Plasma Physics

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Lecture No. # 38
Diffusion in Plasmas

In today's lecture, we will be taking up the topic diffusion in plasmas. Diffusion, why do you need to study diffusion, because it is a very important problem in plasma physics.

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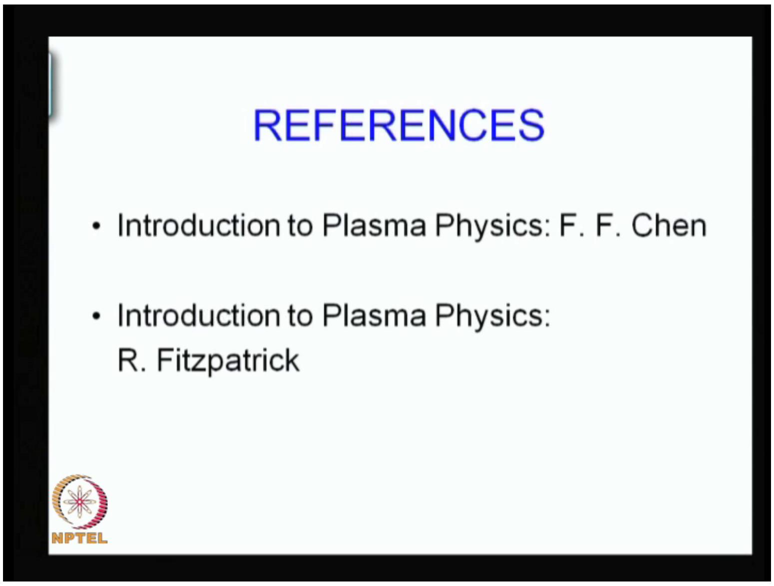

The references, before I go into the details of the problem the I would like to tell you about the references for this particular lecture. These are introduction to plasma physics by F F Chen and introduction to plasma physics by R Fitzpatrick.

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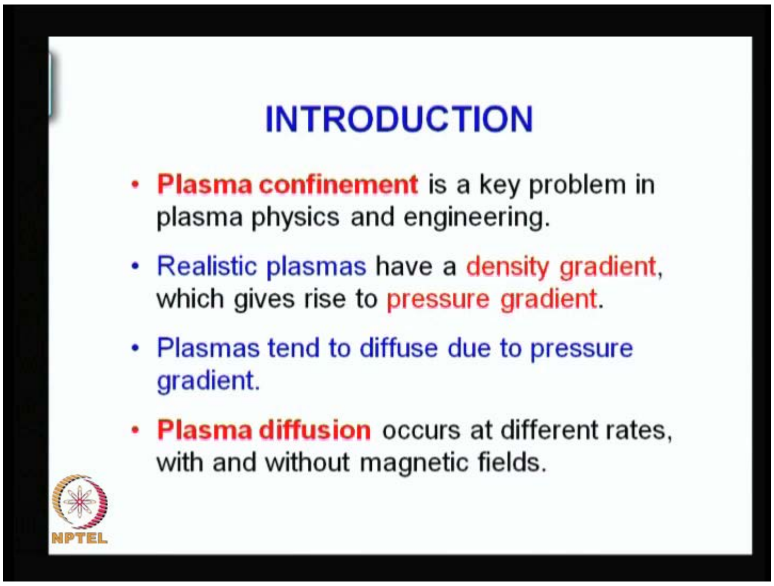

Now, as I said, why do we need to study diffusion phenomena in plasmas. One of the reasons is that plasma confinement is a key problem in plasma physics and engineering. It is very important particularly in controlled thermal nuclear fusion, which is a very active area of research today.

You have learnt in the initial lectures, that realistic plasmas have a density gradient, which gives rise to pressure gradient. Now, plasmas tend to diffuse, because of the pressure gradient. Therefore, it is important when we need to confine plasmas, it is important that we study the diffusion of charge particles in plasmas.

Plasma diffusion can occur at different rates, it is a phenomena phenomenon which happens in the presence of magnetic fields and even in the absence. So with and without magnetic fields, the intend to study the diffusion process in plasmas.

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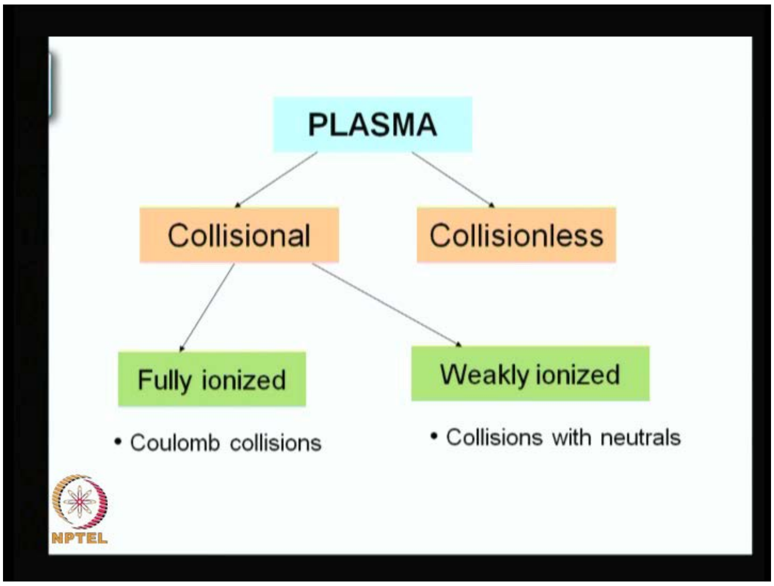

You also know that plasmas are collisional as well as collisionless. Collisional plasmas can be categorized into two types. Fully ionized and weakly ionized. What is the difference between the two types of plasmas, as far as today's lecture is concerned. We will be taking up weakly ionized plasmas, in which we will consider the collisions with of charge particles with neutral atoms. So, we are discussing diffusion in collisional weakly ionized plasmas to begin with in this lecture.

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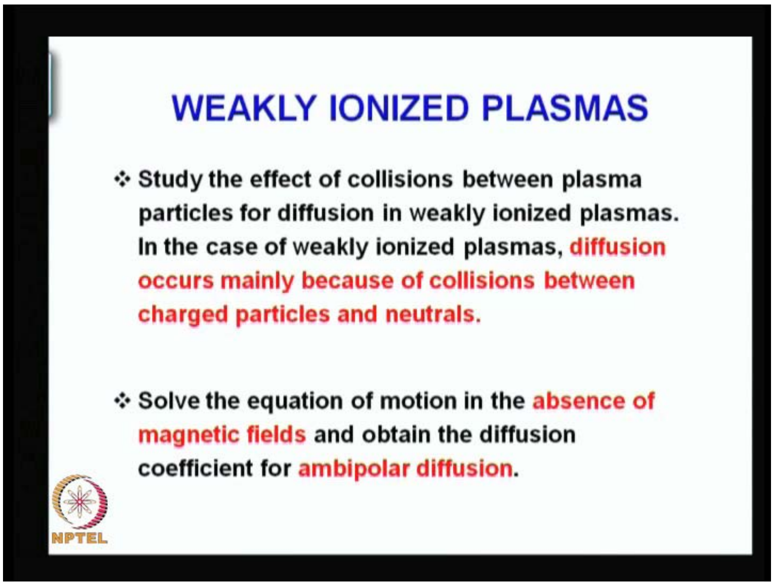

What is a weakly ionized plasma, as I have said, in a weakly ionized plasma collisions occur between charge particles and neutral atoms and so, we study the effect of collisions between plasma particles for the diffusion and diffusion as I said occurs mainly, because of collisions between charge particles and neutral atoms.

In this lecture, we will study the effect of collisions, first define the collision parameters, define the diffusion parameters and solve the equation of motion in the absence of magnetic fields. We will also obtain the diffusion coefficient for ambipolar diffusion and for a very special case of diffusion in laser produced plasmas. Why are weakly ionized plasmas important?

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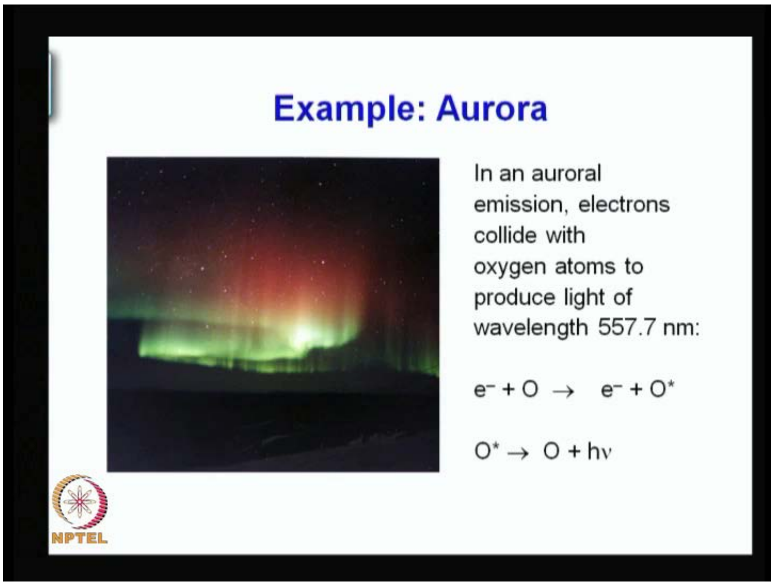

There are many examples of weakly ionized plasmas around us. I have taken up the example of an aurora. In an auroral emission, this kind of aurora, the one that I have taken up. I am showing you the picture. Electrons collide with oxygen atoms to produce light of wave length 557.7 nanometer. This is a weakly ionized plasma.

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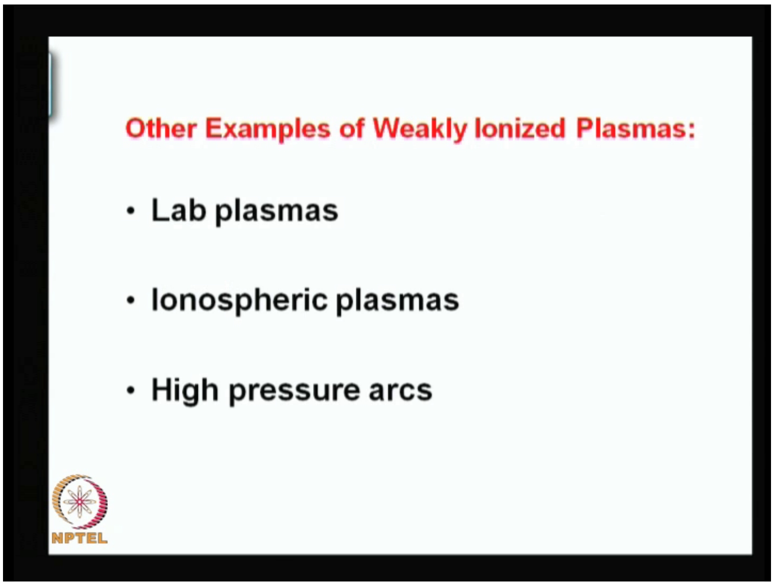

Other examples of weakly ionized plasmas are, plasmas in the laboratory, ionospheric plasmas, high pressure arcs.

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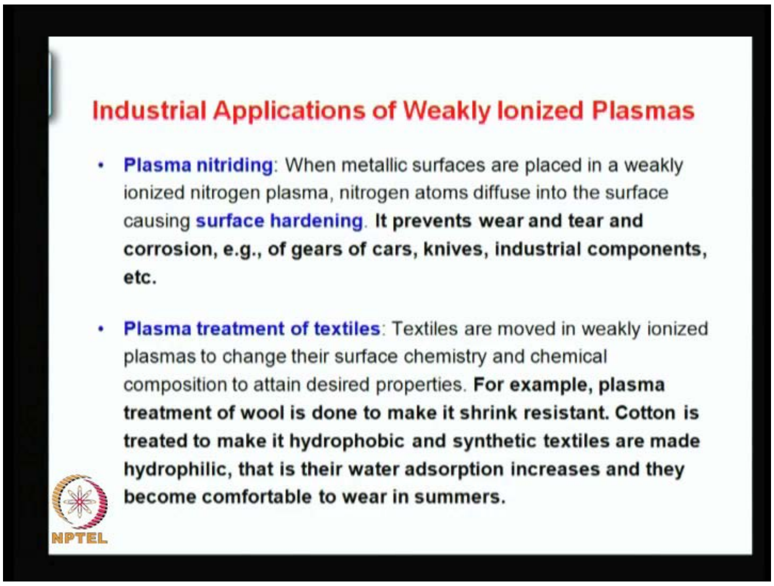

The study of weakly ionized plasmas and diffusion phenomenon in this kind of a plasma is also important, because of the industrial applications of such plasmas. I have chosen two such applications. One is that of plasma nitriding. When metallic surfaces are placed in a weakly ionized nitrogen plasma, what happens is that nitrogen atoms diffuse into the surface of the metal and this causes surface hardening. This kind of a treatment in a weakly ionized nitrogen plasma prevents the wear and tear and corrosion of the metal. And so, we find that the gears of cars, knives and other industrial components are treated with such weakly ionized gaseous plasmas.

Similarly, there is another interesting application, that of plasma treatment of textiles. Now, textiles are moved in weakly ionized plasmas to change the surface chemistry and chemical composition and to attain desired properties in the material in the textile. For example, plasma treatment of wool makes it shrink resistant. Cotton is treated by this kind of a plasma and it makes it hydrophobic, which means that water does not stick to cotton. And synthetic textiles are treated in this kind of a plasma to make them hydrophilic. So, water absorption by a such plasma treated synthetic textiles increases and they have become comfortable to wear in summers.

So, amongst many other applications of weakly ionized plasmas, these are two very interesting industrial applications, where diffusion phenomenon is primarily responsible for these properties.

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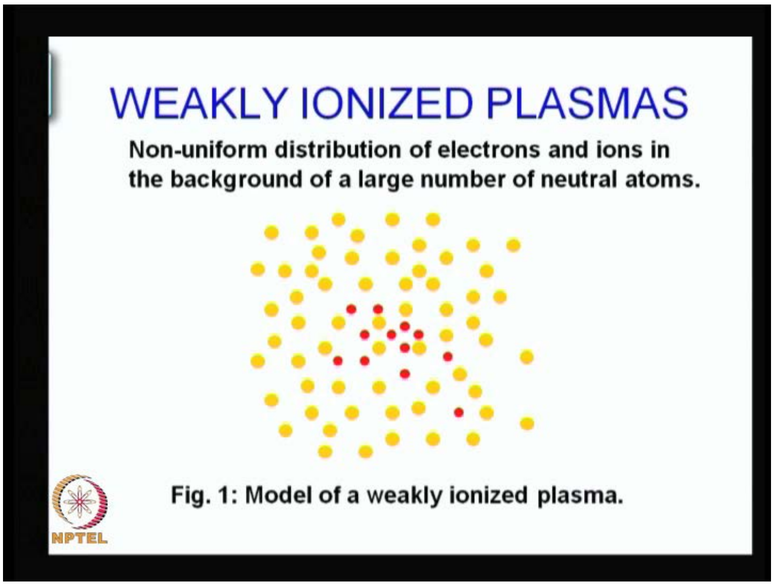

Fig. 1: Model of a weakly ionized plasma.

