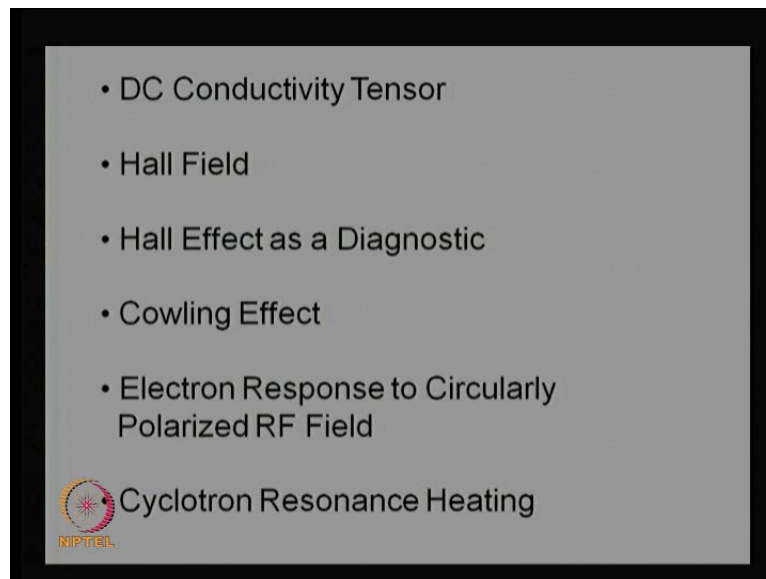


**Plasma Physics**  
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**Indian Institute of Technology, Delhi**

**Lecture No. # 06**  
**Hall Effect, Cowling Effect and**  
**Cyclotron Resonance Heating**

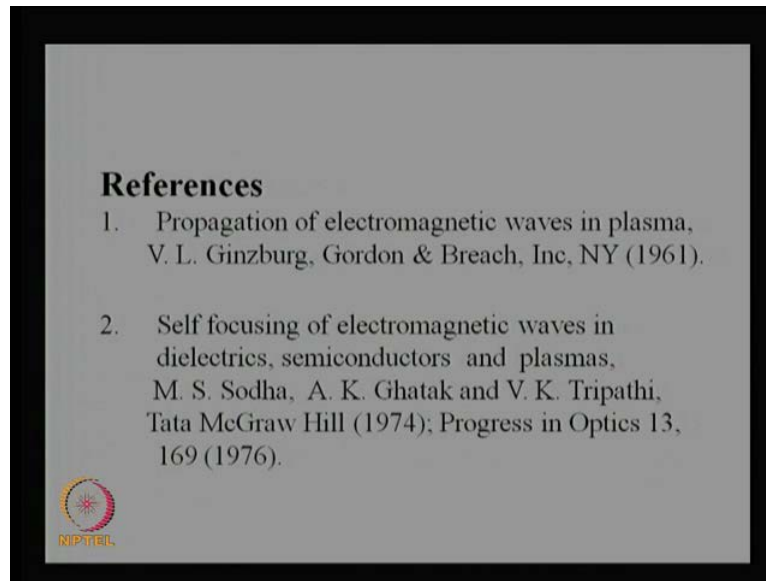
Today I like to discuss Hall Effect, Cowling effect, and cyclotron resonance heating in a plasma.

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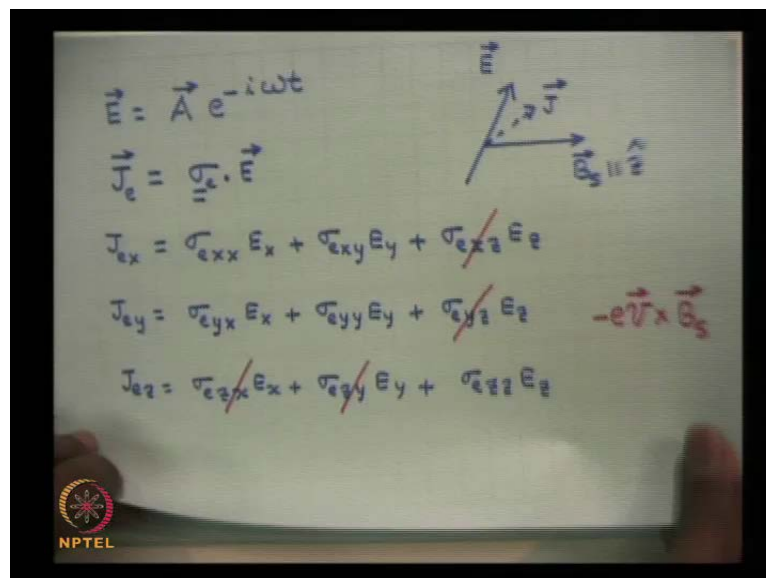
This lecture will include a discussion on DC conductivity tensor: hall field, and hall effect as a diagnostic tool, then I will discuss Cowling effect, electron response to a circularly polarized RF field, and then, Cyclotron Resonance Heating.

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Well the references for today's lecture would be a book by Ginzburg, and another one that he wrote in 1974.

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Well last time we discussed the response of a plasma to radio frequency field, and if the electric field is taken as  $E$  is equal to  $A$  exponential minus  $i$  omega  $t$ , and this could be applied to a plasma, which has a DC magnetic field like  $B_s$  - aesthetic magnetic field  $B_s$ , and I will call this as aligned along the  $z$  axis, and suppose the electric field  $E$  is in some direction like this, then what we found was that if the electric field is in this direction, the current in general

was inclined at an angle to the electric field, and it could be in some other direction, and this direction of  $J$  may not lie in the plane containing  $E$  and  $B$ . This could be in a different plane. In general  $J$  I had written was equal to a tensor which we call as conductivity tensor dot  $E$  this implies, this actually I had written for the case of electron current density.

