Plasma Physics

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Module No. # 01

Lecture No. # 04

RF Conductivity of Plasma

Well, friends today I am going to talk about radio frequency conductivity of a plasma. In this I will discuss plasma response to constant amplitude radio frequency electric field, then electrical conductivity.

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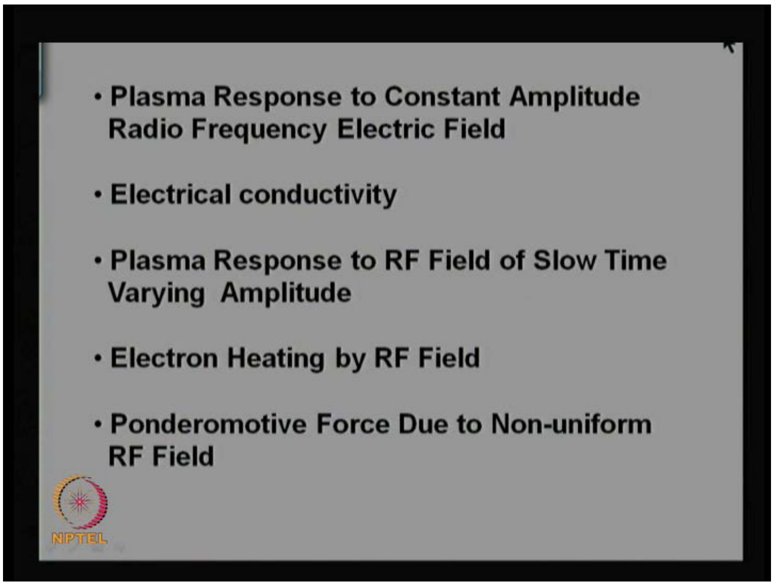

Then I will consider plasma response to a radio frequency field of slow time varying amplitude, then electron heating by a radio frequency field and finally, I would like to discuss a non-linear effect called ponder motive force due to non-uniform radio frequency field.

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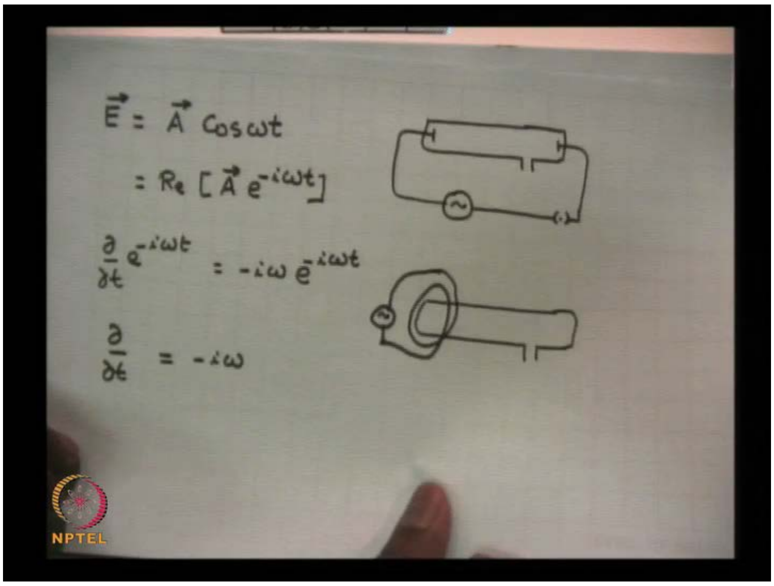

Well, I mentioned in my introduction that one can produce plasma. If I take a plasma tube with connected to a vacuum pump, and if you have two electrodes there, by applying a potential difference between them, the potential difference could be a dc field or a RF or ac field.

This is R F source. Well, R F can also be applied without any context. So, simple you have a tube like this, in which a gas at low pressure is filled and just have a coil like this, R F coil like this and apply R F voltage across the coil.

So, there is no contact with the coil with the space inside the tube or with the gas. But what can happen that when you pass a alternating current in the wire, it produces a time varying magnetic field and then in the region of the space when there is a time varying magnetic field, it will produce an electric field also.

So, you can apply an electric field time dependent electric field into a plasma by means of a contactless coil or by means of a conventional ac source.

Well, the distinction in this case with respect to the dc case is that in the steady state we found that the current in the plasma was a static time dependent, but here because the electric field is time varying, we will have a time varying current and the response could be quite different.

So, now let me first express the electric field of the RF in the simplest possible case. I express the electric field which is a vector quantity is equal to some amplitude A and Cos omega T . This is the simplest representation of a time dependent electric field which is uniform in space, but harmonic in time.

Well, as far as mathematics is concerned, cosine function, if you differentiate it to time, it converts changes its character it becomes sine function. However, if I express this as real part of a exponential minus i omega t, then you may note that this real part of this quantity is certainly equal to A Cos omega t, but the differential coefficient of exponential function preservers the character of the function.

For instance, delta by delta t of e to the power of minus i omega t, is always equal to the minus i omega outside and the exponential function.

So, it is very easy mathematically to handle differential equations if I employ exponential dependence on time. Because if you compare this equation left side to the right hand side, you can say that for exponential functions, delta by delta t operator is equal to minus i omega.

So, a differential operator can be replaced by a scalar operator, algebraic operator, minus i omega simply a multiplier it becomes a simple thing.