So, that is why we study weakly ionized plasmas. As I have said earlier, what is a weakly ionized plasma? It is a non-uniform distribution of electrons and ions in the background of a large number of neutral atoms. In this figure, I have shown the model of a weakly ionized plasma. If the colors distinct to you, then the red colored smaller dots are the charged particles.

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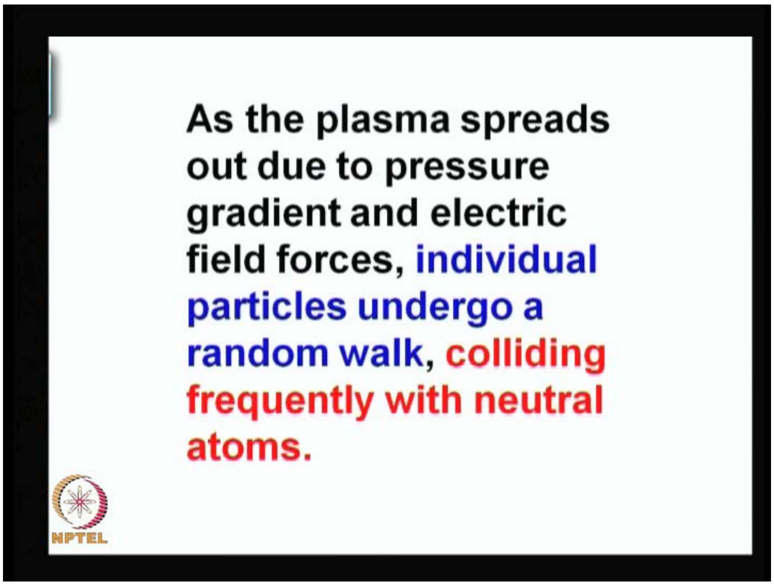

Now, as the plasmas spreads out due to pressure gradient and electric field, individual particles undergo a random walk, colliding frequently with neutral atoms. So, we are essentially treating the diffusion in weakly ionized plasmas as a random walk problem of charge particles amongst a dense background of neutral atoms.

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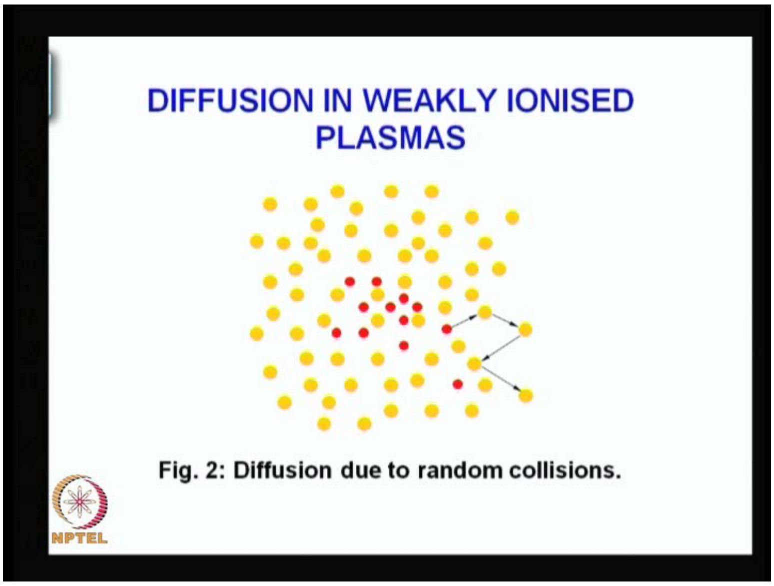

Fig. 2: Diffusion due to random collisions.

This is the module. So, diffusion in weakly ionized plasmas are shown here. In this figure, this takes place due to random collisions of the particles with the atoms.

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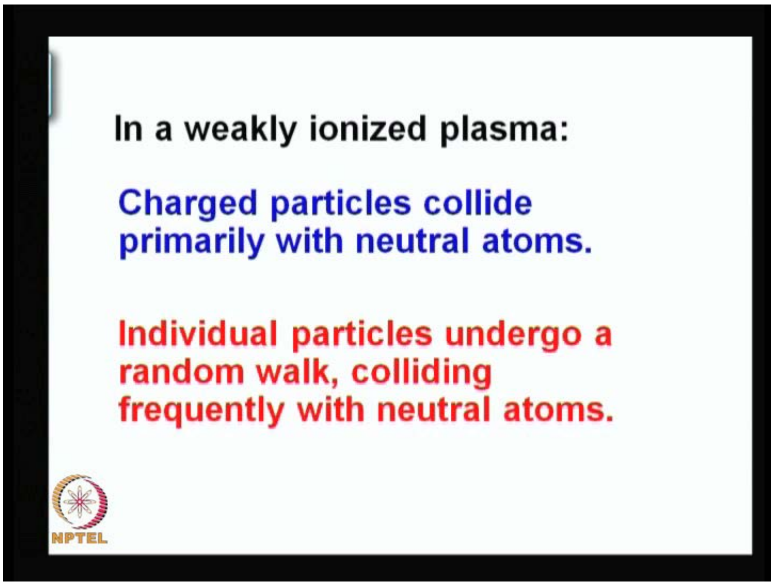

So, you must remember that, in a weakly ionized plasma charge particles collide primarily with neutral atoms and individual particles collide in the manner of a random walk. They collide frequently with neutral atoms and we treat it as a random walk problem. So, we can apply the results, that we already know from our college physics for the random walk problem

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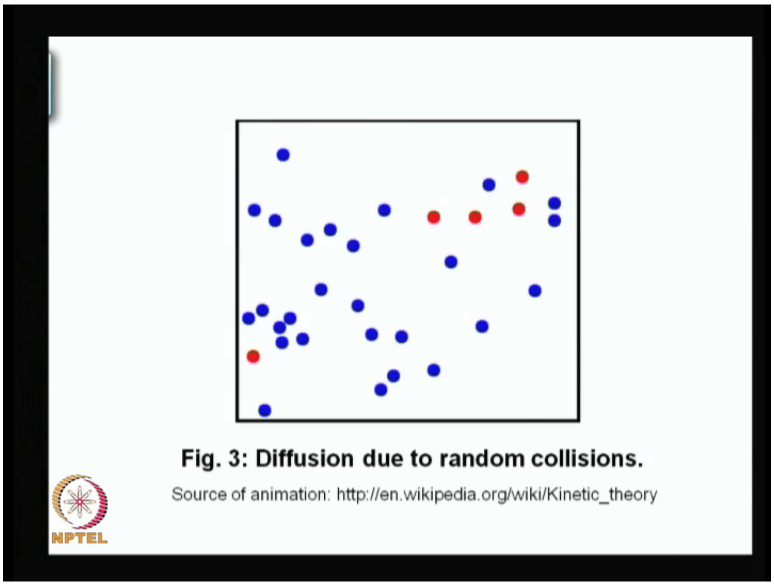

This is the kind of animation, which shows the random walk for a weakly ionized plasma or diffusion due to random collisions. It is available on the common Wikipedia, commons. I have given the source here.

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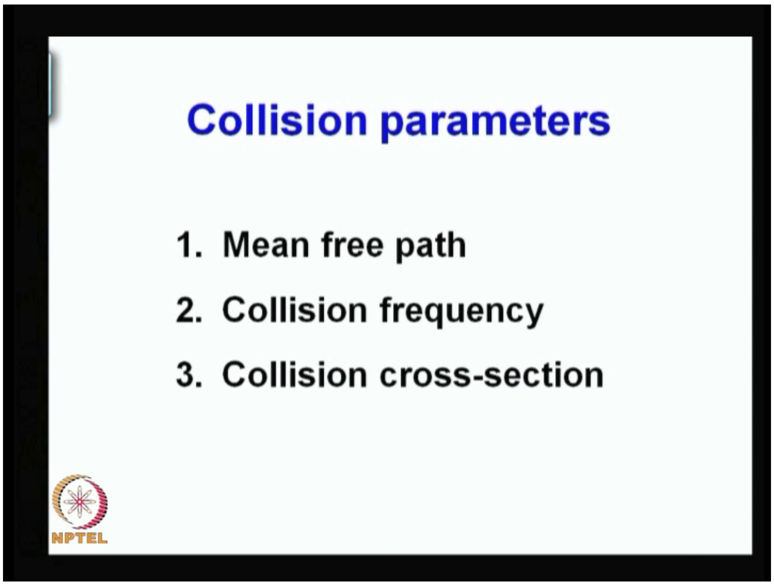

Now, when we treat the collision as a random walk problem, the collision parameters that we need to know are mean free path, collision frequency and collision cross-section.

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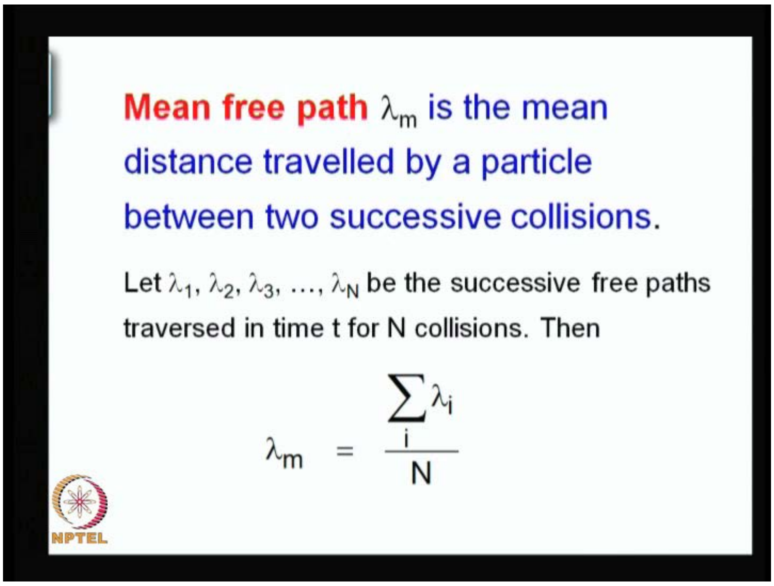

So, let us see, let us go back to the college physics and see how we can obtain the definitions and the expressions, which are relevant to plasma physics. You may recall that, the mean free path is the mean distance travelled by a particle between two successive collisions.

So, if lambda 1, lambda 2, lambda 3, lambda 4, lambda N are successive free paths, which the particle travels in time t in N collisions, then we have lambda m is equal to sigma i lambda i upon N. This is the equation, that defines the mean free path. It is a simple algebric equation.

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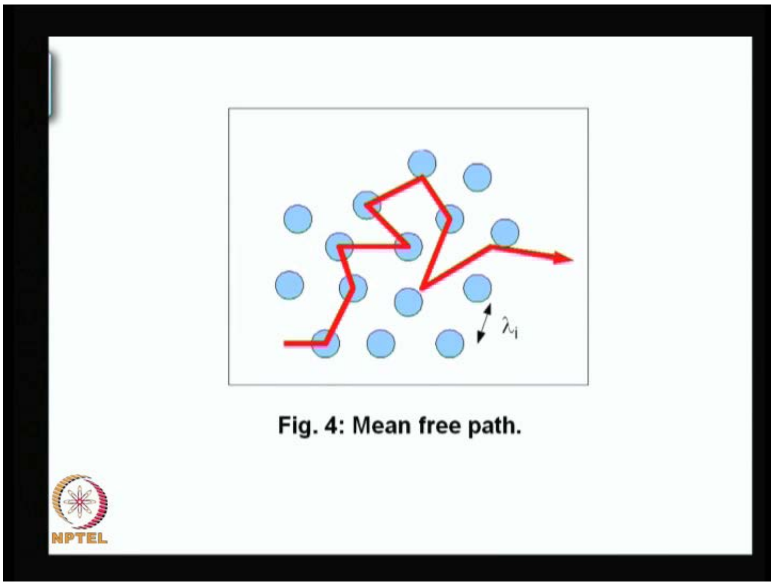

This is a pictorial representation of the mean free path, where the electron or the charge particle is colliding with different atoms in the plasma and in a random manner.

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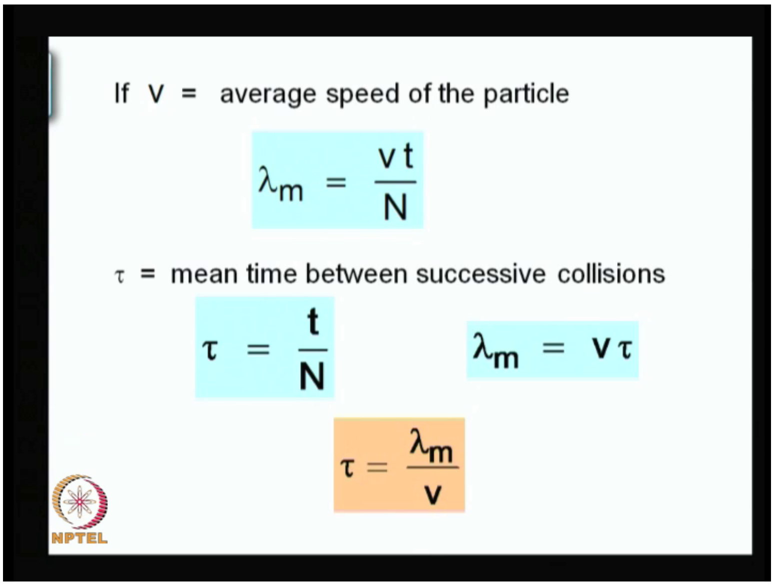

Now, if V is the average speed of the particle, then lambda m which is the sum of all the free paths can be written as V multiplied by the total time t divided by N, the number of collisions. Let us say that, tau is the mean time between successive collisions. Then, if the total time is t and N is the number of collisions, the mean time between successive collisions is nothing but tau equal to t by N.

So, combining the equation lambda m equal to V t by N and tau equal to t by m, we get an expression, lambda m equal to V tau from, where tau is equal to lambda m upon V. This is the mean time between collisions.

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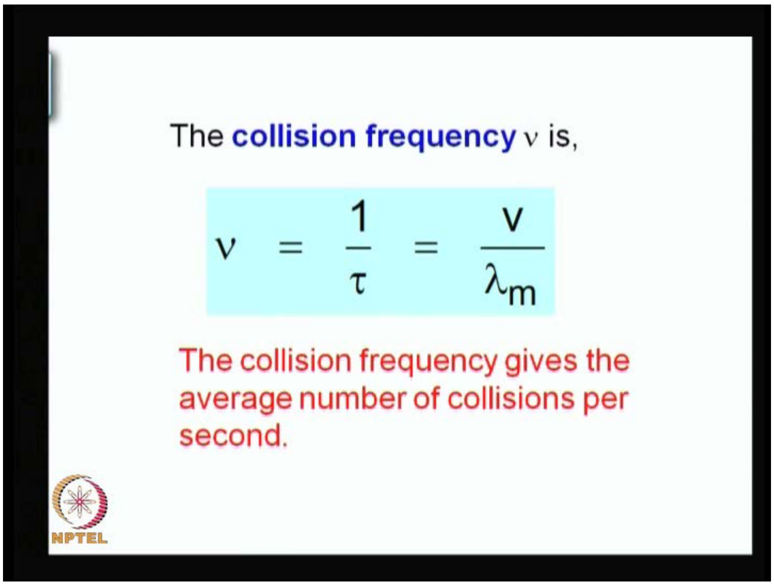

And from here, we get the expression for the collision frequency, which is 1 upon the mean time tau and is given by V upon lambda m. The collision frequency is nothing but the average number of collisions per second.

