

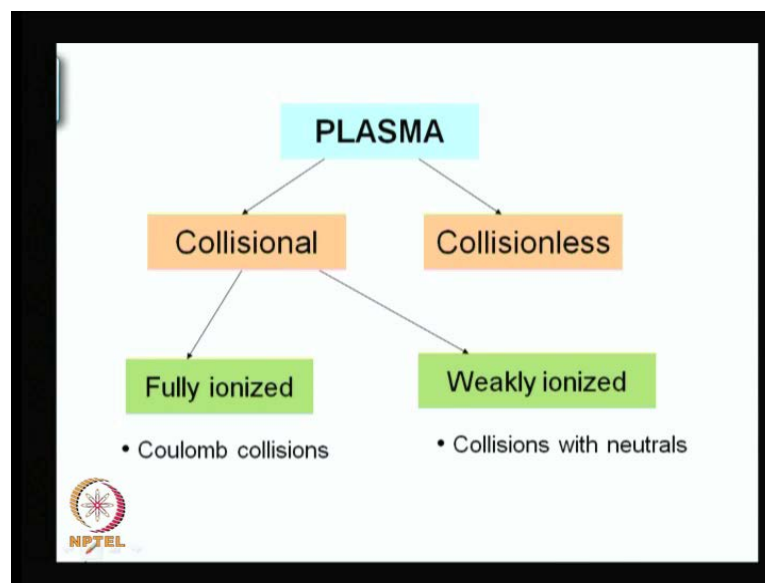
**Plasma Physics**  
**Prof. Vijayshri**  
**School of Sciences**  
**IGNOU**

**Module No. # 01**

**Lecture No. # 39**

**Diffusion in Magnetized Plasmas**

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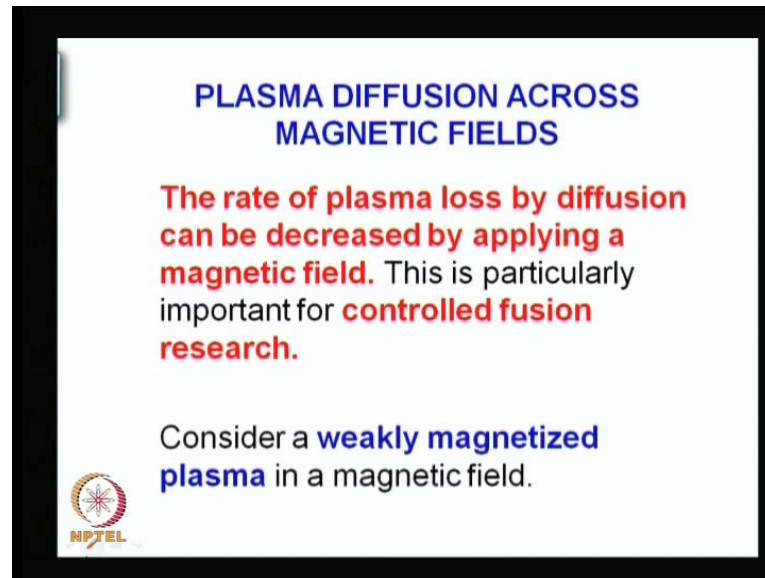
Today's lecture, lecture 39 is about diffusion in magnetized plasmas. If you recall the previous lecture 38 you will remember that we have taken up the case of collisional plasmas, and we have discussed weakly ionized plasmas but, without magnetic fields.

So, we have seen the effect of collisions in weakly un-magnetized plasmas, where the problem was stated like this, that we had considered the collision of charged particles with neutral atoms, there was a dense gas of neutral atoms in which the charged particles exhibited a random walk process.

Today, we are going to take up weakly ionized plasma once again but, now it is a magnetized plasma and if you remember last time, we had not discussed fully ionized plasmas. This time we will also take up fully ionized magnetized plasmas and describe

the effect of coulomb collisions, and find an expression for the diffusion coefficient, which we have already defined in the last lecture.


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**PLASMA DIFFUSION ACROSS  
MAGNETIC FIELDS**

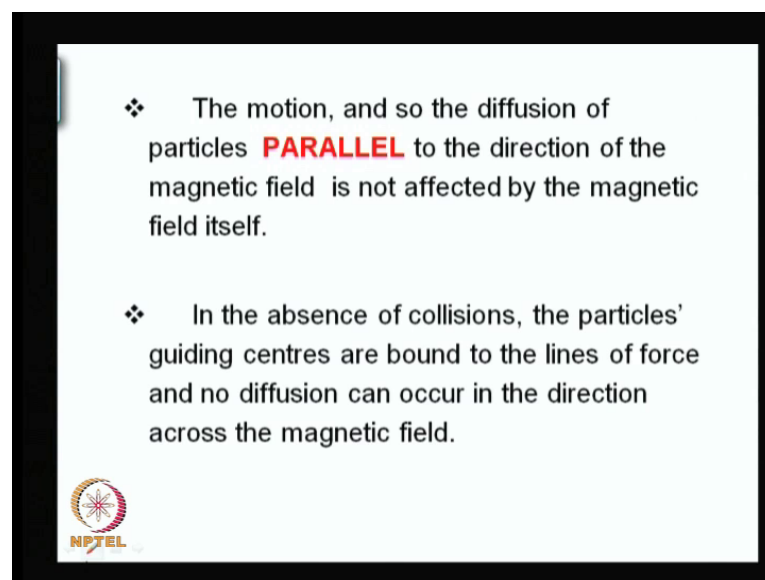
**The rate of plasma loss by diffusion can be decreased by applying a magnetic field.** This is particularly important for **controlled fusion research.**

Consider a **weakly magnetized plasma** in a magnetic field.




So, we will study plasma diffusion across magnetic fields and why is it important? It is very important a problem in controlled fusion research, it has been found that the rate of plasma loss by diffusion can be decreased by applying a magnetic field. Today we are going to see how this is done.

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- ❖ The motion, and so the diffusion of particles **PARALLEL** to the direction of the magnetic field is not affected by the magnetic field itself.
- ❖ In the absence of collisions, the particles' guiding centres are bound to the lines of force and no diffusion can occur in the direction across the magnetic field.



Let us first take up the case of a weakly ionized plasma, a weakly magnetized plasma. So, the weakly ionized plasma is in a magnetic field, qualitatively you know that, because of the magnetic field there is the  $\mathbf{v} \times \mathbf{B}$  force on the charge particles in the plasma.

So, we can say that the motion and therefore, the diffusion of the particles parallel to the direction of the magnetic field is not affected by the magnetic field, because the  $\mathbf{v} \times \mathbf{B}$  force is zero when  $\mathbf{v}$  is parallel to  $\mathbf{B}$ . In the absence of collisions the particles move in circles in the magnetic field and then in a helical path in the magnetic field, and the guiding centers are bound to the lines of force.

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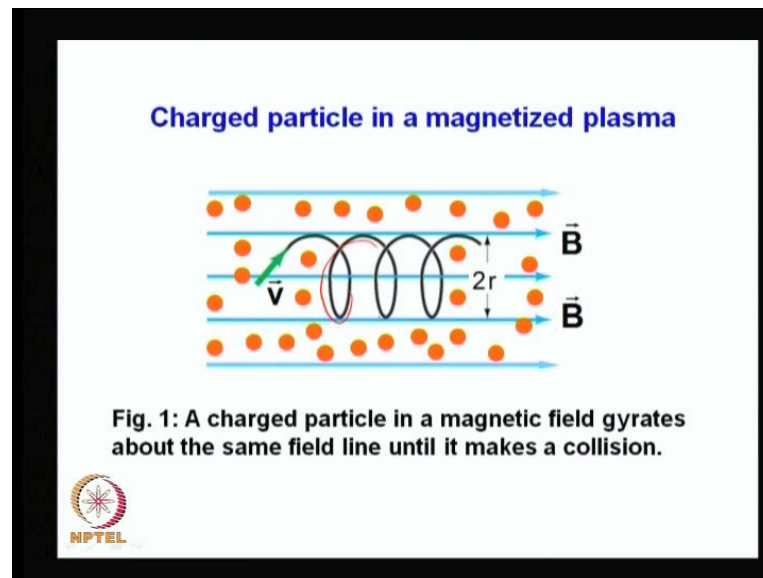



Fig. 1: A charged particle in a magnetic field gyrates about the same field line until it makes a collision.

So, no diffusion occurs in the direction perpendicular to the magnetic field across the magnetic field, it is a situation like this figure. Here I show you charged particles in magnetized plasma, a single charged particle motion in a magnetic field, it is following a helical path, it is bound with the line of force in the magnetic field, the path of the plasma, path of the electron or a charged particle here.

This helix or the helical path is the path of the charged particle, it gyrates about the same field line until it makes a collision, this is the coalitional problem that we are taking up. So, a charged particle will **gyrate** gyrate about the same field line till it makes a collision and what happens when it makes a collision?

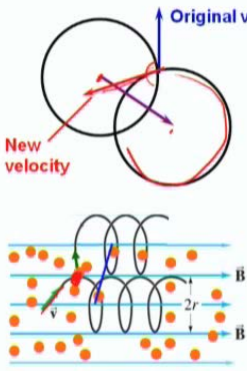
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**Collisions disrupt the particle orbit**  
producing, on average, a random-walk type of diffusion towards the wall. **Collisions abruptly change the velocity of the particle gyrating around the field lines. This causes abrupt change in the direction in which the charged particle travels, that is, it changes the position of the guiding centre.**




The orbit of the particle is disrupted and a random-walk, we can again take it as a random walk type of diffusion. So, what happens is collisions abruptly change the velocity of the particle gyrating around the field line.

