Plasma Physics

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Module No. # 01
Lecture No. # 42
Current Trends and Epilogue

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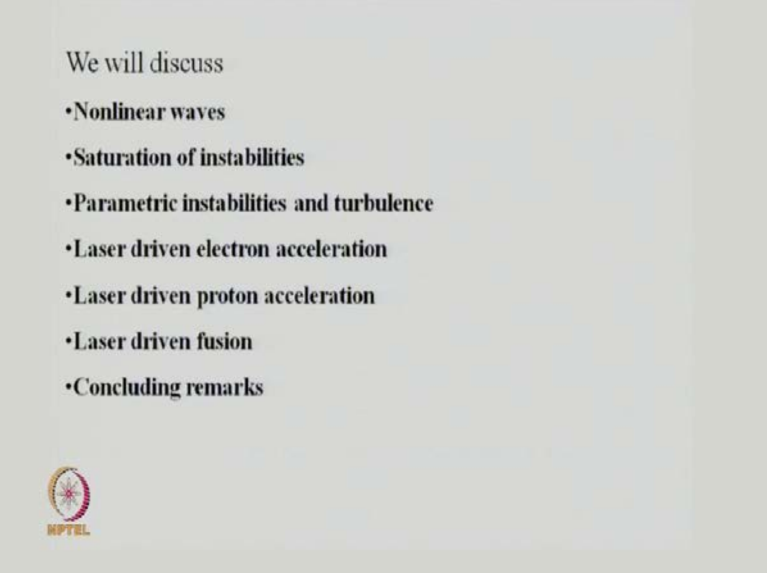

Friends, we have reached the last stage of this course. Today is the last lecture and I plan to give you current trends in plasma physics and with some concluding remark. We I will discuss non-linear waves, saturation of instabilities, parametric instabilities and turbulence, laser driven electron acceleration, laser driven proton acceleration, laser driven fusion and then I will make some concluding remarks.

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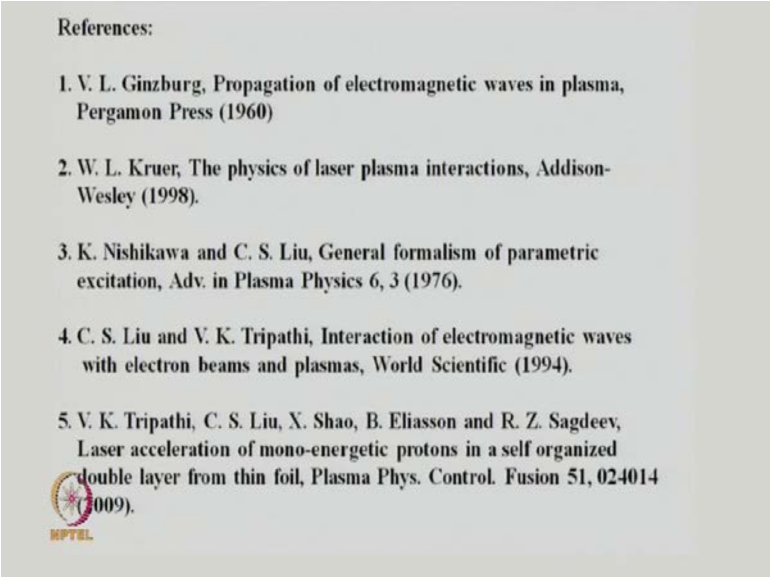
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3. K. Nishikawa and C. S. Liu, General formalism of parametric excitation, Adv. in Plasma Physics 6, 3 (1976).

4. C. S. Liu and V. K. Tripathi, Interaction of electromagnetic waves with electron beams and plasmas, World Scientific (1994).

5. V. K. Tripathi, C. S. Liu, X. Shao, B. Eliasson and R. Z. Sagdeev, Laser acceleration of mono-energetic protons in a self organized double layer from thin foil, Plasma Phys. Control. Fusion 51, 024014 (2009).

The references would be, a classic book by Ginzburg, another 1 by Kruer, a very famous review article by Nishikawa and Liu on parametric instabilities then our book on interaction of electromagnetic waves with electron beams and plasmas and finally, a recent paper that we published on laser driven ion acceleration.

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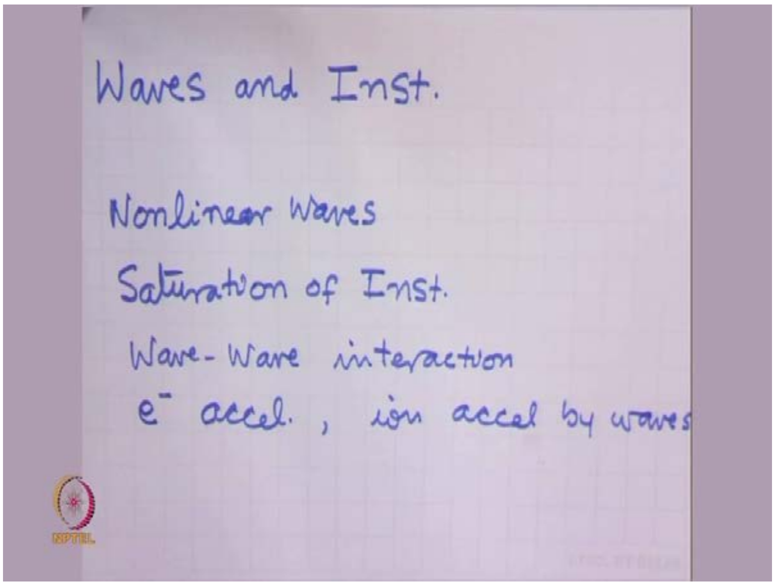

Well, you might have recognized that, in this course our focus has been on waves and instabilities primarily. Well, we essentially discussed electromagnetic wave propagation in plasmas, electrostatic waves in plasmas, surface waves in plasmas and then their applications. And we also talked about 2 steam instability, Relay-Taylor instability, viable instability and so on.

So, both kinds of waves we have studied; the waves that are dumped by collisions or by a landau damping and also the waves that are driven unstable by free sources of energy, sources of free energy. These are valid only in the linear in stage of the instability or when the amplitude of the wave is low. Well, we also discussed in this course some important applications especially related to magnetic confinement of plasmas. Namely the mirror machine and tokamak. And we spend some time on classical transport also.

So, the natural coronary would be that the next stage of this study should be essentially non-linear waves. The waves with that have acquired large amplitude in what way their phase velocity is modified, group velocity is modified in the plasma or can they be self focused or can they travel without distortion via dispersant. So, can be compensate for self distortion of waves by some non-linear effects and such waves are known as then solutions Solitons. So, that is a natural corollary or natural extension of this course to study non-linear waves. And then also, we would like to examine rather 1 should really examine in detail the saturation of instabilities. How the instability saturate?

And in this process of high amplitude wave interaction with plasmas people discovered that, there is a very rich area of plasma physics that talks about wave-wave interaction, non-linear interaction between waves. Like the phenomena of harmonic generation, phenomena of some frequency generation, phenomena of low frequency generation and also half harmonic generation or parametric excitation of 2 waves by a single wave of large amplitude. This is called wave-wave interaction.

So, lot of work on this process, on this, in this area has been done in last three four decades. And then there is a possibility of wave of large amplitude accelerating charge particles to very high energies. So, that gives rise to the phenomenon of electron acceleration and ion acceleration by waves. This is very rich area of physics with very important applications in even in relativity general theory of relativity and understanding the evolution of universe in understanding the radiations coming from pulse (( )).