So, rather than considering the response of the plasma to an electric field given by Cos omega t, I will consider the response of the plasma to an electric field given by an exponential time dependence and Re which tends real part of. I will suppress , I will imply this, but I will not write explicitly

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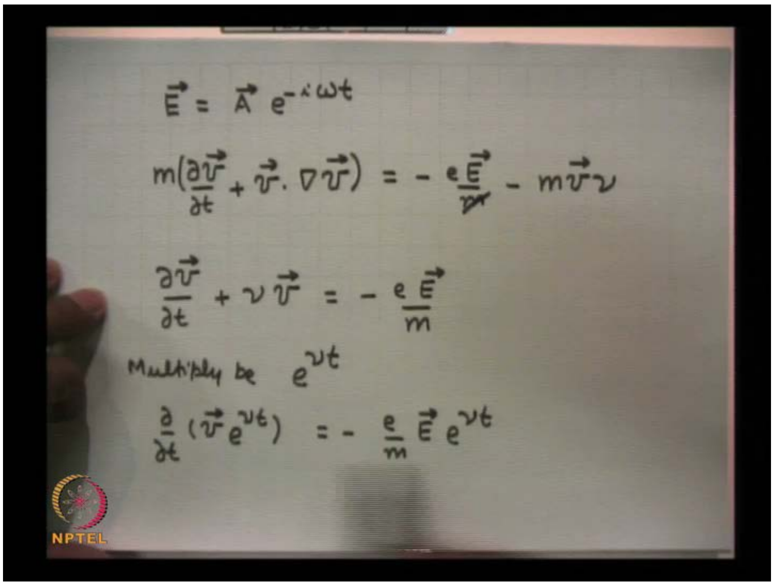

So, now I will consider the response of the plasma to an exponential electric field, oscillatory electric fields rather. I am considering E is equal to A exponential minus i omega t. I may have this dependence on time for a certain time, t greater than 0 for instance and the electric field may be 0 prior to; that means, I may switch on the electric field in the plasma at a given instant of time.

Let us see the response of the plasma. The response of the plasma is largely governed by the behavior of electrons for whom the equation of motion can be written as, m (delta v by delta t plus v dot del v) is equal to the minus e E upon m minus m v nu.

I am not writing the pressure gradient term. Because I am presuming that the field is uniform; temperature is uniform; density is uniform. So, that term does not really count.

Similarly, if my electric field is uniform in its phase, the velocity that will be produced by this field, there also be uniform in its space and hence this term will also not be finite, it will be 0. So, if I ignore this term and divide this equation by m. My equation simply becomes delta v by delta t plus nu v , bring this term to the left hand side is equal to minus, I made a mistake this m should not be here because I already have here. So, this m should not there.

So, now this becomes minus e E upon m. Well, this equation can be solved exactly if I multiply this equation with the integrating factor .

So, multiply by both sides by the integrating factor is exponential of nu t. So, if I multiply this factor here and here and here, then the two terms on the left hand side can be combined and can be written as delta by delta t of v into exponential of nu t is equal to minus e upon m e exponential of nu t.

So, the advantage of multiplying this equation by the integrating factor has been that I could combine these two terms into one term. And then I can integrate both sides on time, because there is no other time dependence; no other dependence. So, I can just integrate this equation and what do I get.

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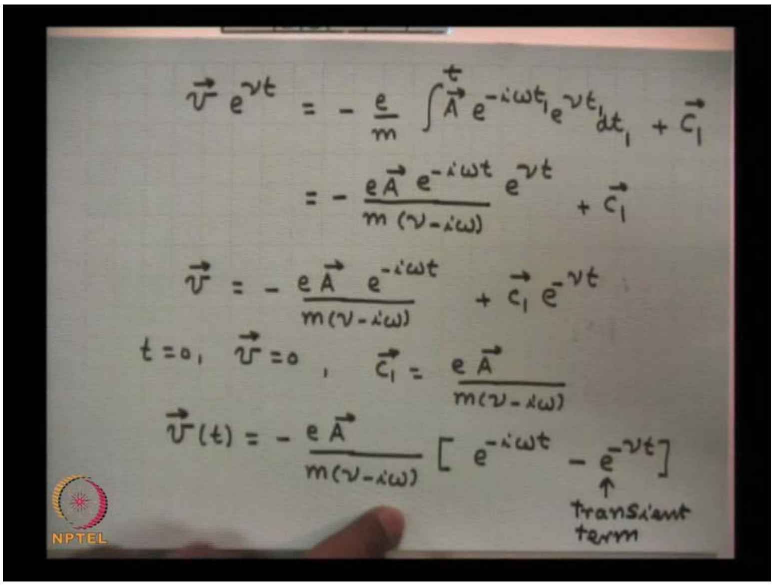

I will get v exponential nu t is equal to minus e upon m , and e is this integral, a exponential minus i omega t, this is the electric field into e to the power nu t into dt plus a constant of integration, I will call the c1.

Well, please understand this integration is over time and the variable of, I can put the limit t, but the variable of integration I can change.

So, it is a t1 here, I can put just t1 there because this is the variable. So, v at time t will depend on the electric field not only at time t, but at a previous time also.

Well, if you integrate this, it turns out to be minus e upon m vector a and this will integrate to give you nu minus i omega and exponential minus i omega t exponential of nu t. Because I am putting a time t this thing, I have to evaluate a time t plus c 1.

Now, this exponential factor on the left hand side can be taken on the right hand side after dividing this equation. And v can be written as minus e A upon m (nu minus I omega) exponential minus i omega t plus c1 exponential minus nu t.

C1 has to be evaluated by the boundary condition. So, suppose my field had just started at t is equal to 0, where I had the drift velocity of electron to be 0.

If I assume this condition, then c1 can be obtained and c1 turns out to be equal to e A, c1 is a vector, please remember. It is a vector equation. So, I should put a vector sign everywhere.

This c1 is a vector. So, this is e A upon m nu minus i omega. And if I substitute this back here, then the velocity of electrons at time t is equal to minus e A upon m (nu minus i omega). I can take this common exponential of minus i omega t minus e to the power minus nu t.

So, this is the general expression for the drift velocity of electrons due to a time varying electric field which is uniform in space.

The second term in the bracket will vanish at large time bigger than one upon nu. Nu is the collision frequency; one upon nu is called the collision time.

So, for time and what is time t? t is the time after the onset of the electric field RF field, because the field has been onset at time t is equal to 0.

So, after a time of the order of or bigger than the collision time, after the onset of electric field RF field, this term can be ignored. This is called transient term transient response or transient term. One can ignore this term, at long time.