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**Fig. 2: Collision of gyrating particles with neutral atoms.**



So, the direction in which the particle moves is changed and therefore, the position of the guiding center is also changed, you can understand this better from this figure. Here, there is a particle gyrating in around field line and let us say the first figure shows you when you are looking at the particle in a transverse direction.

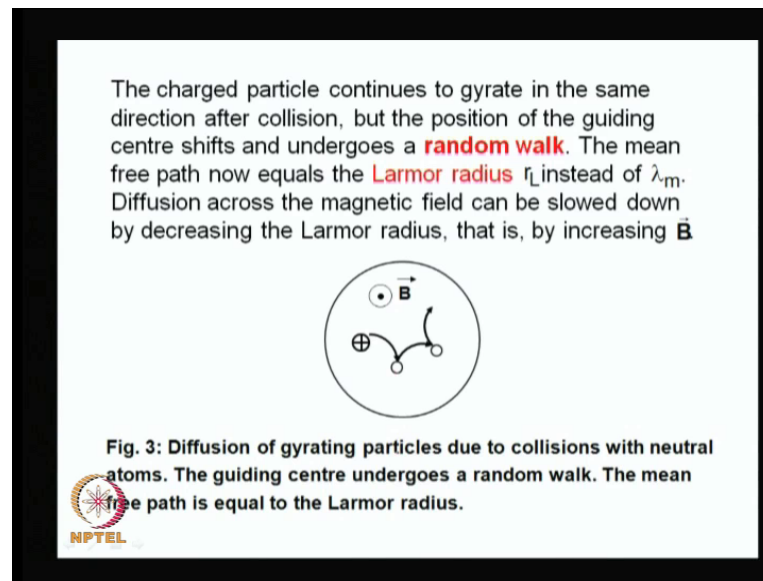
So, you **you** see a circle and you see the original velocity, **at this point** at this point the particle collides with another charged particle so, the velocity changes direction this is the direction of the new velocity the red arrow.

And therefore, the guiding center shifts, this is then the new path around which the particle gyrates, the new circle, this circle and the guiding center shifts from this point to this point. If you look at it from a distance then the lower figure shows you what is happening, here is the original path of the charged particle, it is gyrating about a field line.

Now, look at this particle in the plasma. Say the electron while gyrating about the field line collides with this particular particle, what happens? The direction of its velocity changes, and now it gyrates about a different field line and the guiding center shifts so, the charged particle has moved across the B field, because of the collision. This is how particles diffuse in a collisional plasma in the presence of a magnetic field.

Let me just demonstrate this again to you, this is the path of the original particle, now it collides as it is moving along this path, it collides with the particle here in the plasma and acquires a different velocity in a different direction. So, it starts gyrating in a different path though in the same direction along a different line of force, and its guide the guiding center shifts so, there is a shift, there is a diffusion of the particles across the magnetic field.

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So, this is how diffusion takes place in a magnetized plasma, the charged particle continues to gyrate in the same direction after collision, as you have seen in the previous graphic but, the position of the guiding center shifts, and the guiding center undergoes a random walk.

But the mean free path now is equal to the larmor radius instead of the lambda m, earlier it was the charged particle that was undergoing a random walk, in a magnetized plasma it is the guiding center which undergoes a random walk, and the step length is the larmor radius. Now, diffusion across the magnetic field can be slow down by decreasing the larmor radius that is, by increasing the magnetic field.

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**Equation of motion for a weakly ionized plasma in a magnetic field**

We study the diffusion **across the magnetic field** by taking the equation of motion in the plane perpendicular to the magnetic field for a steady state isothermal plasma for either species:

$$m n \frac{d\vec{v}_\perp}{dt} = \pm e n (\vec{E} + \vec{v}_\perp \times \vec{B}) - k_B T \vec{\nabla} n - m n \nu \vec{v}_\perp$$


So, we have to find out the **the** diffusion coefficient for a weakly ionized plasma in a magnetic field, for the diffusion of particles in a weakly ionized plasma placed in a magnetic field.

As usual, we will start with the equation of motion since, the motion in the direction parallel to the magnetic field is absent, there is no motion in the direction in which  $\vec{V}$  is parallel to  $\vec{B}$  therefore, we will study the diffusion in a direction perpendicular to the magnetic field, across the magnetic field writing the equation of motion in a plane perpendicular to the magnetic field.

Once again, we take the study state isothermal plasma and write down the equation of motion, it is written here  $m n \frac{d\vec{v}_\perp}{dt} = \pm e n \vec{E} + \vec{v}_\perp \times \vec{B} - k_B T \text{grad } n - m n \nu \vec{v}_\perp$  the density gradient minus  $m n \nu \vec{v}_\perp$  the momentum loss.


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**Equation of motion for a weakly ionized plasma in a magnetic field**

$$mn \frac{d\vec{v}_\perp}{dt} = \pm en(\vec{E} + \vec{v}_\perp \times \vec{B}) - k_B T \nabla n - mn v \vec{v}_\perp$$

Assumptions: Large  $\nu$  so that the term  $\frac{d\vec{v}_\perp}{dt}$  can be neglected; electric field zero; density gradient in the positive x-direction.

$$\vec{0} = \pm en(\vec{v}_\perp \times \vec{B}) - k_B T \frac{\partial n}{\partial x} \hat{i} - mn v \vec{v}_\perp$$

$$mn v \vec{v}_\perp = \pm en(\vec{v}_\perp \times \vec{B}) - k_B T \frac{\partial n}{\partial x} \hat{i}$$


So, this is the equation of motion for the perpendicular component of the velocity. Once again, we solve it under certain assumptions, the way we did for un-magnetized plasmas we take the collision frequency to be large so that, in comparison to this term, the term  $D v \text{ perp upon } D t$  can be neglected, for simplicity we take the electric field to be zero and the density gradient in the positive  $x$  direction you will see that the results arrived **are at the same** are the same, but the algebra becomes pretty simple.


So, let us see what this equation of motion, reduces to under these assumptions. The term on the left hand side is zero and we have also put the electric field equal to zero. So, we are left with three terms now the  $V \text{ perp cross } B$ , the density gradient which is parallel to the  $x$  axis, and the momentum last term. From here we can write down an expression for  $V \text{ perp}$  by taking the last term to the left hand side, we are left with the two terms  $V \text{ perp cross } B$  and the density gradient in the  $x$  direction on the right hand side.



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$$mnv\vec{v}_\perp = \pm en(\vec{v}_\perp \times \vec{B}) - k_B T \frac{\partial n}{\partial x} \hat{i}$$

Let us write the **x and y components** of the equation.

$$\begin{aligned}\vec{v}_\perp \times \vec{B} &= (v_x \hat{i} + v_y \hat{j}) \times B \hat{k} \\ &= (-v_x \hat{j} + v_y \hat{i})B\end{aligned}$$
$$mnv v_x = -k_B T \frac{\partial n}{\partial x} \pm en v_y B$$
$$mnv v_y = \mp en v_x B$$


Once again we follow the same process and divide the entire equation by  $mnv$ , but this time the  $v$  perpendicular has two components  $x$  and  $y$  components so, we write the  $x$  and  $y$  components of the equation.

Now here, we have a cross product  $V$  perp cross  $B$ , which you know very well is minus  $V_x \hat{j}$  plus  $V_y \hat{i}$  cap  $B$ , if you apply the definition of the cross product then, you can simply write the equation of motion for the  $V_x$  and  $V_y$  components. You just take the  $x$  component of  $V$  perp and the  $x$  component of  $V$  cross  $B$  term and you get this, the density gradient term is already there, and you get this term from the  $x$  component of  $V$  cross  $B$  term. This is the  $x$  component the  $V_y \hat{i}$  cap similarly, the  $y$  component you do not have in the density gradient.


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The slide contains the following content:

$$v_x = -\frac{k_B T}{m n \nu} \frac{\partial n}{\partial x} \pm \frac{e n v_y B}{m n \nu}$$
$$v_y = \mp \frac{e n v_x B}{m n \nu}$$

We now use the definitions of  $D$  and  $\omega_c$ :

$$\frac{k_B T}{m \nu} = D \quad \omega_c = \frac{e B}{m}$$




So, you are left with only one term, divide by  $m n \nu$  you get expressions for  $V_x$  and  $V_y$ , what is the difference between these two expressions?

From the earlier case, these are now coupled equations you have  $V_y$  in the equation for  $V_x$  and  $V_x$  in the equation for  $V_y$  so, how do you solve this? You have to get an expression for  $V_{\text{perp}}$  and you have to write the equation, continuity equation.

So, what do you do? You just, before getting into the algebra we will also make use of the fact that,  $D$  the diffusion coefficient is  $k_B T$  upon  $m \nu$  and we will use the definition of  $\omega_c$  equal to  $e B$  upon  $m$ , and write  $V_x$  and  $V_y$  in terms of  $D$  and  $\omega_c$ .

