Plasma Physics

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Lecture No. # 37
Anomalous Resistivity in a Plasma

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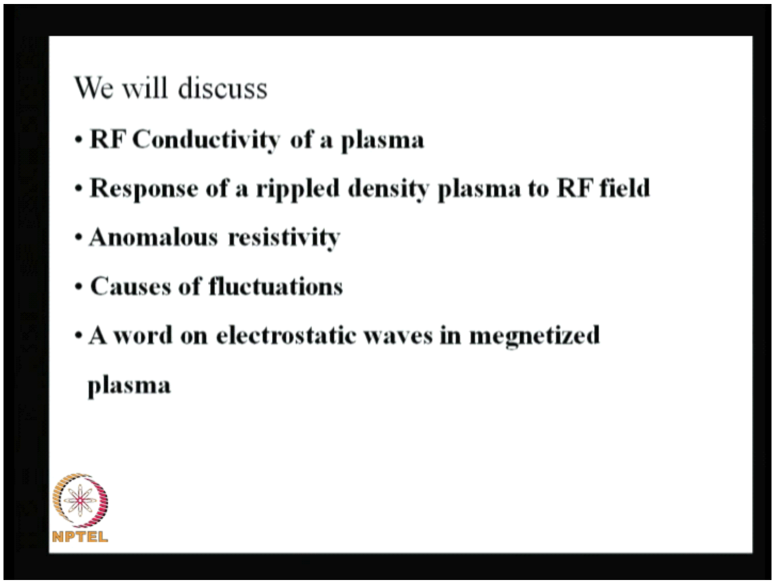

Today, I am going to talk to you about anomalous resistivity in a plasma. We shall discuss a conventional radio frequency conductivity of a plasma, then I will talk to you about response of a plasma, which has a ripple in the density to a radio frequency field; and we will introduce anomalous resistivity, deduce an expression for it, and then we will mention what are the causes that can give rise to density fluctuations, and I would like to mention a few things about magnetized plasmas, because we have discuss the kinetic theory of electrostatic waves in unmagnetized plasmas, but most plasmas are magnetized and hence, I should talk something about magnetized plasmas, though without giving a detail derivation of susceptibility.

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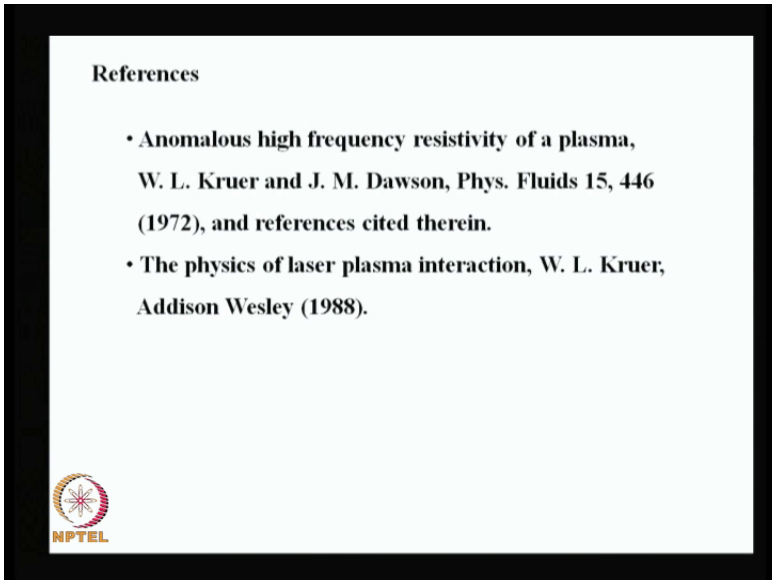

So, let me well for today's presentation, I would like you to refer to two, one paper and one book; one is by Kruer and Dawson, this is a old paper and they have a lot of references sighted in that paper that are relevant for today's discussion, and then there is a book by Kruer, the physics of laser plasma interaction, published in 1988.

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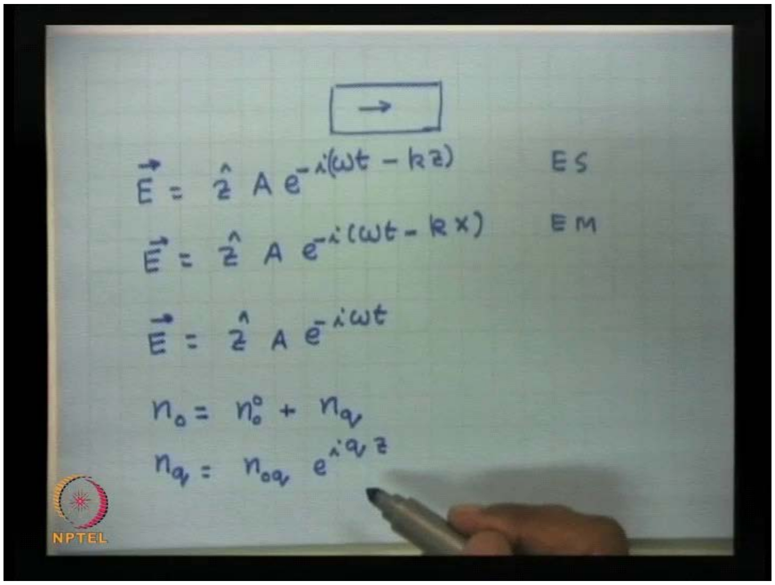