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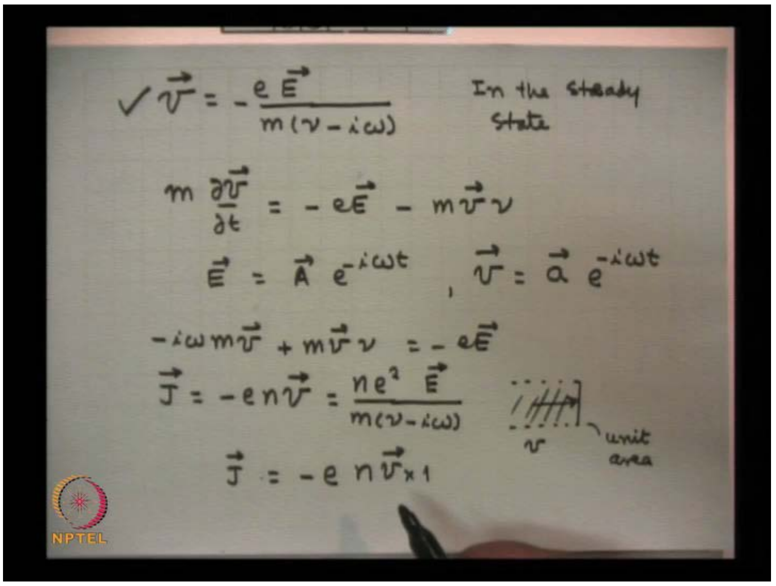

So, in their steady state means after a long time as compared to the collision time, I can ignore this term. And I can write down the electron response simply velocity is equal to E e upon m minus times there, nu minus i omega. This expression you will get.

Now, this is a very simple expression and you can really recover this expression in the steady state in a simpler way. And let me demonstrate this how to do it. In the steady state, let me write this equation, equation of motion again which was m delta v by delta t which was is equal to minus e E , the electric force minus m v nu; this is the momentum loss per second via collisions.

So, this equation have to solve because electric field has a time dependence. Like this one would expect in the steady state, your response should also have the same dependence; we should also have same dependence like a some amplitude exponential minus i omega t.

So, that every term has same exponential dependence just substitute this back in this equation and as I mentioned when you differentiate the exponential function like this, it will simply give delta by delta t is equal to minus i omega and this equation become minus i omega into m v , this term becomes plus m v nu , when it comes from the left hand side and this equal to minus e E when you divide this coefficient of v here, you will recover this expression.

So, it is a simple way. In future largely we will be interested in the study state response of the plasma and hence we will be employing this technique of converting the delta delta t operator by minus i omega. And the equation becomes very simple.

Well, after I have obtained the drift velocity of electrons due to the electric field, RF field, I would like to calculate the current density. Because that is the physical quantity that can be measured. So, now, let's understand what I wanted to calculate J. Yesterday I mentioned that J can be written as minus e and v. But let me recapitulate, but I am considering suppose the, this the direction of flow of electrons, if I consider unit area here; this is unit area.

In one second, the charge crossing this area is called current density. Now the issue is in one second how many particles, how many electrons will cross this area. Because the velocity of average velocity of electron is electrons is v. So, consider a cylinder of length v . All the electron that are filled in this box of area unity and length v they will be able to cross this, which are farther than this; they will not reach because in one second they cannot travel more than v.

So, those electrons will be out. So, in one second all the electrons that are filled in this box will cross this area; fictitious area that I placed here

And the volume of this area is v into cross section 1 and the density of electrons is n. So, so many electrons will cross per second and the charge of each electron is minus e. And hence this is the charge crossing per unit area, and I can write this the same thing as J, but because there is a direction here and velocity also. So, put a direction there; so, simple thing.

Now, the value of v I have already obtained on the top. And if I substitute this value here I get J in terms of the electric field and it turns out to be simply n e square upon m (nu minus i omega) into e; this is what you get. The coefficient of e is called conductivity.

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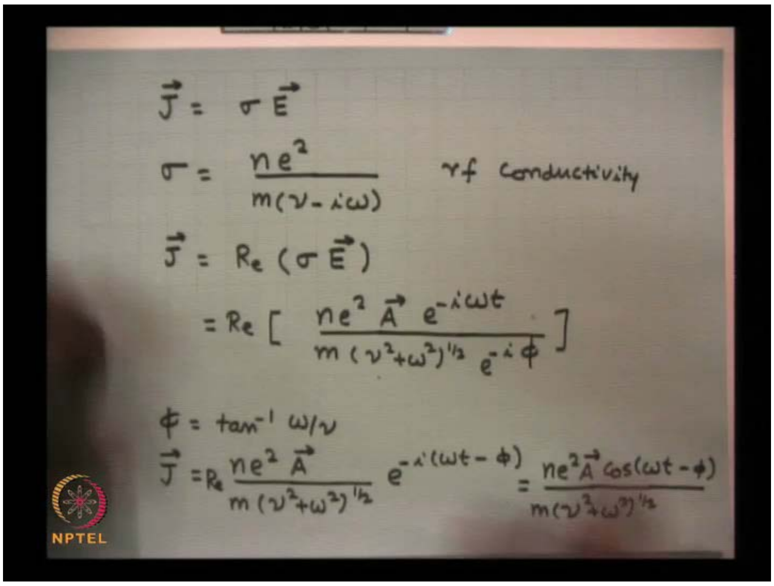

So, let me write this expression with better clarity. So, I can write J is equal to sigma E; and sigma I have obtained as n e square upon m (nu minus i omega). This is called RF conductivity or radio conductivity or ac conductivity of plasma.

I have not included the ion motion here, note here; that if I had calculated the current density due to the ions I will get a similar expression. But I have to replace mass of the electron by the mass of the ion and nu, the collision frequency of electrons by the collision frequency for ions.

But mass of the ion is very heavy. So, sigma due to ions will be very small as compared to electrons and hence can be ignored. This is what we normally we do

So, this is an interesting expression for conductivity, but this is little complex expression to realize its physical significance; I think I should do something different

J is a complex quantity that is obtained here, but actual current density is not complex; it is a physical quantity; it is a real quantity.