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$$\checkmark v_x = -\frac{D}{n} \frac{\partial n}{\partial x} \pm \frac{\omega_c}{v} v_y$$

$$\checkmark v_y = \mp \frac{\omega_c}{v} v_x$$


**Substitute  $v_x$  in  $v_y$**

$$v_y = \mp \frac{\omega_c}{v} \left[ -\frac{D}{n} \frac{\partial n}{\partial x} \pm \frac{\omega_c}{v} v_y \right]$$

$$v_y = \pm \frac{\omega_c D}{nv} \frac{\partial n}{\partial x} - \frac{\omega_c^2}{v^2} v_y$$

So, in terms of D and omega c obviously you can see that the expressions are much simpler. Now, we substitute  $V_x$  in  $V_y$   $V_x$  from this equation in  $V_y$  in  $V_y$ ,  $V_x$  from here in  $V_y$ . What do we get? It is a matter of algebra, we have set  $V_y$  is equal to minus plus omega c upon nu  $V_x$  so, you substitute  $V_x$  here is minus so, minus plus and this is minus this becomes plus minus omega c D upon n nu, del n upon del x minus omega c square upon nu square  $V_y$ .

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$$v_y = \pm \frac{\omega_c D}{nv} \frac{\partial n}{\partial x} - \frac{\omega_c^2}{v^2} v_y$$

$$\left| v_y \left( 1 + \frac{\omega_c^2}{v^2} \right) = \pm \frac{\omega_c D}{nv} \frac{\partial n}{\partial x} \right.$$

$$\underline{\omega_c = \frac{eB}{m}} \qquad \underline{\frac{k_B T}{mv} = D}$$

$$\underline{\frac{\omega_c D}{v}} = \frac{\omega_c}{v} \left( \frac{k_B T}{mv} \right) \frac{eB}{eB} = \frac{\omega_c}{v^2} \left( \frac{eB}{m} \right) \left( \frac{k_B T}{eB} \right) = \underline{\underline{\frac{\omega_c^2}{v^2} \left( \frac{k_B T}{eB} \right)}}$$

So, this equation you have  $V_y$  alone and the density gradient now, you take this term to the left side, minus  $\omega_c^2$  upon  $\nu^2$   $V_y$  and you get  $V_y$  equal to  $V_y$  multiplied by  $1 + \omega_c^2$  upon  $\nu^2$  equal to plus minus  $\omega_c D$  upon  $\nu \frac{\partial n}{\partial x}$ .

Once again, we use the definitions  $\omega_c$  and  $D$  and we write this term  $\omega_c D$  upon  $\nu$  here, the coefficient of the density gradient, do some simple algebra substitute for  $D$ , multiply and divide by  $eB$  so, we can express this as  $\omega_c^2$  upon  $\nu^2$   $k_B T$  upon  $eB$ , this is also that the final expressions look simpler and elegant.

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$$v_y \left(1 + \frac{\omega_c^2}{\nu^2}\right) = \pm \frac{\omega_c D}{\nu} \frac{\partial n}{\partial x}$$

$$\frac{\omega_c D}{\nu} = \frac{\omega_c^2}{\nu^2} \left(\frac{k_B T}{eB}\right), \quad \frac{1}{\nu^2} = \tau^2$$

$$v_y (1 + \omega_c^2 \tau^2) = \pm \omega_c^2 \tau^2 \frac{k_B T}{eB} \frac{1}{\nu} \frac{\partial n}{\partial x}$$

So, we rewrite this equation for  $V_y$  as  $V_y$   $1 + \omega_c^2 \tau^2$ , which is nothing but,  $1 + \omega_c^2 \tau^2$  this is what we have used. And  $\omega_c D$  upon  $\nu$  as  $k_B T$  upon  $eB$ ,  $\omega_c^2$  upon  $\nu^2$   $1 + \omega_c^2 \tau^2$  we have written as  $\tau^2$ .

So, we get an equation for  $V_y$ , which does not contain  $V_x$ , it is a result in terms of the density gradient. We divide the expression by  $1 + \omega_c^2 \tau^2$  and we get an expression for  $V_y$ .

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Slide content:

$$v_x = -\frac{D}{n} \frac{\partial n}{\partial x} \pm \frac{\omega_c}{v} v_y$$

$$v_y = \mp \frac{\omega_c}{v} v_x$$

Substitute  $v_y$  in  $v_x$

$$v_x = -\frac{D}{n} \frac{\partial n}{\partial x} - \frac{\omega_c^2}{v^2} v_x$$

$$v_x (1 + \omega_c^2 \tau^2) = -\frac{D}{n} \frac{\partial n}{\partial x}$$

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Similarly, by substituting  $v_y$  in  $v_x$  here, this now in this  $v_y$  is equal to minus plus  $\omega_c$  upon  $v$   $v_x$  into the expression for  $v_x$ , we can again derive an expression for just  $v_x$  the way we have done for  $v_y$ ,  $v_x (1 + \omega_c^2 \tau^2)$  is equal to minus  $D$  upon  $n$   $\frac{\partial n}{\partial x}$ .

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Slide content:

$$v_x = -\frac{D}{(1 + \omega_c^2 \tau^2)} \left( \frac{1}{n} \frac{\partial n}{\partial x} \right)$$

$$v_y = \pm \frac{\omega_c^2 \tau^2}{(1 + \omega_c^2 \tau^2)} \frac{k_B T}{eB} \frac{1}{n} \frac{\partial n}{\partial x}$$

$$\vec{v}_\perp = (v_x \hat{i} + v_y \hat{j})$$

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So, we get two equations for  $v_x$  and  $v_y$ , which are now uncoupled and they are now given in terms of the density gradient, this is what we had done for un-magnetized plasmas also. We have a term  $D$  upon  $1 + \omega_c^2 \tau^2$  here. Now, we

have to write an expression for the perpendicular velocity vector, which is again reconstituted by adding up the x and y vector components.

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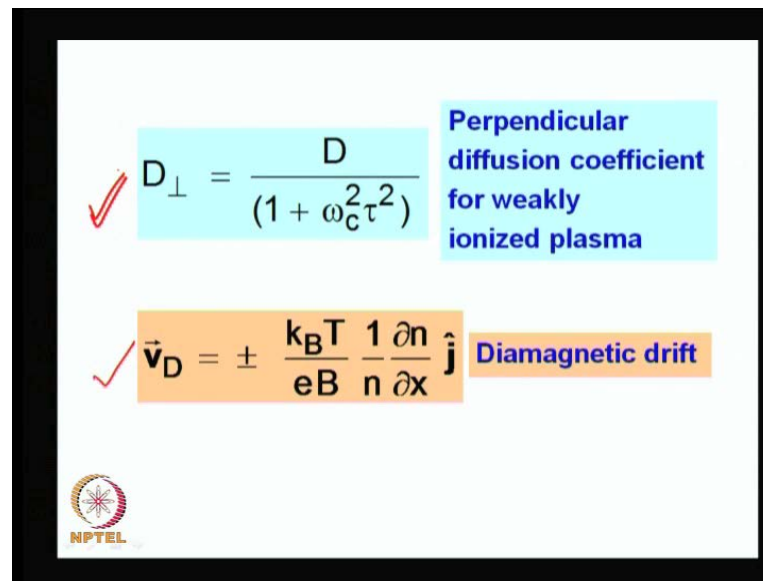
$$\vec{v}_\perp = (v_x \hat{i} + v_y \hat{j})$$

$$\vec{v}_\perp = - \frac{D}{(1 + \omega_c^2 \tau^2)} \left( \frac{1}{n} \frac{\partial n}{\partial x} \right) \hat{i} \pm \frac{\omega_c^2 \tau^2}{(1 + \omega_c^2 \tau^2)} \frac{k_B T}{e B} \frac{1}{n} \frac{\partial n}{\partial x} \hat{j}$$

$$\vec{v}_\perp = - D_\perp \frac{\vec{\nabla} n}{n} + \frac{\vec{v}_D}{(1 + v^2 / \omega_c^2)}$$

So, although the expression looks a bit involved you will find that, this is simply the  $V_x$  term, this is the  $V_y$  term and we define a new quantity called  $D_\perp$  here, and another quantity  $v_D$  here so, that the expression for  $v_\perp$  becomes simple. It becomes minus  $D_\perp$   $\text{grad } n$  upon  $n$ ,  $\text{grad } n$  is simply in the  $x$  direction so,  $\text{del } n$  upon  $\text{del } x$  can be written as  $\text{grad } n$  plus another term  $v_D$  upon  $1 + v^2 / \omega_c^2$ , this is what we have defined to write the second term in a simple expression, this is what we have defined  $D_\perp$  equal to  $D$  upon  $1 + \omega_c^2 \tau^2$  and  $v_D$  equal to plus minus  $k_B T$  upon  $e B$ ,  $1$  upon  $n \text{ del } n$  upon  $\text{del } x$   $\hat{j}$  cap.

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The slide contains two equations, each preceded by a red checkmark. The first equation is  $D_{\perp} = \frac{D}{(1 + \omega_c^2 \tau^2)}$ , with a light blue text box to its right stating "Perpendicular diffusion coefficient for weakly ionized plasma". The second equation is  $\vec{v}_D = \pm \frac{k_B T}{eB} \frac{1}{n} \frac{\partial n}{\partial x} \hat{j}$ , with a light orange text box to its right stating "Diamagnetic drift". In the bottom left corner of the slide, there is a circular logo with a star and the text "NPTEL" below it.