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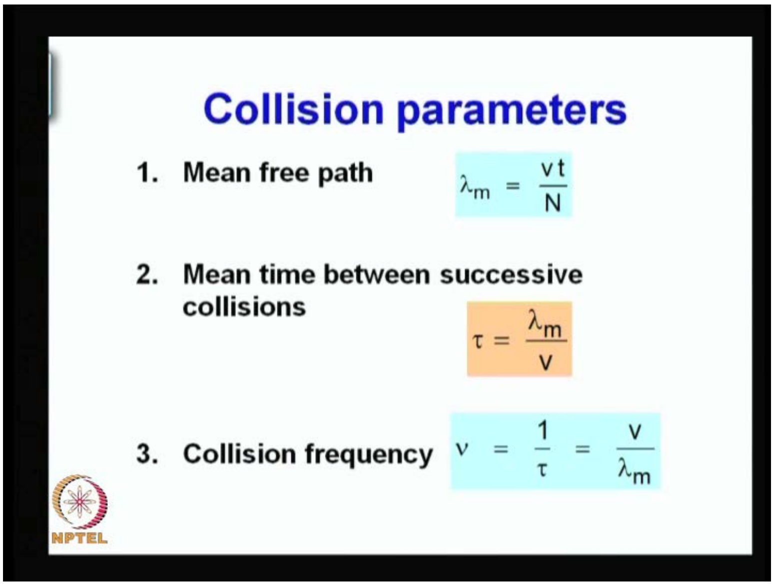

So, these parameters you must be familiar from your usual college physics in the kinetic theory of gases. We are going to apply this now to plasma physics. I have summarize the results in this slide for you, where I have given the mean free path, the mean time between successive collisions and the collision frequency. The mean free path is lambda m equal to V t upon N, mean time between successive collisions is tau equal to lambda m upon V and collision frequency is simply 1 upon tau equal to V upon lambda m.

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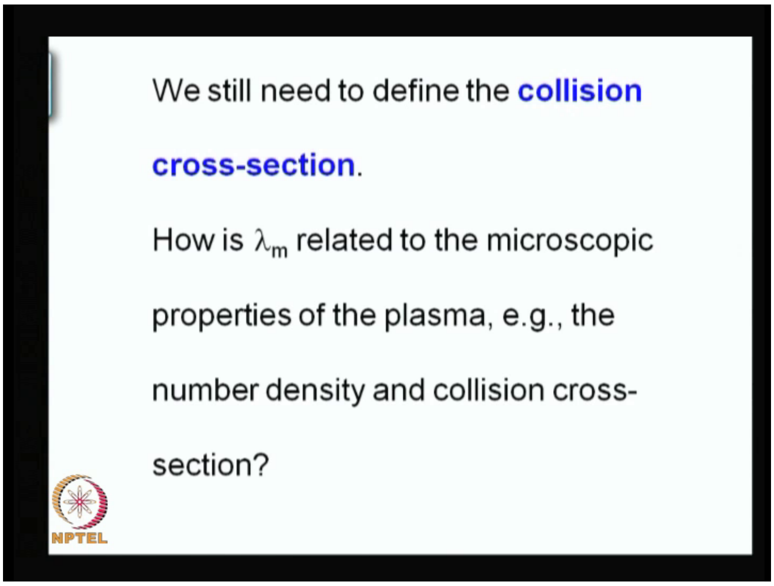

So, these are the collision parameters, which are for any random walk problem. These are the definitions. But we still need to know, what the collision cross section is and we still need to ask, how is lambda m related to the microscopic properties of the plasma. Because we need to apply these concepts to plasma physics. So, what are the properties of the plasma?

The number density and the collision cross section are some of the properties. Two of the properties to which, we can relate the mean free path. So, we first define the collision cross section and then, find the relation between lambda m and the plasma properties the collision cross section and number density.

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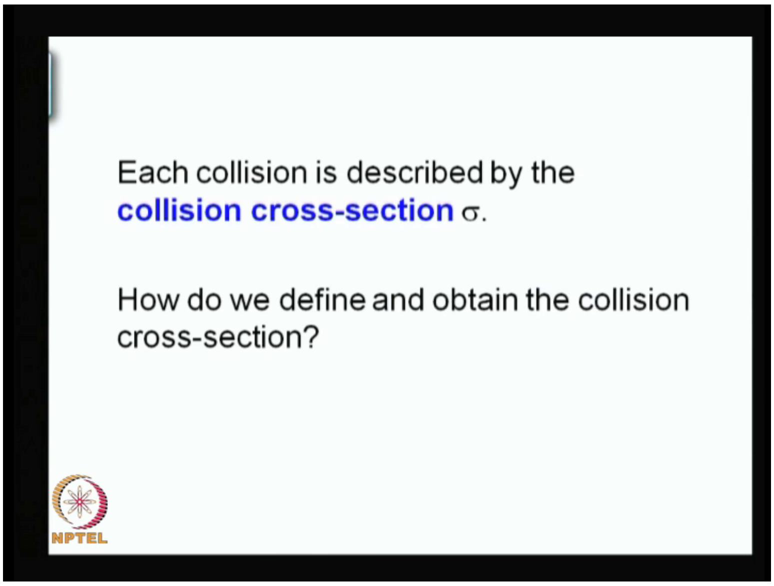

Now in the collision theory, each collision is described by a collision cross sections sigma, what is this what is the definition of the collision cross section?

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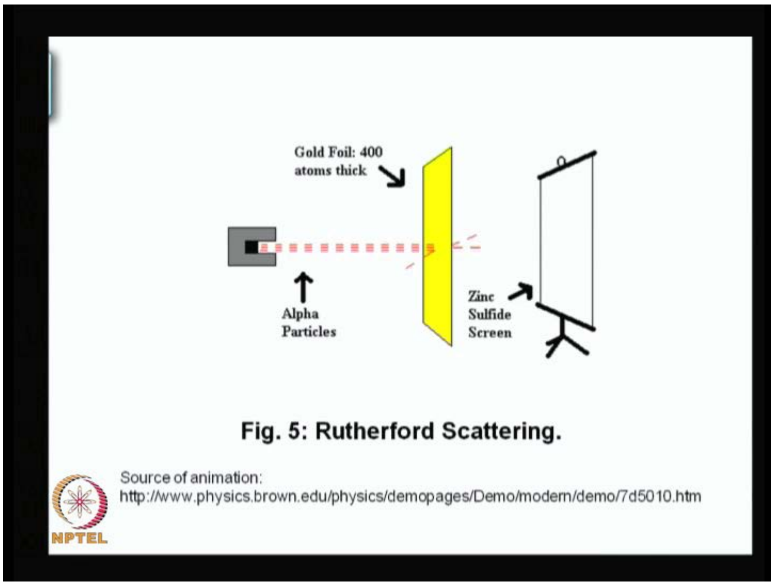

Lets just look at an example of a very famous scattering experiment. This is the animation, we will be discussing this sometime later. The Rutherford scattering, it is a very famous experiment in physics, where alpha particles were bombarded on very thin gold foils and whereas, the existing Thomson's model predicted that all alpha particles would just pass through the gold foil. What was observed was, that some of them just bounced back, there was a back scattering. We will not go into the details but this is the kind of an experiment, in which you can determine the collision cross sections.

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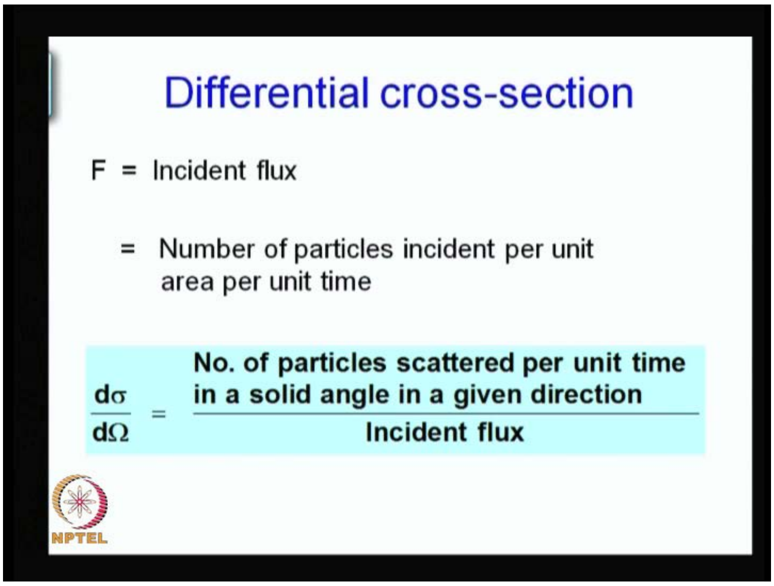

So, what is a collision cross section? There are two kinds of cross sections. One is called the differential cross section and the another the other is called total cross section. We define the differential cross section as, number of particles scattered per unit time in a solid angle in a given direction divided by incident flux. What is the incident flux, it is the number of particles incident on the target per unit area, per unit time.

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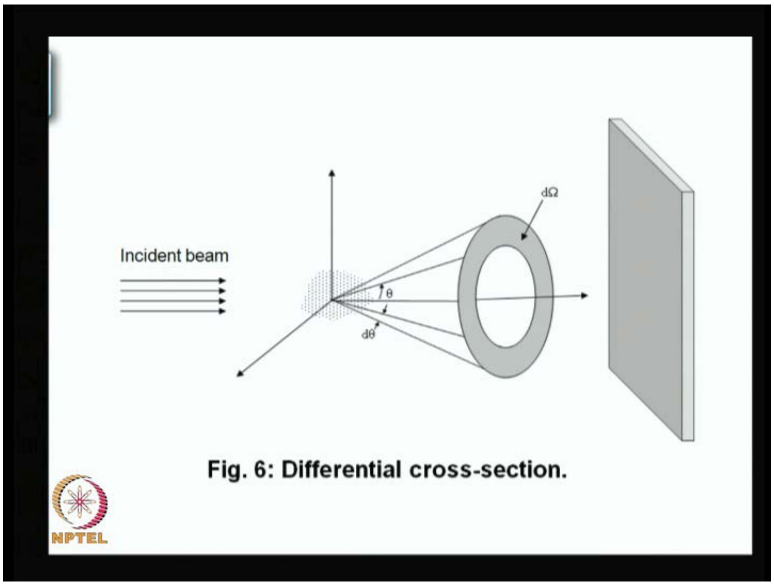

Fig. 6: Differential cross-section.

So, let me just show you the experimental set up. First, this is the experimental set up, where the incident beam is bombarded on the target and then it is observed in a solid angle. You see a solid angle, which is centered at the scattering angle theta. It is the cone, you can imagine it to be a cone centered at the solid at the scattering angle theta and between theta and theta plus D theta. So, if you count the number of particles scattered in this particular solid angle detected by the detector here, then the number of particles scattered per unit time in a solid angle, in the direction theta, divided by the number of particles incident on the target per unit area, per unit time is defined as the differential cross section.

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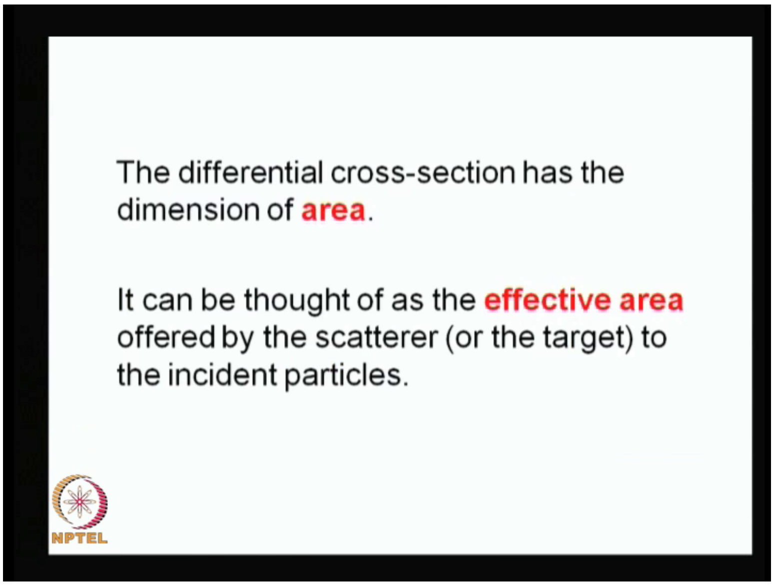

Now, the interesting thing is, that the dimension of the differential cross section is that of area. So, if you want to understand what it is physically, we can say that, it is the effective area offered by this target or the scatterer to the incident beam of particles. It is the effective area offered by the target to the incident beam of particles. This is an important concept, we will be using later in this lecture. So, this is once again the schematics of a scattering process, you are counting the numbers of particles in this solid angle and then you divide it by incident flux.

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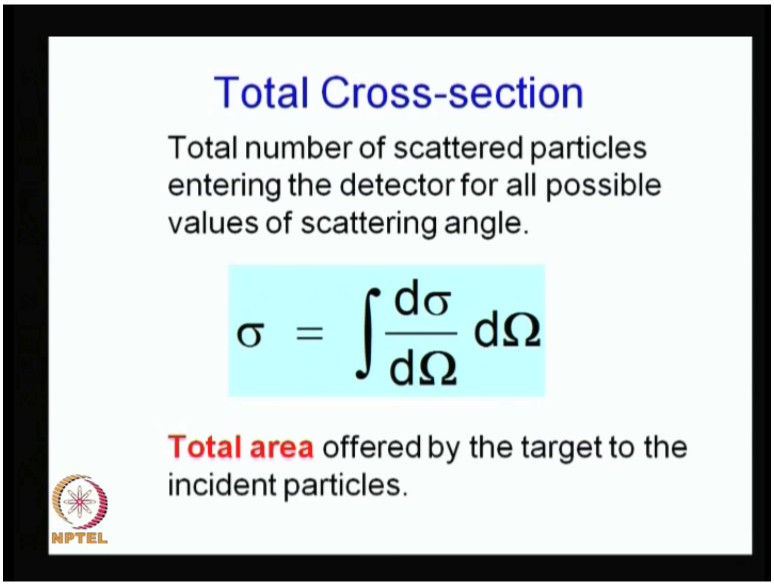

So, there is another parameter called the total cross section, which is you count the total number of particles scattered, you plays the detectors at all angles, all possible scattering angles around the target and you count the total number of particles scattered by the target. You get the total cross section. Mathematically, you just integrate the differential cross section over all values of the solid angle.

Physically it means, the total area offered by the target to the incident particles. These two things, the physical concept is more important for you to remember, as far as the further treatment of this process of diffusion is concerned, that it is the area offered by the target atoms to the incident or the colliding charged particles.