Let me substitute use a symbol subscript  $e$  for electron. This is the electron conductivity, and if I have to write down this in component form. I can call this as  $\sigma_{ex}$ . If I have to write down  $J_{ex}$  for instance then  $x$  component implies a tensor has two indexes the first index has to be kept  $x$ , and the other one has to be common with the electric field index which is a vector with single index, and the common index is a running index or some over that is implied. This becomes  $J_x = \sigma_{ex} E_x + \sigma_{ey} E_y + \sigma_{ez} E_z$  similarly, I could write down  $J_{ey}$  which is equal to  $\sigma_{ey}$ . The first index of  $\sigma_{ey}$  should be same as  $y$  of current density the other one should be running. It be  $x$  there then  $J_x = \sigma_{ex} E_x + \sigma_{ey} E_y + \sigma_{ez} E_z$  and  $J_y = \sigma_{ey} E_x + \sigma_{yy} E_y + \sigma_{yz} E_z$  and  $J_z = \sigma_{ez} E_x + \sigma_{zy} E_y + \sigma_{zz} E_z$ . I can write down a starting  $\sigma_{ex}$  the first index should be  $x$ , and other one should be running,  $x E_x$  there plus  $\sigma_{ey} E_y$  plus  $\sigma_{ez} E_z$ .

Well one may note here that the anisotropy is coming largely or essentially because of the magnetic field. If there was no magnetic field then, we know that in plasma current density is in the direction of  $E$ , and this is related, and they are related to each other by a scalar coefficient  $\sigma$ .  $J$  and  $E$  are in same direction which means that  $J_x$  will depend only on  $E_x$ ,  $J_y$  will depend on  $E_y$  and  $J_z$  will depend only on  $E_z$ . All other these diagonal terms will be finite in this representation, and all other components will be 0. Now, what the magnetic field does magnetic field exerts a force on the electrons which is charged times velocity cross  $B$ . If you recognize the character of this force, you can quickly tell which components of this must vanish. This force is perpendicular to  $B$  field  $B$  is the aesthetic magnetic fields I will put  $B$  s there.

And it is also perpendicular to the velocity. So, for instant if I had applied a magnetic electric field in the direction of magnetic field there and then, electron wants to move in the direction of magnetic field then, this velocity in the  $z$  direction  $v_z$  is also in the  $z$  direction then this force will vanish. Magnetic field will have no consequence if the electric field is in  $z$  direction on the other hand if the electric field is transverse to magnetic field then the electrons will like to move in the direction of electric field. Whenever, they acquire a velocity in that direction the  $v \times v$  force will be perpendicular to  $B$   $S$  means the velocity will always remain perpendicular to magnetic field there is a important message in this that an

electric field in the z direction will not cause current in the x or y directions and similarly, an electric field in the x y plane perpendicular plane this is our x y plane this will not produce any electric field in the x y plane will not produce current in the z direction.

Consequently,  $J_e x$  should not depend on  $E_z$ , and hence, this should vanish  $J_e y$  should also not depend on  $E_z$ , and hence this must vanish, and  $\sigma_{Ez} J_e z$  should not depend on  $E_x$  and  $E_y$ . So, these terms must vanish. Means out of the nine coefficients 1, 2, 3, 4, 5, 6, 7, 8, 9, 4 are 0. Only 5 terms 1, 2, 3, 4, 5 in the conductivity tensor survive, and they are to be examined last time we noted last time that these two diagonal terms are equal. These two of diagonal terms differ by a negative sign and hence, the conductivity tensor.

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The image shows handwritten mathematical derivations for the conductivity tensor components. At the top, the conductivity tensor  $\underline{\sigma}_e$  is written as a 3x3 matrix:

$$\underline{\sigma}_e = \begin{vmatrix} \sigma_{xx} & \sigma_{xy} & 0 \\ -\sigma_{xy} & \sigma_{xx} & 0 \\ 0 & 0 & \sigma_{zz} \end{vmatrix}$$

Below this, the components are defined:

$$\sigma_{e,xx} = \frac{n e^2 i (\omega + i\nu)}{m [(\omega + i\nu)^2 - \omega_c^2]}$$

$$\sigma_{e,xy} = \frac{n e^2 \omega_c}{m [(\omega + i\nu)^2 - \omega_c^2]}$$

$$\sigma_{e,zz} = -\frac{n e^2}{m i (\omega + i\nu)}$$

Additional notes include the definition of cyclotron frequency  $\omega_c = \frac{e B_s}{m}$  (electron cyclotron freq.), and the relationship between the tensor components:  $\underline{\sigma} = \underline{\sigma}_e + \underline{\sigma}_i$  and  $\underline{\sigma}_i = \underline{\sigma}_e$  with substitutions  $m \rightarrow m_i$ ,  $\omega_c \rightarrow -\omega_{ci}$ , and  $\nu \rightarrow \nu_i$ . An NPTEL logo is visible in the bottom left corner of the slide.

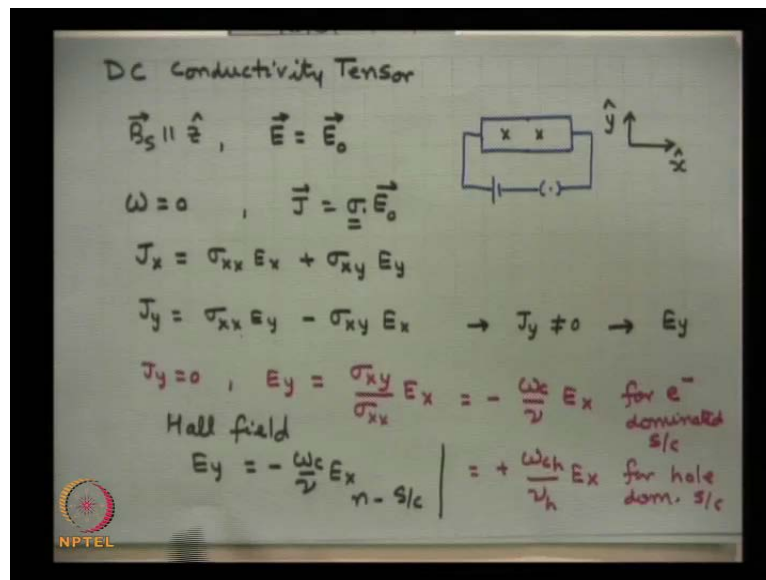
Let me write down the conductivity tensor, sigma for the electrons is equal to sigma x x sigma x y 0 minus sigma x y sigma x x 0 0 0 sigma z z, for the sake of brevity I have not written the subscript e here though its implied, otherwise one should write e also there. Now, what is sigma x x let me use this abbreviation anywhere. x x would be n e square upon m here you get a term like this omega plus i nu square minus omega c square, and here it was i omega plus i nu. Sigma e x y is equal to n e square omega c upon m omega plus i nu square minus omega c square. And sigma e z z is equal to n e square over m with the negative sign i omega plus i nu. Here, n is the electron density, minus e is the electron charge, m is the