So, actual current density J is the real part of this (complex number sigma into e). Well, you can do two things to take the real part. First thing is, first express the sigma e itself into a convenient form; what was this I want the real part of sigma was n e square upon m nu minus i omega, I can write down as nu square plus omega square under root into e to the power minus i phi. Any complex number a plus i v or a minus i v can be written as a square plus b square under the root exponential minus i phi, where phi , I will call as tan inverse of imaginary part upon real part. So, omega upon nu.

And as far as e is concerned, I will write down a vector and that is exponential of minus i omega t. So, what you have to do this exponential i phi term I can take in the numerator, because rest of the things are real here.

Then take the real part and you will get actual J; is equal to when you take the real part is n e square upon m nu square plus omega square to the power half , a vector and then exponential of minus i omega t minus phi; and you can take the real part of this quantity.

So, let me put real part here Re, and if you take the real part; this will be simply n e square A upon m nu square plus omega square to the power half into Cos of omega t minus phi.

So, there is a phase difference between the electric field which was Cos omega t and the current density which is Cos omega t minus phi.

So, if you can measure the phase difference between current and the electric field that you apply. Then from there you can measure the collision frequency because phi if you can measure experimentally; and you know omega nu can be reduced. I will give you an example

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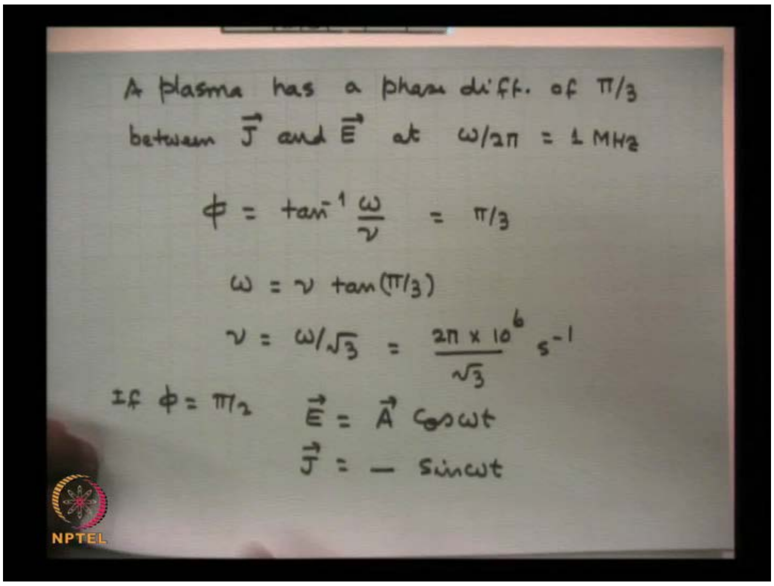

Suppose I have a plasma; a plasma for instance, has a phase difference of say pi by three between current density J and electric field E at frequency ; if I write frequency and hertz is equal to one mega hertz per instance.

Suppose, I apply a RF electric field of one mega hertz to a plasma, and I measure the phase difference between phi and E; J and e and suppose this is pi by 3.

So, I will simply say that phi is equal to tan inverse omega upon nu is equal to pi by 3.

So, this will give me omega is equal to nu times tan of (pi by 3); tan of pi by 3 is root 3. So, nu is equal to omega upon root 3 ; omega is 2 pi into ten to the power 6 radian per second upon root 3.

So many collisions per second is the value of collision frequency. So, you say useful quantity the phase difference between the current density and the electric field. It has a very profound physical significance. Also the thing is that if phi were equal to pi by 2, then you would say that if the electric field is, if phi is equal to pi by 2, then your electric field would be a Cos omega t and current density would be equal to some constant here into sin omega t. Because they are out of phase by pi by 2. If you multiply the 2, to calculate the heat dissipation or the work done by the electric field per second, then the product of the 2 J dot e will give you sin omega t to coos omega t and time average of that is 0.

So, there is no net energy absorption, when J and e differ by pi by 2. So, it is phi which is no need not equal to pi by 2.

Only then there is a net energy transfer from the wave of on the RF field to the electrons.

So, heating takes place only when J and E are not out of phase by phi by 2. So, phi is the significant quantity that determines the power absorption by the electrons from the RF field.

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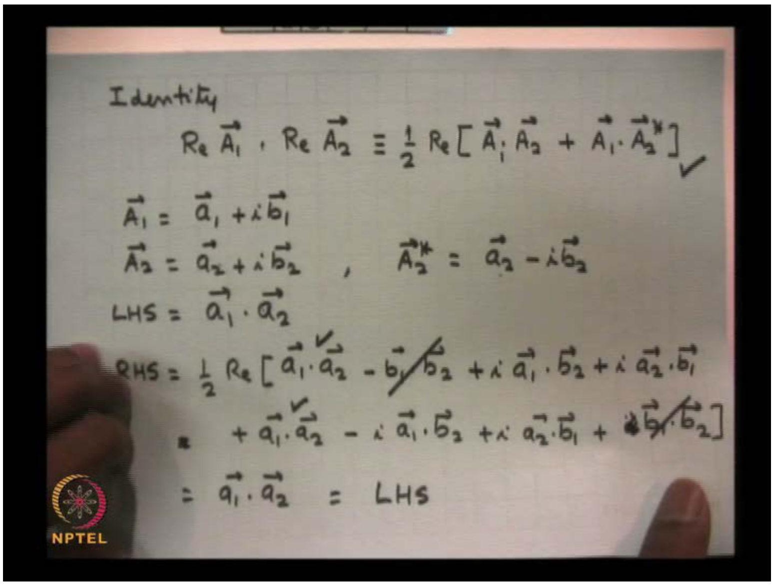

I think, elaborate on this is issue. Before I employ a complex representation for the power dissipation, I would like to bring to your attention an identity; a complex number identity which says that if there are 2 complex quantities, A1 vector and another complex quantity is A2 vector. If you multiply their real parts, then always this is equal to half real part of A1 dot A2 and plus A1 dot A2 star, where star represents the complex conjugate.