So, very rich physics is revealed by these processes. And then, there are some very major application that emerged out of high power wave plasma interaction and 1 of the major applications is laser driven fusion. We already discussed the magnetically confined plasmas employed in tokamak and mirror machine for thermonuclear fusion purposes. But, there is another scheme that could be realized by high power lasers is the scheme of laser driven fusion.

So, today in my concluding lecture I would like to draw some, your attention to some of these processes. I will discuss some physics of these processes.

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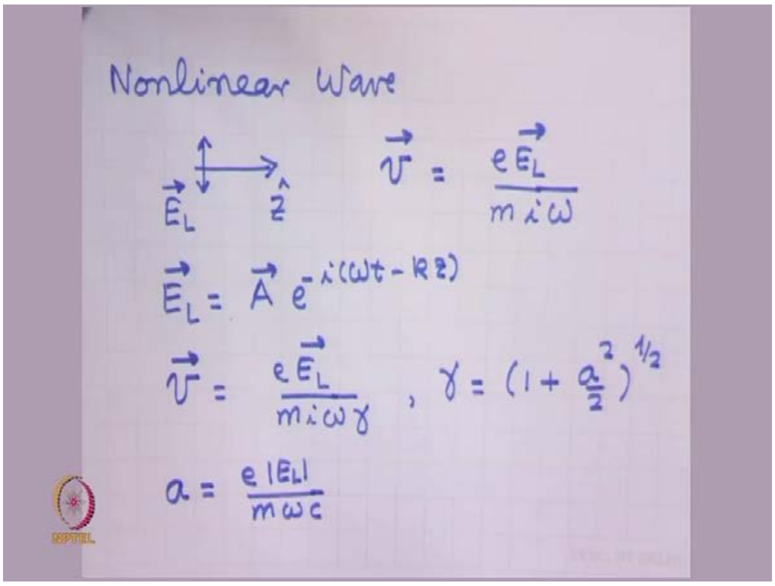

Let me begin with the fundamental problem of non-linear wave. Let me begin with the propagation of a high power laser in a plasma. The laser field is like this. This is a direction of propagation of the laser say, z direction. This is the electric field of the laser or it could be an electromagnetic wave of radio frequency wave also. Consider the propagation of a wave in a plasma. When the wave acquires large amplitude, well, it will give a large oscillatory velocity to electrons which we have already learnt that this is equal to e E L upon m into I omega where, I am assuming that my laser field goes as some amplitude a exponential minus I omega t minus k z.

So, this is the linear response of electrons. Then we linearize the equation of motion. Then we get this sort of velocity. Now, what happens? If this velocity approaches c, the velocity of light in free space then the electron mass will get modified. And when we solve the equation little more carefully, we find that the oscillatory velocity is modified to v is equal to e E L upon m I omega multiplied whether relativistic gamma factor and this gamma factor turns out to be equal to 1 plus a square by 2 to the power half if the laser is linearly polarized.

Where a is called the normalized velocity of the electron magnitude which is e E L upon m omega. Same quantity, modulus of this normalized to c. This quantity could be bigger than 1 or comparable to 1 and hence the effective mass of the electron is modified from m by a factor gamma. And gamma if it is larger than the refractive index of the plasma will be modified because that depends on the mass of the free electrons.

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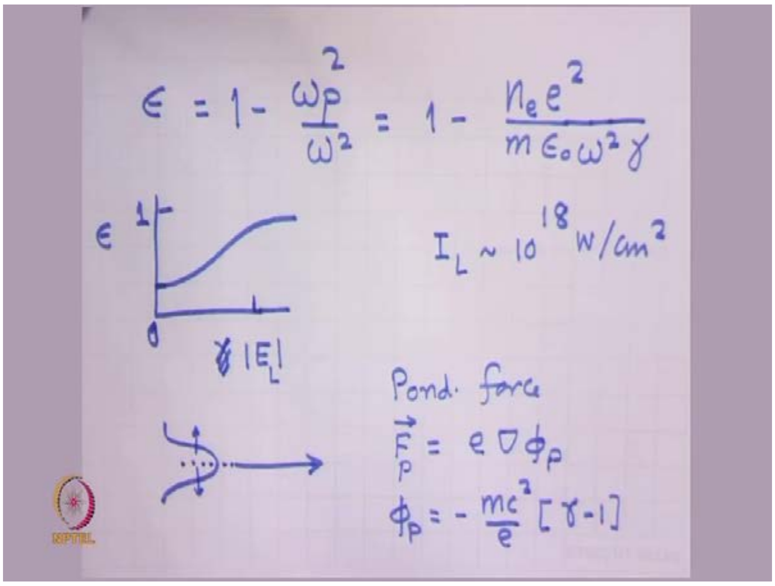

So, if I in view of this, if I write down the effective plasma permittivity then, plasma permittivity at frequency omega is given as 1 minus omega p square by omega square. But, now let me write down omega p explicitly which is 1 minus electron density into e square upon m epsilon 0. This is the value of omega p square into omega square. Here e is the magnitude of electron charge, m is the rest mass of the mass of the electron. But, now it should be modified because of the gamma factor Lorentz factor. So, put a gamma here.

Now, this gamma depends on the electric field amplitude and hence this permittivity becomes a function of electric field. And if you plot e versus gamma epsilon versus gamma then, if gamma is close to 1, it starts from 1, rather let me plot this as a function of laser field amplitude. Laser field amplitude if I plot, it starts from very low amplitude and then go on increasing. The value, this will start from some small value because this is one. So, this, a value of epsilon, but, has as gamma increases or laser amplitude increases, this saturates to this value. This is unity here.

So, refractive index of a plasma is an increasing function of electric field and the kind of fields that you plot here if you calculate the intensity corresponding to this fields then, this occurs at laser intensity of the order of ten to the power eighteen watt per centimeter square. So, at high intensities just by the relativistic mass effect the plasma permittivity becomes a function of intensity. Another important thing occurs. Suppose I am launching a beam, laser beam of this sort of intensity, Gaussian laser beam in a plasma then the laser amplitude is large on the axis of the beam and it decreases perpendicular, in the perpendicular direction.

So, the radiation pressure is more here, less outside and their plasma experiences a force, electrons experience a force in the relatively outward direction and there is a density depletion. So, the density that appears here can also be modified by this radiation pressure force which is also called ponder motive force. And this force if you calculate for the laser, it turns out to be F P, time average ponder motive force or radiation pressure force is equal to e grad phi p. And phi p is called the ponder motive force. And it turns out to be phi p is equal to minus m c square by e into gamma minus 1. Gamma is the same expression that I have written earlier.

So, the laser exerts a radial transverse force on the electrons. And the electrons move out. As they move out, they carry the ions also with them if you allow them enough time and this is we call as the m bipolar diffusion of plasma. So, there is a reduction in density that also brings further enhancement in permittivity. So, permittivity of the plasma is a function of the laser field amplitude by 2 causes; 1 is the increase in electron effective mass and second is the radiation force induced reduction electron density.

Now, let us see the consequence of this. It is a fascinating consequence this result in the process of self focusing. Let me explain the physics.

