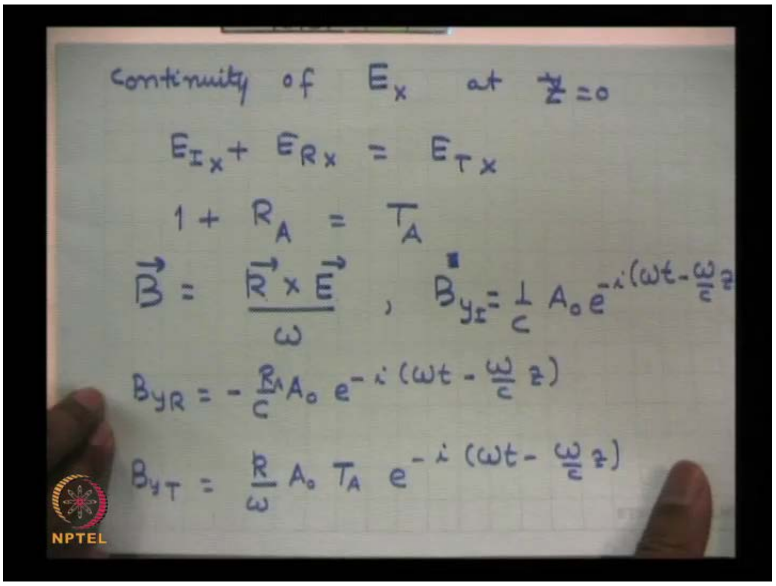

So, let me apply this to boundary conditions. the first boundary condition continuity of E tangential E x at boundary which is z equal to 0, in these expressions put z equal to 0 and so the on the left hand side, the field is E I incident field plus reflected field x component of this x component of this should be equal to x component transmitted field x z equal to 0 this gives me 1 plus R A is equal to T A.

Second thing is magnetic field continuity of magnetic field. Now, magnetic field we already know for a wave v is equal to k cross E upon omega, what you get from here? For the incident wave B, well please understand k is in the x z direction E is in the x direction so k cross E will be in the y direction. So, I will write down simply v y is equal to for the incident wave this would be k is omega by c so this will be 1 upon c, A 0 exponential minus i omega t minus omega by c into z is the incident.

Let me call this as I B y of the reflected wave I can write down similarly, as minus 1 by c because k reverse sign into A 0 exponential minus i omega t minus omega by c into z. And B y of the transmitted wave would be this R also there I forgot R A this will be k upon omega into A 0 into T A exponential minus i omega t minus omega by c z. Just I have used this expression to write the fields due to three waves in three different forms.

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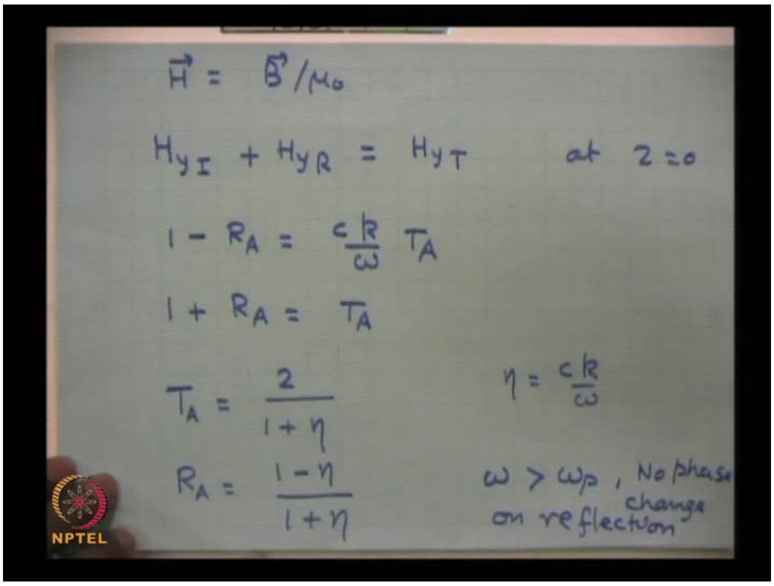

Apply form there you can write down the H which is equal to v upon mu 0 and the continuity condition is that H y due to the incident wave plus H y due to the reflected wave should be H y due to the transmitted wave apply this at z equal to 0.

The condition turns out to be 1 minus R A is equal C k upon omega into T A the other equation was 1 plus R A is equal to T A these two equations can be added to obtain amplitude transmission coefficient, which is equal to 2 upon 1 plus eta. Where, eta is equal to C k upon omega of the refractive index and similarly, amplitude reflection coefficient is 1 minus eta upon 1 plus eta, so you get amplitude reflection coefficient to be positive because eta is less than 1. So far, omega bigger than omega p, if collisions are weak then eta is nearly real and R A is less than 1 but, positive. there is no phase change on reflection on the other hand, so no phase change on the reflection on the reflection on the other hand.