This is a general relationship. If the sign between these two vectors is a cross product, then there will be cross product here and a cross product there. If it is a scalar product then scalar products on both sides. This is always true and we can just check it.

Suppose my A1 is equal to a small a1 plus i times b1 ; and A2 vector they are all vector quantities, A2 vector suppose is a2 plus i times b2. Now let us substitute these two quantities on the left as well on the right and see whether this is verified. What will happen? As far as the left hand side is concerned, real part of A1 is a small a1. So, simply a1 will come. Then dot product; this dot product will come here. Real part of A2 is simply small a2. So, a2 will come.

How about the right hand side? Half real part, I will write, a1 dot a2 in complex form multiply them. If I multiply a1 dot a2 what do I get? a1 plus a2 if you multiply, you will get four terms a1 dot a2 then minus b1 dot b2; when I multiply the imaginary parts, then cross multiply the terms. So, plus i times a1 dot b2 plus i times a2 dot b1.

And second term will give me, I could take a2 star. So, a2 star I will write down as, a2 star is equal to a2 minus i b2 , because I have to write the complex conjugate.

So, replace i by minus i , what sorry. So, I have to add this term there. So, multiply these two quantities; I will get plus a1 dot a2 then minus i a1 dot b2, then I multiply this quantity. So, plus i a2 dot b1 , then plus i times b1 dot b2 ; not i, i is gone , b1 dot b2.

Please note here, this b1 b2 term will cancel out with this; a1 a2 term will add up. These are all imaginary terms, so, when you take the real part they do not count.

So, only this term will count and this will count because they are the real term or they are imaginary terms.

So, when you take the real part they do not count. So, this is simply equal to a1 plus a2 ; a1 dot a2 , which is the same thing as LHS, left hand side.

So, this a very important identity complex number identity, which will be useful in understanding the or in dealing with the propagation of waves in plasmas in general ; or plasma response to any time dependent fields.

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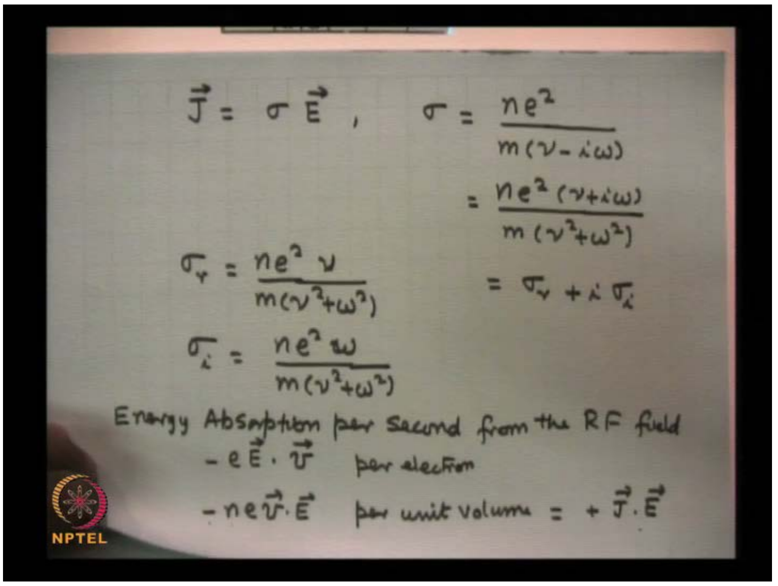

So, I will employ this identity to understand the response of a plasma to RF field. We have learnt that J is equal to sigma E and sigma I have already written as a complex quantity ; n e square upon m nu minus i omega.

I can rationalize this quantity sigma and write this as n e square upon m. To rationalize multiply the complex conjugate of this number in the numerator and the denominator. So, it becomes nu plus i omega there and nu square plus omega square is the denominator.

So, there is a real part of sigma and the imaginary part of sigma. So, sigma has a real part which is equal to n e square omega upon m nu square plus omega square into nu. And sigma i, the imaginary part of sigma is n e square omega upon m nu square plus omega square. So, I have written this is equal to sigma real plus i times sigma i.

Well , if you calculate the heat dissipation for instance, in a minute we will calculate that and we learn that sigma r is responsible for heat dissipation in the plasma ; sigma i does not count.

Though it plays a very important role in storing the energy inside the plasma, or when we study the propagation of electromagnetic waves, or plasma waves in the plasma, then this plays a very important role.

But as far as the energy dissipation is concerned this does not play any role. So, let us examine this issue.

So, my issue is energy absorption by the electrons from the electric field, energy absorption per second. Well, energy absorption per second is also known as power absorption from the error; from the RF, from the radio frequency field.

Let's understand what is the force on the electron due to the RF field minus e E is the electric force on the electron due to the radio frequency field and the distance travelled per second is v.

So, force into distance travelled by the particle in the direction of force is called the work done per second.

So, this is the work done on each electron per second or energy dissipation per second per electron; and in a volume, unit volume there are n electrons. So, power dissipation per unit volume would be minus n e v dot E ; I can just write v earlier E later. So, this quantity per unit volume this is the power absorption from the radio frequency field by the electrons per unit volume.

Now, n e v with the negative sign by definition is called J. So, I can write this quantity as minus J dot E ; a plus J dot e. So, the energy absorption by the electrons per unit volume is J dot E; it is a very important quantity. But please remember here we are multiplying the real part of J with the real part of E, because J is the physical quantity and actual J has to be written and so, is the case with electric field?