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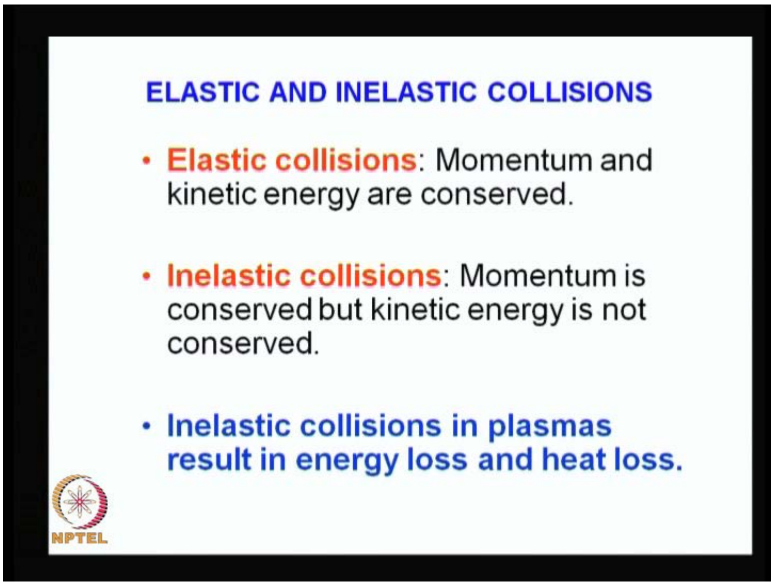

Now, interesting an interesting point as for as the plasmas are concerned is that collisions are of two types, elastic collisions and inelastic collisions. Elastic collisions are those in which, the momentum and kinetic energy of the system of incident particles and target particles are conserved. So, the total linear momentum and the total kinetic energy are conserved in this process.

Inelastic collisions are the ones in which, the linear momentum or momentum, it could be angular momentum also, as we will see in the scattering of charge particles by atoms in a plasma, momentum is conserved but kinetic energy is not conserved. And so, there can be energy loss due to heat loss in plasmas. Inelastic collisions taking place in plasmas give rise to energy loss by way of heat loss

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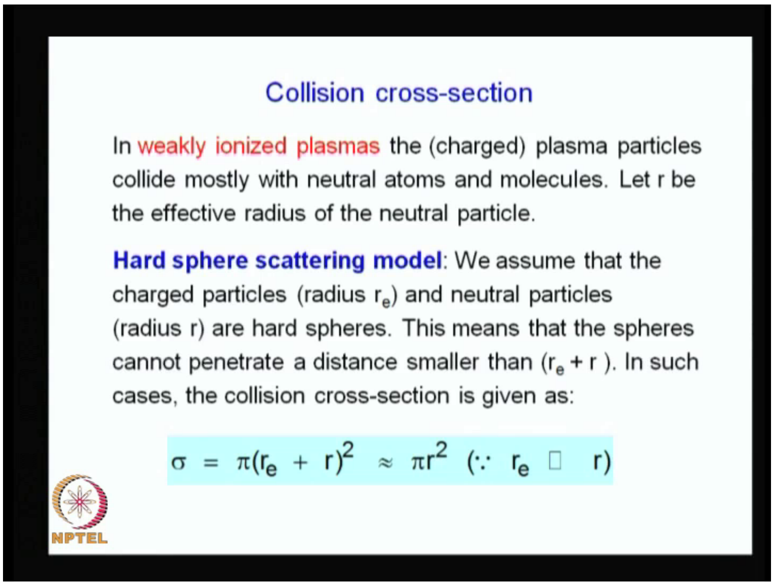

Now, having defined the collision cross section as an effective area or the total area offered by the target atoms to the colliding electrons or ions in a plasma. Let us find out, what its value is for a weakly ionized plasma.

As you have learnt earlier on in this lecture, in weakly ionized plasmas, charge particles collide mostly with neutral atoms and molecules. So, let us say that they are colliding with neutral atoms and let r be the effective radius of the neutral particle.

We can describe this scattering process using the hard sphere scattering model. And we in the hard sphere scattering model, we assume that the charged particles, the incident charged particles say the radius is r e and the neutral particles of radius r are hard spheres. What does this mean? This means, that this spheres cannot penetrate each other. So, they cannot penetrate a distance, that is smaller than r e plus r. If this is one sphere of r e and an another sphere of r, they cannot get into each other. This will be the distance, smallest distance between them r e plus r.

Without going into the derivations, I am just giving you the result of the collision cross section for hard sphere scattering, which is shown here.

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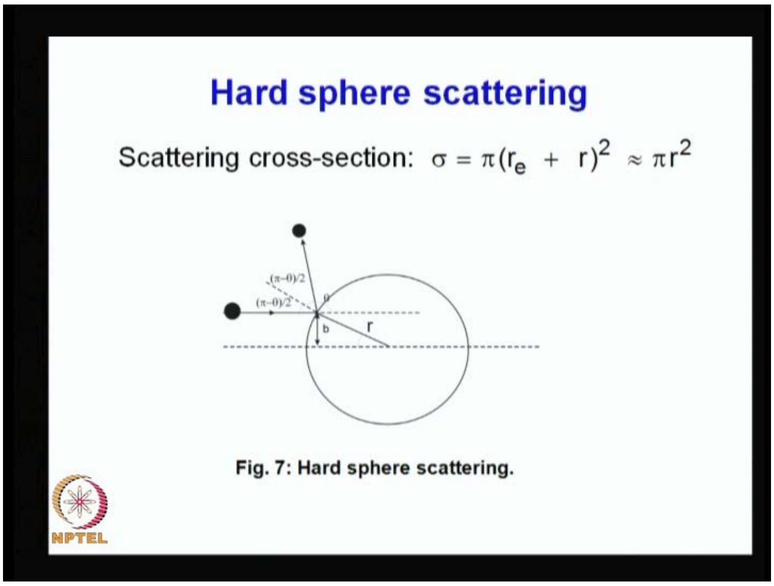

And it is shown, I have also written it down for you, it is sigma equal to pi r e plus r whole square.

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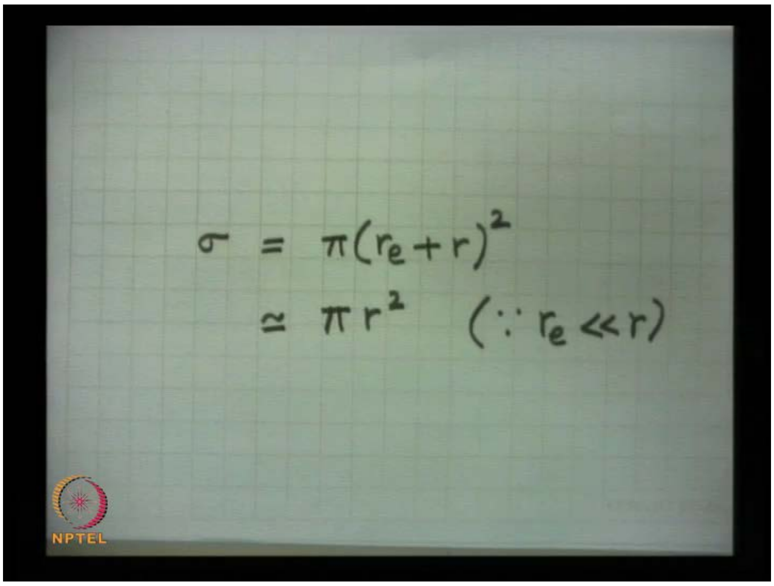

Now, since r e is much less than r, the radius of the charge particle which is either an electron or an ion is much less than the radius of the atom or the molecule. We can say that sigma is equal to pi r square. So, pi r square is the effective cross section of the target atom or molecule.

So, assuming the hard sphere scattering model, we have arrived at an expression for the scattering cross section, that sigma is pi r square. This is the effective area offered by the target atom of molecule to the charged particles in the plasma.

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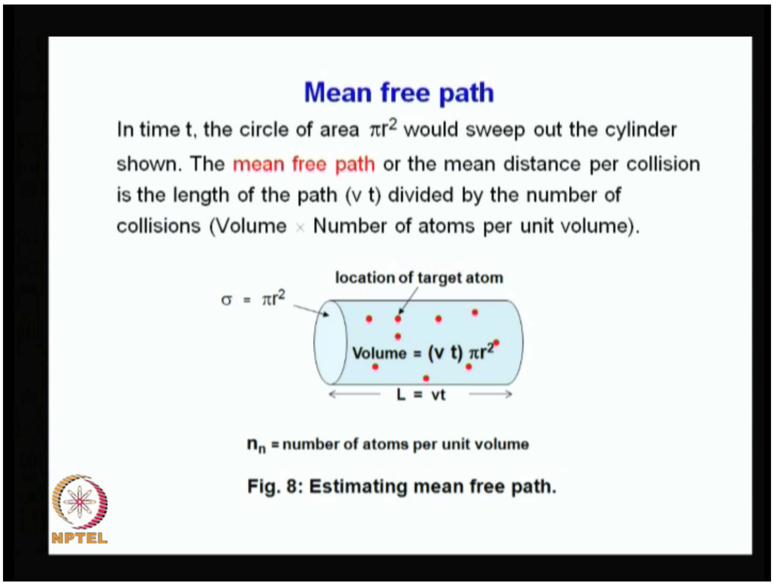

Now, how do we express the mean free path, in terms of this collision cross section and number density. Imagine a cylinder in the plasma and there are certain atoms of molecules and a charged particle collides with those atoms and molecules in that particular cylinder.

Let, the cross sectional area of the cylinder be pi r square, which is the area offered by an atom or a molecule to the incident charged particle. So, if we take a cross sectional area pi r square in time t, let us say it sweeps out the cylinder shown here on the slide.

Then, what is the mean free path as per our original definition. It is the mean distance per collision. So, it is equal to the length of the path divided by the number of collisions. This is what we had done in our basic definitions sigma lambda i upon n. What is the length of the path travels by a charged particle? It is the speed multiplied by the time, in which we have taken the cylinder divided by the number of collisions is just volume of the cylinder multiplied by number of target atoms per unit volume.

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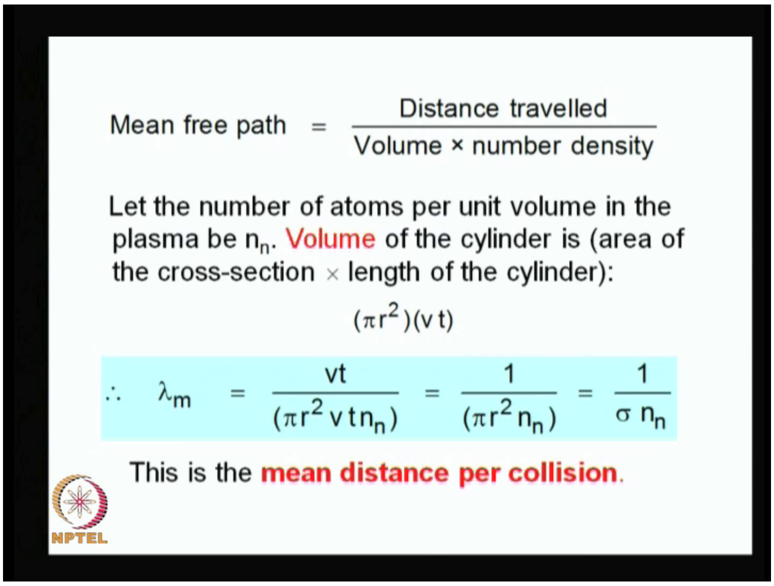

So, this is what it is, mean free path is distance travelled divided by volume into number density. Distance travelled is some simply V t, you can see it in the expression of lambda m V t and volume is pi r square multiplied by the length V t and number density is n n. We have denoted it by n n, number of atoms per unit volume in the plasma.

So, V t and V t cross out and we get lambda m as 1 upon pi r square n n or 1 upon sigma n n. So, this is nothing but the mean distance per collision. We have obtained an expression for the mean free path in terms of sigma the collision cross section and the number density

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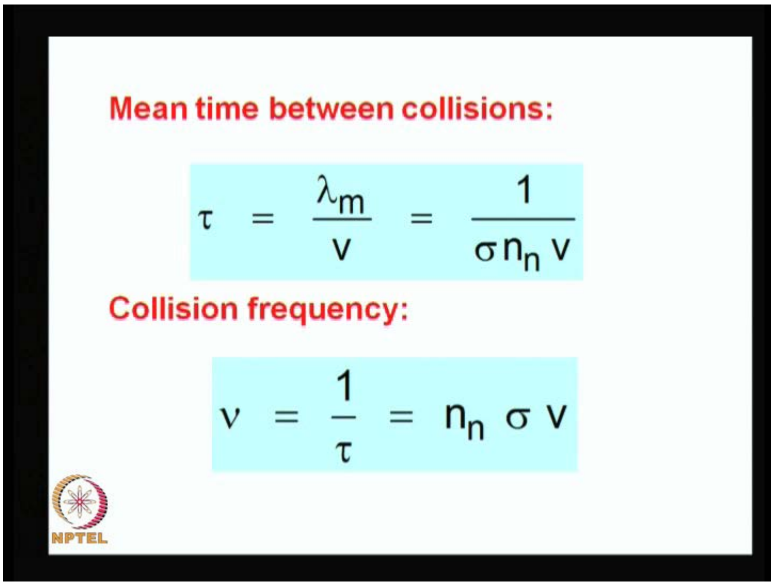

From there it is simple to get the mean time between collisions, which is tau and lambda m upon V. This is what is given here, tau is equal to lambda m upon V is equal to 1 upon sigma n n V and collision frequency is simply 1 upon tau is equal to n n sigma V. Remember, V is the average velocity.

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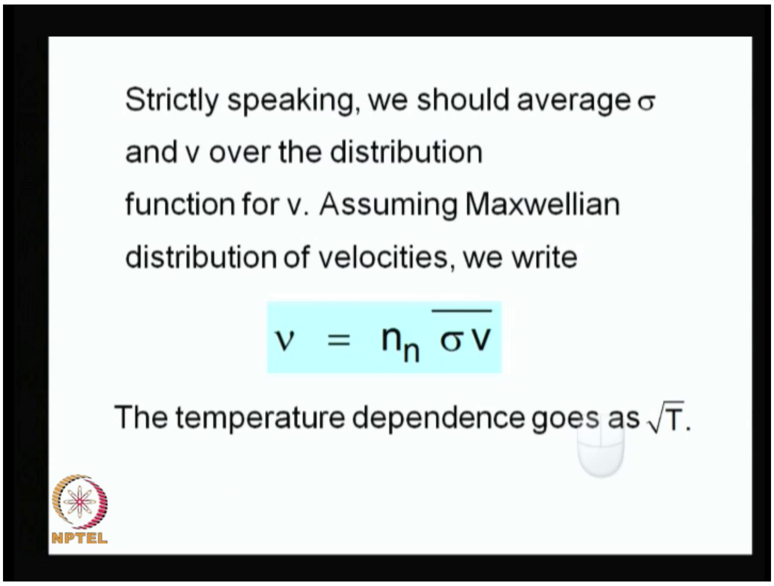

So, strictly speaking, we should average both sigma and V over the distribution function for V. So, if we assume Maxwellian distribution of velocities, we can write nu is equal to n n sigma V bar, where V bar and sigma are obtained from Maxwellian distribution.