electron mass,  $\omega_c$  we defined as the magnitude of electron charge into magnitude of magnetic field upon electron mass, and this was called the electron cyclotron frequency.

And  $\nu$  is the collision frequency. Collision frequency usually in high temperature or strongly ionized plasmas is small as compared to the magnetic field that we employ. That  $\nu$  is if you ignore  $\nu$  then, this denominator will be of the form  $\omega^2 - \omega_c^2$ , and at  $\omega = \omega_c$  you see, you expect a large enhancement in these components of conductivity tensor two components of conductivity tensor. We like to understand, what is the implication of this resonance on say plasma heating. However, before I go over to discuss that effect I would like to mention one thing that the actual conductivity of plasma is made of conductivity of electrons, and conductivity of ions. Actual conductivity is conductivity due to electrons plus conductivity due to ions, and what is conductivity due to ions similar expressions will hold just replace in conductivity expression. This is the same thing as conductivity due to electrons, but you have to make a change  $m$  has to go with the mass of the ion  $\omega_c$  has to go with minus  $\omega_c$  of the ions because charge of ions is opposite to the charge of the electron, and  $\nu$  has to go as collision frequency of ions.

With these changes these expressions will hold for  $\sigma_i$  add component to component for instance  $\sigma_{xx}$  means  $\sigma_e$  plus  $\sigma_i$  and so on. This is the total conductivity tensor of plasma. I would like to go over to discuss the simplest possible case when the electric field is a DC electric field, and we are more familiar with semiconductors. I would like to consider the case of electric field DC electric field applied to a semiconductor.

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Consider a semiconductor like this to which suppose I apply a DC electric fields like this. This is my semiconductor here, and I apply a magnetic field perpendicular to the plane of the paper. Suppose this is my x direction, this is my y direction, and I say that  $B_s$  is parallel to z axis, and I am talking about the DC conductivity tensor. Let us see, what does it tell us. In this case if I put in the expression for conductivity  $\omega$  is equal to 0, then my electric field becomes simply  $E_0$  for instance DC electric field so,  $E_0$ . Here initially I apply the electric field in the x direction. Certainly, I am not having any  $E_z$ . In this case if I write down the current density in x direction using the expression  $J$  equal to  $\sigma \cdot E$  or  $E$  is equal to  $E_0$ . Let us, write down the components  $J_x$  will be is equal to  $\sigma_{xx} E_x$  plus  $\sigma_{xy} E_y$   $\sigma_{xz}$  is 0. I want to write it and similarly,  $J_y$  would be  $\sigma_{xy} E_x$  minus  $\sigma_{xx} E_y$ , where, I have used the equality that  $\sigma_{yx}$  is equal to minus  $\sigma_{xy}$ , and  $\sigma_{yy}$  is equal to  $\sigma_{xx}$ .

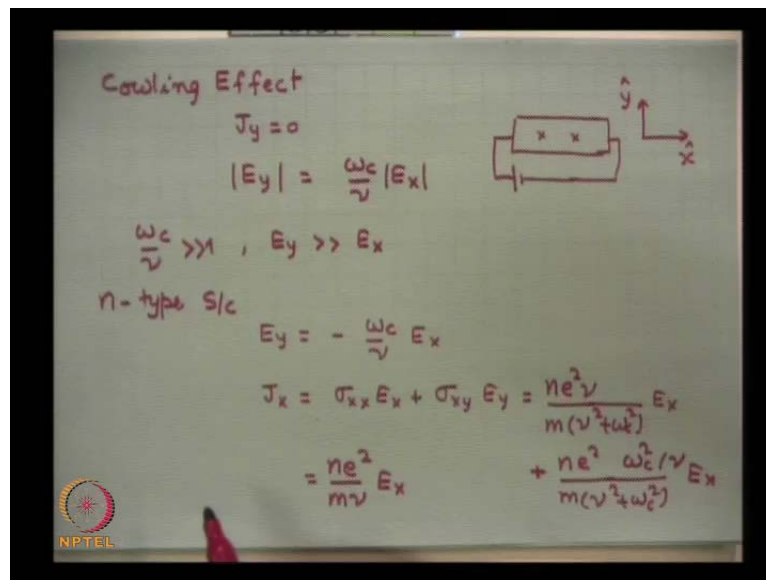
You may note one thing in here initially, you have applied the electric field in the x direction. Because the potential difference between these two have created. As a result if  $E_x$  is finite  $E_y$  is 0 initially, this term is not there, and this term is not there. But that electric field will produce not only  $J_x$  it will also produce a  $J_y$ . This gives rise to  $J_y$  which is non 0 charges move, and when current moves then obviously, we have not allowed the current the electrons are holds to move out of the semiconductor in the y direction, because this is limited by the size of the semiconductor, the charges will build up over there. And when charges build up over there they will produce an electric field. As a result  $E_y$  will be produced. This gives rise

to  $E_y$ , and a situation will arise a steady state will arise when the current stops in the  $y$  direction because electrons if they do not move out, and the charges build up over there, then the net current will vanish eventually.