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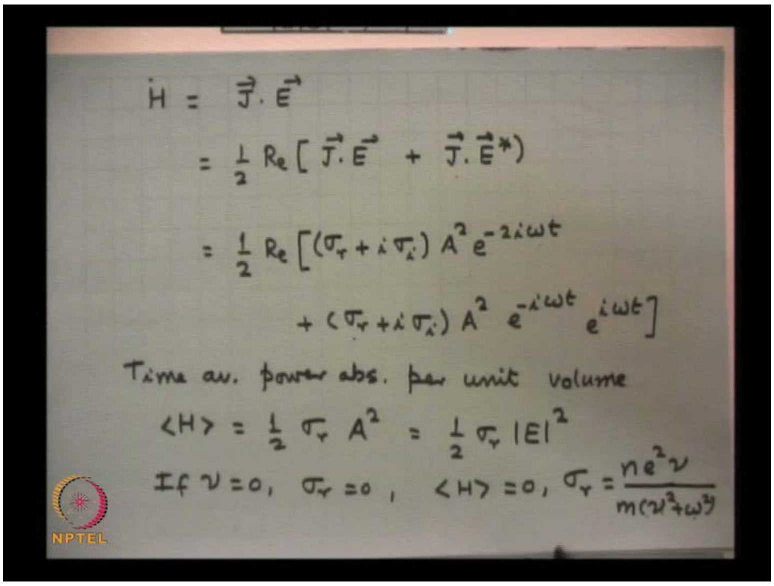

So, now let me calculate this quantity. I will call this quantity power dissipation as H which is J dot E; which implies this is equal to half real part of J dot E plus J dot E star. Put J equal to sigma E , real part of J equal to sigma e which is sigma r plus i times sigma i; this is the value of sigma into E. So, E dot E is E square ; e square is A square exponential minus twice i omega t.

And second term will give me J dot, so, J again ; I have to write which is sigma r plus i sigma i that becomes E dot E star; E dot E star if I write, it becomes simply a square. Because exponential of minus i omega t with E and E star is exponential of plus i omega t. So, they will cancel each other.

So, what you get here ; this is the power absorption by electrons per unit volume from the radio frequency field.

Now, this quantity depends on time , this term does not depend on time. You have to take the real part. So, this when you take the real part of this quantity , it will have either Cos 2 omega t term or Sin 2 omega t term . And when you take the time average it will be 0.

So, time average heat dissipation or power absorption per unit volume; I will call this H average is simply equal to half , I will ignore this because time average quantity is 0 and this is time independence. So, I do not bother about it this is real ; this is real ; this is purely imaginary.

So, when I take the real part it does not count, where I take only real part. So, sigma r A square or this can be written as half sigma r of electric field square , modulus of electric field square.

So, you have seen that sigma I, imaginary part of sigma plays no role and sigma i if you recall depends on omega and sigma r depends collision frequency. Well, both depend in their denominator by on nu also, but primarily this quantity when this is when there is no collision.

So, if nu equal to 0, sigma r vanishes and hence there is no power dissipation. This is a important result. So, if you want to heat a plasma, collisions are mandatory; is necessary. And higher the collision frequency, but the dependence of sigma r on collision frequency, you must note is equal to n e square nu upon m nu square plus omega square.

Usually in plasmas the radio frequency that fields that you apply have omega bigger than nu. So, you can ignore this new square here, and hence power dissipation will increase with collision frequency; stronger the collision frequency higher the power dissipation.

So, collisional plasmas absorb more power than low collisonal; low collisonal plasmas. it is an important issue

Well. So, far we have learnt the response of a plasma to a field which is time independent amplitude, whose amplitude is time independent. I would like to look into the possibility that if I build an electric field, RF electric field in a plasma gradually in time what will happen?

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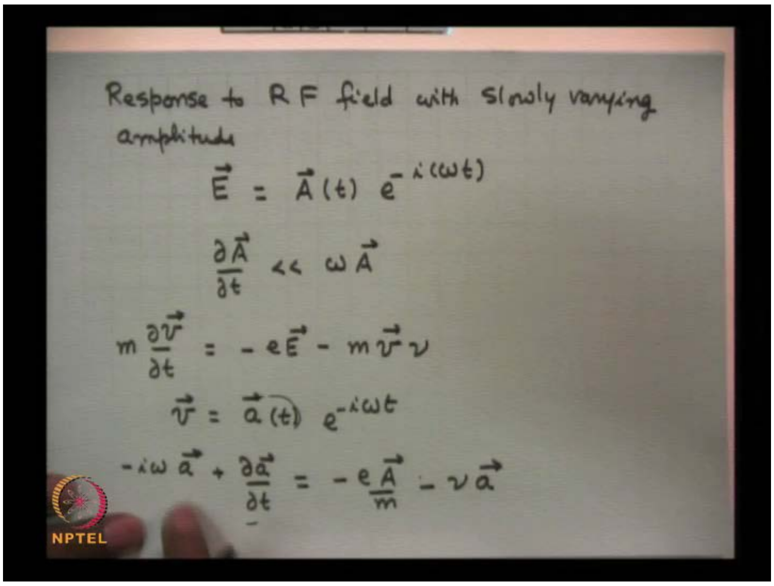

So, I will consider now the response to RF field with slowly varying amplitude. I think this will reveal a very important mathematical technique and some interesting physics as well.

So, I am considering a simple problem that electric field is still is independent of position. And suppose this is a which is a gradually varying function of time slowly varying function of time exponential minus i omega t , means a rate of change of A with time. I am presuming delta A by delta t is much shorter than omega into A. This is called slow dependence of amplitude on time. So, I am presuming this . I have to solve again the same equation.

Equation of motion which I can write as delta v by delta t m rather, m delta v by delta t is equal to minus e E minus m v nu ; this is by equation of motion .

Well, I can assume a solution. I assume a solution v is equal to a, but now this a should depend on time, because this depends, this amplitude of the field depends on time. So, I would expect this also depends on time ; exponential minus i omega t. Not necessarily this has same dependence on same as this one, because the time dependence of this a should be satisfying this equation. So, let us evaluate the time dependence of small a, in terms of time dependence of big ; that is the issue.

If I substitute this in this equation, what do I get? First of all divide this equation by m and then substitute. So, you will get minus i omega into a, where you differentiate this term by parts, I will get this one. And then you will get plus delta a by delta t , then you will get is equal to minus e A upon m minus nu is there and v into a. Please remember this is omega times a this is delta by delta t of a.

a is slowly vary function of time. So, I accept this to be weak. So, there is a technique of iteration that we can employ that implies that ignore these small quantities in the first 0 the order. And obtain the value of a ; use that value of a in the quantity that you ignored and evaluate the a again . This is called the technique of successive approximations or iteration.