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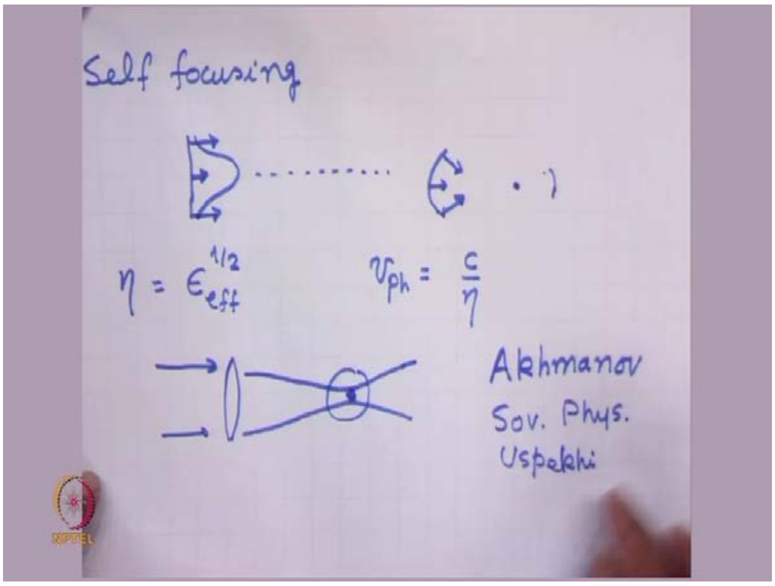

Now, consider a plasma in which a laser beam is travelling. This is the wave front of the laser beam and this is the intensity distribution. Normally, people choose take it to be Gaussian laser beam with maximum intensity on the axis and a monotonically decreasing function of transverse distance r.

Now, this is called the axial region. We had permittivity of the plasma has become larger and the refractive index is defined as effective plasma permittivity to the power half. Actually I should be use a symbol epsilon effective rather than epsilon. But, it is implied even if I do not use this subscript effective that, this is their effective relative plasma permittivity. And a refractive index of the medium is relative to this quantity. So, the plasma, when the laser travels in a plasma, the portion of the wave front which is near the accessible travel with a lower phase velocity because phase velocity of the wave is equal to c upon refractive index.

And the marginal waves will travel faster with larger phase velocity. As a result this wave front will acquire a curvature because this travels less distance and this travel more distance. So, they will acquire a curvature and then the waves will travel converge further in a avalanche manner. And this will reduce to a very small size. This is called the phenomenon of self focusing. However when the laser converges; two things are happen, the diffraction divergence increases because, when the size of the wave front is comparable to wave length then strong diffraction divergence occurs.

And secondly, as the intensity increases, the permittivity saturates. So, one has to balance these two things. And the laser, after acquiring some minimum spot is starts diverging again. But, this is a non-linear phenomenon, non-linear stage. Beyond this point actually laser acquires some sort of an average spot size and continues to travel without convergence or divergence. The important issue is that, in a non-linear medium, in plasma at high intensity, the laser has a tendency to self converge. And this phenomenon is very important because, all non-linear effects depend on the laser intensity if self focusing can cause enhancement of intensity of radiation then, the non-linear, the yield of non-linear phenomena would be very large.

One thing more, normally we use lenses to focus radiation. If I shine laser light on a lens, it will converge to a point. And the typical size of this spot is of the order of a laser wave length. But, then the rays comes come over here and then they diverge like this. So, the region where the intensity is large is very small.

In plasma the region where this laser is focusing, then continue to be like this for a very long distance may be tense ten's of wave lengths. So, the spot size could be quite large. And hence, large intensity region of a self focused laser beam could be much longer than one would realize by using a lens. So, it is a very important, non-linear phenomenon. And Akhmanov and coworkers did pine lingering pioneering work on this. They published a review article, a soviet physics Uspekhi in late 60's. And I think, the references that I have mentioned, our book also has as utilized their paraxial formalism to study this phenomenon.

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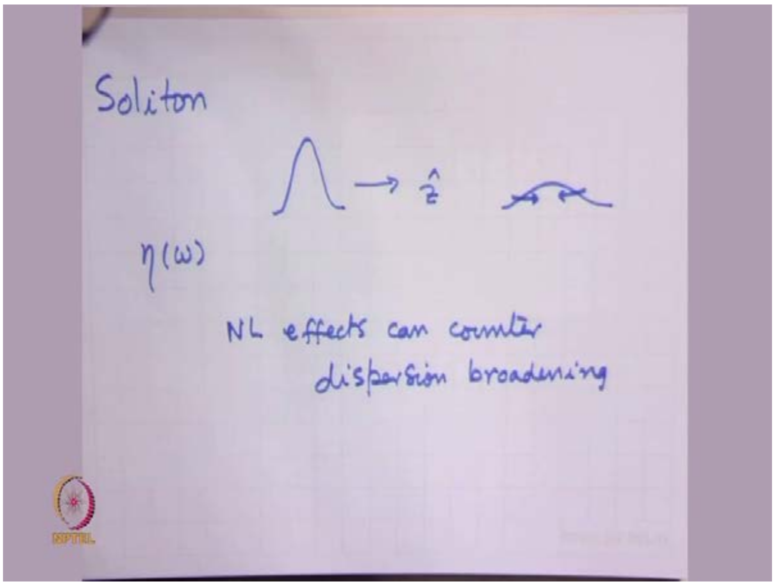

Effect in 1 dimension due to this non-linear permittivity is called solution Soliton formation. You know if I send a pulse which is limited in time, suppose, this is my pulse travelling in a medium in z direction. But, it is localized in its space and time. A laser may be having uniform intensity in the x y plane. But, suppose the pulse is of finite duration and limited in its space and time is travelling like this. Then this pulse is equivalent to many frequencies. A pulse of finite time duration is equivalent to a super position of waves of many frequencies. And a plasma being a dispersive medium, different frequency components travel, but, different velocities because omega refractive index depends on the frequency.

So, a wave of finite time domain, finite time finite time pulse direction would have a self broadening as it travels because different frequency components travel with different velocities, different phase and group velocities and consequently this will broaden. Now, the when the pulse are broadens the intensity falls down. So, it has always it is always desirable to conquer this effect. We have just seen that the nonlinearity has a tendency to converge energy in the regions of higher intensity.

In this case also, non-linear effects will have a tendency to converge energy from here to here and bring it from here to here and hence to retain the original shape of the pulse. So, if you choose laser intensity of proper value then the laser pulse may you can avoid the pulse distortion, you may counter completely the diffraction the dispersant broadening.

So, non-linear effects can counter dispersant broadening. The mathematical analysis is very similar to self focusing and then the pulse will go will propagate without distortion. In such a pulse shape, which propagates loud distortion is called solution. So, this is another interesting field. Lot of work in this field as been done, currently optical fibers are also using solutions for communication. Well, another important effect is non-linear mixing between waves or parametric instabilities. Let me just mention a few words about what is a parametric instability.