The temperature dependence of the collision frequency then goes as square root T. Because sigma is a constant and V bar goes as square root T.

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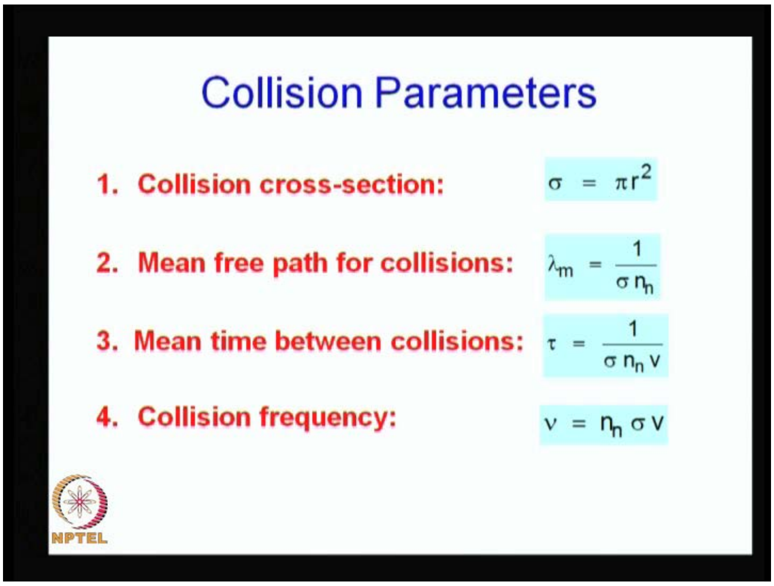

To summarize the collision parameters, we have established four of them. The collision cross-section, the mean free path for collisions in terms of sigma and n n, mean time between collisions and the collision frequency. We shall be using these parameters.

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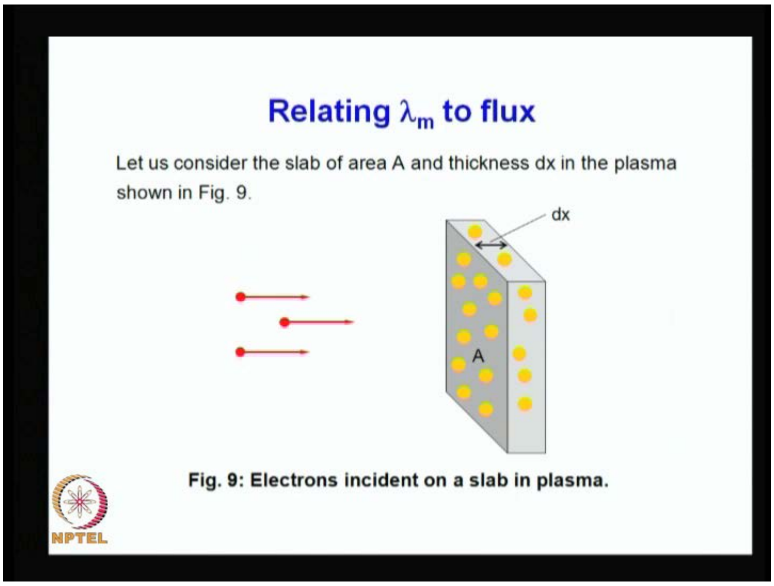

The next category of parameters, okay before we go to the next category the diffusion parameters, we it is an interesting thing to understand physically what this mean free path is as for as plasmas are concerned.

To do that, we need to relate the mean free path to the flux of the particles. So, as shown in the diagram here, let us consider a slab of area A and thickness d X in the plasma. And these are the incident charged particles, incident on the slab. They could be electrons. They could be ions.

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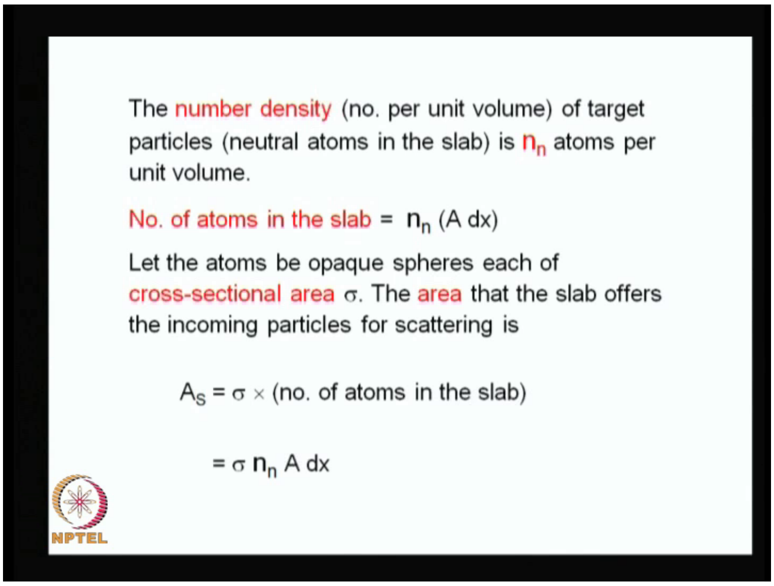

You know that, the number density is n n atoms per unit volume. So, the number of atoms in this slab is n n multiplied by the volume of this slab, which is A dX. If the cross sectional area of these atoms is sigma, then the area that the slab offers the incoming particles for scattering is sigma multiplied by the number of atoms in the slab, which is sigma n n A dX. A s we are denoting it by A s, this is sigma n n A dX.

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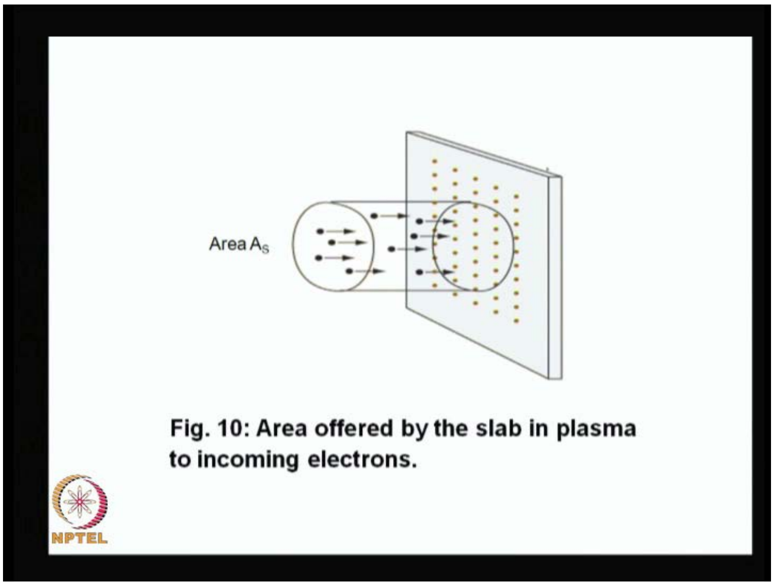

This is the pictorial representation of this process area offered by the slab in the plasma to incoming charged particles.

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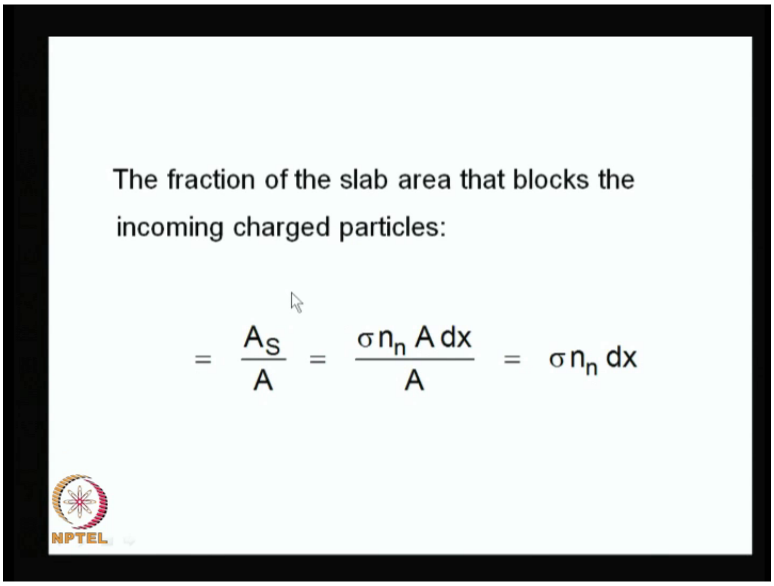

Now, this is the fraction of the total area. So, the fraction of this slab area that blocks the incoming charged particles is simply A s upon A, which is sigma n n dX. Because A cancels out. So, this is the fraction.

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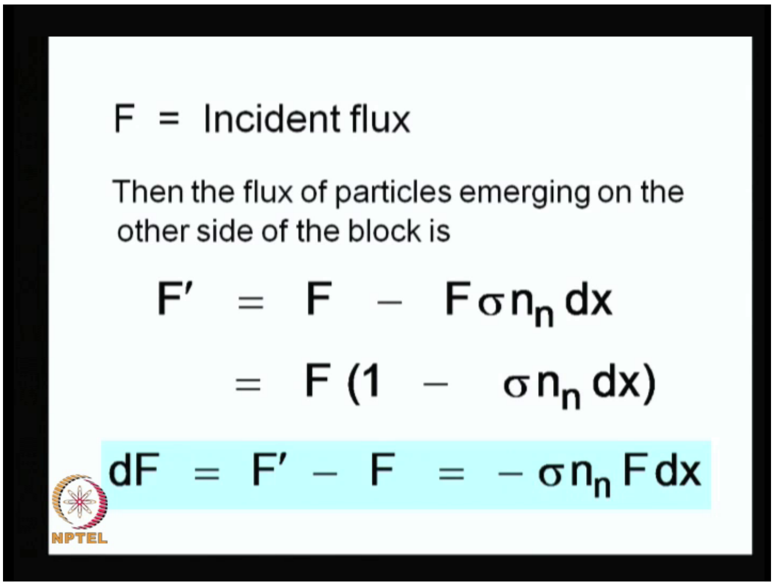

Now, we come to the incident flux, what is the expression for the incident flux. If, F is the incident flux and F is incident on the slab, then some part of the incident beam is blocked, which is sigma n n dX. So, that much flux will then be blocked by the slab. Sigma n n dX is the fraction multiplied by the incident flux, that much flux will not emerge from the other side of the block. So, the flux of particles emerging on the other side of blocks is simply F minus F sigma n n dX. Now, this gives me an expression for the change in flux. This is dF, this is the change in flux is equal to F prime minus F is equal to minus sigma n n F dX.

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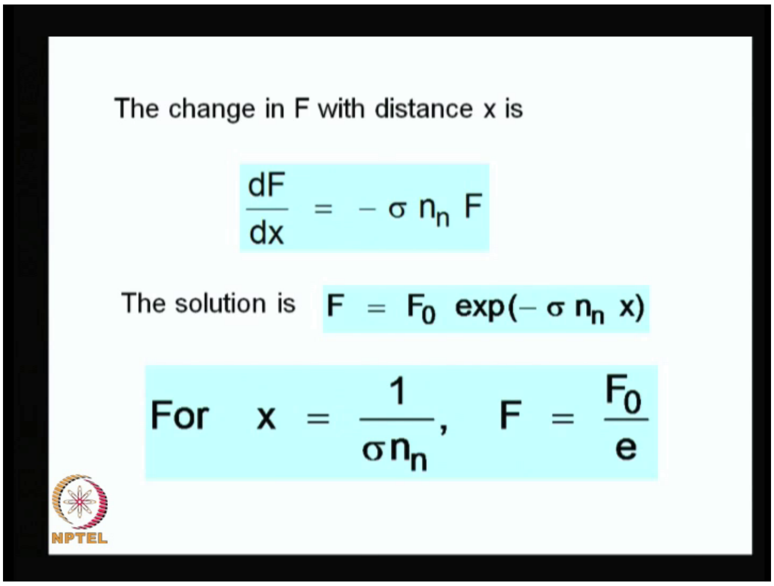

So, the change in flux with distance comes out to be a first ordered differential equation, d F upon d X is equal to minus sigma n n F and its solution is an exponential solution, exponentially decane solution. You have come across these kinds of equations, elsewhere at many places in physics, I will tell you in a moment. Now, for this kind of an exponential, linear exponentially decane function, you can see that when X is equal to 1 upon sigma n n this, then F is equal to F 0 upon e, F is reduced to 1 by e of its initial value.

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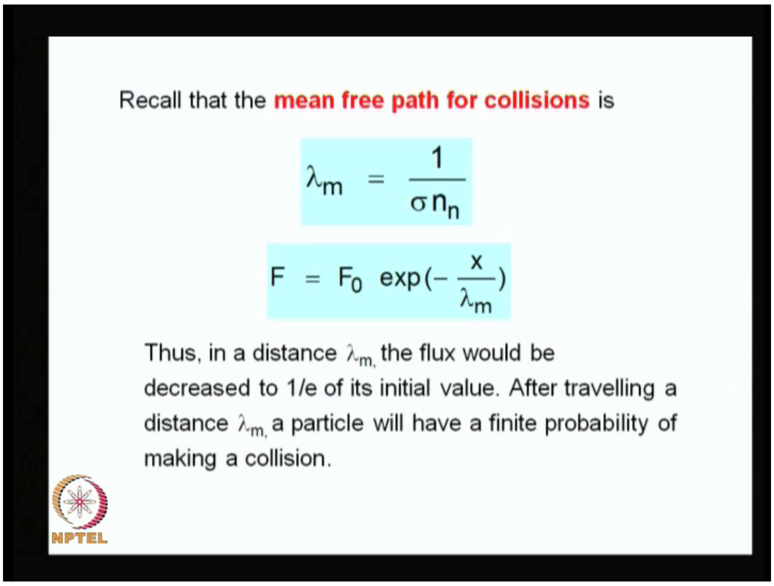

So, this is the length, in which the flux is reduced to 1 by e of its initial value and you will recall, we have just define, we have just obtained an expression for lambda m as 1 upon sigma n n. So, F becomes F 0 exponential minus X upon lambda m, here. So, what does this equation tell us. It tells us, that in a distance lambda m, the flux would be decreased to 1 upon e, of its initial value. After travelling a distance lambda m, a particle will have a finite probability of making a collision.

Such equations, you have come across in radioactive decay as well as, these are also equations that you will come, you come across in radiation biology, where it gives you the depth to which radiation, electromagnetic radiation can penetrate biological systems. But that is a different story.