So, this particular term except for this the frequency dependent term can be written as  $D_{\perp}$ , and if you divide this whole thing by  $\omega_c^2 \tau^2$  you can write the expression as  $\frac{D}{1 + \omega_c^2 \tau^2}$ .

So, the  $D_{\perp}$  is the perpendicular diffusion coefficient for weakly ionized plasmas, and what is this term  $\vec{v}_D$ , that we have defined. It is actually a diamagnetic drift term, write now we are not going into the diamagnetic drifts across a magnetized plasmas, we will be concentrating on the diffusion coefficients.

So, we have found a diffusion coefficient for magnetized, weakly ionized plasmas as  $D_{\perp}$ , you can see that it depends on the collision frequency, because you have  $\tau$  here or  $\frac{1}{\nu}$  and also on  $\omega_c^2$ .


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**Diffusion Across a Magnetic Field for Weakly Ionized Plasmas**

$$\checkmark \vec{v}_\perp = -D_\perp \frac{\vec{\nabla}n}{n} + \frac{\vec{v}_D}{(1 + v^2/\omega_c^2)} \quad D_\perp = \frac{v^2 D}{(v^2 + \omega_c^2)} \checkmark$$

The perpendicular velocity of either species is composed of two parts:

- The **diamagnetic drifts**, perpendicular to the gradients of potential and density. These drifts are **slowed down** by the collisions with neutrals.
- The **diffusion drifts**, **parallel** to the gradients in potential and density. These are reduced as compared to the case when **B** is zero. These are **enhanced** by the presence of collisions (these terms would be zero in absence of collisions because the guiding centers will stay on the same line of force).



So, let me summarize this discussion on diffusion across a magnetic field for weakly ionized **pro/plasma** plasma, we have derived an expression for the perpendicular velocity term.

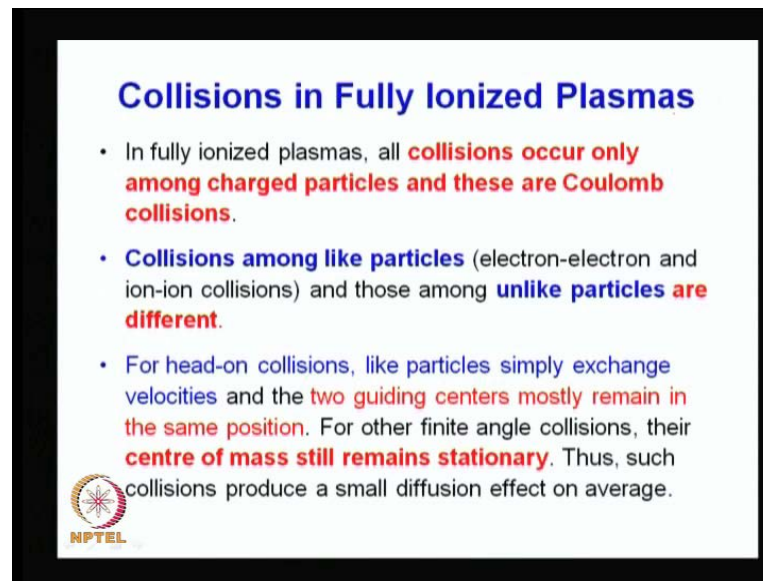
We have found that, there is an extra term due to the diamagnetic drift, write now we do not pay any attention to that, and we have defined the perpendicular diffusion coefficient

The diamagnetic drifts are perpendicular to the gradients of potential and density, and these are slowdown by collisions with neutral atoms, because there you have the collision frequency. The diffusion drifts on the other hand are parallel to the gradients in density, the diffusion drifts are parallel to the density gradients these are in the direction of the density gradient, and these are reduced as compare to the case when B is zero, when you have B you have a term  $\omega_c$  here so, the diffusion drift is reduced.

However, the presence of collisions enhances this phenomenon of diffusion, because we can see that, this term would be zero if the collisions were absent, you have  $\nu^2 D$  upon  $\nu^2 + \omega_c^2$ . So, if  $\nu$  was zero,  $D_\perp$  would be zero there would be no diffusion across the magnetic field, this is in the direction of the density gradient it is reduced as compare to the case when the magnetic field is zero, and these are enhanced by the presence of collision, the diffusion processes enhanced. So, this in a nutshell is diffusion across a magnetic field for a weakly ionized plasma.




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**Collisions in Fully Ionized Plasmas**

- In fully ionized plasmas, all **collisions occur only among charged particles and these are Coulomb collisions.**
- **Collisions among like particles** (electron-electron and ion-ion collisions) and those among **unlike particles are different.**
- For head-on collisions, like particles simply exchange velocities and the **two guiding centers mostly remain in the same position.** For other finite angle collisions, their **centre of mass still remains stationary.** Thus, such collisions produce a small diffusion effect on average.

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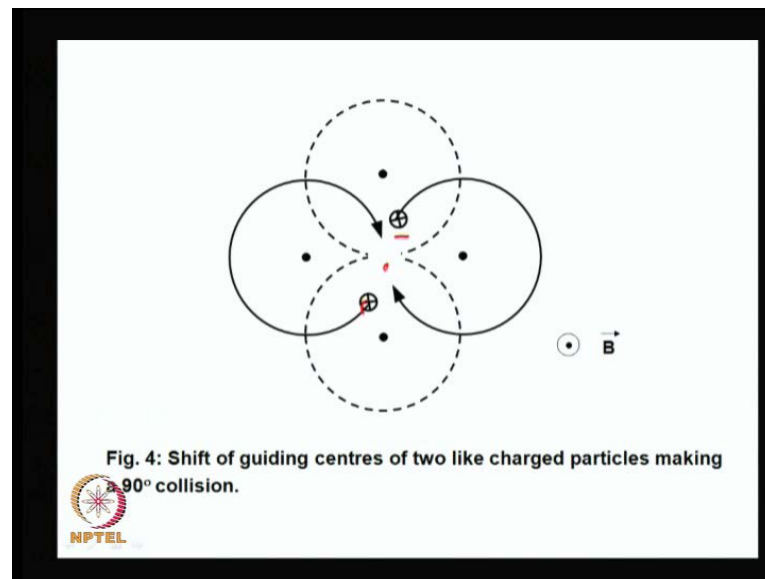
Now, we come to the case of collisions in fully ionized plasmas, in fully ionized plasmas all collisions occur only among charge particles, in weakly ionized plasmas you will remember the collisions were between charged particles and neutral atoms now, the collisions are between charged particles and these are the coulomb collisions or the Rutherford scattering that I have shown you last time.

Now, if we want, if we look at the collisions between charged particles, there is a difference between, collisions between, like particles and collisions between unlike particles, for example, if it is an electron, electron collision or an ion, ion collision the masses are about the same, what happens is simply that the velocities get exchanged and the guide, and the center of mass of the system remains just where it was, the guiding centers mostly remain in the same position.

So, for our calculations we can neglect this, there is no diffusion as such for or very small diffusion for like particles collisions, particularly for head-on collisions you can remember for, you can go back to your college physics and you can see that the two particles of equal mass just exchange their velocities.

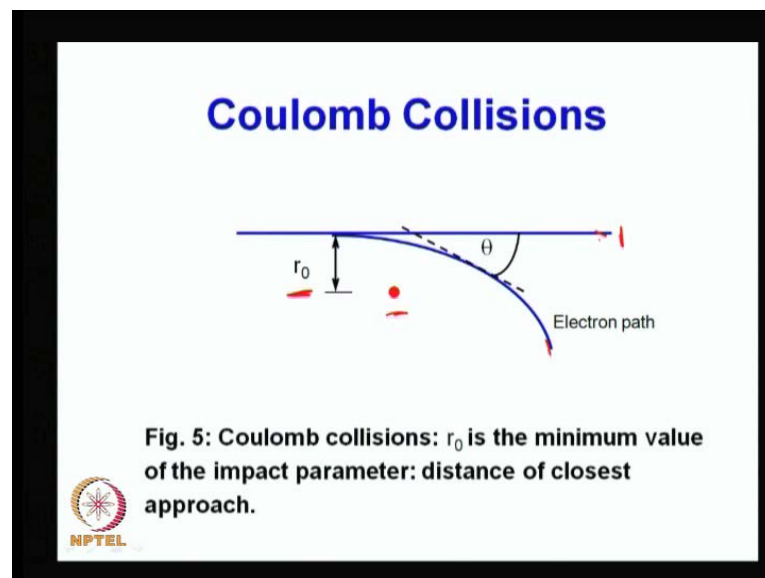
And for other finite angle collisions, the center of mass of the system remains stationary and so on, an average such collisions produce small diffusions, which we can neglect.

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This is the figure which shows the collision of like particles, this is one particle and this is the other particle, they are gyrating, they collide and their paths shift in a manner that, the center of mass remains the same it is stationary, it is a 90 degree collision between the particles.

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Now, what is of interest here in the diffusion phenomena, is the collision of unlike particles, collision of electrons with ions or ions with electrons in the plasma fluid. And here we take records to coulomb collision and coulomb collisional cross section.