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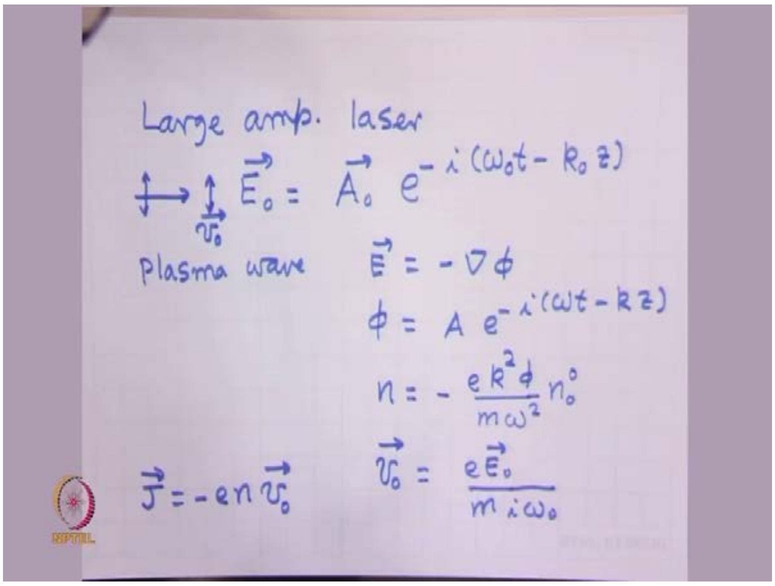

Suppose I have a large amplitude e m wave or laser, you may take a laser wave or plasma wave or any large amplitude wave. Let me denote this field by e 0 amplitude a 0 exponential minus I omega 0 t minus k 0 z. Subscript 0 I may am using for the wave of large amplitude. Suppose a wave is traveling in a plasma like this, what can happen that, this wave may coupled with a low amplitude wave that may pre emphasis in the system and that low amplitude wave may be a suppose, a plasma wave.

Suppose, there exists a plasma wave in the system and the electric field of the plasma wave is minus grad phi. And phi I write down as a exponential minus I omega t minus k z for instance. Suppose there exists a wave of frequency omega in wave number k in the system and my laser is travelling. What would happen? It is possible that these two can couple. How would they couple? This plasma wave whenever travels, it will give rise to density perturbation and if you use cold plasma theory that we did earlier it turns out to be equal to minus e k square phi upon m omega square this is the density perturbation caused on electrons where n 0 is the equilibrium n 0. 0 is the equilibrium electron density of the plasma.

So, this density perturbation is seen by this electromagnetic wave. Now what is if this electromagnetic wave will do? It will produce a oscillatory velocity to electrons. So, if the wave is going like this, the electro field is like this and it will cause an oscillatory velocity of few electrons called v zero. So, v 0 is produced by this field of the pump wave which is equal to e E 0 upon m I omega 0. What is the consequence? If you calculate the current which is the produce of density and velocity and charge , J simply charge of the electron into density of the electrons into velocity. What would they do? This n is oscillating with the frequency omega, electron velocity is oscillating with frequency omega zero. So, the product will give rise to quantities that will vary with frequency omega 0 plus omega and omega 0 minus omega. So, a system will produce a non-linear current J at sum and a difference frequency. But, built in automatically and they will produce radiation it was frequencies.

Obviously, radiation will be of large amplitude if the current that you produce satisfies a proper frequency wave number relationship. Otherwise, this will be non this will be a non resonant to driven. So, it can be seen, it can be proved that, the wave of frequency omega 0 minus omega z can be resonantly driven and omega 0 plus omega frequency wave is not resonantly driven. So, we ignore it.

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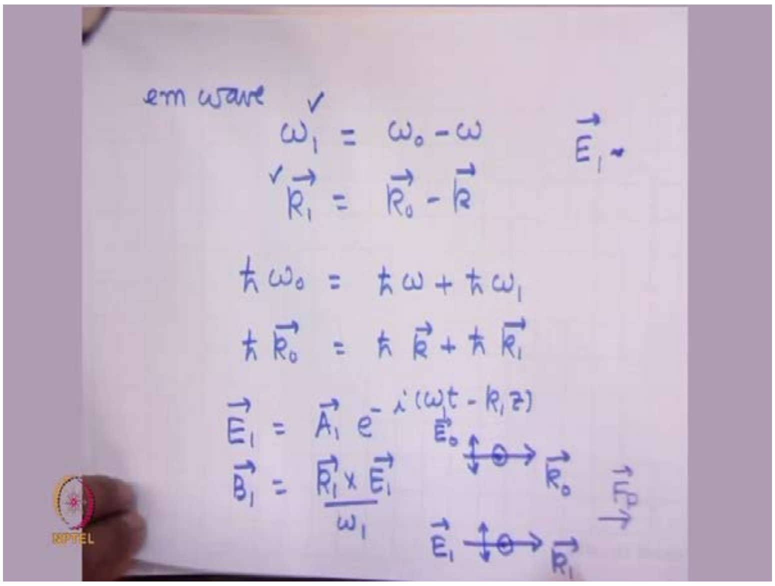

So, now look at the process. A large amplitude laser couples with a plasma wave and gives rise to an e m wave of frequency omega 1 for instance wave number omega 0 minus omega sorry frequency and wave number k 1 which is equal to k 0 minus k.

You can visualize this you can write this in different manner, h cross omega 0 bring this omega and omega 1 together on 1 side and multiplied by the Planck's constant upon 2 pi. So, this is h cross omega plus h cross omega 1. This equation can be written as h cross k 0 is equal to h cross k plus h cross k one.

What is happening here? You can visualize this process of coupling, as if a pump wave of photon h cross omega 0 energy is decaying into 2 photons one of energy h cross omega called a Plasmon and another photon of energy h cross omega 1. And similarly, the momentum of the photon or the pump wave h cross k 0 is a splitting into a 2 photon in momentum. This is the momentum given into the momentum of the Plasmon and this is the momentum of the photon which is just scattered or generated quantum. Mechanically, this is the energy and momentum conservation equation otherwise they are also known as phase matching conditions. Obviously, each omega in decade must satisfy a linear dispersant relation for the wave that we are talking about. Omega 1 k 1 will satisfy dispersant relation for the electromagnetic wave, omega k must satisfy the dispersant relation for a plasma wave, omega 0 k 0 must satisfies the dispersant relation for the pump electromagnetic wave.

Now, the question arises what will happen to the plasma wave, waves? This omega 1 frequency and k 1 wave number, wave is generated then the electric field of this wave will be there in the system, which will, let me write down how much this would be. E 1 will be varying now with amplitude a 1 exponential minus I omega 1 t minus k 1 z. What is it will do? This wave will also be magnetic field from Maxwell's equations, you can show that this is equal to k 1 cross e 1 upon omega one.

Similarly, the pump wave will also have a magnetic field. So, now, look at the situation. You are having a pump wave of wave number k 0. You are also having a decay wave or a scattered wave of wave number k 1. The electric field of this wave in this direction, this is my e 0 and this is my e 1. And the magnetic fields of this wave will be perpendicular to this plane. magnetic field of this wave will be perpendicular to this plane. So, what will happen?

The electrons which were oscillating in the direction of e 0 they will see the magnetic field of omega 1 wave and they will experience a Lorentz force also known as ponder motive force. And that will be in this direction called F P and how much this f e is? Let me just write down.