If I put  $J_y$  equal to 0, in that case if I equate this to 0. I get a  $E_y$  is produced in the steady state of the value  $\sigma_{xy}$  upon  $\sigma_{xx} E_x$ , and if you look at the expressions for a entire semiconductor in which only electrons exist this ratio turns out to be simply equal to minus  $\omega_c$  upon  $\nu$  into  $E_x$ , for electron dominated semiconductor, and for holes if it is a  $p$  type semiconductor then, this will be equal to minus  $\omega_c$  of plus  $\omega_c$  of holes, because charge of holes is positive, and divide by the collision frequency of holes, and put  $E_x$  there for hole dominated semiconductor. There is an important message in here that the electric field that is produce there could be negative or positive depending on whether, it is an  $n$  type semiconductor or a  $p$  type semiconductor.

This is a very important inference that you can easily find out by noting down whether, you generate a positive potential here with respect to this phase or negative potential on the basis of that you can tell whether the semiconductor is  $n$  type or  $p$  type. This effect is called Hall Effect. The production of electric field in the transverse direction to the initial electric field is called Hall Effect. This is an important thing, it is a very simple phenomenon, but it is very profound because it can tell about the character of the semiconductor. Another important thing you may note that cowling absorbed was that if  $\omega_c$  is bigger than  $\nu$  then, this induced electric field  $E_y$  could be bigger than  $E_x$ . So, important issue is that cowling noted was let me mention. So, this was actually, this field is called Hall field  $E_y$  is equal to minus  $\omega_c$  upon  $\nu E_x$  in a  $n$  type semiconductor and similarly, in a  $p$  type this expression. This hall field is very valuable its magnitude could be bigger than  $E_x$ .

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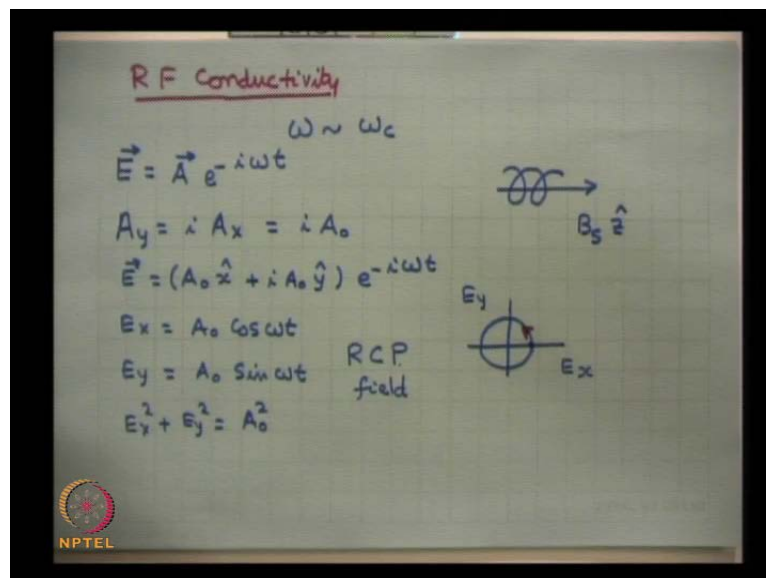
And Cowling what did he observe, and this effect that he discovered is known as Cowling Effect. He noted that, if  $J_y$  is 0 in the semiconductor I have my semiconductor here, and this is my  $x$  direction, this my  $y$  direction, and I am applying a magnetic field transverse to this plane. Then, if  $J_y$  is 0 I find  $E_y$  magnitude wise is equal to  $\omega_c / \nu$  times  $E_x$ , and this is a strong field if  $\omega_c$  is bigger than  $\nu$  then,  $E_y$  could be much bigger than  $E_x$ . Another thing that you can do suppose I am taking a  $n$  type semiconductor or simple plasma in which ion motion you can ignore, and electron motion is considered in that case, what do you will see, that  $E_y$  is equal to minus  $\omega_c / \nu$  into  $E_x$ . How about  $J_x$  then,  $J_x$  is equal to I had written  $\sigma_{xx} E_x + \sigma_{xy} E_y$ , please put the values of  $\sigma_{xx}$  which is equal to  $ne^2 \nu / m(\nu^2 + \omega_c^2)$  into  $E_x$ .

Then, this  $\sigma_{xy}$  multiplied by minus  $\omega_c / \nu$ , and if we put this it becomes is equal to plus  $ne^2 \omega_c / m(\nu^2 + \omega_c^2)$  one  $\omega_c$  is there, one  $\omega_c$  is there. It becomes  $\omega_c^2 / \nu$  into  $E_x$ , and if you add these two they give you simpler expression  $ne^2 / m \nu$  into  $E_x$ , means in a semiconductor if you apply a electric field like this then, in their steady state the current here current density will not be influenced by the magnetic field. What so, ever magnetic field you apply this result is independent why because the Cowling effect produces a vertical electric field, the cumulative effect of the field on the current is you have to write down the current density due to the  $E_x$  component of electric field, and  $E_y$  component of electric field, which is induced. When you add the two this is as if there is no effect of magnetic field. This is the important effect.



Recently professor (audio not clear) has employed this effect that  $E_y$  could be much bigger than  $E_x$ , and in the lower ionosphere at low frequencies actually, the plasma behaves more like a collisional plasma, and if wave frequency is less than collision frequency, and electrons synods frequency then, the behavior is very similar to the semiconductor, and he demonstrated that because of this cowling effect one can have a more efficient energy radiation or radio frequency radiation from an antenna ground based antenna. This is called Cowling Effect, and which is currently being used to enhance the efficiency of radiation of extremely low frequency antenna. Now, I would like to discuss the implications of cyclotron effect on conductivity RF conductivity.

