**Plasma Physics**

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**Lecture No. #07**

**Electromagnetic Wave Propagation in Plasma**

Today, I begin a new topic that is electromagnetic wave propagation in plasma, in fact plasma is known for very to be very rich for wave phenomena, and hence I would like to discuss this in some detail.

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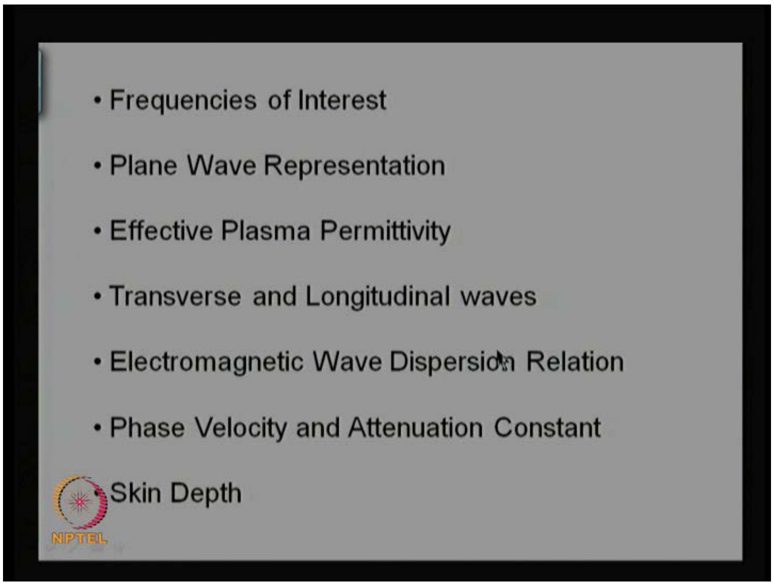


And, I will begin my presentation with elaborating on the frequencies of interest, then I will talk about plane wave representation of an electromagnetic wave. Then I will talk about effective plasma permittivity that brings equivalence of plasmas to dielectrics, then we will discuss transverse, and longitudinal waves as two possible solutions of Maxwell's equations. And then, we will discuss propagation of electromagnetic waves, and we will derive an expression called dispersion relation, and then we will discuss phase velocity and attenuation constant. And finally, we will discuss the propagation of waves in over dense plasmas, and hence we introduce a term called is skin depth.

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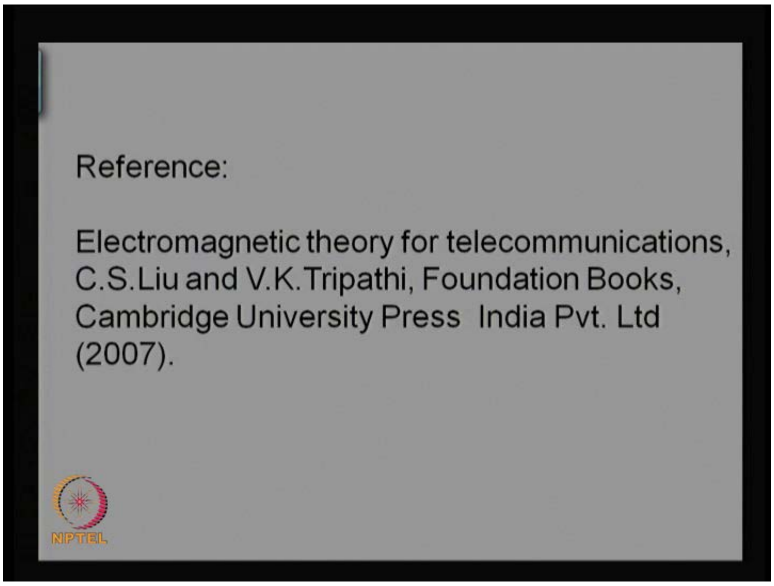


Well, the reference for today's presentation is this book electromagnetic theory for telecommunications that professor C S Liu and I wrote this was published by foundation books.

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Let me begin with the frequencies of interest; well these frequencies of interest range from very low frequencies called ultra low frequencies U L F, which has a range from 1 to about 30 hertz, hertz means cycle 1 cycle per second this is the frequency.

Now, these waves are used for underground exploration suppose this is our earth this is earth and this is air and there may be some object buried inside the earth it may be a oil reservoir or it may be a mineral and we would like to probe at what depth is this mineral and what is the identity of the mineral. So, what you have to do? Because, earth is a conducting medium in m t s units earth conductivity is about one and in wet soils it can (( )) as much as four in m t s units. So what you have to do you have launch those waves that can penetrate deeper and go to the buried material.

So you have to launch waves from here and then the waves are reflected back at the surface you measure a quantity called ratio of the electric field of the wave to magnetic field of the wave this E by H ratio can provide information about the material its the depth and its conductivity or effective plasma conductivity.

So these waves are very useful how the plasma arrives in this domain is in the generation of these waves. These ultra low frequency waves obviously one can produce by (( )) an antenna on the ground but, the efficiency is very low. So what we do? There are some natural sources in the earth's ionosphere which is above the surface of the earth.

So in the ionosphere there are already some natural sources that produce these radiations. So, we employ those U L F waves to examine that E by H ratio on the surface of the earth, second frequency band of interest is called E L F waves this E L F wave range is from 30 hertz to about 300 hertz. Let me just check 30 hertz to about 1 kilo hertz rather this band of frequency is useful in communicating with submarines.

So, this is suppose the surface of the ocean and there is a submarine sitting underneath or any objects submerged in water. And if we want to communicate with this then what people do there is a ionosphere also up in the atmosphere of the earth so this is ocean and this is ionosphere. Ionosphere has some currents you can modulate those currents and generate these waves those waves will penetrate into the water. And you can send signals command signals to the person sitting in these objects like submarines. So, E L F waves are used for submarine communication.