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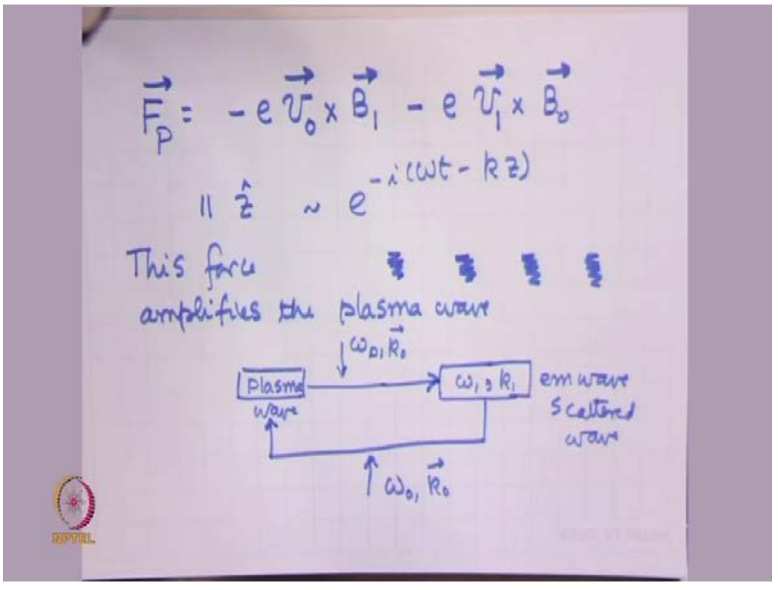

This ponder motive force is the charge of the electron into velocity of the electron due to the one wave cross magnetic field of the second wave, e m wave. And similarly, you can multiply this v due to the scattered wave and magnetic field due to the first wave. These are were the terms which that were ignored in the process of linearization when we solve the response of electrons to the to the e m wave.

But, now if you retain these neglected terms they are called the non-linear force or ponder motive force and they give rise to the direction of these this force is parallel to z axis. Because this is in the x direction, this is the y direction. So, x close y is z. This is a longitudinal force. The force is in the z direction and has a variation also in z direction phase variation is in this direction. That is the beauty. So, what will this do? This force will cause compression, rarefaction, compression of electrons, rarefaction, compression of electrons. And this will essentially reinforce the original plasma wave or amplify the plasma wave.

So, this force amplifies the initial plasma wave. So, now, look at the beautiful process. There was a plasma wave and you brought in e m bump of frequency omega 0 k 0 that produces a current and that current then produced an omega 1 frequency and k 1 wave number e m wave.

So, plasma wave generated an electromagnetic wave in the presence of a pump wave. And this again interacted with the pump wave omega 0 k 0 waves and generated a plasma wave. So, this is the feedback process that, plasma wave of larger amplitude will produce a e m wave of even larger amplitude. And this e m wave, called scattered wave will again beat with the pump wave and generate of plasma of even larger amplitude. So, by this feedback mechanism both these waves grow in amplitude, the plasma wave and the scattered wave at the expense of pump energy and this process is known as stimulated Raman scattering.

The coupling could also take place with the ion acoustic wave. This plasma wave if we’re not there suppose there is a sound wave even then this process is possible. So, this is a fundamental process in plasmas where a large amplitude wave gives rise to generation of two lower frequency waves in systems with electron beams even a more fascinating process takes place.

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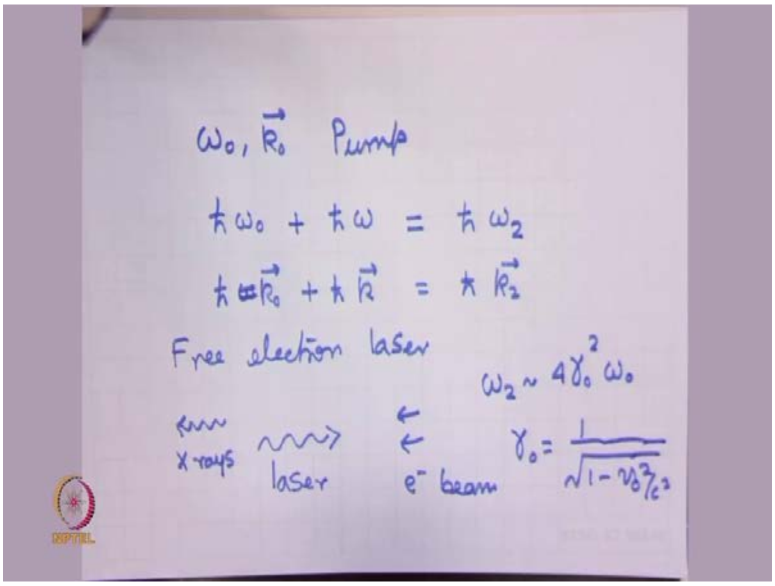

What can happen? That the omega 0 k 0 frequency wave pump, a large amplitude wave is called pump, can give a quantum of energy h cross omega 0 and can pick if there is an electron beam in the system, can pick a quantum of energy from the electron beam and can produce radiation of higher frequency. We call as omega 2. And the momentum of this has to be compensated also. h cross k 0 plus h cross k is equal to h cross k 2.

This process in which some momentum has to be brighten brought from the beam and added to a quantum of momentum with the pump wave photon to produce a photon of higher frequency and higher momentum, this process is known as free electron laser instability. You can convert by this process, a laser radiation by employing a counter propagating electron beam. You can produce x-rays. This is my laser, sorry yeah if you are having a counter propagating electron beam, then this produce as radiation x-rays.

Coherent x-rays will be produced by the coupling between these two. This is a very important scheme and the frequency of radiation if you workout little theory for this, the frequency omega 2 turns out to be of the order of four gamma 0 square into omega 0 where gamma 0 is the Lorentz factor of the electron beam. Gamma 0 is related to beam velocity by this quantity v gamma 0 is equal to 1 upon under root of 1 minus v 0 square by c square where, v 0 is the beam velocity.

So, you can produce coherent x-rays by this parametric process. Lot of work in last three, four decades as gone into parametric interaction between waves magnetized plasmas are particularly very rich in wave phenomena and they can give rise to very useful sorts of instabilities. Some instabilities are very bad because, they can remove the energy from the plasma return back to outside a space, but, some instabilities are very helpful.

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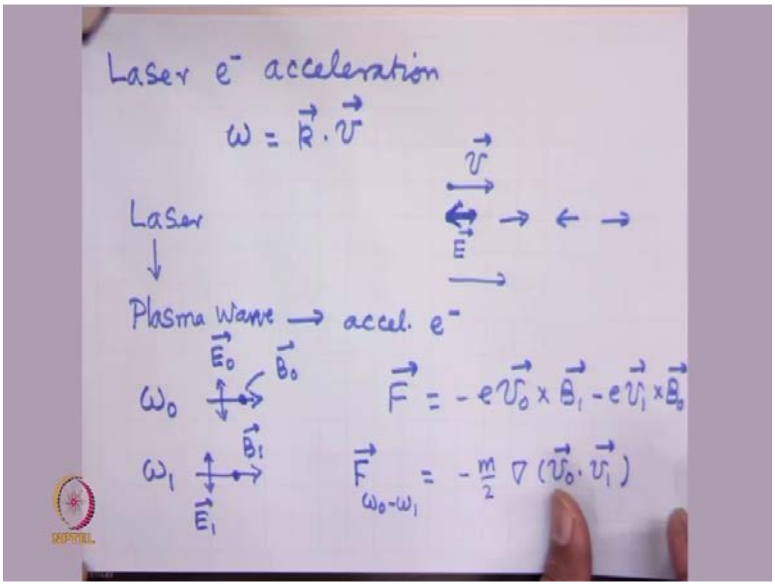

Now, a word about electron acceleration how does an a laser accelerate electrons to energies approaching g v. Well, a simple understanding about wave particle interaction is and that we have seen in when we were discussing the free electron laser or Cerenkov free electron laser or landau damping that, the maximum interaction or strongest interaction of particles with waves occurs when the condition omega is equal to k dot v beam.