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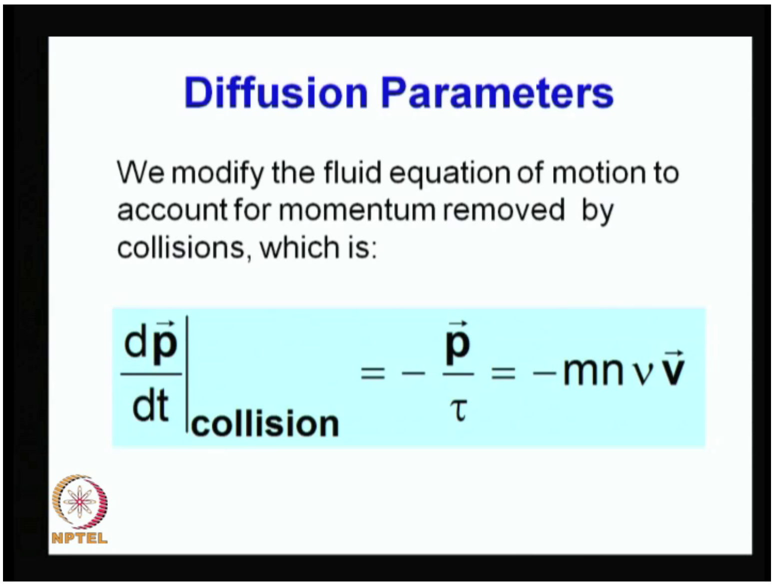

Now, having understood the collision parameters, we come to the diffusion parameters and we define the diffusion coefficient and mobility. For introducing the diffusion parameters in collisional plasmas, we have to account for the momentum removed due to collisions. So, we have to modify the fluid equation of motion and the momentum that is removed by a collisions is given by this equation dp upon dt is equal to minus t upon tau is equal to minus m n nu V, nu comes from 1 upon tau.

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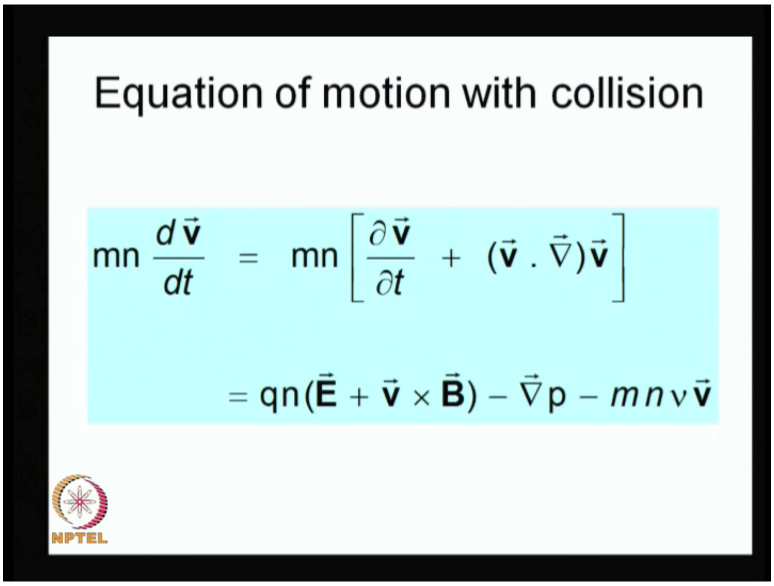

When we write this particular term in the equation of motion, the collision term, the equation of motion becomes this. m n d V by dt is equal to m n del V by d del t plus V dot del, V the convective term and in the presence of electric and magnetic fields and the pressure gradient, we have to add the collisional term. We solve this equation of motion under certain simplifying assumptions and those are, that firstly, we consider the case of un-magnetized plasma.

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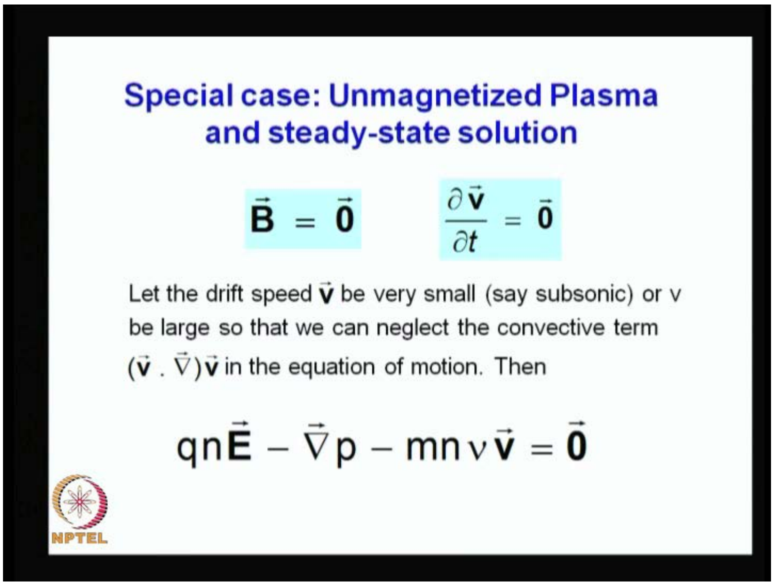

We will take up magnetized plasmas later and take the V cross B term also but not today. So, for un-magnetized plasma B is equal to 0. Further, we take only the steady state solution of the equation of motion. So, we put del V upon del t also 0. And we make another assumption, physics is all about simplifying the problems. So, we make another assumption that the drift speed or drift velocity is very small. So, that this second, order term or the convictive term can be neglected in the equation of motion, the V dot grad V term.

So, we are left with these three terms q and E minus grad p minus m n nu V is equal to 0. This is the equation of motion, that we need to solve.

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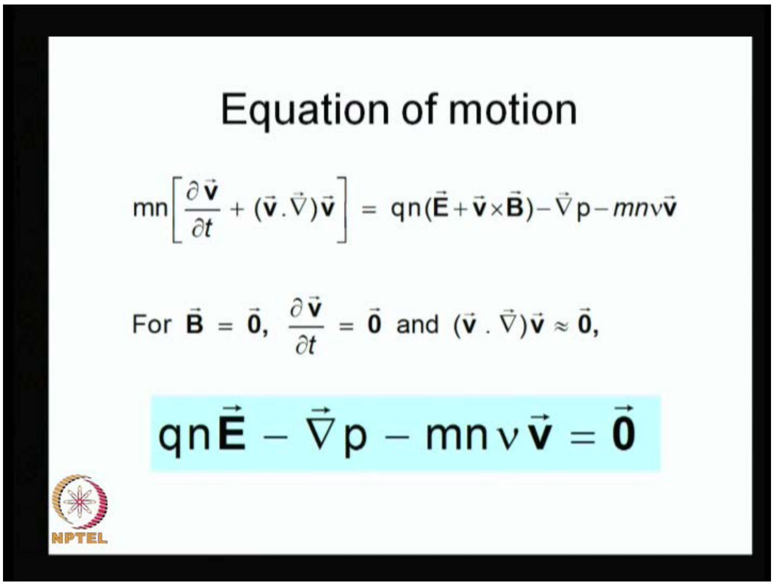

So, here again, I have written the assumptions and the equation of motion.

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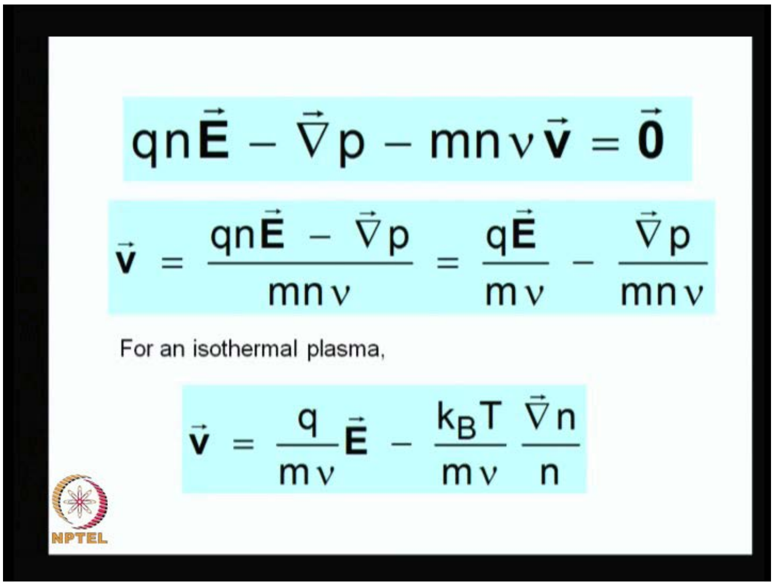

Now, from this equation of motion, we can immediately write an expression for V, which is q and E minus grad p upon m n nu and we get two terms, we can express them in terms of two terms and for the case of isothermal plasmas, the pressure gradient term can be expressed as V is equal to k B T grad n. And so we get, we have V is equal to q upon m nu E minus k B T upon m nu grad nu grad n upon n. This is the equation, solution for drift velocity.

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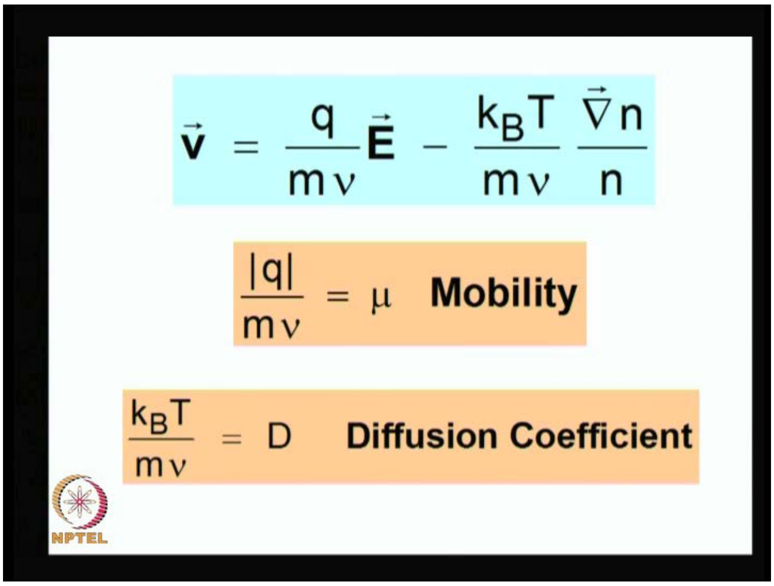

In this equation, the coefficient of the electric field q, the modulus of q, we do not include the sign of the charge q upon m nu is defined as mobility and k B T upon m nu is defined as diffusion coefficient. So far, we have written it only for one kind of charged particles, this is just so that you recognize the expressions.

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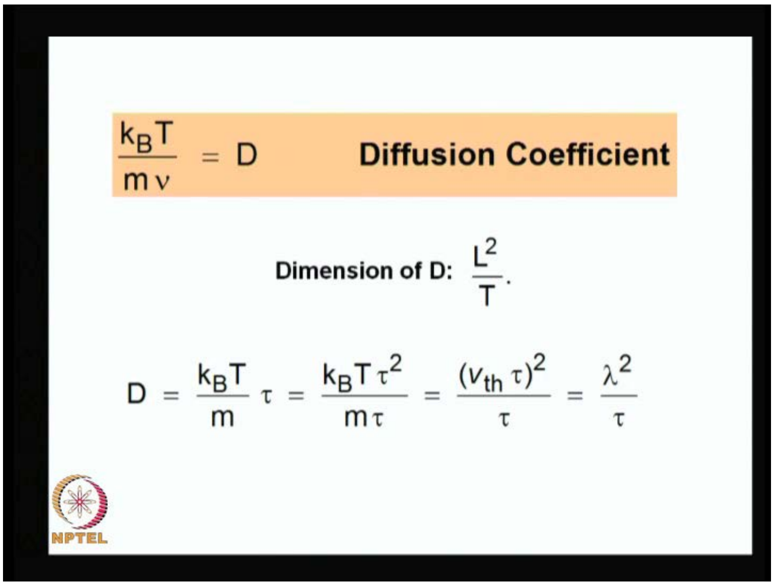

Interesting point about that diffusion coefficient is that, it has the dimension of L square upon t. You can readily show this yourself by writing the dimensions of all the quantities. You will find that it comes out to be lambda square upon tau, which has the dimension length square upon t.

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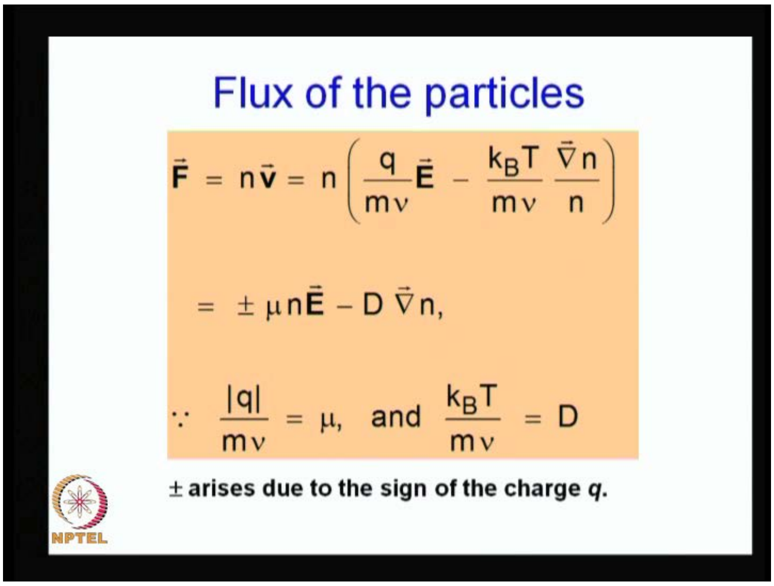

Now, we express the flux in terms of mobility and diffusion coefficient flux is n V is equal to n. You substitute for V, you get an expression straight away, which can be written as plus minus mu n E minus D grad n. This is a simpler simple expression, the sign plus minus comes due to the sign of the charge q.

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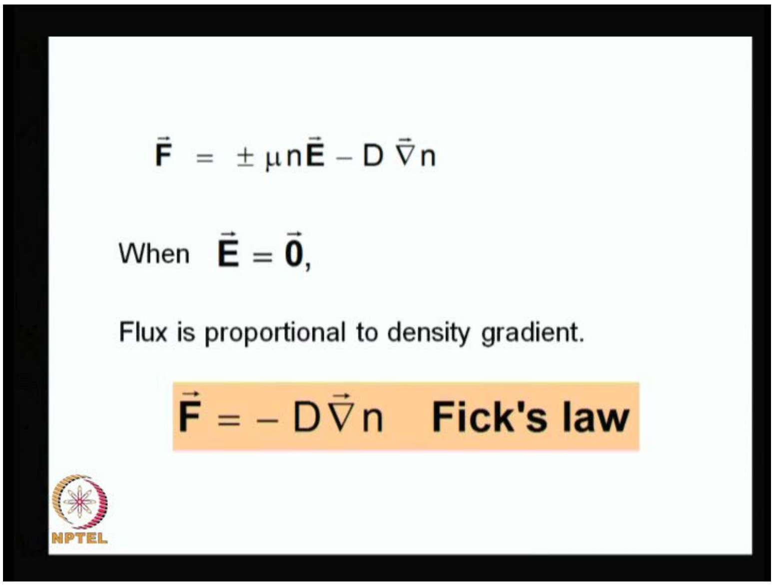

So, we get an expression for the flux in terms of the electric field and the grad density gradient. And when we put the electric field equal to 0, what do we get? We get that, the flux is proportional to density gradient. F is equal to minus D grad N, this result is known as fick's law.