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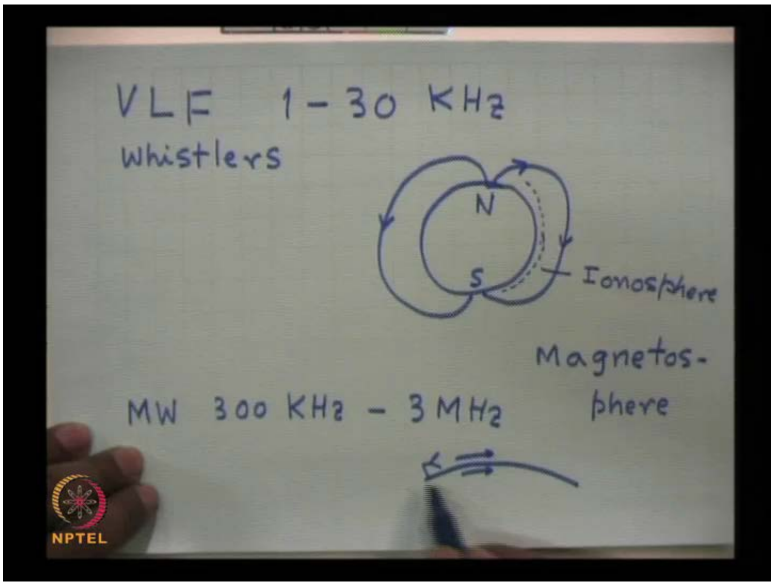


Then there is another frequency of interest to plasma and that is called V L F waves, the frequency range is between one to 30 kilo hertz, you know our earth is like a sphere. If, north pole is somewhere here and south pole somewhere here, so there is a line magnetic lines of force like these.

These are the lines of force magnetic lines of force, what happens that? In this region of space which is about 90 kilometers and above the earth surface of the earth, the air is ionized and we call this region as ionosphere and when you move to distances of the order of earth radius and larger than the region of space is called magnetosphere. So, in order to probe those regions of space ionosphere and magnetosphere waves of these frequencies between one to 30 kilo hertz have been found to be very useful they are given a name called whistlers.

So whistler waves have been used for exploration of ionosphere and magnetosphere then comes the famous frequency meant for radio communication called medium frequency waves; medium frequency waves M W, I will call and these waves have a frequency range between 300 kilo hertz to 3 mega hertz.

Now, we have earth which has a curvature. If, you have a antenna placed somewhere here this will emit waves these waves travel along the surface of the earth when earth bends these waves also bend and they can carry undistorted or noise free signal to long distances up to a few 1000 kilometers in this frequency range and they have been used for radio communication.

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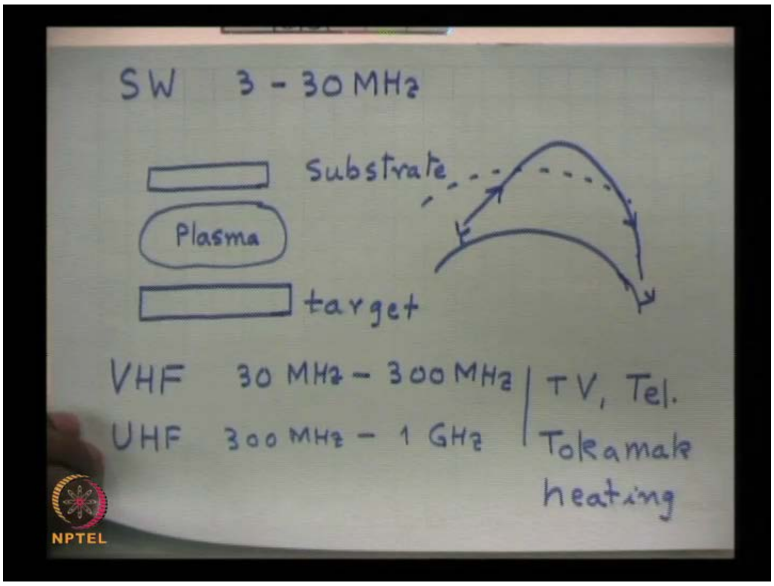


Then comes the short wave communication, the frequency used are called short waves, the frequency bend is 3 mega hertz to 30 mega hertz in this bend also ionosphere plays a very important role. Suppose, this is my earth surface, this is the ionosphere, and I want to communicate from here, and and send my signal to far away distances like somewhere here, what should I do? I will launch a wave in the ionosphere, and we learn that ionosphere is a optically rarer medium.

So when the wave enters the ionosphere it gets turned like this and it can reach a far distant point like this so ionosphere plays a very important role in short wave communication. Then we have a very important industrial application of waves in this frequency or radio frequency fields. In this band and that is called material processing. So what you do in material processing you have a substrate here like gold whose film you have to deposit on a substrate.

So, this is a target and this is substrate, you bring in a plasma you create a plasma here by means of a radio frequency field in this frequency range typical frequency that is used in most of these systems is about 13.6 megahertz. So, it is a plasma here, what plasma does? The plasma ions which come in contact with the target they cause very rapid ablation of material top clear of the material here and then the ions which are released they pass through here and the film is deposited over the substrate.

So, plasma plays a very important role in increasing the sputtering or ablation yield of target material and deposition of the film on the substrate. Then comes the V H F band very high frequency band and ultra high frequency band, this is between 300 between 30 megahertz to 300 megahertz and this goes from about 300 megahertz to about 1 gigahertz; gigahertz means 10 to the power 9 cycles per second.

These waves have been used for television communication, telephones and also used for plasma heating in a big device called tokomak tokomak heating. Let me also mention that the waves of frequencies below 30 megahertz called short waves are also useful are also used in plasma heating. So, plasma heating or rather heating the fusion devices a very major application of radio frequency fields.