What happens in a coulomb collision is that, a charged particle, as a charged particle approaches an ion, it undergoes scattering and follows a hyperbolic path, if we go back again to our college physics, we can find out the path of charged particles scattering. In this figure, we have shown the collision at the distance of closest approach, this distance  $r_0$  here is the impact parameter, it is the minimum value of the impact parameter. So, it is called the distance of closest approach and this is the electron path the solid line, this is the original path of the electron this, horizontal line and it gets deflected in the collision, because of the presence of the ion.


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In an electron-ion collision,  $m_i \gg m_e$  and we may assume that the ion remains fixed during the collision. The Coulomb force is:

$$F = -\frac{e^2}{r^2}$$

This force acts for a time  $r_0/v$ , where  $r_0$  is the minimum value of the impact parameter.

Change in momentum:  $|Ft| = \frac{e^2}{r_0 v}$



Now, in an electron ion collision, the mass of the ion is much larger than the mass of the electrons so that, we can assume that the center of mass is at the ion **and at the ion** and the ion remains almost fixed during the collision.


The coulomb force is simply between the two, if we do not take the atomic number into account, to keep things simple we can write the coulomb force as  $F$  equal to minus  $e$  square upon  $r$  square. If the velocity is  $V$  then and the distance of closet approaches  $r_0$  then, this force acts for a time  $r_0$  upon  $V$ .

So, the change in momentum is just the average force multiplied by the time so, you multiply  $e$  square upon  $r$  square, by  $r_0$  upon  $V$ , we can put  $r$  equal to  $r_0$  there so, this is the change in momentum  $e$  square upon  $r_0 V$ .

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**Large angle scattering**


When the Coulomb potential energy  $-e^2/r$  equals the electron kinetic energy  $m_e v^2/2$ , the scattering angle is  $90^\circ$ .



And we are right now discussing large angle scattering, when the coulomb potential energy is equal to the kinetic energy of the electron, this scattering angle is ninety degrees and at those scattering angles the change in the momentum is of the order of momentum itself.

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For  $90^\circ$  collisions, the change in momentum is of the order of momentum itself. Thus,

$$\Delta (mv) \cong mv \cong \frac{e^2}{r_0 v}, \quad r_0 = \frac{e^2}{m v^2}$$
$$\sigma = \pi r_0^2 = \frac{\pi e^4}{m^2 v^4}$$
$$v_{ei} = n \sigma v = \frac{n \pi e^4}{m^2 v^3}$$


So, we have an equation which gives us the change in momentum as  $e^2$  upon  $r_0 v$ , from where we get an expression for the impact parameter as  $r_0$  equal to  $e^2$  upon  $m v^2$ .

And once again using the classical total collisional cross section as  $\pi r_0^2$ , we simply put  $r_0^2$  here we get the collisional cross section as  $\pi e^4 / m^2 v^4$  and therefore, the collisional frequency is simply  $n \sigma v$ , we have defined it earlier and this is equal to  $n \pi e^4 / m^2 v^3$  so, we have our collision parameters.


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### Debye shielding

**Debye shielding** imposes a maximum value on the impact parameter.

In a plasma, the positive charges attract the electrons and repel the ions, making a cloud of net negative charge around themselves, reducing (**shielding**) the electric field the positive charges make.


But the electrons have too much **thermal energy** and they cannot just collapse onto the ions to completely neutralize them.



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Now, there is something which we need to discuss here is Debye shielding which you have studied earlier in these lectures. Debye shielding imposes a maximum value on the impact parameter, as you know in a plasma, the positive charges attract the electrons and repel the ions and so a net negative charge accumulates around the positive charges, thus producing or shielding the electric field due to the positive charges, on the other hand electrons also have thermal energy.

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**Debye shielding:** The situation when due to inter-particle collisions, the competition between Coulomb attraction and thermal motion comes to an equilibrium.

**Debye length  $D$ :** it is the “**screening**” distance, or the distance over which the usual Coulomb  $1/r$  field decays exponentially due to the polarization of the plasma.

So they cannot just collapse on to the ions so, there is an equilibrium which is established between the attraction, force of attraction of the ion on the electron and, because of the collision, due to collisions, because of the movement of electrons away.

So, due to inter particle collisions, coulomb attraction and thermal motion comes into equilibrium and the field of the plasma is shielded from any external fields, this is the Debye shielding. You also studied the parameter Debye length, which is the screening distance over which the coulomb field decays exponentially.

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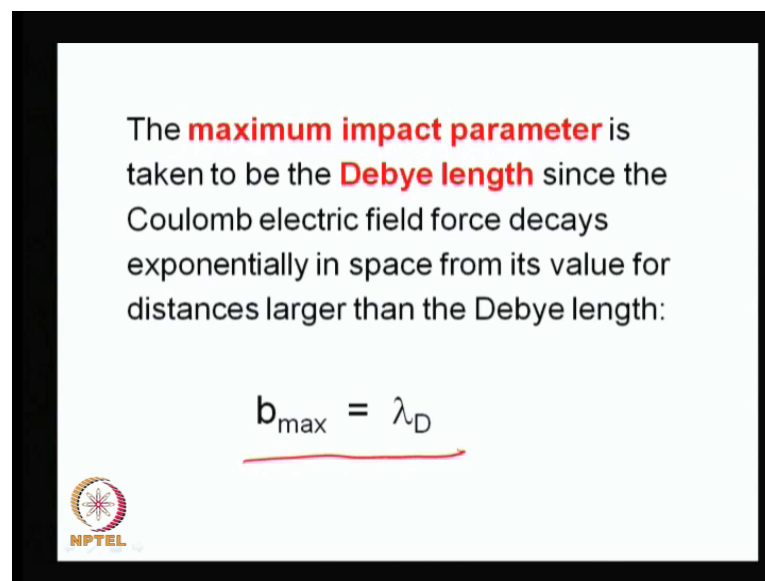


If the **Debye length is smaller than the size of the gas**, as occurs in various types of plasmas, then **electric fields applied to such plasmas don't penetrate into them any deeper than a few Debye lengths.**


So, beyond the Debye length the coulomb field of the charged particle falls very fast and so, within the plasma electric fields applied to the plasmas beyond a few Debye lengths do not matter, the plasma is shielded so the upper limit of the impact parameter then becomes equal to the Debye length.

So, we are considering a collision between  $r_0$  and the Debye length  $\lambda_D$ , and we have seen the parameters for ninety degrees collisions for small **angle**, so the  $b_{\text{maximum}}$  impact parameter is simply the Debye length.

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The **maximum impact parameter** is taken to be the **Debye length** since the Coulomb electric field force decays exponentially in space from its value for distances larger than the Debye length:


$$b_{\text{max}} = \lambda_D$$


We have already written the collision frequency for 90 degrees or large angle scattering for the small angle scattering effects, a small correction which is called as spitzer correction needs to be added.

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Correction to collision cross-section due to Debye shielding: small angle scattering.  
 Small angle scattering effects turn out to be much larger than the large-angle collisions.

**Spitzer correction:**

$$\ln \Lambda \quad \text{where } \Lambda = \frac{\lambda_D}{r_0}$$


And we will not going to the calculations, because these are pretty involved, with this we can now once again try and derive the coefficient of diffusion in a fully ionized plasma.


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**Equations of motion with charged particle collisions**

We neglect the convective term and the terms due to like particle collisions, which do not give rise to much diffusion as explained earlier:

$$\left\{ \begin{aligned} Mn \frac{d\vec{v}_i}{dt} &= en(\vec{E} + \vec{v}_i \times \vec{B}) - \vec{\nabla} p_i + \vec{P}_{ie} \\ mn \frac{d\vec{v}_e}{dt} &= -en(\vec{E} + \vec{v}_e \times \vec{B}) - \vec{\nabla} p_e + \vec{P}_{ei} \end{aligned} \right\}$$

The last term in each equation represents the respective momentum gain caused by collisions of ion fluid with electrons and electron fluid with ions.




Once again, we write the equations for both this species,  $m n \frac{d\vec{v}_i}{dt}$  is equal to  $e n \vec{E} + \vec{v}_i \times \vec{B}$  minus  $\text{grad } p_i$  plus  $\vec{P}_{ie}$  and similarly, the equation for the electrons. The last term in these equations  $p_{ie}$  and  $P_{ei}$  are the momentum terms, momentum gain caused by collisions of ions with electrons and electrons with ions. Once again, we will be making certain simplifications and we will study steady state and isothermal plasmas.



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$$Mn \frac{d\vec{v}_i}{dt} = en(\vec{E} + \vec{v}_i \times \vec{B}) - \vec{\nabla} p_i + \vec{P}_{ie}$$
$$mn \frac{d\vec{v}_e}{dt} = -en(\vec{E} + \vec{v}_e \times \vec{B}) - \vec{\nabla} p_e + \vec{P}_{ei}$$

For **steady state** and **isothermal plasmas**:

$$\vec{0} = en(\vec{E} + \vec{v}_i \times \vec{B}) - k_B T_i \vec{\nabla} n + \vec{P}_{ie}$$
$$\vec{0} = -en(\vec{E} + \vec{v}_e \times \vec{B}) - k_B T_e \vec{\nabla} n + \vec{P}_{ei}$$


So, the term on the left hand side then becomes zero and for isothermal plasmas, we can write the pressure gradient in terms of the density gradient so, the equations are somewhat simplified.