When the velocity of electron v is such that, the phase velocity of the wave as seen by moving electron in this direction that quantity they they are equal then the electron can resonantly interact with the wave. Because the Doppler shifted frequency of the wave becomes 0 for the electron.

The thing is that, if I have a moving electron going in this direction and I want to accelerate it by a by a wave then, I need a wave that has a component of electric field in this direction, this is the direction of electron velocity. And I want the electric field also to be in the same direction or rather opposite direction. So, they have the force is in the forward direction.

If I want to accelerate this electron, I would like the electric field of the wave to be in the opposite direction. And as the electron moves, this field should also move with the same velocity, nearly same velocity. Otherwise if this this is a wave going in this direction, the electric field is for any wave that is travelling in the forward direction, if it is a plasma wave then the electric field will be in half region. It will be in the backward direction, half will be forward direction, half will be backward direction, half forward direction.

And if this entire structure is moving with the same velocity as the electron then, the electron will field as if the field is always accelerating it. Otherwise if this stars starts seeing this phase then, this electron will be retarded. So, what you require that, the electric field of the wave should have a component in the direction of electron velocity and secondly, the phase velocity of this wave should be very close to the velocity of the particle. These two things are required. For electromagnetic waves you know, in plasmas they have a phase velocity more than c, this condition cannot be satisfied.

Moreover, the electric field is has no component in the direction of wave propagation. So, electromagnetic waves are not of much use as for as acceleration of electrons is concerned. But a plasma wave certainly can satisfy this condition. So, the issue is that laser can excite efficiently a plasma wave. So, laser, let it excite a plasma wave by some process and this wave then accelerate electrons. This wave can do this job.

How about this laser excite a plasma wave? Just now I discussed the Raman scattering process in which, a large amplitude laser but, exciting a plasma wave and a side bend electromagnetic wave. But, I am saying here rather than sending one laser you may send two lasers.

So, send two lasers of frequency omega 0 and omega 1. This laser is travelling here, this laser is travelling here with this is be the electric field e 0 and magnetic field is perpendicular to this page. This is called B 0 and this is my second laser of electric field e 1 and there is a magnetic field B 1 also here.

So, what will happen? This omega 0 frequency laser will impart a velocity to electrons in the direction of e zero. But, will beat with B 1 through the Lorentz force minus e v 0 cross B 1 and minus e v 1 cross B 0 and recognize that the real part of v 0 is to be multiply with the real part of B 1. Whenever you are multiplying complex quantities in this fashion, real part of the complex quantity is the actual velocity. Real part of this complex B 1 is the actual magnetic field. So, when you multiply the real quantity is actually little careful and this gives rise to a force at frequency omega 0 minus omega 1 and its value turns out to be simple, it is simply equal to minus m by 2 gradient of v 0 dot v 1.

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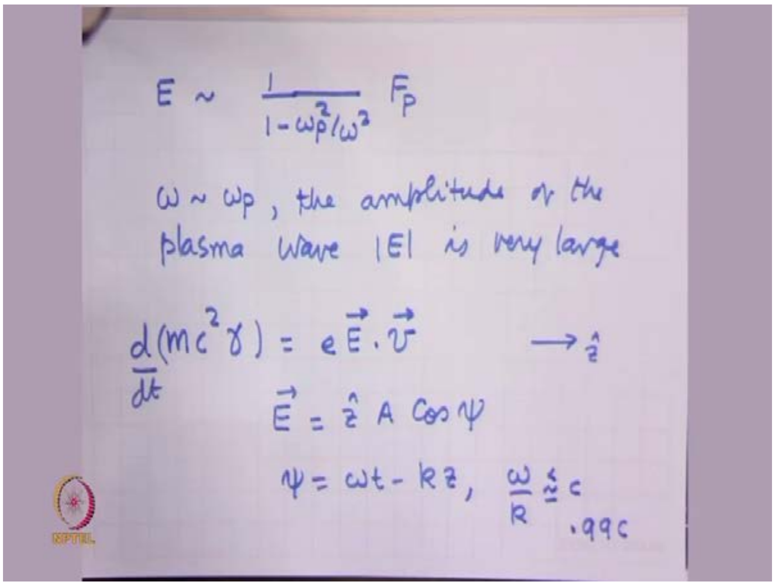

And if you calculate the electric field of the plasma wave generated in this process e of the plasma wave goes as 1 upon 1 minus omega p square by omega square. Obviously, it depends on F P. This is the kind of electric field that is produced by this longitudinal force and this can be resonantly enhanced by this quantity whenever omega is close to omega p.

So, for the omega of the order of omega p the plasma frequency, the amplitude of the plasma wave generated could be very large. The amplitude of the plasma wave is very large. It could be when larger than the amplitude of the lasers that were producing it, much larger. So, you have a system in which a plasma wave is generated by non-linear mixing of lasers. Professor Liu and Professor Rozenworth carried pioneering work in this field and they demonstrated the existence of such a plasma wave or rather excitation by lasers.

Now, the issue is how will this plasma wave accelerate electrons? That is a simple physics. You can just write down the equation of motion well in the form of energy. Energy equation for electron, m c square gamma is the energy of an electron moving with the realistic velocity gamma is the Lorentz factor.

So, rate of change of this quantity is equal to the electric field of the wave which is e E. I am taking to be in the z direction multiplied by the work done by velocity. So, if I take my electron motion in the z direction, forget the x and y motions. Then this equation can be solved and one rather than solving the evolution of energy as a function of time, people want to understand that in the wave phase because people write electric field of the plasma wave as amplitude a cos psi where psi is omega t minus k z. And people can gain lot of insight into energy exchange between electron and wave especially when, omega by k is very close to c, slightly more than c or slightly less than c, just slightly less than c maybe, 0.95 c or 0.95 c of that order.

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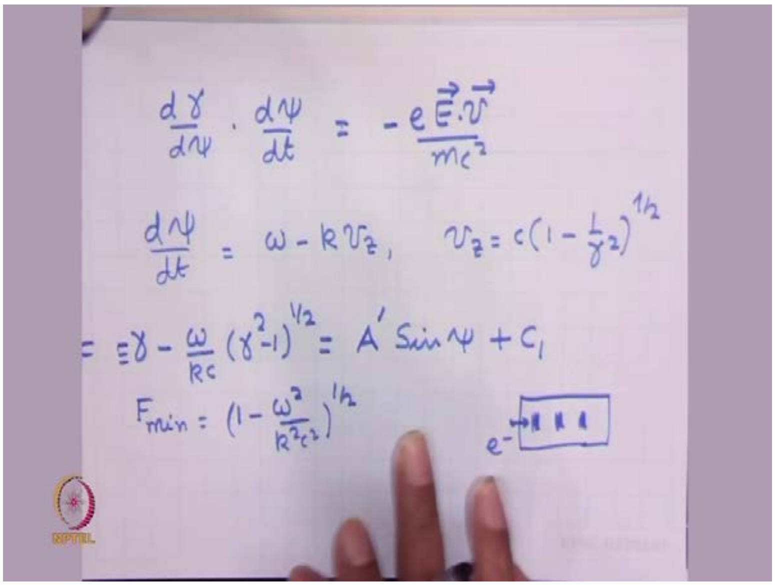

What happens that, if you solve this equation which is very simple to solve rather than, solving d d t, you solve the equation for d gamma by d psi and d psi by d t where psi d psi by d t if I differentiate the expression for psi, it turns out to be omega minus k v z and v z is related to gamma by this relation c into 1 minus 1 upon gamma square to the power half.