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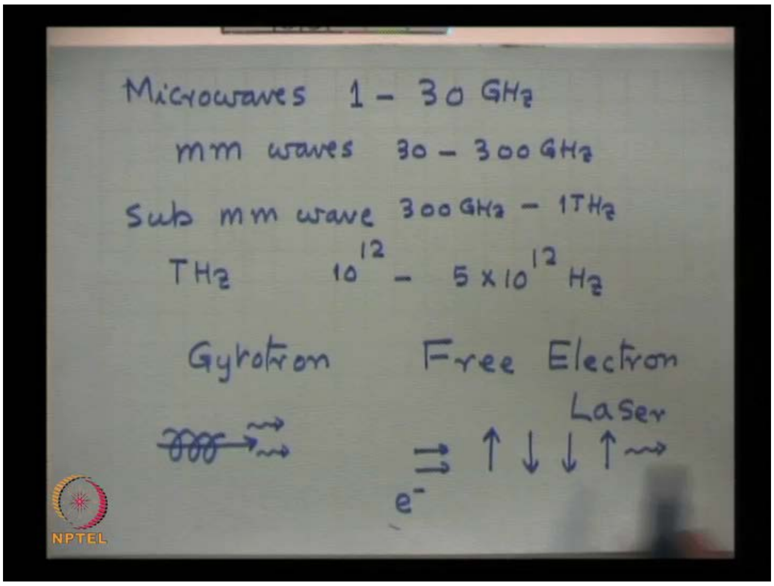


Then comes the famous microwave range usually we talk this about 1 to 30 gigahertz. Where, the at 30 gigahertz the wave length of these waves in free space is 1 centimeter and at 1 gigahertz it will be 30 centimeter. These waves have lot of applications in domestic for domestic use like heating, microwave heating, micro oven is very famous but, then they are also very useful in material processing in plasma heating and in communication.

And then we go to higher frequencies or shorter wavelengths called millimeter waves then sub millimeter waves whose wavelength is in the millimeter range and these wave have wavelength in the sub millimeter range less than 1 millimeter and then we go over to what we call as terahertz waves, whose frequencies vary from about 10 to the power 12 hertz to about may be 5 times 10 to the power 12 hertz.

Well, in both these bands this goes from 30 gigahertz to about 300 gigahertz and this goes from 300 gigahertz to about 1 terahertz these frequencies are not produced by conventional sources to produce these waves one employs very special kind of plasma sources called gyrotrons and free electron laser.

Gyrotron and free electron laser much of plasma physics research has gone into these two devices in a gyrotron the basic principle is that if you can launch an electron beam in a magnetic field like this, then the electrons gyrate in the field lines like this and millions and trillions of such electrons gyrates in free lines. And if, they can radiate coherently then we get microwaves at the cyclotron frequency or two time cyclotron frequency.

So this is a very important device and in free electron laser you launch an electron beam electron beam and you pass them through a very special kind of magnetic field structure where lines of force are like this alternately up and down. This region of space we call having a wiggler magnetic field, when electron beam travels through them they produce radiation and this radiation is tunable. The frequency of this radiation can be controlled by controlling the energy of these electrons and this could be starting from sub millimeter waves to optical frequencies it is a very versatile and very efficient device.

So, collective behavior coherent oscillation coherent radiation by electron beam induced by a wiggler magnetic field is a very important plasma physics subject and this device is very useful and it has got a promise for plasma heating in tokomak as well as for communication. So, with these applications in mind. Now, I would like to go over to discuss the propagation of electromagnetic waves in plasmas, first of all I like to mention a few things regarding what is a wave.

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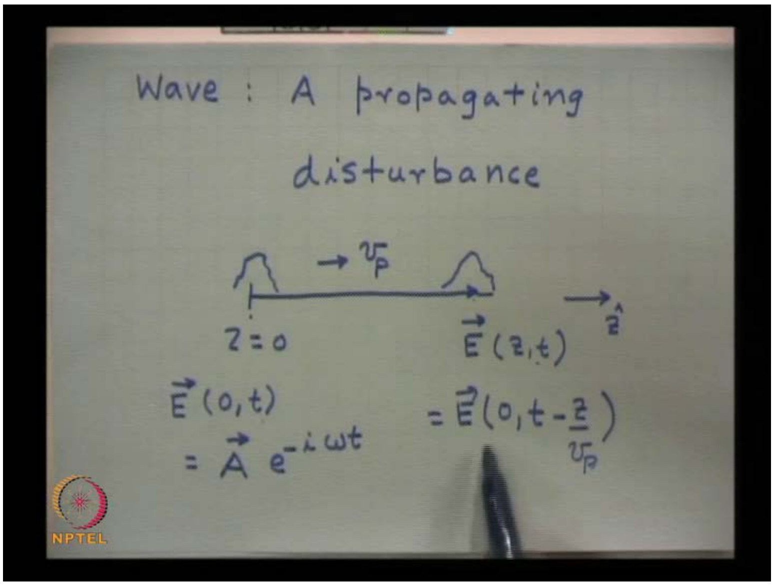


So, let me start with a very simple basic definition of a wave. Normally, wave is called a propagating disturbance a propagating disturbance what is this mean? In the simplest possible terms suppose this is the direction of propagation of my disturbance this is my position Z is equal to 0. If, I have some signal here may be disturbance in the case of sound wave it could be a pressure disturbance for the case of electromagnetic waves it could be an oscillatory electrical magnetic field.

So, suppose this is my signal here, I have no electric field, magnetic field here or there only in this region this is there after a while the same thing appears here little later this will appear somewhere here, then I call this as a propagating disturbance. A field that is confined in certain region of space here after a while it appears here, then after a while it appears there then there then we say its called a propagating disturbance for a wave.

The velocity of propagation of a signal suppose is v p. Now, the disturbance in the case of electromagnetic wave is in electric disturbance or a magnetic disturbance or a combination of both.

So, let me consider this to be the electric field at Z equal to 0. so let me the first space here is allocate for the Z coordinate and then the time coordinate. So electric field in general in the simplest possible situation depends on position and time and at this suppose this is simplest representation. Suppose, I say is A exponential minus i omega t.