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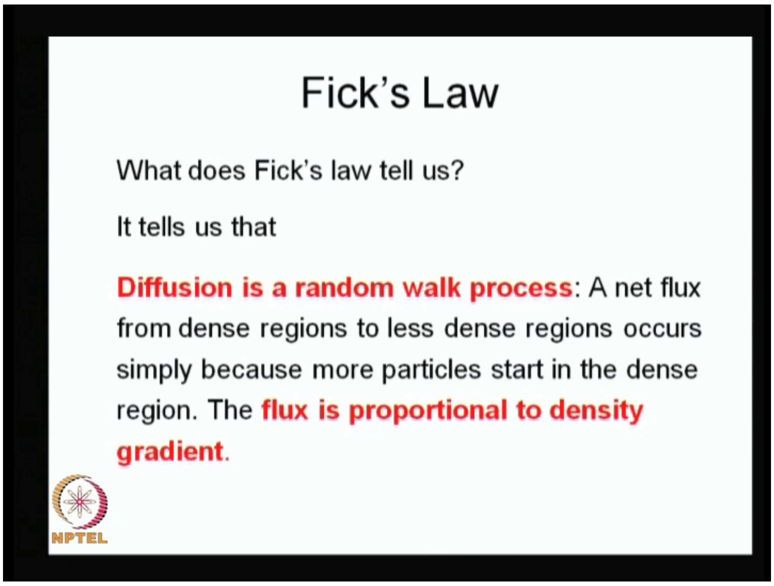

And what does Fick's law tell us. It tells us that, diffusion is a random walk process and this arises because a net flux from dense regions to less dense region happens because more particles start in the dense region, the flux is proportional to density gradient.

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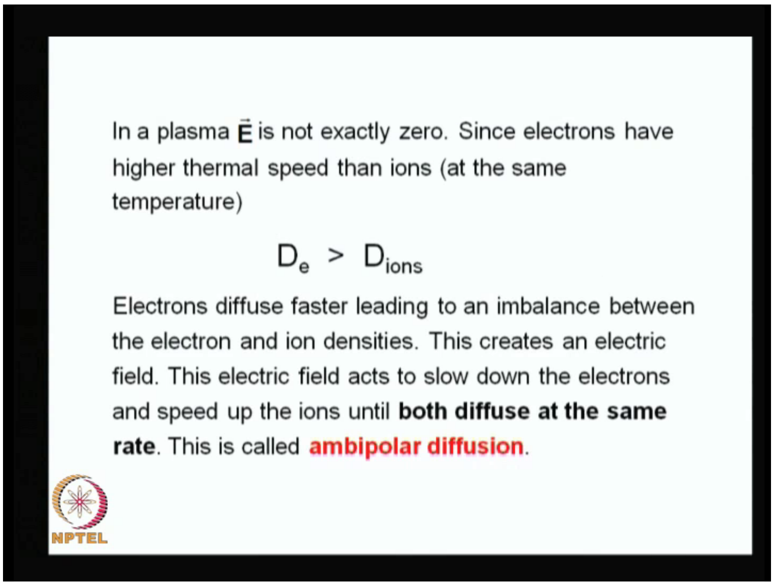

However, you will you do know that, electric field is not exactly 0 in a plasma. Because electrons have higher thermal speeds than ions. The diffusion coefficient at the same temperature, the diffusion coefficient for electrons is greater than the diffusion coefficient for ions. And so, electrons diffuse faster in the plasma. This leads to an imbalance between electron and ion densities creating an electric field.

However, what happens is, that the electric fields so created, acts in a direction, that slows down the electrons and speeds up the ions. So, until the time that both of them diffuse at the same rate. This phenomenon is called ambipolar diffusion, in which the electrons and ions diffuse at the same rate both. Ambi means both and so both kinds of charged particles diffuse at the same rate.

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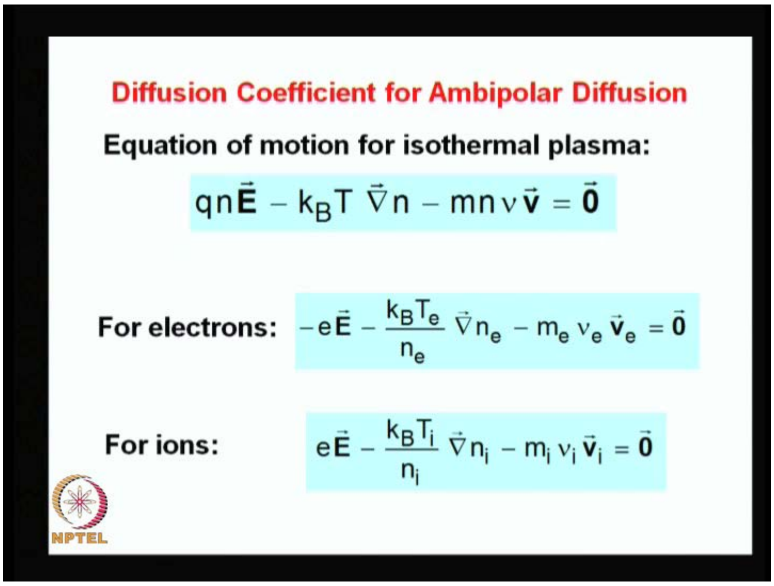

Now, we will solve the equation of motion for ambipolar diffusion to obtain an expression for the diffusion coefficient. Lets go back to the equation of motion for isothermal plasma, which was q and E minus K B T grad n minus m n nu V is equal to 0. We can write it for electrons and for ions. For electrons, this becomes minus e E, we divide the equation by n, so we get minus e E minus K B T e upon n e grad n e minus m e nu e V e is equal to 0. Similarly, we write this expression for ions. The charges plus e, so we get e E minus K B T i upon n i grad n i minus m i nu i V i is equal to 0.

So, what have we done? We have simply written the equation of motion for isothermal plasma for both electrons and ions. We have divided the equation by the number density and we have used the charge minus e for electrons and then the temperature for electrons is T e number density is n e and collision frequency is nu e and mass is m e. Similarly for ions, the charge is plus e, so we have e E. The temperature is T i, the number density is n i, the mass is m i, the collision frequency is nu i and the velocity is V i. We have done, we have written the equation of motion for electrons and for ions.

Now, we would like to obtain an expression for the diffusion coefficient for ambipolar diffusion. How do we do that? What we do is, simply add the equations. Add the two equations that I have shown you for electrons and for ions.

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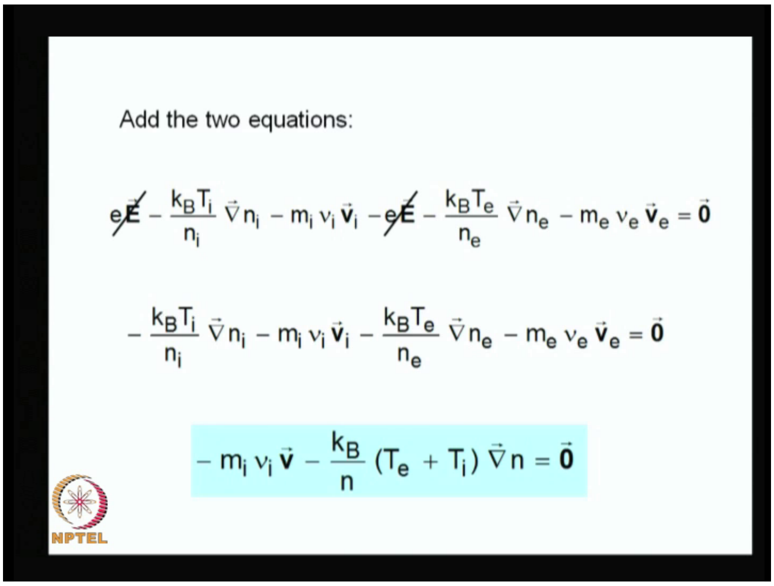

So, what do we get? We get an expression e E minus K B T i upon n i grad n i minus m i nu i V i minus e E minus K B T e upon n e grad n e minus m e nu e V e is equal to 0. The term e e and minus e e cancel out. e e and minus e e cancel out and we are left with minus K B T i upon n i grad n i minus m i nu i multiplied by V i minus k B T e upon n e grad n e minus m e nu e V e is equal to 0. It is still looks a pretty complicated equation. And so, we solve it, we solve it under certain assumptions, which are valid for the case, that we are considering, which are because the ions and electrons are diffusing at the same rate. We can say that V i is approximately equal to V e.

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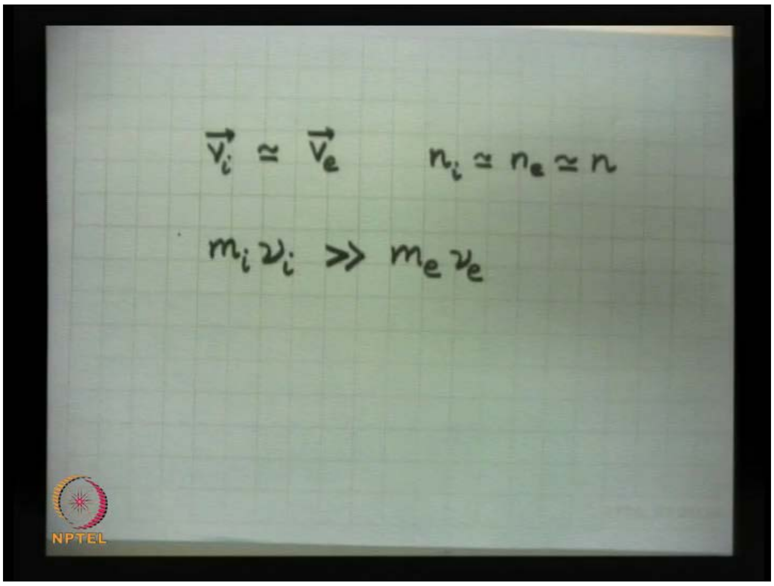

I have written it down for you. V i is approximately equal to V e, the number density we can assume to be approximately equal and we put it equal to n. But the product since ion mass is much greater than m e. The electron mass, the product m i nu i is much greater than m e nu e.

So, we neglect the term m e nu e V e in this equation and we put n i equal to n e equal to n and we put V i equal to V e equal to V. So, we are left with this equation. Minus m i nu i V minus k B upon n multiplied by T e plus T i within brackets grad n is equal to 0. This is a much simpler equation compare to the earlier one and from here, we can immediately write a solution for the velocity.

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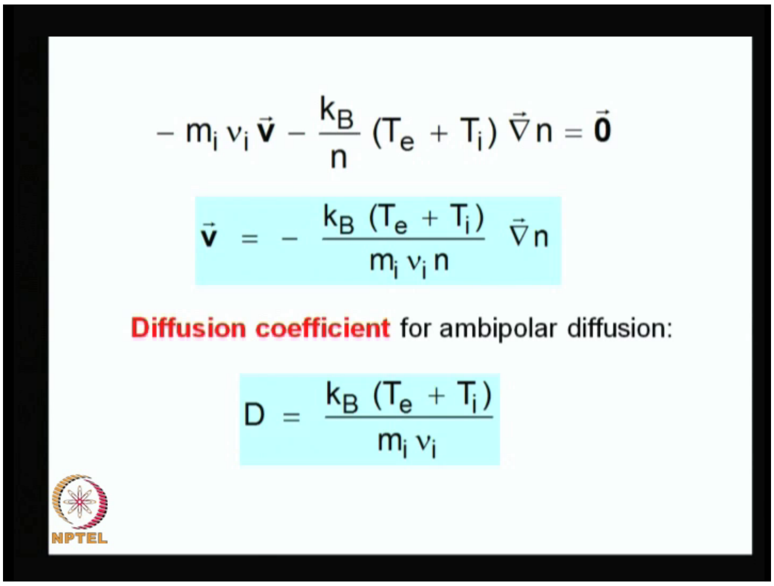

I have repeated the equation in this slide for ready reference. The expression for velocity then becomes, V is equal to minus k B T e plus T i upon m i nu i n grad n. So, you can readily see that the diffusion coefficient for ambipolar diffusion is simply D is equal to K B T e plus T i upon m i nu i. This is a simple enough expression for the diffusion coefficient for ambipolar diffusion.

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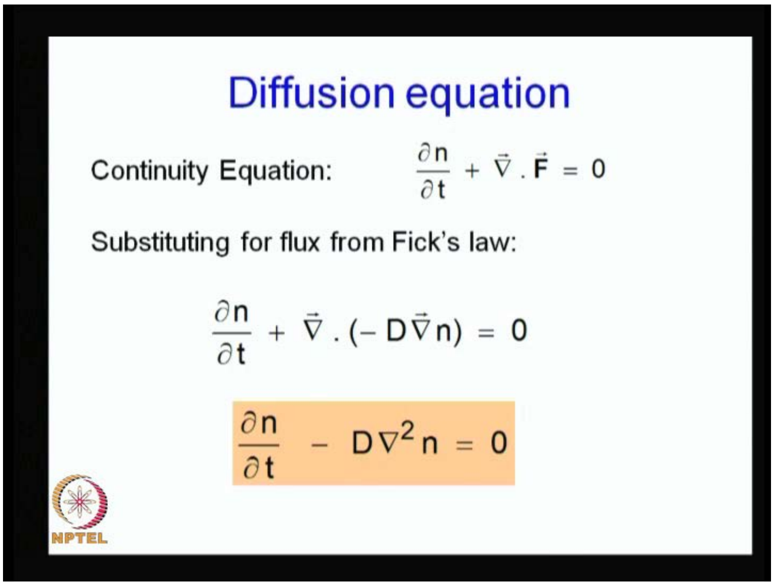

Now, using the information, we using whatever we have learnt so far, we can write down the continuity equation or the diffusion equation and try and solve it for some special cases.

As you know the continuity equation is, you are familiar with it is del n upon del t plus del dot F is equal to 0. This is a scalar equation, because the divergence of F is a scalar.

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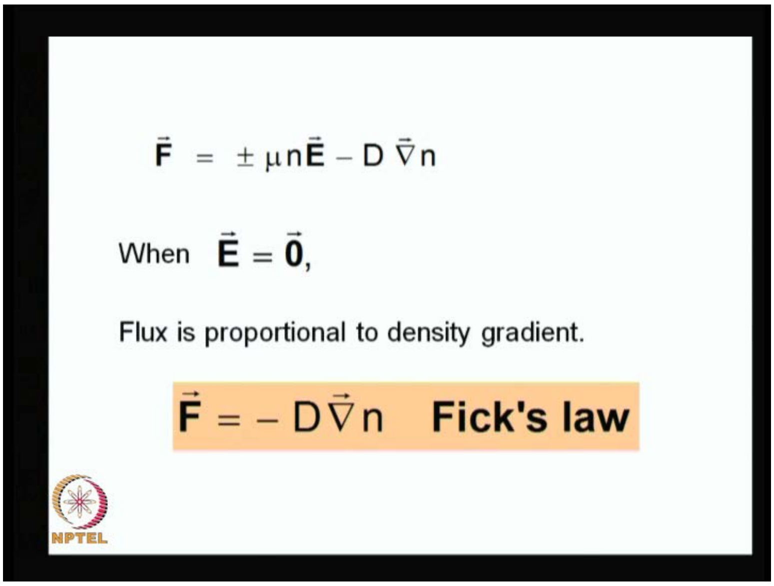

Now, a little while ago, I have shown you that for plasma weakly ionized plasma. In this special case, when e is equal to 0, the electric field is 0, which happens to be the case for ambipolar diffusion also. The flux is simply negative of diffusion coefficient multiplied by the density gradient. This is the fick's law.