So, this equation which is equal to minus e E dot v upon m c square can be written a gamma versus psi equation. And the solution turns out to be of this form, gamma minus omega by k c into gamma square minus 1 to the power half is equal to some normalized amplitude of the of the plasma wave.

Sin psi plus a constant of integration that depends on the initial conditions. The left hand side you may note omega by k is slightly less than c. So, this quantity is less than 1. Gamma square minus 1 under root is always less than gamma square under root one. So, this quantity is certainly less than this gamma and hence this left hand side it has a, is positive definite. And the minimum value of the left hand side is let me call this as f. So, f minimum value of this left hand side is, turns out to be equal to 1 minus omega square by k square c square under root. You can just check it.

So, this is the left hand side which has a minimum value. Now this c 1 depends on the initial phase and initial energy of the particle initial phase of the wave that is enters. So, if suppose there is plasma here in which a plasma wave exists, this plasma wave which is moving in the forward direction and electrons enter here.

I want to accelerate this electron. So, depending on at what instant of time I enter, the electron in the plasma into the plasma wave then, c 1 will be determined by that. If c 1 is less than a prime then, all values of psi will not be permitted. Because psi, the phase of the wave can change between 0 and 2 pi. So, for half values of psi this will be positive half psi will be negative.

So, if c 1 is less than a prime plus f minimum, all values of psi are not permissible and particles are trapped in the wave, they can be accelerated. So, this equation is very rich in revealing that there are electrons which are trapped in the wave and if you plot gamma versus psi you get lot of insight in to particle acceleration. Let me just mention that.

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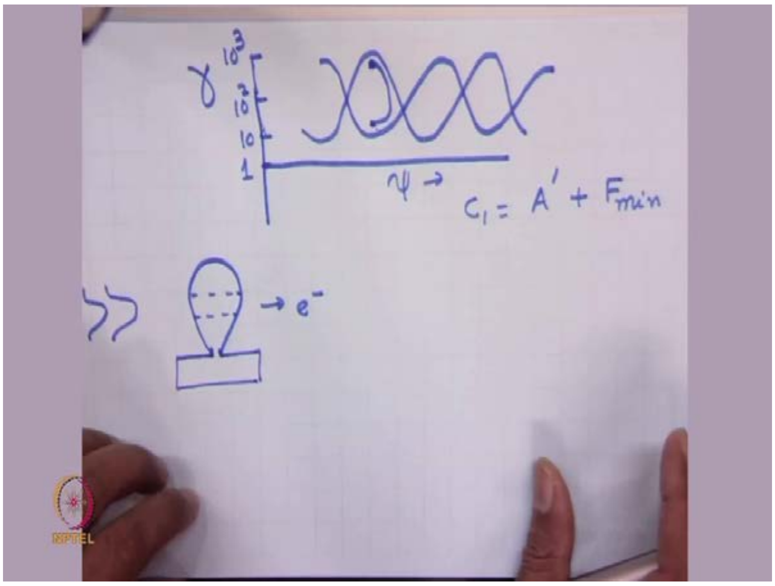

Just a simple solution equation reveals, this equation is like this. This is equation of a separate tricks where, c 1 I have chosen is equal to normalized wave amplitude plus f minimum. And the this I have plotted gamma here, this is psi here and this is a logarithmic plot 1 is there, ten is here, hundred is here, thousand is there and. So, on

This is a logarithmic plot. So, the electron which enters a wave somewhere here as it moves it can go to the peak. It can gain a huge amount of energy gamma, if you multiply by m c square, you will get the energy. So, initially the energy if suppose, the particles is like ten gamma is ten it can acquire a gamma like thousand when it goes from here to here. So, just the when the wave travels, the electrons trapped inside the wave can gain huge amount of energy in going from top of this gamma versus i curve to the, from bottom to top.

And that is the scheme people have employed in last thirty years or so, to study electron acceleration a simple physics, physical picture of the scheme is like this: you choose a gas jet target a plume comes out from here. This is a gas filled in here and there is a small nozzle here, gas plume comes out here, you shine laser light here and you see energetic electrons coming out from there. This is the plasma formed here if wave is excited in this region plasma wave may be you can send two lasers. They will nonlinearly combine with each other to produce a difference frequency plasma wave. And that plasma wave accelerates, a electrons and electrons come out. It is a simple physics simple picture. And this can be done by using table table top lasers. That is a interesting scheme.

A word about ion acceleration or proton acceleration; proton acceleration are important for cancer treatment. You know people are using chemotherapy and radiation therapy. But, they have harmful effects on human body. Protons do not have. Even people can use c six plus ions also for that purpose.

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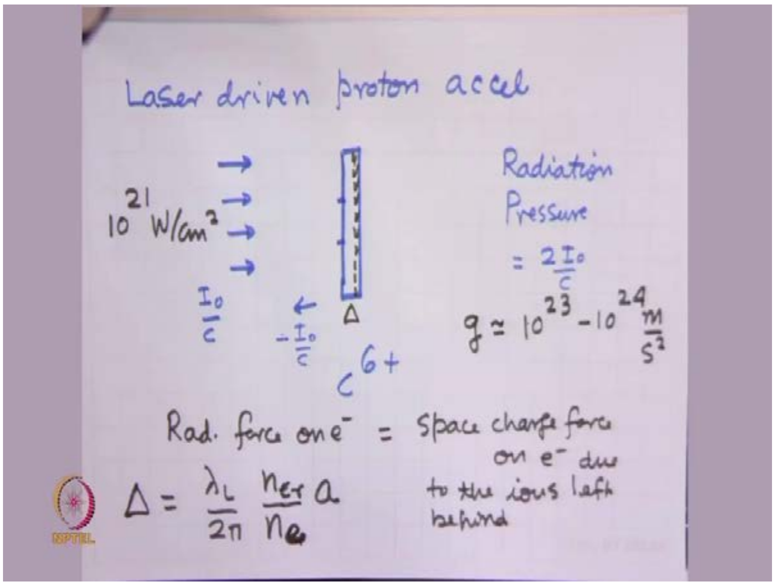

So, I am talking of laser driven proton acceleration. This physics is also very simple, very interesting. You choose a thin foil of some material maybe, choose a thin foil of say hydrogen, it is hydrogen plasma. What will happen? If you shine laser light on this actually hydrogen in the foil form is not available. But, what happens is that if you take any thin foil of some material some water vapors are there deposited on this phase and when this you converted into a plasma you can suddenly find a significant number of hydrogen ions or protons in the plasma they want to accelerate.

But for the sake of mathematical simplicity or physical picture, let us consider theoretically a proton foil. People have done experiments with carbon foils, diamond like carbon foil. And so, rather than accelerating protons they can accelerate c 6 plus ions. But, now let see what the its physics is.