Suppose, the electric field that is produce by any antenna or any source in this region Z equal 0 is having this dependence well, why I am choosing this dependence? Because, any general time dependence of electric field can be represented as a sum of many such monochromatic fields by using Fourier series or Fourier integral representation.

So, if we can understand the propagation of a one frequency component, we can study the propagation of a group of such frequencies. So, most of the electromagnetic theory is really concentrated about the propagation of a monochromatic or a single frequency wave. So I will consider this kind of dependence there is another reason for this as we shall learn plasma is a dispersion medium.

If you send a time bound signal like this, when it travels in a plasma it gets distorted and time bound signal if you carry out the Fourier analysis is equivalent to having many frequencies and different frequencies signals travel with different velocities and consequently the signal gets distorted. So, in order to avoid such subtle effects to begin with I consider single frequency so that we can assign a single velocity of propagation to this field. So, now consider an a signal consider a signal at Z equal to 0 having this kind of field and my issue is that if the signal is travelling along Z axis this by Z direction, so what is the value of the signal at position Z at time t.

Well, I do not want to solve any equation for this I just want to use this concept of a propagating disturbance introducing this value. I know that the signal had arrived from Z equal to 0, the time it has taken is equal to Z by v p, because the distance it has travelled is Z and time is v p.

So whatever was the value of the signal is here at time t must had been here at earlier time, so this should be the same as E at 0 at time t minus this, because this is the additional time it has taken to travel.

So whatever field you observe at Z at time t should be same as at Z equal to 0 at time t minus this quantity. But, here I have already stated that if you know that at time t the field is so much then in place of t, replace this t by t minus Z by v p, and you will get the field there so I will write this as such.

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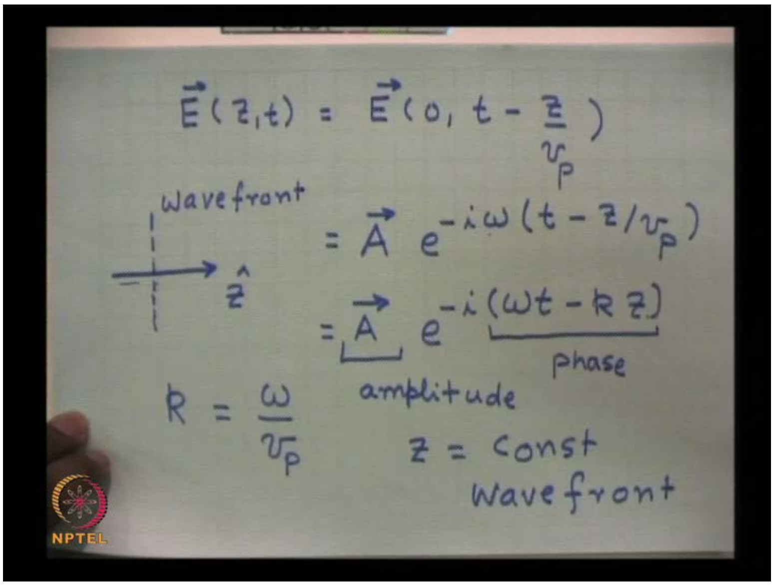


You will get E at Z t is equal to E at 0 at time t minus Z upon v p. And if I put the expression for E in terms of a then this becomes E to the power minus i omega in place of t, I will write t minus Z upon v p. I can multiply omega in the interior and I can write this as A exponential minus i omega t minus k Z, where I have defined k is equal to omega upon v p this by definition.

So on physical grounds I have written down a dependence of electric field on position and time, if my wave is travelling in the positive Z direction with velocity v p and the wave frequency is omega, this entire exponent from here to here is called phase of the wave. And the quantity which is here is called amplitude, one may note here in this particular case phase does not depend on x and y.

So, if I consider a plane perpendicular to Z axis means over which Z is constant then this is the wave front equation of the wave front. So, a plane perpendicular to Z axis on which Z is constant my wave was propagating like this. So any plane this is my Z axis, any plane perpendicular to this will have same value of electric field here, here here here here because on this planet Z does not change only x and y change but, x and y do not appear in the phase and hence phase is constant.

So this is called the equation of the wave front Z is equal to constant. Because, if Z is constant then phase will not change at a given time. So, the wave one may note here my wave was travelling in Z direction, a plane perpendicular to the direction of propagation is called wave front this is wave front. Well, this is what we call as the one dimensional representation of a plane wave, the wave front is a plane surface and hence this wave is called plane wave but, here I assumed that my wave was travelling around Z direction.

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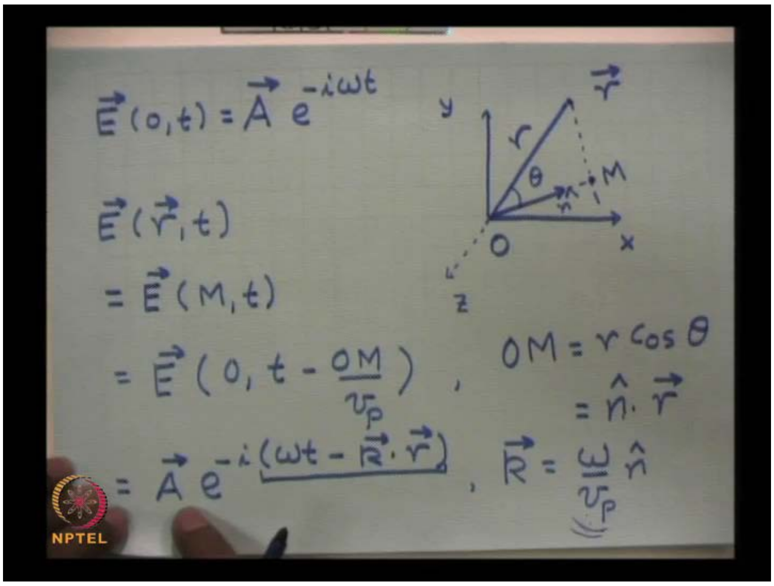

Now, I would like to write this in a little more general case suppose my wave is travelling at an angle to Z axis, suppose this is my x axis, this is y axis and Z axis is perpendicular (( )) of the board (( )) of the paper so this is my x axis y axis and Z axis I may denote like this. Suppose my wave is traveling at an angle like this, I call this as unit vector n. So, I want to find out if field at position O is given to me or the origin is given to me and it is also given to me that my signal is travelling in the direction of this arrow, what is the field at a point here at position r from the origin.