We have  $e n E$  plus  $v_i$  cross  $B$  minus  $k_B T_i \text{grad } n$  plus  $P_{ie}$  equal to 0, and in the same way we can write the equation for the electron with  $v_i$  replace by  $v_e$ ,  $T_i$  by  $T_e$  and  $P_{ie}$  become instead of  $P_{ie}$  we write  $P_{ei}$ .

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From conservation of momentum

$$\vec{P}_{ie} = -\vec{P}_{ei}$$

In a fully ionized plasma

$$\vec{v}_e \text{ nearly equal to } \vec{v}_i$$

(not exactly equal)



From conservation of momentum  $\vec{P}_{ie}$  is simply minus  $\vec{P}_{ei}$  and in a fully ionized plasma  $v_e$  nearly equals  $v_i$ , it is not exactly equal you will see the implications, if you remember  $p$  is  $m n v$  nu  $i$ .

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$$\vec{P}_{ie} = - mn (\vec{v}_i - \vec{v}_e) v_{ei}$$

$$\vec{P}_{ei} = mn (\vec{v}_i - \vec{v}_e) v_{ei}$$

On physical grounds, we expect the momentum term to be proportional to:

1. The Coulomb force since the collisions are Coulomb collisions;
2. Density of electrons and density of scattering centres (ions), which are equal; and
3. Relative velocity of the two fluids.

$$\vec{P}_{ei} \propto e^2 n^2 (\vec{v}_i - \vec{v}_e) = \eta e^2 n^2 (\vec{v}_i - \vec{v}_e)$$



So, in terms of the velocities you have  $\vec{P}_{ie}$  minus  $m n$  multiplied by  $v_i$  minus  $v_e$  nu  $e i$ .  $\vec{P}_{ei}$  is minus  $\vec{P}_{ie}$  so, you write that expression.

Now, on physical grounds we expect the momentum term to be proportional to three factors, the coulomb force; since the coulomb collisions are coulomb collisions, density of electrons; density of scattering centers which are equal and relative velocity of the two fluids.

So,  $\vec{P}_{ei}$  lets take  $\vec{P}_{ei}$  is proportional to  $e^2 n^2$  and the difference of velocities here, we replace the proportionality constant by  $\eta$  here.


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$$\vec{P}_{ei} \propto e^2 n^2 (\vec{v}_i - \vec{v}_e) = \eta e^2 n^2 (\vec{v}_i - \vec{v}_e)$$

$$\vec{P}_{ei} = m n (\vec{v}_i - \vec{v}_e) v_{ei}$$

$$v_{ei} = \frac{n e^2}{m} \eta \quad \eta = \frac{m v_{ei}}{n e^2}$$

The constant  $\eta$  is the **specific resistivity** of the plasma.



So, we write  $P_{ei}$  as  $m n v_i$  minus  $v_e$  nu  $e I$ , and we have on physical grounds an expression for  $P_{ei}$  in terms of  $\eta$ , and if we compare the two we get  $v_{ei}$  in terms of  $\eta$  or  $\eta$  in terms of the collision frequency.


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Substituting for  $v_{ei}$  we can write the **specific resistivity** for large angle collisions:

$$v_{ei} = n \sigma v = \frac{n \pi e^4}{m^2 v^3}$$

$$\eta = \frac{m}{n e^2} v_{ei} = \left( \frac{m}{n e^2} \right) \left( \frac{n \pi e^4}{m^2 v^3} \right)$$

$$\eta = \frac{\pi e^2}{m v^3}$$



What is this constant  $\eta$ ? We can see that it is nothing but, this specific resistivity of the plasma. How it is so? Let me explain very quickly but, before that there is one expression for  $\eta$ , which we can also write by substituting for the expression of the


collision frequency in terms of the mass, and the charge, and the velocity we get an expression for eta equal to pi e square upon m v cube.

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**Accounting for small angle collisions, we include the Spitzer correction to the specific resistivity :**

$$\eta = \frac{\pi e^2}{m v^3} \ln \Lambda = \frac{\pi e^2}{m v^3} \ln \frac{\lambda_D}{r_0}$$

**Why is  $\eta$  the specific resistivity?**



And if we account for the small angle collisions, we have to multiply by a factor log lambda which is the spitzer correction again, lets come back quickly to the point why we call eta as the specific resistivity.

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**Why is  $\eta$  the specific resistivity?**

$$m n \frac{d\vec{v}_e}{dt} = -en(\vec{E} + \vec{v}_e \times \vec{B}) - \vec{\nabla} p_e + \vec{P}_{ei}$$

Let us take  $B = 0$  and  $k_B T_e$  to be zero in the electron equation of motion. Then for steady state, it reduces to

$$en\vec{E} = \vec{P}_{ei}$$

Since  $\vec{j} = en(\vec{v}_i - \vec{v}_e)$ , we can write

$$\vec{P}_{ei} = \eta e^2 n^2 (\vec{v}_i - \vec{v}_e) = \eta en \vec{j} \quad \vec{E} = \eta \vec{j}$$



If you go back to the equation of motion and put  $p$  is equal to 0 and  $K_B T_e$  equal to 0 in this equation of motion for electrons, for steady state you get a simple expression  $e$  and  $E$  equal to  $P_e i$ . And since  $j$  is  $e n v_i$  minus  $v_e$ , we can write  $P_e i$  is equal to  $\eta e^2 n^2 v_i$  minus  $v_e$  equal to  $\eta e n j$ , from where we get  $E$  is equal to  $\eta j$ . This is nothing but, the ohm's law.

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**Why is  $\eta$  the specific resistivity?**

$$\vec{E} = \eta \vec{j}$$

**This is simply Ohm's law and the constant  $\eta$  is just the specific resistivity.**



And so  $\eta$  is nothing but, the specific resistivity, with this we can now make a calculation for the diffusion coefficient for fully ionized plasma in terms of the specific resistivity. We write the equations of motion, for steady state isothermal plasmas, for electrons let us say we get this particular equation, we have the electric field term the  $\nabla \times \mathbf{B}$  the density gradient and the momentum terms.

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### Diffusion coefficient for fully ionized plasma

The steady state equation of motion for electrons in **fully ionized** and **isothermal** plasmas:

$$\vec{0} = -en(\vec{E} + \vec{v}_e \times \vec{B}) - k_B T_e \nabla n + \vec{P}_{ei}$$

$$e(\vec{v}_e \times \vec{B}) = -e\vec{E} - \frac{k_B T_e}{n} \nabla n - m v (\vec{v}_e - \vec{v}_i)$$

$\vec{E} \perp \vec{B}, \nabla n \perp \vec{B}, \vec{v}_e, \vec{v}_i \perp \vec{B}$



We do a simple algebra and we write  $\vec{E} + \vec{v}_e \times \vec{B}$ , in terms of  $e \text{ gradient } n$  and substitute for  $\vec{P}_{ei}$  here,  $\vec{P}_{ei}$ . We say that  $\vec{E}$  is perpendicular to  $\vec{B}$ , the density gradient is perpendicular to  $\vec{B}$  and the electron and ion velocities are perpendicular to  $\vec{B}$ .


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$$e(\vec{v}_e \times \vec{B}) = -e\vec{E} - \frac{k_B T_e}{n} \nabla n - m v (\vec{v}_e - \vec{v}_i)$$

$\vec{E} \perp \vec{B}, \nabla n \perp \vec{B}, \vec{v}_e, \vec{v}_i \perp \vec{B}$

We take the cross product of this equation and the magnetic field:

$$\vec{B} \times e(\vec{v}_e \times \vec{B}) = -e(\vec{B} \times \vec{E}) - \frac{k_B T_e}{n} (\vec{B} \times \nabla n) - m v [\vec{B} \times (\vec{v}_e - \vec{v}_i)]$$

$$\vec{B} \times e(\vec{v}_e \times \vec{B}) = e(\vec{E} \times \vec{B}) + \frac{k_B T_e}{n} (\nabla n \times \vec{B}) + m v [(\vec{v}_e - \vec{v}_i) \times \vec{B}]$$


Under these assumptions I have written down the equations again for ready reference, under these assumptions we write the equation and take the cross product of this equation and the magnetic field from the left,  $\vec{B} \times e, \vec{v}_e \times \vec{B}$  and from the left you

have this, you have this, and you have this, in all three terms you take the cross product of  $\mathbf{b}$  from the left.

So, on the right hand side minus  $e \mathbf{B} e$ ,  $\mathbf{B}$  cross  $e$  is simply  $e$ ,  $e$  cross  $\mathbf{B}$  similarly, we observe this minus sign in the second term by interchanging the cross product, and in the third term too so, we interchange the vectors in the cross product and now we have to calculate the vector triple product.