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Let me, go over to the RF conductivity again. As I mentioned to you the RF conductivity has a character that when  $\omega$  is of the order of  $\omega_c$  one would accept the enhancement in electrical current. In order to appreciate this lets understand what is really, why this resonance is coming we are having in the system a DC magnetic field or a static magnetic  $B_s$  along  $z$  axis then, we had learnt that if I release an electron with some finite velocity. The electron will have tendency to gyrate over the line of force like this. The electron generates in the right handed sense that if I take a right handed screw like this, and if I rotate the right handed screw then, the direction of advancement of the screw will be in the  $z$  direction. If I point my screw along  $z$  axis, and rotate in the  $x y$  plane in a clockwise sense the screw will advance in the right hand direction.

Similar is the motion of an electron, if I put an electron somewhere here this is my z direction. The electron will generate along the line of force like this. The same way as a right handed screw moves. Now, let's me consider if I have a right circularly polarized electric field the electric field that I apply normally, I write this as  $A e^{-i\omega t}$  let me allow it will be complex its independent of time it may be dependent on space it may not be depend on space. But I am trying to understand the response of plasma in a local region where I can treat  $A$  to be uniform. Let me, find out what is the consequence of this. If I consider  $A_x A_y$  rather is equal to  $i A_x$  then  $A_x$ , I choose to be real suppose this is equal to  $i$  times  $A_0$   $A_x$  is equal to  $A_0$ . My actual electric field is x component of this will be  $A_0$  in the x direction plus  $i$  times  $A_0$  in the y direction. This is my actual electric field which means that, if I write down  $E_x$ , and take the real part of the right hand side, it will be  $A_0 \cos \omega t$ , and  $E_y$  will be  $A_0 \sin \omega t$ .

There is a beauty in plasma response to such sort of a field which is called right **E y**. y it is called circularly polarized, because if you take  $E_x^2 + E_y^2$ . It is a constant, magnitude of  $E_x$  does not  $E$  does not change with time. And secondly if I plot on the x axis I plot  $E_x$ , and on the y axis I plot  $E_y$  at time  $t$  equal to 0  $E_x$  is equal to  $A_0$ , and  $E_y$  equal to 0. If I can draw a point here this represents the location of  $E_x$  is equal to  $A_0$ , and  $E_y$  equal to 0. At a little later time when this quantity increases  $\cos$  will decrease below one, and  $\sin$  will increase, but please remember  $E_x^2 + E_y^2$  is equal to constant this is an equation of a circle. This will be like this, and the electron the electric field rotates the tip of the electric field rotate on the circle like this in this sense.

So, this is the rotation of the electric field is if I take the right handed screw rotate in the direction of rotation of the electric field. This will advance in the same direction as the z axis electric field direction, and if you examine the character of electron rotation, which is the same way. Electron rotates in the same way as the electric field rotates, if I have written the electric field like this. This is called the R C P right circularly polarized field is written like this. Now, in this particular case what is  $J$ , let me, write down  $J$ . You can write in two ways either you use the conductivity tensor which many times we do not remember, we forget. Otherwise it is simple to write down the equation of motion.

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Handwritten derivation on a grid background:

$$m \frac{\partial \vec{v}}{\partial t} = -e \vec{E} - m \vec{v} \times \vec{v} - e \vec{v} \times \vec{B}_0$$

$$(\nu - i\omega) \vec{v} + \vec{v} \times \omega_c = -\frac{e \vec{E}}{m}$$

$$(\nu - i\omega) v_x + v_y \omega_c = -\frac{e E_x}{m}$$

$$(\nu - i\omega) v_y - v_x \omega_c = -\frac{e E_y}{m}, \quad E_y = i E_x$$

$$v_y = i v_x$$

$$v_x = \frac{e E_x}{m i (\omega' - \omega_c)}, \quad v_y = \frac{e E_y}{m i (\omega' - \omega_c)}, \quad \omega' = \omega + i \nu$$

A small diagram shows a vector  $v_x$  pointing right and  $v_y$  pointing up, with a circular arrow indicating a counter-clockwise rotation.

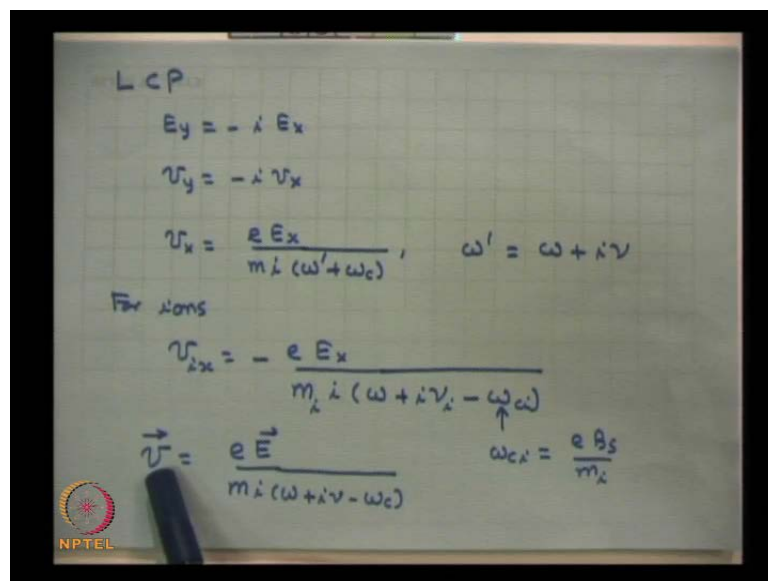
And write from there equation of motion is  $m \frac{d\vec{v}}{dt} = -e \vec{E} - m \vec{v} \times \vec{v} - e \vec{v} \times \vec{B}_0$  divide this equation by mass, and replace  $\frac{d}{dt}$  by  $-i\omega$ . Then, this equation gives you  $-i\omega \vec{v} + \vec{v} \times \omega_c = -\frac{e \vec{E}}{m}$ .  $\nu$  can be taken from here to the left. It becomes left into  $\vec{v}$  this term also can be brought back it becomes plus  $\vec{v} \times \omega_c$  is equal to minus  $\frac{e \vec{E}}{m}$ , if we write down the x component of this equation, it becomes  $\nu - i\omega v_x + v_y \omega_c = -\frac{e E_x}{m}$ , and write down the y component of this equation.