So my point is that E at the origin at time t is suppose A exponential minus i omega t then question is what is E? At position r at time t, what you can do first of all consider a draw a plane passing through point r. So, if I draw a plane passing through r but, perpendicular to this direction of wave propagation. So then I will draw a plane like this and extend this unit vector n, this will hit this unit vector n at a point M.

So, what I except is that the electric field at this point is the same at this point because this plane is perpendicular to direction of propagation and hence this is the wave front and on the wave front, the value of the electric field at every point is the same so E at r at time t will be same as E at M point at time t. Now, the field at M is reaching from the origin along this path, so this should be same as E at the origin at time t minus that distance O M by the velocity of propagation of the signal v p.

Now, O M I can easily calculate because the distance from here to here, this distance is r if this angle between the two is theta, I can write O M is equal to r Cos theta but, r Cos theta can be vectorially written as n unit vector along O M dot r. So, O M I will write down a simply n dot r and then I use this stop expression I will get this is equal to A exponential minus i omega t minus k dot r. where, k I have written as magnitude wise is equal to omega upon v p as before but, I have put a unit vector n.

So, k is known as the propagation vector whose magnitude is omega upon v p, the velocity of propagation of the signal and direction is the direction of wave propagation n. So this is called a 3 dimensional representation of a plane electromagnetic wave but, we do not know what is the connection between the direction of k and direction of a and what is the value of v p, the entire information about the medium is contained in this quantity v p this is the medium which gives rise to specific values to phase velocity to v p.

Now, please understand here we are considering the signals whose amplitude is constant only phase changes. So this is the velocity of phase propagation amplitude is not changing. So, v p is really not called the velocity of signal propagation this is called the velocity of phase propagation. Because, only phase moves only phase changes with time so v p is the velocity of phase propagation or phase velocity.

So, I think we have obtained on physical grounds an expression for electric field E as a function of position and time, I will call this is the simplest possible expression for field that represents a propagating disturbance of constant amplitude and constant frequency.

Now, let us see under what conditions this solution satisfies the Maxwell’s equations. So, before I undertake the general conditions under which this solutions satisfies the Maxwell’s equations.

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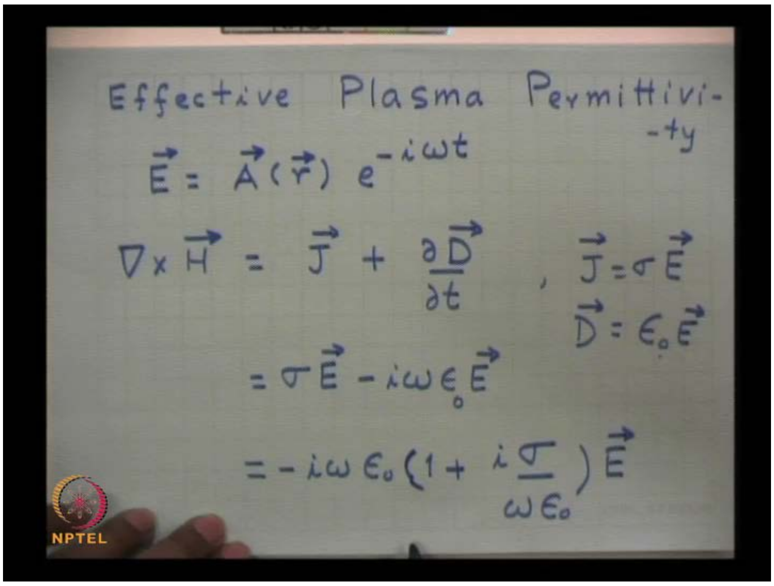


I would like to introduce a quantity called effective plasma permittivity. let me examine the Maxwell's equations from this perspective well, in general my electric field is a function of position and time, well I will like to keep my r dependence little more general then the plane wave representation suppose like this. Well, I can write down this as omega t minus k dot r then a can taken as a constant but, there is a very special space dependence it could be more general dependence. So suppose I am specifying only by time dependence and to keep my space dependence of electric field is more general.

Let us see, in terms of this representation how do the Maxwell's equation look, let me begin with the last Maxwell equation.

The generalized ampere's law which says that curl of H is equal to J plus delta D by delta t, when I write down the time dependence of electric field like this I also imply the time dependence of J, H and D also of the same form. Because, these equation should be true in the steady state and every term should have same time dependence. So, whenever the D or J or H they vary with time like this it means delta delta t of this function if you take you replace delta delta t by minus i omega and J, I write J for plasma is equal to sigma E conductivity times, the electric field and D for plasma is written as epsilon 0 into E.

So, just substitute them here and you will get sigma E minus i omega for this operator delta delta t and D is epsilon 0 into E, epsilon 0 is called free space permittivity. I can take minus i omega epsilon 0 common, in the interior you will get one plus i sigma upon omega epsilon 0 into E. So this is a single term, so these two terms have been combined into a single term which is a coefficient of E, the term in this bracket is given a symbol epsilon effective.