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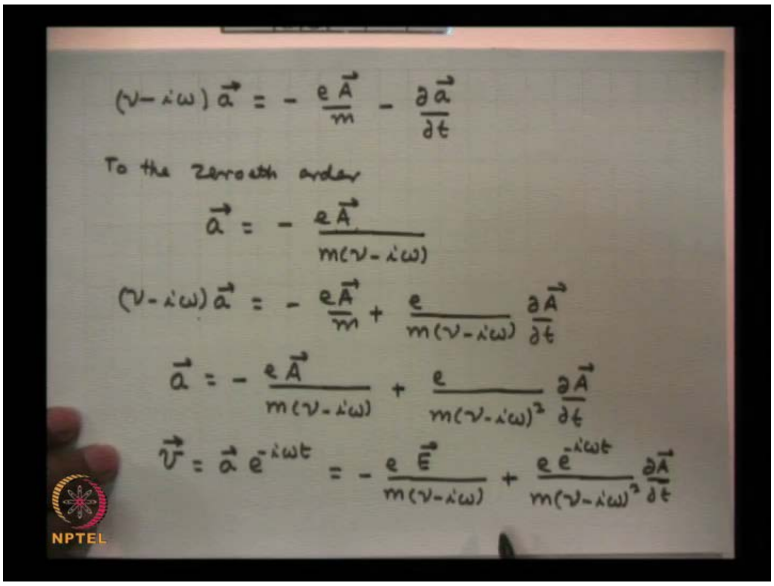

So, I am going to employ the technique of iteration. So, if I take the delta a by delta t term on the right hand side , my equation becomes minus i omega ; actually nu term also you can take on the left, this into a can be written as minus e A upon m minus delta a by delta t . No approximation, so far.

I have just rewritten the equation that I had written just a minute ago. So, now, to the 0 the order ignore the last term, because this is small term. So I get a is equal to minus e A upon m nu minus i omega. Use this value in here, then my equation becomes nu minus i omega a is equal to minus e A upon m minus this expression; so, becomes plus and this can be written as e upon m nu minus I omega and delta a by delta t.

Because , a I substitute; so, this is the only quantity that depends on time and this will what you'll get.

So, a you can obtain now by dividing this equation by nu minus i omega and you get a is equal to two; minus e A upon m nu minus i omega and then plus e upon m nu minus i omega whole square delta a upon delta t.

To this if you multiply the exponential minus i omega t, you will get the velocity. So, the velocity of the electron which was a e to the power minus i omega t, if you just multiply here. I can write this equation as minus e upon m nu minus i omega ; and this become simply E because a when multiply by this exponential factor , its simply E.

And second term is plus e upon m nu minus i omega whole square delta a by delta t into exponential minus i omega t.

So, there is explicit dependence of v ; v does not depend on E only. It depends on the derivative of amplitude also. It is a very important thing and let us see what is the consequence of this on energy dissipation.

In order to understand the energy dissipation, I will consider the case when nu is very smallest compared to omega, a high frequency which is a very reasonable approximation. Just to avoid mathematics, I will consider the case when nu is very smallest compared to omega.

So, let us see what happens ? So, I am talking about the heat dissipation. Well, before I do that let me write down the current density.

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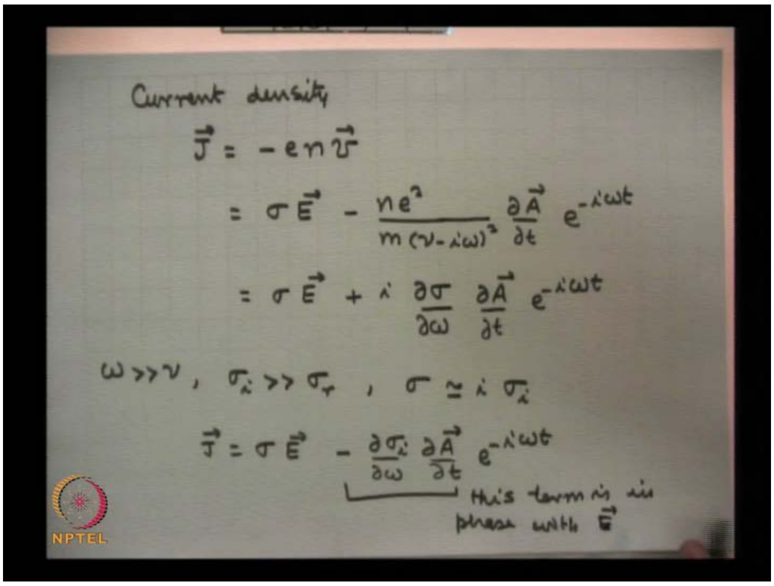

J is equal to minus e n v. If you multiply this, you will get this is equal to sigma e and then you get another term which is equal to minus n e square upon m nu minus i omega whole square into delta A by delta t exponential minus i omega t . This is the additional term that you get, because of derivative of amplitude of the RF field.

Well, if you look at the expression of sigma ; this is simply the derivative of sigma with respect to omega. I can write this as sigma E , this turns out to be equal to plus itimes delta sigma upon delta omega into delta a by delta t into exponential minus i omega t.

Please note, in a plasma where omega is much bigger than nu , sigma imaginary is much bigger than sigma real.

So, second term is primarily having sigma imaginary here . And sigma, I can write simply is equal to i times sigma i ; sigma i is very tiny. If I substitute this here, I get the second term is equal to sigma e minus delta omega upon sigma upon delta omega into delta a by delta t l this is sigma I exponential minus I omega t.

If you take the real part , please understand this quantity is purely real; this is real I have taken. So, real part of J now has , because of sigma i , a term in phase with the electric field like Cos omega t if you take , the real part of it will be Cos omega t in phase with the electric field.