And so, if we substitute this expression of the flux from fick's law, what do we get? We get del n upon del t minus D del square n is equal to 0. Again, this is a Three-dimensional equation and it can be solved for special cases. And, one of the interesting special cases, there are there are many cases that have been taken up in the reference books, that I have sighted. F F Chen as well as fitzpatrick and any other text book on plasma physics that you will be studying in this course. But we have for this particular lecture, I have chosen a special case of laser produced plasmas.

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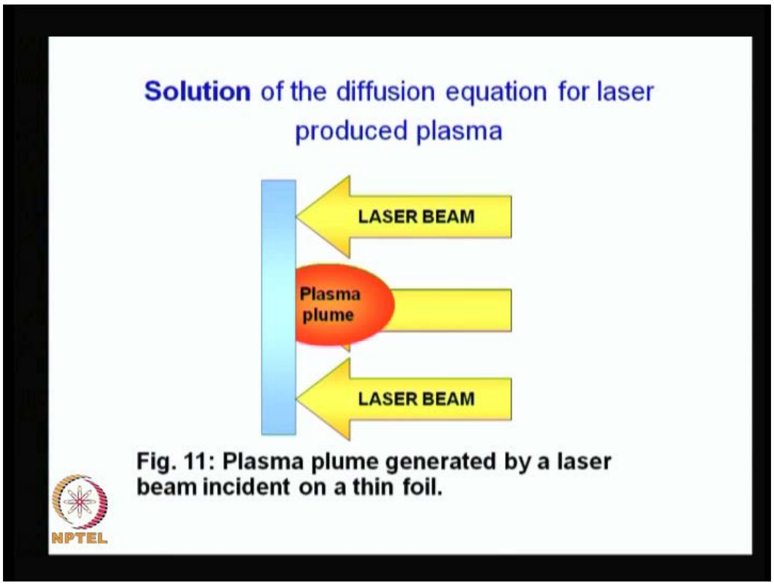

So, for a very special case of laser produced plasma, we will solve the equation and obtain an expression for the diffusion coefficient. What happens in this process? You have a thin foil, which is shown on the screen and you bombard that with laser beams, what happens is that a plasma plume is produced. This is the plasma plume that is produced, when laser beams bombard a thin foil.

What are we interested in now, is how the plasma particles will diffuse in this plasma plume? What will be the diffusion time? This is a case of a weakly ionized plasma. This is a kind of a Gaussian density distribution.

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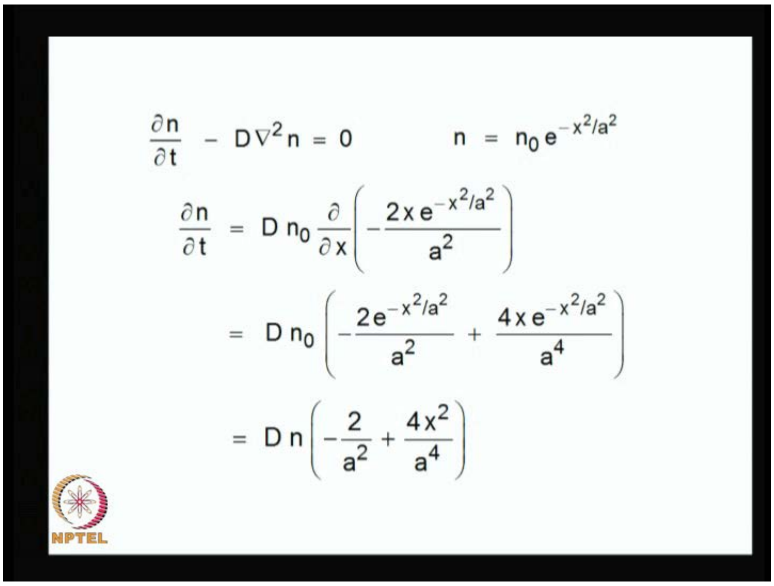

And we can solve the equation for by assuming a distribution for the number density, which goes as n is equal to n 0 e raise power minus X square by a square. This is a reasonably good approximation for the plasma plume that is formed, when laser beams are bombarded on a thin foil.

So, if we assume this, we have assumed a one-dimensional function for the number density, it is e raise power minus X square upon a square. So, this then turns into a one-dimensional differential equation in time and length say X. So, del square n is simply D square n upon d X square, I have not written it here.

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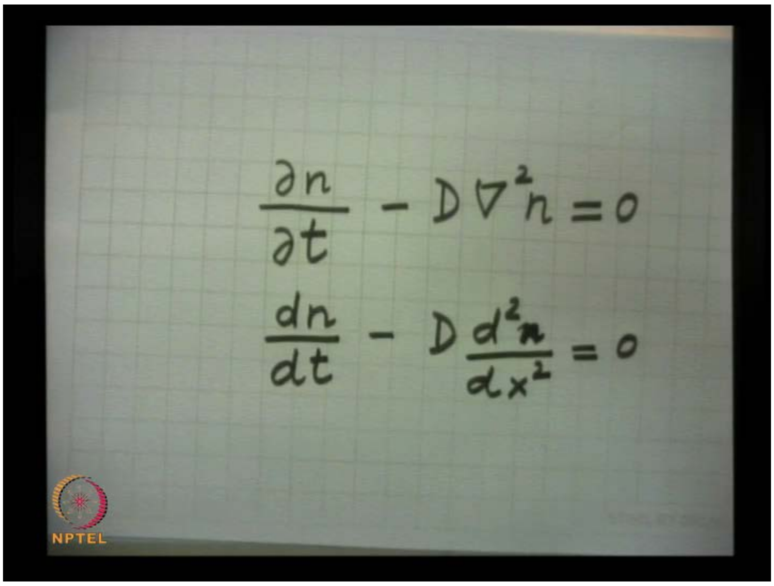

Del n by del t, I can write this equation may be here on the craft paper. Del n, this is the original equation, del n upon del t minus D del square n is equal to 0. We have assumed a one-dimensional expression for the number density. So, essentially we get what? Let us say, d n by d t minus D d square X d square sorry D square n by d X square is equal to 0. This is n.

So all we need to do, is find out D square n upon d X square, I hope it is clear to you that this is N. We find out the value of d square n upon d X square and let us see what happens? So, what we need to do is, differentiate this expression with respect to X twice and this is what we get. These are three terms shown here. When you differentiate it once you get these 2 terms and differentiate it differentiated once you get minus 2 X e raise power minus X square by a square upon a square. This is first derivative. Differentiate it again, you get these 2 terms, right.

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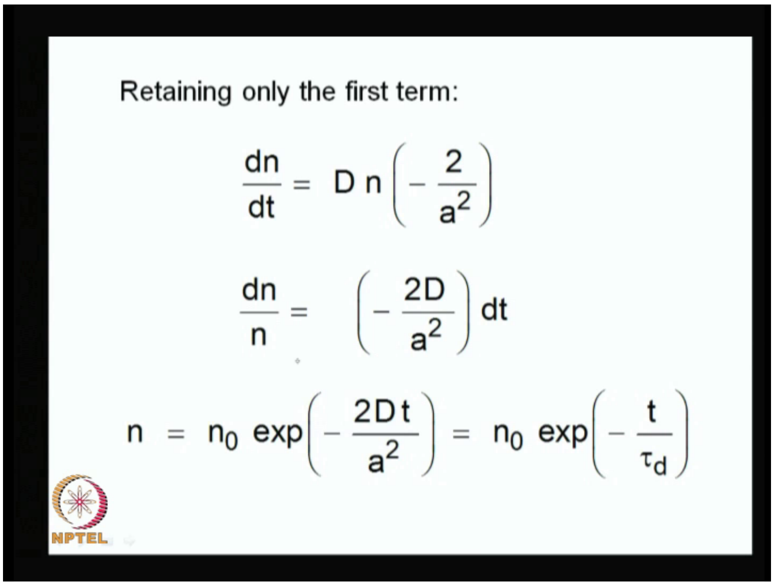

So, when you simplify, you can write it as this. n 0 n 0 e raise power minus X square upon a square is nothing but n here. n is n 0 e raise power minus X square upon a square. So, you get you just take this common factor out and put n 0 e raise power minus X square upon a square equal to n and within brackets, you are left with minus 2 upon a square plus 4 X square upon a 4. No, this should have been X square.

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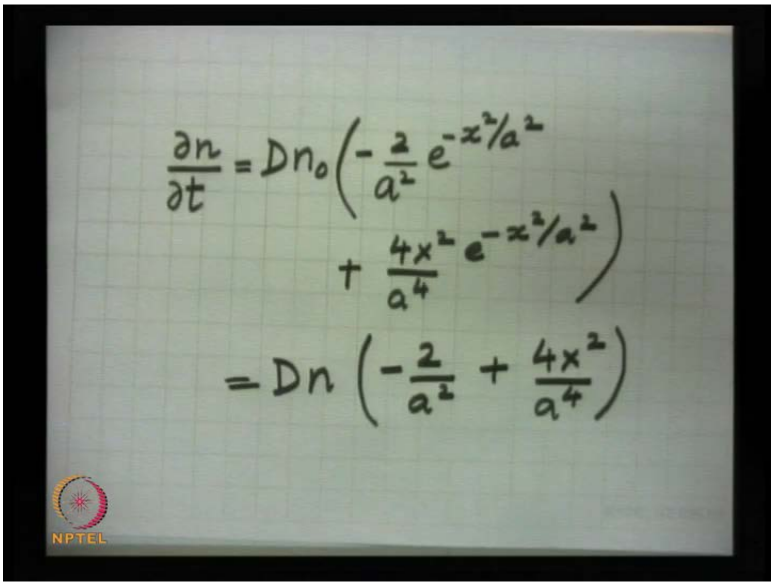

So, I am sorry, this should have been X square. Shall I write this equation again for you. So that, for the sake of correctness, del n upon del t is equal to D n 0 minus because it is the second derivative of this particular expression or the derivative of this. So, you differentiate X, you get this and you differentiate here, you get X square. So, this becomes minus 2 upon a square e raise power minus X square upon a square plus it should be 4 X square e raise power minus X square upon a square and the whole thing divided by a 4.

So, this should be the expression and so we take e raise power minus X square upon a square out of the bracket, multiplied by n 0, we get n and within the bracket, we get minus 2 upon a square plus a second order term in X upon a 4. This is what, is the correct expression.

Now, if we neglect the second order term 4 X square upon a 4 and retain only the first term, the constant term. We can write the derivative D n upon D t as D n minus 2 a 2 upon a square and you can easily solve this equation, to get an expression of n is equal to n 0 exponential minus t upon tau d.

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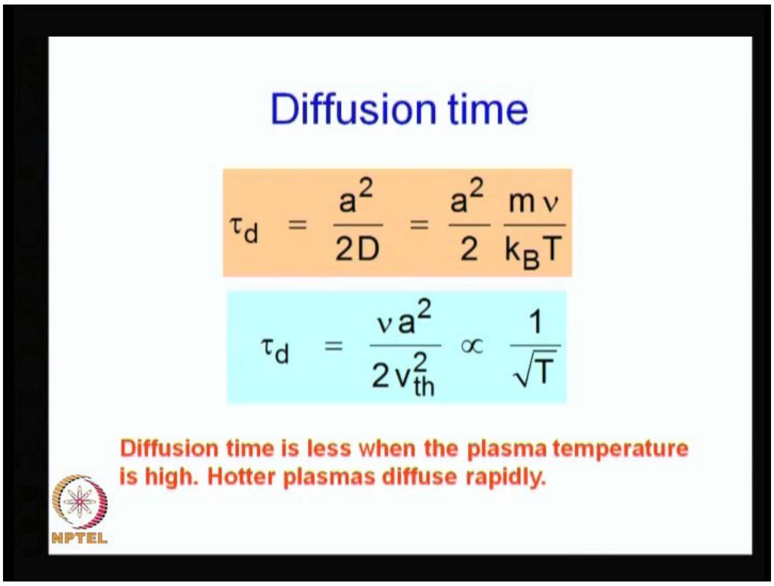

Tau D is nothing but the diffusion time is equal to a square upon 2 D, tau d is a square upon 2 D and we get that as a square 2 m nu upon K B T or nu a square upon 2 V thermal square, which goes as 1 upon square root t. The diffusion time in laser produced plasmas is 1 upon square root t. So, it is less, when the plasma frequency is high diffusion time and therefore, we say that hotter plasmas diffuse rapidly.

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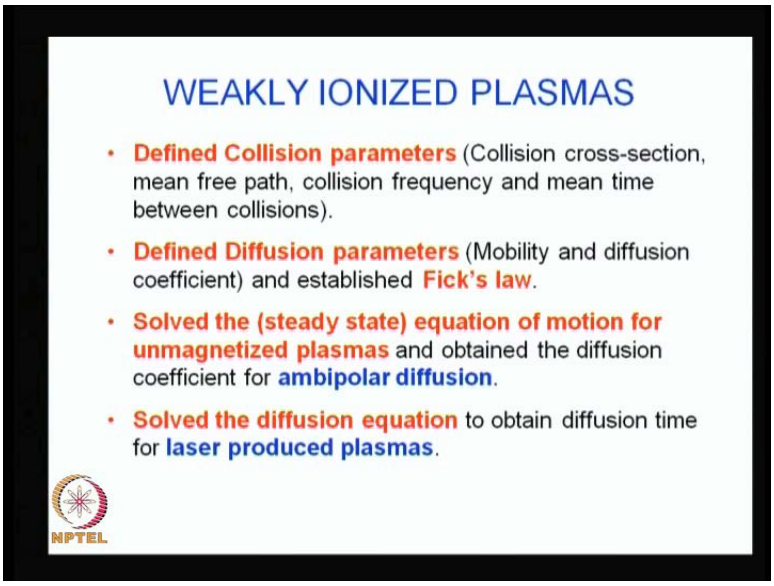

So, this is where, we come to an end of this discussion. Let me quickly summarize this for you. What we have done today, in our discussion on diffusion in weakly ionized plasmas, we have defined collision parameters like collision cross section, mean free path collision frequency and mean time between collisions. We have then defined diffusion parameters, mobility and diffusion coefficient and established fick's law. We have solve the steady state equation of motion for un-magnetized plasmas and obtain the diffusion coefficient for ambipolar diffusion.

Lastly, we have solved the diffusion equation to obtain diffusion time for laser produced plasmas. So, this is an important process of diffusion in weakly ionized plasmas, that we have studied today for ambipolar diffusion and laser produced plasmas.