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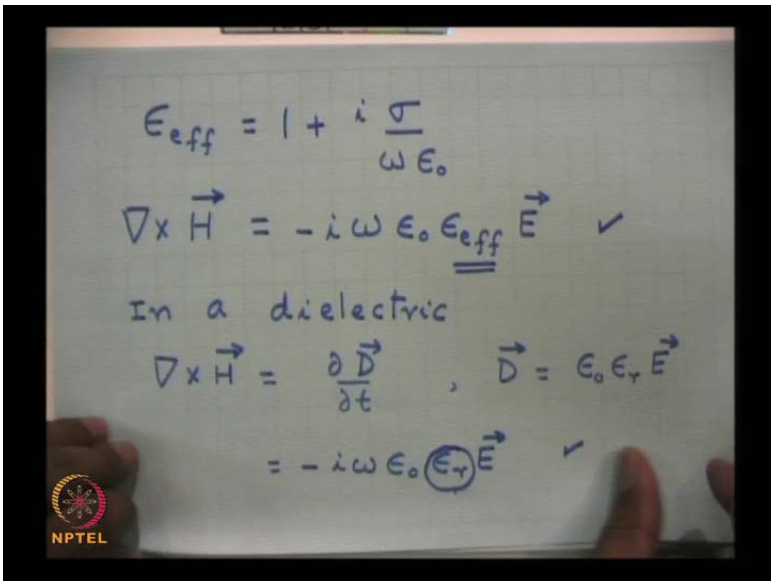


And I can write down this as epsilon effective is equal to 1 plus i sigma upon omega epsilon 0, in terms of this my Maxwell equation becomes curl of H is equal to minus i omega epsilon 0 epsilon effective into E.

If you look at the generalized amperes law in a dielectric, what would you have? In a dielectric, what do you get curl of H is equal to J but, there is no current density, so J is 0 is equal to delta D by delta t but, in a dielectric D can be written as free space permittivity into relative permittivity epsilon r into E. And delta delta t for monochromatic fields can be written as minus i omega, so it becomes minus i omega epsilon 0 epsilon r into E.

Now, compare this equation with this equation the two equations are exactly same with the only difference that epsilon r, in the case of a dielectric is replaced by epsilon effective in the case of a plasma. So plasma behaves mathematically as a dielectric with only difference that wherever epsilon r occurs, relative permittivity occurs, you just put epsilon effective so epsilon effective is called also effective relative permittivity of the plasma. So this quantity epsilon r is a very important quantity here and equivalent of this is epsilon effective.

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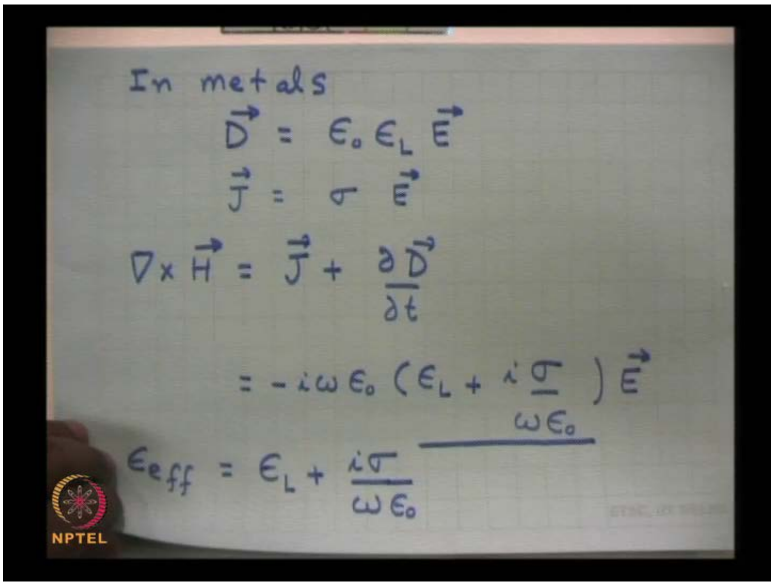


In case of metals or semiconductors, what you have? In metals you have D is equal to epsilon 0 into epsilon L into E, where epsilon L is the relative permittivity due to lattice in case of solids and J of course, we write as sigma E. So when you look at the fourth Maxwell equation, curl of H becomes J plus delta D by delta t, if you put J equal to sigma E delta delta t is minus i omega. I can write down this is minus i omega epsilon 0 into epsilon L plus i sigma upon omega epsilon 0 into E.

So, this quantity from here to here is called effective permittivity of metal relative permittivity of metal. So epsilon effective is equal to lattice permittivity plus conductivity into I upon omega epsilon 0.

So, the only difference in conductors and plasma is in this term epsilon L for plasmas this term is unity for lattice, for solids its epsilon L for intense for gold at optical frequency, infra red frequency. Lower frequencies epsilon L is like 9, for silver its 4, for microwave at microwave frequencies for germanium like 14, for indium antimonite at microwave frequencies the value of this expression is like 70.

So, this varies from material to material but, for plasmas this quantity is unity. So, we have found some equivalence of plasma to a dielectric or a conductor to a dielectric. But this is only one equation. I would like to see whether this epsilon effective brings the equivalence in other equations also.

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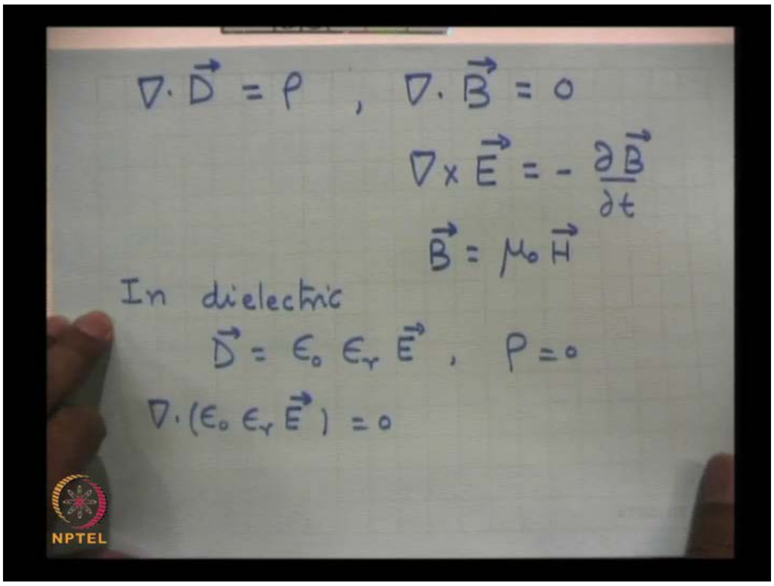


Now, the Maxwell's equations other Maxwell equations are divergence of D is equal to rho then divergence of B is equal to 0 and curl of E is equal to minus delta B by delta t, for non magnetic materials B is written as mu 0 into H, so no dielectric properties involved in these two equations.