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$$\vec{\mathbf{B}} \times e (\vec{\mathbf{v}}_e \times \vec{\mathbf{B}}) = e(\vec{\mathbf{E}} \times \vec{\mathbf{B}}) + \frac{k_B T_e}{n} (\vec{\mathbf{v}}_n \times \vec{\mathbf{B}}) + m v [(\vec{\mathbf{v}}_e - \vec{\mathbf{v}}_i) \times \vec{\mathbf{B}}]$$

$$\checkmark \vec{\mathbf{A}} \times (\vec{\mathbf{B}} \times \vec{\mathbf{C}}) = (\vec{\mathbf{A}} \cdot \vec{\mathbf{C}}) \vec{\mathbf{B}} - (\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}) \vec{\mathbf{C}}$$

$$[\vec{\mathbf{B}} \times (\vec{\mathbf{v}}_e \times \vec{\mathbf{B}})] = (\vec{\mathbf{B}} \cdot \vec{\mathbf{B}}) \vec{\mathbf{v}}_e - (\vec{\mathbf{B}} \cdot \vec{\mathbf{v}}_e) \vec{\mathbf{B}}$$

$$\checkmark \vec{\mathbf{v}}_e, \vec{\mathbf{v}}_i \perp \vec{\mathbf{B}} \quad [\vec{\mathbf{B}} \times (\vec{\mathbf{v}}_e \times \vec{\mathbf{B}})] = B^2 \vec{\mathbf{v}}_e \checkmark$$

$$\checkmark e \vec{\mathbf{v}}_e B^2 = e(\vec{\mathbf{E}} \times \vec{\mathbf{B}}) + \frac{k_B T_e}{n} (\vec{\mathbf{v}}_n \times \vec{\mathbf{B}}) + m v [(\vec{\mathbf{v}}_e - \vec{\mathbf{v}}_i) \times \vec{\mathbf{B}}]$$

From your college physics you know that  $\mathbf{A}$  cross,  $\mathbf{B}$  cross,  $\mathbf{C}$  is  $\mathbf{A}$  dot  $\mathbf{C}$   $\mathbf{B}$ , minus  $\mathbf{A}$  dot  $\mathbf{B}$   $\mathbf{C}$  so, you can apply that result, and what do you get under the condition that  $v_e$  and  $v_i$  are perpendicular to  $\mathbf{B}$ , you get the triple product simply as  $B^2 v_e$  write, it is a simple expression. Substitute that in the original equation you get a simple expression on the left hand side, the right hand side remains the same.


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$$\checkmark \quad \vec{v}_e = \frac{(\vec{E} \times \vec{B})}{B^2} + \frac{k_B T_e}{enB^2} (\vec{\nabla} n \times \vec{B}) + \frac{m v}{e B^2} [(\vec{v}_e - \vec{v}_i) \times \vec{B}]$$

For ions:

$$\vec{0} = en(\vec{E} + \vec{v}_i \times \vec{B}) - k_B T_i \vec{\nabla} n + \vec{P}_{ie}$$

$$\vec{0} = en(\vec{E} + \vec{v}_i \times \vec{B}) - k_B T_i \vec{\nabla} n + mn v (\vec{v}_e - \vec{v}_i)$$

$$-e (\vec{v}_i \times \vec{B}) = e \vec{E} - \frac{k_B T_i}{n} \vec{\nabla} n + m v (\vec{v}_e - \vec{v}_i)$$


You divide then by e B square the entire equation so, you get this equation in which you have divided the previous equation by e B square so, e and E cancelled out in the first term, and here you have e B square in the denominator.


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$$\checkmark \quad \vec{v}_i = \frac{(\vec{E} \times \vec{B})}{B^2} - \frac{k_B T_i}{enB^2} (\vec{\nabla} n \times \vec{B}) + \frac{m v}{e B^2} [(\vec{v}_e - \vec{v}_i) \times \vec{B}] \quad (1)$$

$$\checkmark \quad \vec{v}_e = \frac{(\vec{E} \times \vec{B})}{B^2} + \frac{k_B T_e}{enB^2} (\vec{\nabla} n \times \vec{B}) + \frac{m v}{e B^2} [(\vec{v}_e - \vec{v}_i) \times \vec{B}] \quad (2)$$

Subtracting (1) from (2):

$$(\vec{v}_e - \vec{v}_i) = \frac{k_B (T_e + T_i)}{enB^2} (\vec{\nabla} n \times \vec{B})$$

$$\vec{v}_e = \frac{(\vec{E} \times \vec{B})}{B^2} + \frac{k_B T_e}{enB^2} (\vec{\nabla} n \times \vec{B}) + \frac{m v}{e B^2} \frac{k_B (T_e + T_i)}{enB^2} [(\vec{\nabla} n \times \vec{B}) \times \vec{B}]$$


This is simple algebra, you can repeat the same procedure for ions and write an expression for the ion velocity in terms of E cross B gradient, and the momentum terms. These are the two terms for electron velocity and ion velocity, in these you have an expression for v e minus v i so, we have to still do some more algebra, what we do is we



subtract  $v_i$  from  $v_e$  and we get an expression for  $v_e$  minus  $v_i$  in terms of  $\text{grad } n \times B$ , which we substitute again in these two equations.

So, we get  $v_e$ , all other terms remaining the same the last term has another vector triple product  $\text{del } n \times B, \times B$ , you use the same method to simplify the last term but, the triple product is this time different, it is a cross  $B$  then cross  $c$ , which is a different product apply this and you get a simple answer  $\text{del } n \times B, \times B$  is equal to minus  $\text{del } n B^2$ .


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$$\vec{v}_e = \frac{(\vec{E} \times \vec{B})}{B^2} + \frac{k_B T_e}{enB^2} (\vec{\nabla} n \times \vec{B}) + \frac{m v}{e B^2} \frac{k_B (T_e + T_i)}{enB^2} [(\vec{\nabla} n \times \vec{B}) \times \vec{B}]$$

$$\checkmark (\vec{A} \times \vec{B}) \times \vec{C} = -\vec{A} (\vec{C} \cdot \vec{B}) - \vec{B} (\vec{C} \cdot \vec{A})$$

$$\checkmark [(\vec{\nabla} n \times \vec{B}) \times \vec{B}] = -\vec{\nabla} n (\vec{B} \cdot \vec{B}) + \vec{B} (\vec{B} \cdot \vec{\nabla} n)$$

$$\checkmark [(\vec{\nabla} n \times \vec{B}) \times \vec{B}] = -(\vec{\nabla} n) B^2$$

$$\checkmark \vec{v}_e = \frac{(\vec{E} \times \vec{B})}{B^2} + \frac{k_B T_e}{enB^2} (\vec{\nabla} n \times \vec{B}) - \frac{m v}{e^2} \frac{k_B (T_e + T_i)}{nB^2} (\vec{\nabla} n)$$


So,  $v_e$  becomes, just substitute this term in the third term on the right hand side you get an expression for  $v_e$ , all other terms are the same. Having found  $v_e$ , we substitute it in the continuity equation the way we did earlier, and simplify  $\text{del } \text{dot } n v$ .

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**Continuity equation**

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0$$

$$\vec{v}_e = \frac{(\vec{E} \times \vec{B})}{B^2} + \frac{k_B T_e}{enB^2} (\vec{\nabla} n \times \vec{B}) - \frac{m v}{e^2} \frac{k_B (T_e + T_i)}{nB^2} (\vec{\nabla} n)$$

$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot n \left[ \frac{(\vec{E} \times \vec{B})}{B^2} \right] + \vec{\nabla} \cdot n \left[ \frac{k_B T_e}{enB^2} (\vec{\nabla} n \times \vec{B}) \right] - \vec{\nabla} \cdot n \left[ \frac{m v}{e^2} \frac{k_B (T_e + T_i)}{nB^2} (\vec{\nabla} n) \right] = 0$$

$$\vec{\nabla} \cdot (\vec{\nabla} n \times \vec{B}) = 0 \quad [ \because \vec{\nabla} \cdot (\vec{\nabla} n \times \vec{B}) = \vec{B} \cdot (\vec{\nabla} \times \vec{\nabla} n) - \vec{\nabla} n \cdot (\vec{\nabla} \times \vec{B}) = 0 ] \checkmark$$


$$\vec{\nabla} \cdot (\vec{E} \times \vec{B}) = 0 \quad [ \because \vec{E} \times \vec{B} \text{ is constant} ] \checkmark$$


This is the equation when we substitute  $v_e$ , here you will know from the vector identities that  $\text{del dot del } n \text{ cross } B$  is 0. I have shown you the vector identity  $\text{del dot } E \text{ cross } B$  is also 0 since,  $E \text{ cross } B$  is constant so, this term is 0 and this term is 0, and we are left with two terms here on the right hand side.