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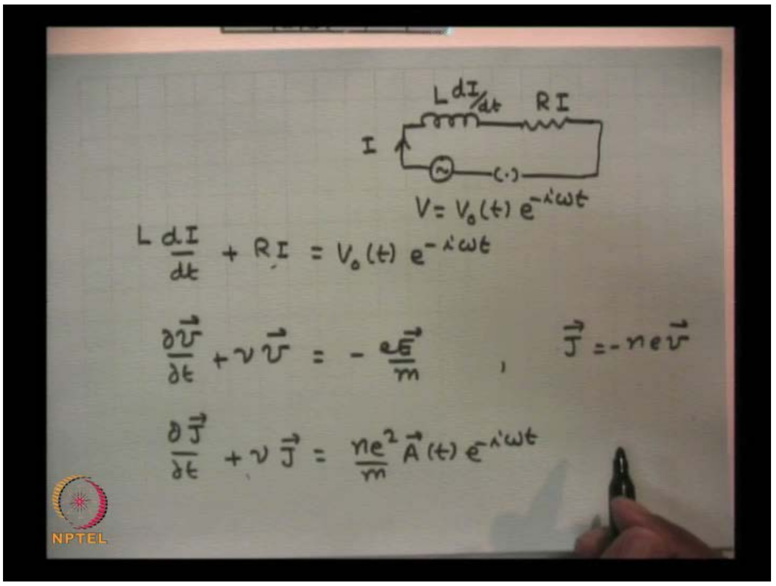

So, this term is in phase with E and gives rise to energy absorption. This is very similar to a LR circuit. Let me just mention this in LR circuit, what do you have?

If I have a inductor L and a resistor R and apply a RF voltage or ac field to this. And at any instance there's a current I here, the emf across this inductor would be L dI by dt and across the resistance will be R I. So, if the voltage is V ; is equal to say V0, which is a slowly varying function of time exponential minus i omega t.

If you write down the equation governing current in the circuit, then the total potential difference across the whole circuit should vanish , because of Kirchhoff's law . You will get L dI by dt plus R I is equal to V0 , which is a function of time exponential minus i omega t; very similar to the equation that I had written my equation was delta v by delta t plus nu v is equal to minus E e upon m , is by equation of motion.

Actually this equation can be written for current also. Because we know J is equal to minus n e v. So, multiply this equation by minus n e, then this can be written as delta J by delta t plus nu J is equal to n e square upon m e , which is a exponential minus i omega t. Look at the character above the equations.

The collisional term is equivalent to resistor term and this term is equivalent to inductive term . This is like voltage kind of thing. The two are very similar ; in inductor you know, if you gradually build up the field in a inductor if you turn on your RF field at time t equal to 0 then the ; the energy is stored in the capacitor and you remember that this is equal to half L I e square.

Similarly, energy is stored in the plasma particles ; plasma electrons whenever we apply RF. So, this expression that I gave to you , delta sigma I upon delta omega into delta I by delta t that really represents the energy stored in the plasma particles. Let me evaluate this quantity

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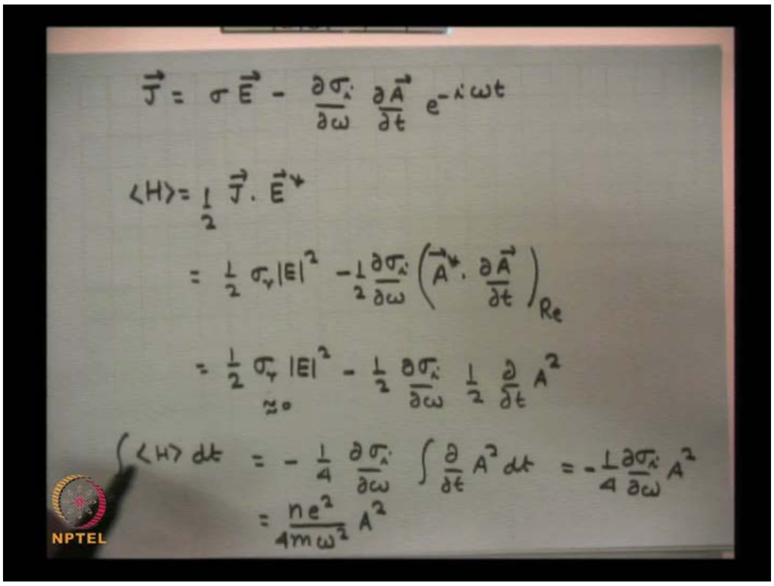

I will make a simple calculation for that. So, what I have done ; I have written J is equal to sigma E minus delta sigma i upon delta omega into delta a by delta t exponential minus I omega t . Then H if I calculate, time average H would be J dot E star and real part half by the identity the that I gave to you.

So, multiply this and take the real part , which is implied here it turns to be equal to half sigma r E square because of the first term . Second term will give me minus delta sigma i upon delta omega, half will also be there , into A star dot delta A by delta t and real part I have to consider here.

So, write down the real part. Real part of a complex number is half of it is the number plus its complex conjugate. So, I can write down this is equal to half sigma r E square minus half delta sigma i upon delta omega and this is half of delta by delta t, of actually this is real here. So, I do not worry about, this is simply A square by 2.

Well, sorry and I have taken it to be real. So, I do not have to worry about this; this is what we really get.

So, this is responsible for; this is actually energy taken by the particle as long as the amplitude evolves with time. It will be finite and when the amplitude has stabilized when saturated, then this will becomes 0

So, if I have to calculate and this term is tiny , because as I mentioned for a weekly collisional plasma , this is small.

So, if I want to calculate the energy stored, I have to integrate this quantity ; I will get minus 1 by 4 delta sigma i upon delta t omega delta by delta t of A square dt. And just integrate this you will get minus 1 by 4 delta sigma i by delta omega into A square. Please remember delta sigma i by delta omega is a negative quantity. So, this whole quantity is positive. If you put the numbers here, actual expression for sigma i, this turns out to be equal to n e square upon m omega square, I think 4 is there into A square. This is a very interesting expression.

So, I think I have derived an expression for the current density due to a time varying amplitude , and I think I will stop at this stage . And well, next time we will discuss the response of plasma to electric fields in the presence of magnetic fields. Thank you.