So, this is the only equation which may be different from in different media, from conductors to semiconductors plasmas they may be different then from dielectrics but, we will see that if you use the concept of effective plasma permittivity this equation can also be written in the same form as in a dielectric, let me write down this. In dielectric, D is equal to epsilon 0 epsilon r into E, where as rho equal to 0, so this equation simply becomes divergence of epsilon 0 into epsilon r into E equal to 0. how about in the plasma? In a plasma rho comes, so let me write down in a plasma.

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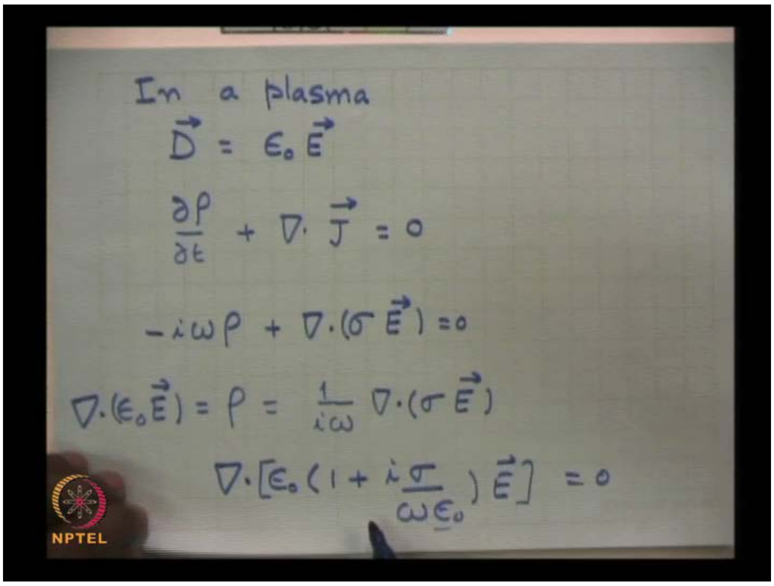


In a plasma or a conductor, what we do? For plasmas we write D is equal to epsilon 0 E, how about rho we know that plasma in equilibrium is quasi neutral, so charges appear only when there is flow of current so and rho is governed by the equation of continuity which is delta rho by delta t plus divergence of J is equal to 0.

J, I am writing sigma E and E varies in time as exponential minus i omega t, so rho must also depend on time in the same exponential fashion, so replace this delta delta t after minus i omega plus divergence of sigma E equal to 0. So we obtain rho from here, which is rho is equal to 1 upon i omega divergence of sigma E and this I am putting in this equation divergence of D which is equal to epsilon 0 E.

So, I can combine these two divergence terms take this from right to the left and what you get is divergence of epsilon 0 if you take common in the interior you will get 1 plus i sigma upon omega epsilon 0 into E equal to 0. The equation is the same as in a dielectric, because this term I call as epsilon effective, so epsilon effective comes in automatically there.

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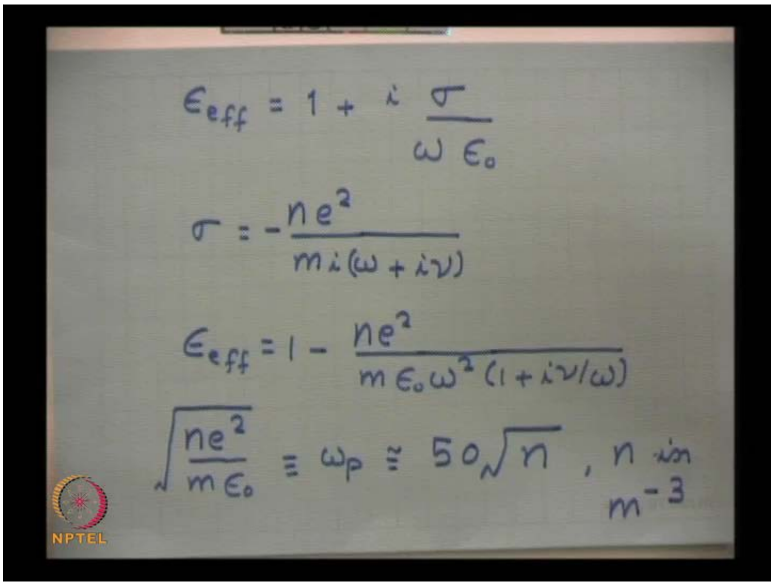


So, we have shown that epsilon effective which is written as 1 plus i sigma upon omega epsilon 0, makes a plasma equivalent to a dielectric. Now, conductivity expression if I substitute rf conductivity we derived an expression in a un magnetized plasma it was n e square upon m negative sign here, I there omega plus i mu this is the conductivity substitute this back in there then it becomes epsilon effective is equal to 1 minus n e square upon m epsilon 0 omega square into 1 plus i nu upon omega.

Here, n is the electron density, E is the magnitude of electron charge, m is the electron mass, epsilon 0 is free space permittivity and the value of this expression we already seen earlier, n e square upon m epsilon 0 under the root this quantity has the dimension of frequency and it is defined as omega p, if you put the value of E m and epsilon 0 then this equal to about 50 times under root of electron density, when n in per meter cube n in per meter cube.