When laser of intensity i 0 falls on a foil and gets reflected back if the foil is become a super dense plasma over dense plasma, then the forward momentum carried by the laser is I 0 by c, backward momentum it comes is minus I 0 by c. So, its exerts a force on the foil per unit area 2 I 0 by c. So, radiation pressure force on the electrons per unit area is 2 I 0 by c if it is stationary. But, this radiation pressure does not apply on the entire foil, it applies only on the electrons of the foil.

So, the electrons which were initially filled in the entire foil, they are pushed backwards and they are pushed backward to the small region, here something like this. So, in this a small thin region, the electrons are pushed. They cannot be pushed further because, the space charge of ions left behind will pull them back. So, you choose a thickness of the foil such that the radiation pressure force on electron exactly balances the space charge force due to ions.

So, what I am looking forward is, radiation pressure force on electrons is equal to the space charge force on electrons due to the ions left behind. And what will happen? Beyond this point, the electrons will not move. Rather these electron electrons will pull the ion layer with it. So, this whole thing moves as a double layer.

Electron layer and ion layer they form a double layer and this entire system moves under the radiation pressure force without charge separation. And how much is the delta to be chosen for this? If you workout this equality, then delta turns out to be typically equal to laser wave length upon 2 pi multiplied by the electron density of the foil without compression. Before compression divide by the critical density multiplied by the laser amplitude a, normalized laser amplitude that I wrote down.

And typically values of a are like ten or hundred. n upon n critical, is also like hundred sorry this is n critical upon, I am sorry I made a mistake here. This is n critical upon just a second just a second let me just see yeah this is n critical upon n e.

So, what happens? Typically if I choose a like 100 which corresponds to an intensity of the order of ten to the power twenty 1 watt per centimeter square, choose electron density in the foil to be about hundred times the critical density. So, that this ratio is around unity. Lambda is of the laser employed in this experiment is about 1 micron wave length. So, this is like 1 tenth micron.

So, if you choose a foil of about 1 tenth micron width, shine laser light of intensity ten to the power 21 watt per centimeter square then, you can accelerate electrons with acceleration. That will be called as g equivalent g of the order of 10 to the power 23 to 10 to the power 24 meter per second square. Compare this with acceleration. Due to natural gravity like 10 meter per second or 9.8 meter per second square.

So, this is huge acceleration that you are really producing here. And people have seen in the stimulations that this scheme can give rise to 1 g e v ions in a very short distance of about twenty thirty micron. Very strong acceleration is possible. So, this is a very fascinating field of research.

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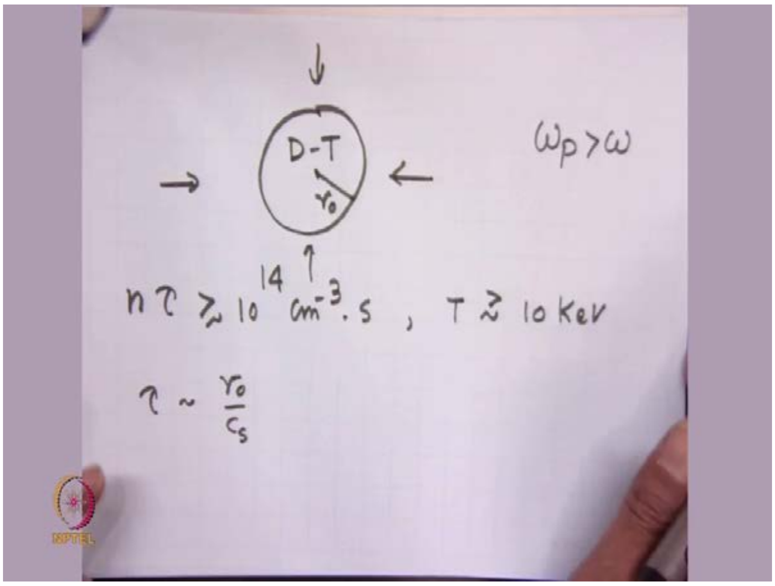

Well a word about laser driven fusion. Well, I am running out of time. But, a word about laser driven fusion, laser driven fusion is a very fantastic scheme. Well, when we were discussing the magnetic fusion we discussed that the goal is to achieve for breakeven n tau, the product of density of the plasma multiplied by the consignment time of the plasma to exceed ten to the power fourteen per centimeter cube into second and electron temperature or ion plasma temperature has to be of the order of ten k e v or more. This is the goal.

In case of laser driven fusion, people do not confine plasma by any magnetic field or any external fields. They believe that, if I have a initially a plasma of radius r 0. If you heat the plasma it will expand the sound velocity and tau will be of the order of r 0 by c s. c s is the sound speed which is typically ten to seven per second centimeter per second.

So, if you have r 0, a small, then, tau can be small. But, to reach this number ten to fourteen, you require density to be quite large. So, what people do? You shine laser light for all directions here and here and here. This laser will quickly convert this deuterium-tritium fuel into a plasma. But, as soon as the plasma is formed, this laser does not penetrate because omega p of the plasma is more than the frequency of the laser.

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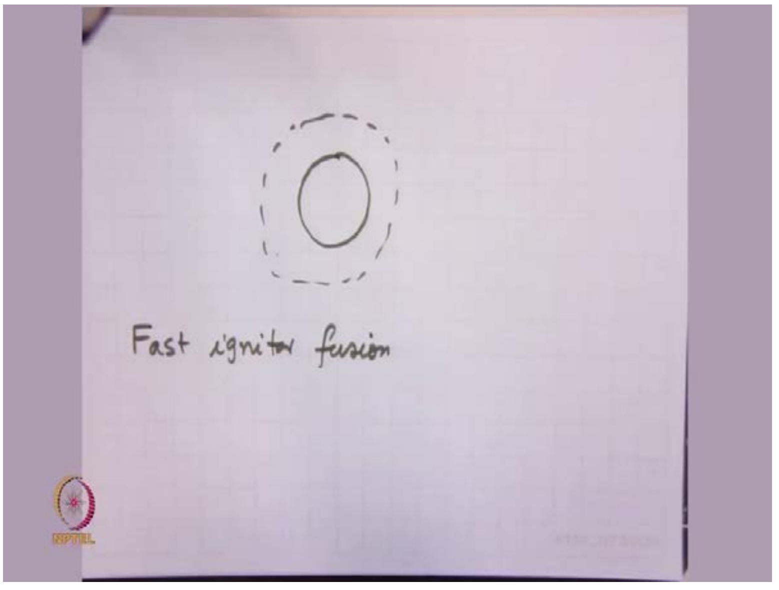

As a result, outside region becomes a plasma and core becomes like this. And outside region is a plasma. This plasma compresses the core and laser is observed in the outer region called corona. The issue is that, from the corona the heat has to be transported to the core and core has to be heated. So, there are some schemes, very fantastic schemes have come up for efficient energy transfer from laser to ions.

Yesterday I, in the last lecture, I talked to you about coulomb explosion in which a lasers can efficiently transfer energy into ions. Here also, the same process can be exploited and this new scheme of laser driven fusion is called fast igniter fusion. And one can achieve very high temperature and one can reach the Vlasov criteria I think a in near future.

So, we have seen that, there is lot of fascination in plasma activities, plasma research these days. And the basic physics that we discussed in this course, I think, it should give you a fare analytical ability to handle these fascinating phenomena. It has been wonderful being with you in these lectures. Thank you very much.