It becomes  $\nu - i\omega v_y - v_x \omega_c = -\frac{e E_y}{m}$ , but  $E_y$  is for RCP wave  $E_y = i E_x$ . This should be if I multiply this should be equal to  $i$  times this quantity which is  $\nu - i\omega$  into  $v_x$  plus  $v_y$  into  $\omega_c$ . If you simplify this equation you will find  $v_y$  is equal to  $i$  times  $v_x$  which is similar to this equation,  $v_y$  is equal to  $i v_x$  means in the velocity space if I plot  $v_x$  here, and  $v_y$  there then. What you get, simplify this equations I will plot this in a minute but you can solve this equations also. Because  $v_y$  is equal to  $i v_x$  put this in here, and you can obtain  $v_x$ . What you get is  $v_x$  is equal to from here  $\frac{e E_x}{m i (\omega' - \omega_c)}$ . Similarly, for  $v_y = \frac{e E_y}{m i (\omega' - \omega_c)}$ . This is a very simple way of writing things,  $v_x$  express belongs in terms of  $E_x$  because though there was a  $E_y$ , but  $E_y$  being equal to  $i$  times  $E_x$  i could write down both the terms in terms of  $E_x$  alone, and the simple result is that the denominator is very similar to the case when, there is no magnetic field, but frequency is replaced by  $\omega - \omega_c$ .

Omega c is the frequency with which the electron would like to gyrate if there is no electric field. The electron gyrates with the cyclotron frequency about the magnetic field. This is the modification caused by the magnetic field, and similarly, in v y if you take the real part of this expression, and plot your electron will start from here, and will rotate on the circle like this. The same way as the magnetic as the electric field RF electric field that you have applied rotates. The rotation of the electron is in the same sense as the electric field both are R C P right circularly polarized, and the response appears as if the effective frequency of the wave has been omega minus omega c. Omega is the actual frequency of the radio frequency field, and omega c is the electron cyclotron frequency.

This when collision frequency is ignored actually, let me put this omega prime here, where omega prime is equal to omega plus i nu. I forget to write this omega prime is actually this omega prime because collision frequency is finite. This is an important thing.

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In case, I had chosen the left circularly polarized wave L C P, where E y is equal to minus i times E x. We will obtain v y is equal to minus i v x, and the response if you calculate would be v x is equal to e E x upon m i omega prime plus omega c, where omega prime I am writing as omega plus i nu. There is no resonance here. When the wave is rotating in the anti clock wise sense, when viewed or when the electric field rotates about the line of force in the anti clock wise sense. In that case, there is no resonance here. However, for the ions the velocity of ion in the x direction would be charge of the ion is opposite to that of electron. I will write

down minus  $e E_x$  upon  $m$  of the ion into  $i$ , this becomes  $\omega + i\nu$  of the ion minus  $\omega_c$  of the ion.

This is the magnitude of ion cyclotron frequency a negative sign comes because the charge of the ion is opposite to the rate of the electron. This is  $\omega_c$   $i$  is denoted as  $e v_s$  upon  $m_i$  ion mass. If a quantity much smaller in magnitude then, the electron cyclotron frequency, but it has a resonance character here, because the ions being positively charged they rotate in the anti clock wise sense about the lines of force, and hence you get a resonance for L C P polarization for the ions, and for the R C P for the electrons. This is a basic difference.

Now, one can employ because if the electrons undergo a drift motion in the presence of an electric field they give rise to they observe energy from the electric force, and the electric field then, causes heating of the particles, and we will just look in to the possibility how could you efficiently heat the electrons, and ions by a wave. A circularly polarized wave whether, left circularly polarized or R C P right circularly polarized has the advantage that the velocity drift velocity can be written as  $e$  times  $E$  for the electrons upon  $m$  times  $i\omega + i\nu$  minus  $\omega_c$ , as if the response is in a isotropic medium, but its implied in only in special case when the electric field is circularly polarized, right circularly polarized  $v$  can be written in terms of  $e$  like a simple expression. That is a big advantage.

There is a phase difference between  $v$ , and  $e$  that is a separate matter, but the direction is concerned this is R C P right circularly polarized. This also right circularly polarized, and that is a beauty in here otherwise you have write this in terms of a tensor.

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Heating

RCP

$$\vec{E} = A_0 (\hat{x} + i\hat{y}) e^{-i\omega t}$$

$$\vec{J} = \sigma \cdot \vec{E}$$

$$\vec{v} = \frac{e \vec{E}}{m i (\omega + i\nu - \omega_c)}$$

$$H = -e \vec{E} \cdot \vec{v}$$

$$\langle H \rangle = -\frac{e}{2} \text{Re}[\vec{v} \cdot \vec{E}^*] = \frac{e^2 \vec{E} \cdot \vec{E}^* \nu}{2m [(\omega - \omega_c)^2 + \nu^2]}$$