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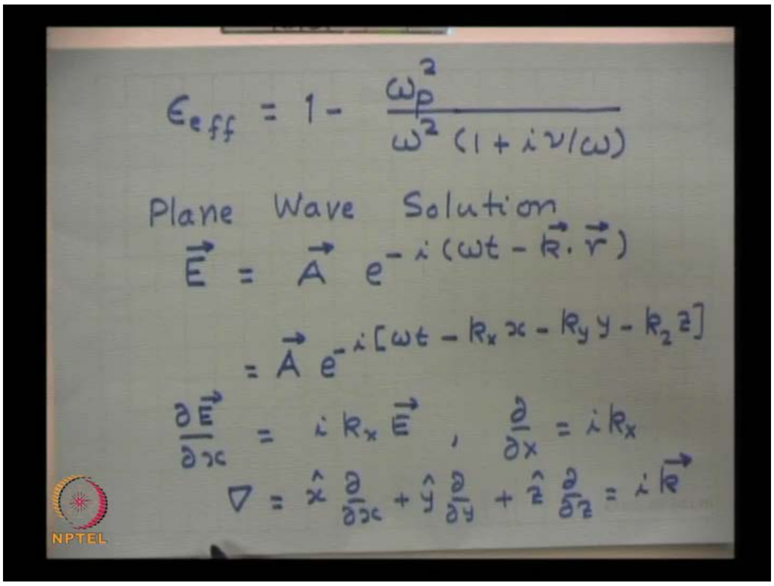


So, this is a simple expression I get and let me rewrite this effective plasma permittivity. As epsilon effective is equal to 1 minus omega p square upon omega square into 1 plus i mu by omega.

In most dielectrics, imaginary part of relative permittivity is small, so in plasmas also if you are talking of high frequency waves through a plasma mu is quite small so character is very similar however there is a big difference. Usually, the dielectric constant of a dielectric or relative permittivity of a dielectric is bigger than 1, in case of plasma it is less than one so that is a big difference and it has lot of implications on wave propagation. With this term let me examine the validity of our plane wave solution under what conditions this will satisfy Maxwell’s equations.

So, I am examining the plane wave solution, two Maxwell’s equations my electric field I am taking as A exponential minus i omega t minus k dot r, well in my definition of epsilon effective I kept the r dependence of electric field very general. But, when I am trying to study the plane wave propagation I am writing my r dependence like a plane wave so writing like this. So, using A as a constant amplitude. Now, this is a dot product of two vectors,so if I elaborate this it will comes out to be a exponential minus i omega t minus k x into x minus k y into y minus k z into Z this is the meaning of this dot product.

Explicitly, I have written this for a purpose. Because, if I ever encounter the derivative of E with respect to x for instance then I will simply get i k x into E. Because, all field quantities like E, H, B, D, J etcetera all of them for a monochromatic wave response of propagation they depend in the same way. And hence whether the derivative occurs over E or H or v we must replace this del operator delta delta x operator by i k x or simply I am saying that is del operator is written as which is written as x component of this delta delta x plus y component is written as delta delta y plus z component is written as delta delta z, it turns out to be simply i into k vector.

So, in subsequent discussion when we examine the validity of this solution from the perspective of Maxwell's equations, wherever del operator comes in the Maxwell's equations. I will replace this by i k vector and similarly, wherever time operator comes I will replace by minus i omega and let us see, how the Maxwell's equations look.

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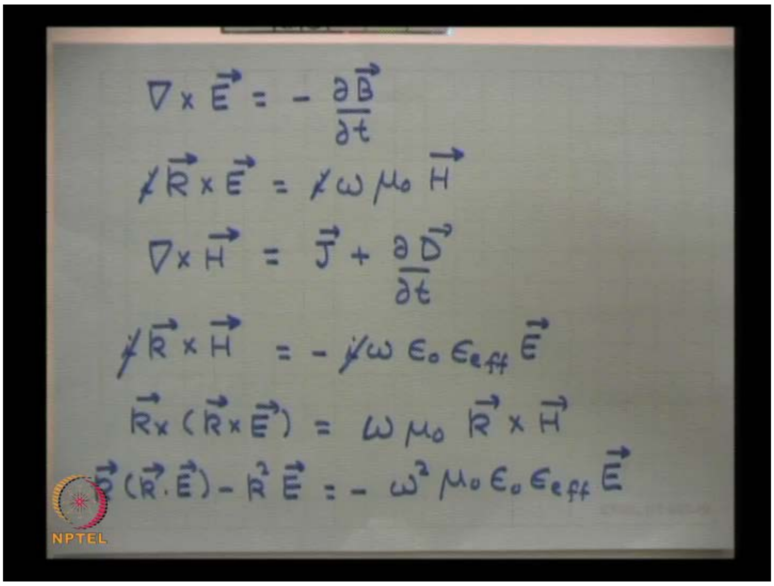


I will write down third and fourth Maxwell's equations, curl of E is equal to minus delta B by delta t; this is known as the third Maxwell's equation or faradays law of electromagnetic induction. Just replace this by i k cross E delta delta t by minus i omega, so it becomes plus i omega B is mu 0 into H or a plasma I can cancel this I from here, there the fourth Maxwell's equation is curl of H is equal to J plus delta D by delta t but, we just note down this equation as minus i omega epsilon 0 epsilon effective into E and curl of H, I can write down as i k cross H.

I can cancel this i from here with this i there, what I can do? I can take k cross of this equation. So, I will get k cross of k cross E is equal to omega mu 0, k cross H then use k cross H from the lower equation it becomes is equal to minus omega square mu 0, E 0, epsilon 0, epsilon effective into E. And k cross k cross E if you use vector repul product can be written as k vector into k dot E minus k square E vector, this is the equivalence of wave equation it is an algebraic equation, because I have already replaced del operators by i k, otherwise this like a wave equation.

The character of this equation will reveal that this equation supports two kinds of waves electrostatic waves, and electromagnetic waves, and also for electromagnetic waves this will give you a dispersion relation, and entire information about the waves is contained in this equation. I think, we are close to closing this lecture for now. In our next lecture, we shall discuss the implications of this equation, and elaborate on the character of electromagnetic waves. Thank you.