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$$\frac{\partial n}{\partial t} + \vec{\nabla} \cdot n \left[ \frac{(\vec{E} \times \vec{B})}{B^2} \right] + \vec{\nabla} \cdot n \left[ \frac{k_B T_e}{enB^2} (\vec{\nabla} n \times \vec{B}) \right] - \vec{\nabla} \cdot n \left[ \frac{m v}{e^2} \frac{k_B (T_e + T_i)}{nB^2} (\vec{\nabla} n) \right] = 0$$

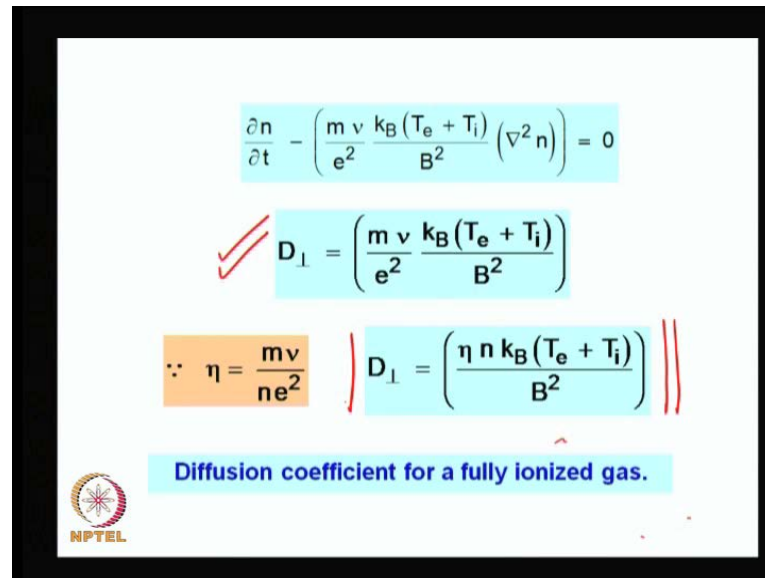
$$\vec{\nabla} \cdot (\vec{\nabla} n \times \vec{B}) = 0 \quad \text{and} \quad \vec{\nabla} \cdot (\vec{E} \times \vec{B}) = 0$$

$$\frac{\partial n}{\partial t} - \left( \frac{m v}{e^2} \frac{k_B (T_e + T_i)}{B^2} (\nabla^2 n) \right) = 0$$


We are left with two terms, I have repeated this expression, in this huge expression two terms are zero this term and this term so, we **we** are left with two terms one of which

gives us the coefficient of del square n, and that is nothing as you know but, the diffusion coefficient, this is nothing but, the perpendicular diffusion coefficient.

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The slide contains the following mathematical derivation:

$$\frac{\partial n}{\partial t} - \left( \frac{m \nu}{e^2} \frac{k_B (T_e + T_i)}{B^2} (\nabla^2 n) \right) = 0$$

$$D_{\perp} = \left( \frac{m \nu}{e^2} \frac{k_B (T_e + T_i)}{B^2} \right)$$

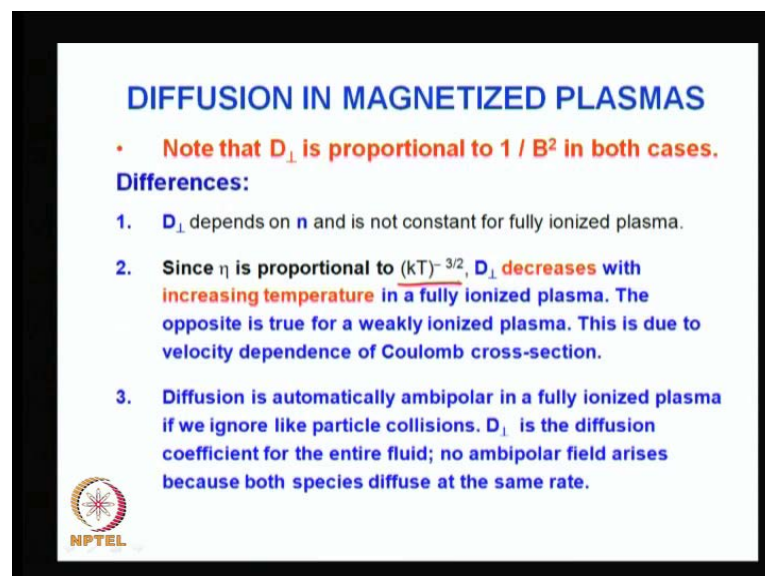
$$\because \eta = \frac{m \nu}{n e^2} \quad \left| \quad D_{\perp} = \left( \frac{\eta n k_B (T_e + T_i)}{B^2} \right) \right|$$

Diffusion coefficient for a fully ionized gas.



And since eta is m nu upon n e square, we can write this diffusion coefficient in terms of eta as eta n K B, T e plus T i upon B square this is the perpendicular diffusion coefficient for a fully ionized gas.

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**DIFFUSION IN MAGNETIZED PLASMAS**

- Note that  $D_{\perp}$  is proportional to  $1 / B^2$  in both cases.

**Differences:**

- $D_{\perp}$  depends on  $n$  and is not constant for fully ionized plasma.
- Since  $\eta$  is proportional to  $(kT)^{-3/2}$ ,  $D_{\perp}$  decreases with increasing temperature in a fully ionized plasma. The opposite is true for a weakly ionized plasma. This is due to velocity dependence of Coulomb cross-section.
- Diffusion is automatically ambipolar in a fully ionized plasma if we ignore like particle collisions.  $D_{\perp}$  is the diffusion coefficient for the entire fluid; no ambipolar field arises because both species diffuse at the same rate.

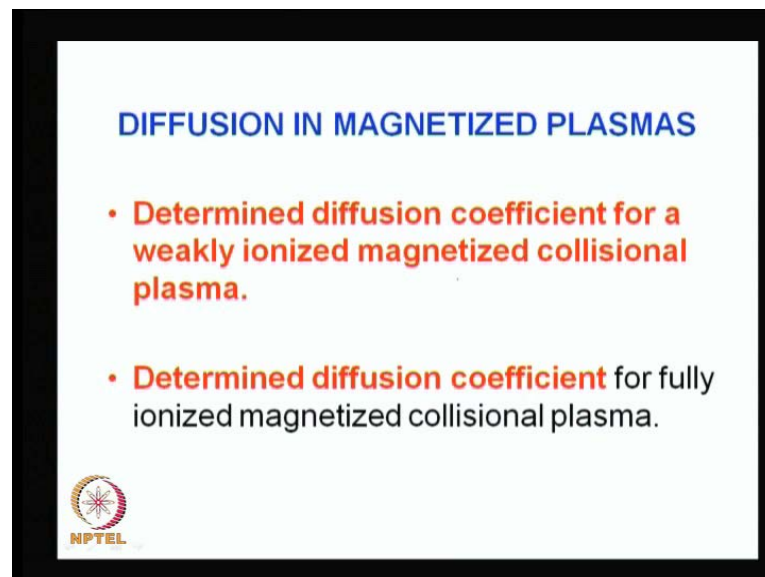


So, we have been able to derive an expression for the perpendicular diffusion coefficient in fully ionized plasmas and weakly ionized plasmas in the presence of magnetic fields.

What, we let us compare quickly the  $D_{\perp}$  in both cases you will note that, they are proportional to  $1/B^2$  so, as you increase the magnetic field diffusion across the field is reduced. However, the difference is that for a fully ionized plasma  $D_{\perp}$  is not constant, it depends on the density. Moreover, the specific resistivity is proportional to  $kT^{-3/2}$  so,  $D_{\perp}$  in a fully ionized plasma decreases as the temperature increases and the opposite is true for the weakly ionized plasmas, this occurs because of the velocity dependence of coulomb cross section.

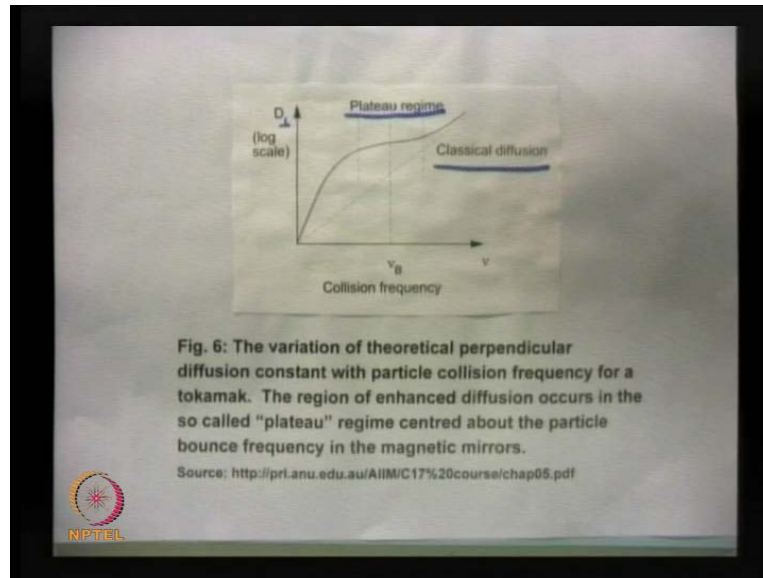
The third difference is that, diffusion is automatically ambipolar in a fully ionized plasma if we ignore like particle collisions, it is the diffusion coefficient for the entire fluid, no ambipolar field arises, because both species diffuse at the same rate so, these are the differences between diffusion in a weakly ionized magnetized plasma and a fully ionized magnetized plasma.

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For the sake of interest, I would like to show you graphic, showing the theoretically determined perpendicular diffusion coefficient  $D_{\perp}$ , this is  $D_{\perp}$  on a logarithmic scale. This is the classical diffusion, which is the expected diffusion we are taking a logarithmic scale and showing  $D_{\perp}$  versus collision frequency.

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There is an enhanced diffusion in this so called plateau regime, it is a, it is an experimentally observed situation and it is still an area of active research for explaining this particular diffusion phenomena, which appears in a tokamak in controlled thermo nuclear fusion.

So, with this we come to an end of the discussion today on diffusion in magnetized plasmas. In this particular lecture, we have determined the diffusion coefficient for a weakly ionized plasma, in a magnetic field the plasma is also collisional so, we have taken into account the effect of collisions in a weakly ionized plasma in a magnetic field then, we have also determined the diffusion coefficient for fully ionized magnetized collisional plasma. So, this is all for today, thank you very much.