$$= \frac{e^2 A_0^2 \nu}{m [(\omega - \omega_c)^2 + \nu^2]}$$

NPTEL

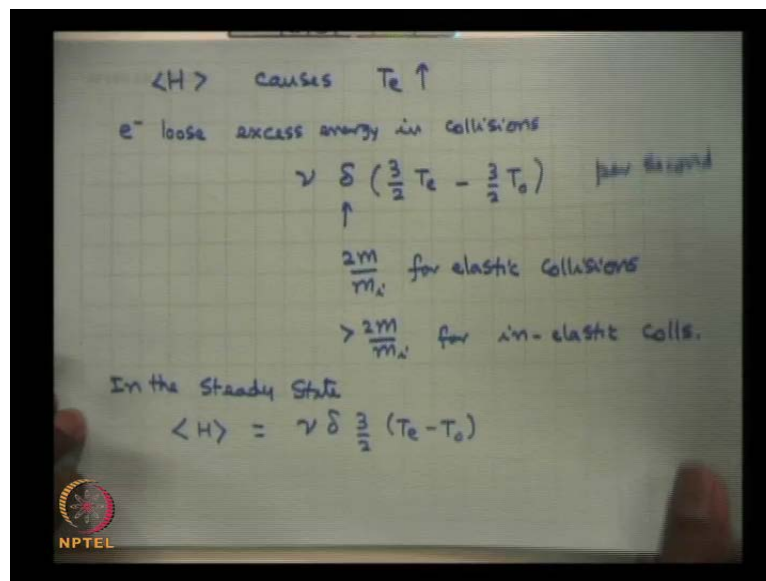
Now, I would like to go over to discuss the heating of electrons in the presence of magnetic field. I will be considering plasma in a magnetic field  $B_s z$ , and electric field may be R C P. For instance, I just consider R C P because I am interested in heating the electrons. Let me, apply a right circularly polarized wave with electric field  $E$  is equal to  $A_0 x$  plus  $i y$  minus  $i \omega t$ . This produces a current density  $J$  is equal to  $\sigma \cdot E$  that we already seen or in terms of velocity if I want to write down; this causes a velocity to gives a velocity electrons which is equal to  $e E$  upon  $m i$  outside  $\omega$  plus  $i \nu$  minus  $i \omega c$ . This is the drift velocity this electric field will produce. Heating rate means the work done by the electric field per second on an electron the force on the electron due to the electric field is minus  $e E$  multiply this by the distance traveled by the electro in one second which is equal to velocity. This is the energy given to each electron by the electric field per second or power absorption by the electron from the RF field.

I am not interested in finding out the total value of  $H$ . I want only time average value. What is the net power dissipation per second time average, Time average if I have to calculate then, I should take the please always remember here, I imply real part of  $e$  to be multiplied with the real part of  $v$  this is gives you minus  $e$  by two real part of you can write down this  $v \cdot e^*$ . The other term with  $v$  into  $E$  as I discuss earlier will vanish when you take time average. But this value in here, and you will get this expression  $e^2$  upon twice  $m$ . You will get a quantity here  $\omega$  minus  $\omega c$  whole square plus  $\nu$  square multiplied by  $e \cdot E^*$  into  $\nu$  just like this, and simply you will get this expression. And  $e \cdot e^*$  if you evaluate

from here it will give you simply two  $A_0$  square so, it becomes is equal to  $e$  square  $A_0$  square upon  $m \omega$  minus  $\omega_c$  whole square plus  $\nu$  square into  $\nu$ .  $\nu$  is the collision frequency in plasmas collision frequency usually is very low.

When, one wants to heat the plasma very efficiently obviously, you would like to have  $H$  as large as possible which will be largest when  $\omega$  equal to  $\omega_c$  to tune your frequency of the radio frequency, which you are applying somewhere here in this plane its rotating like this, in the close wise sense then, choose the frequency close to  $\omega_c$ . So that,  $H$  is maximum. However, even if you have chosen  $\omega$  slightly half  $\omega_c$  because  $\nu$  usually is very a small **or** eventually smaller than  $\omega_c$ . If suppose you are within one percent of  $\omega_c$  still this term could be bigger than  $\nu$  square, and you can ignore  $\nu$  square, and still you will have a significantly large  $H$ . The issue is if to heat plasma like this, how long you can heat the plasma. When the heating occurs the electrons temperature will goes up.

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So,  $H$  causes enhancement in electron temperature, and the average electron energy becomes larger than the average energy of the background particles. So, the enhancement the energy the electrons loose excess energy in collisions. Average kinetic energy of an electron at temperature  $T_e$  is  $\frac{3}{2} T_e$ , where as if this were not heated the temperature would have been the average energy would have been  $T_0$ . So, this I call as the excess energy. In one collision electron normally, loose a fraction  $\delta$  fraction of energy; excess energy, and if

there are  $\nu$  collisions per second then,  $\nu$  times  $\Delta$  into this quantity is the energy loss per second.

Where  $\Delta$  is  $2 m$  upon  $m_i$  the ratio of electron mass to ion mass for elastic collisions, and its bigger than this for inelastic collisions greater than  $2 m$  upon  $m_i$  for inelastic collision. Inelastic collision is the one in which part of the electron energy goes into excitation of electrons bound electrons of the atoms from one orbit to another orbit or in ionizing atoms that is called inelastic collisions. Their number is smaller than the inelastic collisions, but average you have to take out so, usually this term is around  $10$  to minus  $3$  to  $10$  to minus  $4$   $\Delta$  means whatever excess energy electrons have gained they do not lose all the excess energy one collisions to ions or neutral atoms. They lose very small fraction of energy  $\Delta$  fraction of energy in collision, as a consequence in each collision momentum gets randomized, but only very little energy is transferred. Electrons get heated, and as a result what happens, in the quasi in the steady state the net time average energy that the electron gains from the RF field should be equated due to this loss.

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The whiteboard contains the following handwritten content:

$$T_e = T_0 \left( 1 + \frac{e^2 A_0^2}{m \frac{3}{2} \delta (\nu^2 + (\omega_c - \omega)^2)} T_0 \right)$$

$$|\omega - \omega_c| > \nu$$

$$T_e = T_0 \left( 1 + \frac{A_0^2}{E_p^2 (\omega_c - \omega)^2 / \omega^2} \right)$$

$$E_p = m \frac{3}{2} \delta \omega^2 T_0 / e^2$$

Next to the equations is a graph of  $T_e$  versus  $\omega$ . The graph shows a resonance curve with a peak at  $\omega_c$ .

In the bottom left corner, there is a logo for NPTEL (National Programme on Technology Enhanced Learning).