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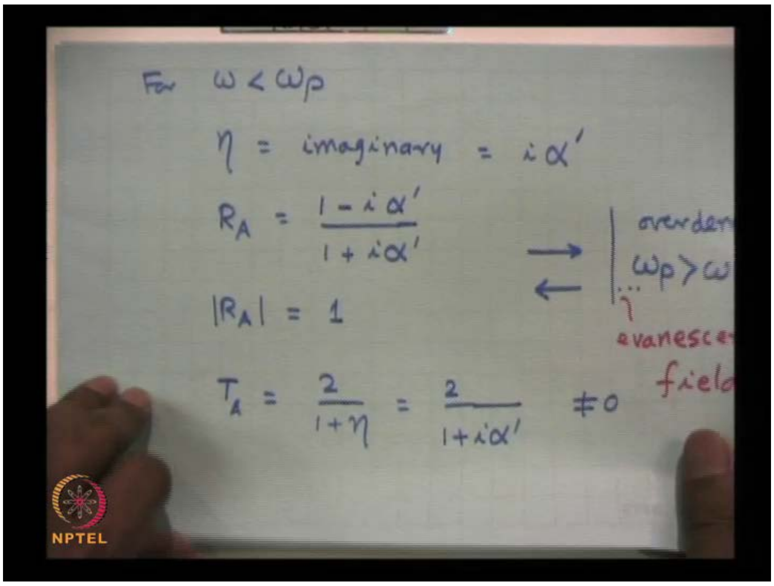

If, omega is less than omega p in that case eta is imaginary, when eta is imaginary R A this can be written as suppose i alpha prime for instance, that becomes 1 minus i alpha prime upon 1 plus i alpha prime. So modulus of this quantity will be one take modulus of quantity which is 1 plus alpha prime square under root denominator will also have the same magnitude so there is 100 percent reflection but, there is a phase change on reflection.

Well, we shall elaborate on this issues later another important issue is that in this case t is non zero amplitude transmission coefficient is 2 upon 1 plus eta, if eta I put i alpha it becomes 2 upon 1 plus i alpha prime which is non zero; means if, I have a plasma over dense which is omega p bigger than omega a wave is coming in here, wave will be reflected back from there there is no power transmission, because k is imaginary there but, field is not zero, there is always a field in this region and that is called evanescent field; So evanescent field exists here and how much is the evanescent field.

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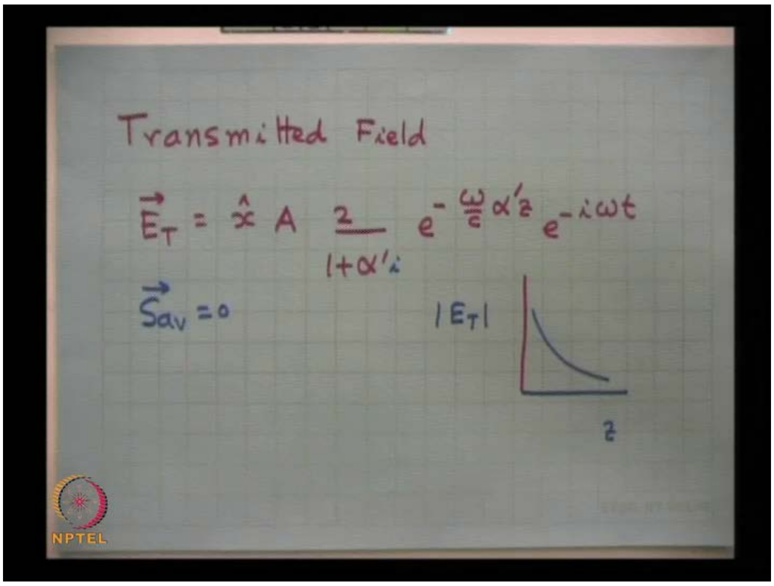

Let me write down this expression, in terms of T i will write down my transmitted field is E T, x direction that is the direction of polarization, amplitude was A into T A. T A, I have taken as 2 upon 1 plus alpha prime, and in the second medium the this goes a exponential minus omega by c into alpha prime, this is the value of k imaginary into exponential of minus i omega t.

So your wave into z also there, its having a amplitude which is decaying with z. So, if you have the boundary here. If, I plot magnitude of this quantity which is called amplitude of the wave, this is i alpha actually i is there. So, if I plot here say modulus of E T as a function of z, because of this factor it will fall down. So this is a wave whose real part of k is 0, so as average is 0 in the transmitter medium, I am talking about the steady state, and eventually. So, there is no power transfer over there, if there is no collision. I think further details of such situations, we will discuss when we consider specific cases; that is for that is all for today. Thank you. .