Put this is equal to  $\nu \Delta$  three by  $2 T_e$  minus  $T_0$ , and you can get the expression for rise in electron temperature, and that turns out to be  $T$  turns out to be equal to  $T_0$  into  $1$  plus  $e$  square  $A_0$  square  $m$   $3$  by  $2 \delta$ . Well, you get here  $\nu$  square plus  $\omega_c$  minus  $\omega$  whole square into  $T_0$ . This shows resonance at  $\omega$  equal to  $\omega_c$ . However,  $\nu$  in general depends on temperature of the electrons. This equation can be solved depending on

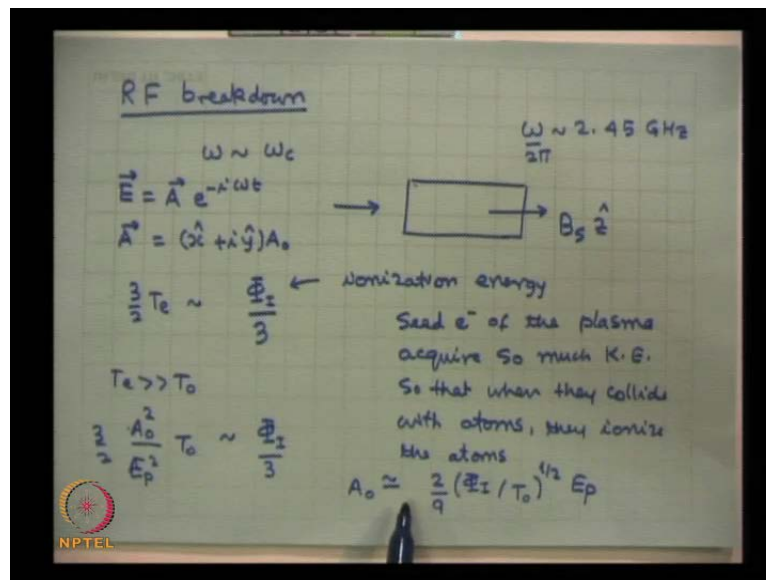


the nature of collisions. If I choose for the sake of simplicity suppose  $\omega - \omega_c$  magnitude wise though it is very small but bigger than  $\nu$ .

I am choosing  $\omega$  close to  $\omega_c$ , but not exactly  $\omega_c$ . So that, this difference is still bigger than  $\nu$  then, electron temperature is equal to  $T_0 (1 + A_0^2)$ , and this I will call it as  $E_p^2$  multiplied by a factor  $\omega_c - \omega$  square upon  $\omega^2$ , where I am defining  $E_p$  as that field is given by  $m \frac{3}{2} \frac{\Delta \omega^2 T_0}{e^2}$  upon  $e^2$ . This is the plasma field if I choose  $\omega_c$  equal to 0 then, this is equal to 1 but cyclotron resonance effect, I have explicitly retained over here. I plot  $T$  as a function of  $\omega$  then at around  $\omega$  equal to  $\omega_c$ . You see, a large enhancement in temperature, and that can give rise to very strong heating at modest values of radio frequency field amplitude, and this is a very important way of heating plasma and producing plasma.

I think, before I close my discussion of RF conductivity of magnetized plasma. Let me, mention if two things about plasma production by RF radio frequency production of plasmas or RF breakdown of plasmas a comment on this.

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Normally, a high frequency field of frequency  $\omega$  of the order of  $\omega_c$ , you can apply to plasma in the form of a wave. So, you have plasma, you have a gas chamber somewhere, and you launch a wave there. And suppose the gas chamber has a magnetic field a static magnetic field somewhere there. You are launching a wave into a gas chamber, and you are expecting the plasma to be produced. So, what happens? If there are a few seed electrons in

the gas, then those electrons in the presence of the RF, which I am choosing to be of the order of  $E$  is equal to  $A \exp(-i \omega t)$ , and  $A$  circularly polarized. This is  $x + iy$  into  $A_0$ , suppose this is the kind of electric field, I have of the wave, then this wave will heat the electrons to a temperature that I have given to you.

And whenever, average kinetic energy becomes typically of the order of ionization potential  $\phi$ , I called ionization potential, ionization energy rather the energy required to ionize an atom. Not only is this because the distribution functions of electrons maxwellian. So, average energy may be less than this, but there may be some electrons in the maxwellian tail. People found that whenever, this is of the order of one third or one fourth that is good enough temperature for production of plasma for ionization.

What you want that is the seed electrons of the plasma, acquire so much kinetic energy. So that, when they collide with atoms, they ionize the atoms. It is typical, I am just giving you an order of magnitude estimate that whenever, the average temperature of electrons is about given by this condition. Then, the RF will cause break down of the plasma, and if I put the expression  $T_e$ , I get the breakdown field. What do I get, if I assume that  $T_e$  is much bigger than  $T_0$ . The background temperature of the gas in that case,  $T_e$  will be if I put this condition in the previous expression; I will get  $A_0^2$  should be upon  $E_p^2$  multiply by  $T_0$  into  $3/2$ . This should be of the order of  $\phi I$  by  $3$ .

You get the breakdown field here.  $A_0$  should be of the order of put this here  $2/9 \phi I$  upon  $T_0$  to the power half into  $E_p$ . I think, this is a very interesting expression; very simple expression that tells you that if your radio frequency field had an amplitude bigger than this. It will cause brisk ionization or breakdown of the gas, and plasma of high density can be formed. In fact, when you choose  $\omega$  around  $\omega_c$ . Normally, people employ microwaves or frequencies typically 2.45 gigahertz. They are choosing  $\omega$  of the order of 2.45 gigahertz obviously, I am talking about  $\omega$  upon  $2\pi$  equal to so much and corresponding magnetic field you can calculate this turns out to be in a few hundred gas in that case,  $\omega$  equal to  $\omega_c$ ; this condition can be achieved, and one can have a plasma production of very strongly ionized plasma one can produce.

Plasma of  $\omega_p$  bigger than  $\omega_{plasma}$  frequency could be even bigger than the frequency of the microwave that we employ. I think this is a very important technique, well

when we discuss the plasma production techniques, we shall elaborate on this mechanism at that time.