Well, let me briefly formulate the problem. The problem is that suppose I have a plasma and if I apply a radio frequency field to the plasma, the electric field may be oscillating. Suppose, the electric field I subject to the plasma is suppose z and amplitude is A exponential minus i omega t, it may be z dependence also but that z dependence I am considering for the moment to be weak or may be if you like, you can have this is suppose k z also here. but actually I would like to keep this general because if I specify my z dependence like this, then this means the electric field is corresponding to a longitudinal wave, because the electric field in the same direction as z variation. but in general a plasma could have an E M wave like this E that can have electric field in the z direction but the phase variation may be in the x direction for instance.

So, this is a transverse electromagnetic wave, the wave is travelling in x direction but the field is polarized in the z direction. So, this is the electrostatic wave and this is the E M wave, electromagnetic wave. So, rather than a specifying the dependence of space on a space for E, I will simply say that I am considering a case where E is equal to say z cap A exponential minus i omega t. but in the system has a density, plasma has a density, equilibrium density is suppose n 0 0 plus there is a ripple in the density, I will call this as n q, the density ripple, which I will write down n q as some amplitude n 0 q exponential of i q z and I will presume, for the sake of simplicity that q is much bigger than k.

So, the dependence on a space of current etcetera that are produced is primarily coming because of n q rather than it because of E. but the time dependence is coming from the electric field. So, my issue is that if I have an electric field in a system like this, then the plasma will respond by creating a current in the system and normally, we solve the fluid equation, which says that m delta v by delta t plus v dot del v is equal to minus e E minus m nu v, this is the collisional momentum loss per second, this is the momentum electron gains from the electric field, where minus e is the electron charge, m is the electron mass and this is the d v by d t, rate of change of velocity. So, what we do? We normally write v as a exponential minus i omega t, because of the electric field being having time dependence like this, we write like this and linearise this equation and solve it, we obtain v is equal to e E upon m i omega plus i nu.

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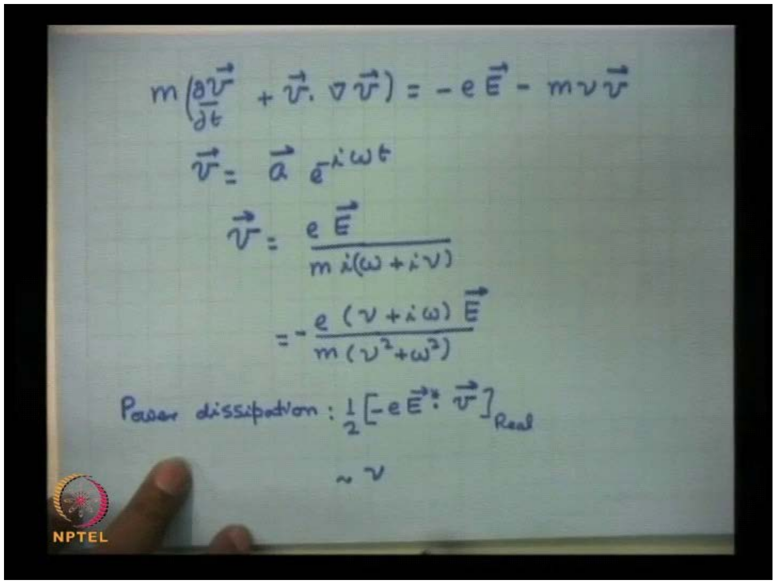

I can write this in two parts. Rationalize this, it becomes e upon m nu square plus omega square, actually minus times will be here and you will get nu plus i omega into E. Now, this equation tells you that v has a part in phase with the electric field because of nu, because the real part of nu. but then there is a phase a part of velocity, which is out of phase by pi by 2, because of this i term.

And if you calculate the power dissipation, power dissipation per second or energy dissipation per second per particle, then we have mention this earlier, is equal to the electric force on the electron is minus e E dot the displacement of the electron per second. So, this is the energy dissipation by the wave on the electron per second, where it is implied the real part of the first term and real part of second term to be multiplied and which turns out to be half complex conjugate of the real part. So, this is the. So, real part of this product if you take where star represents the complex conjugate, then power dissipation you get and this is proportional to nu. That is why this new part is called resistive part of conductivity or mobility and omega is called the reactive part.

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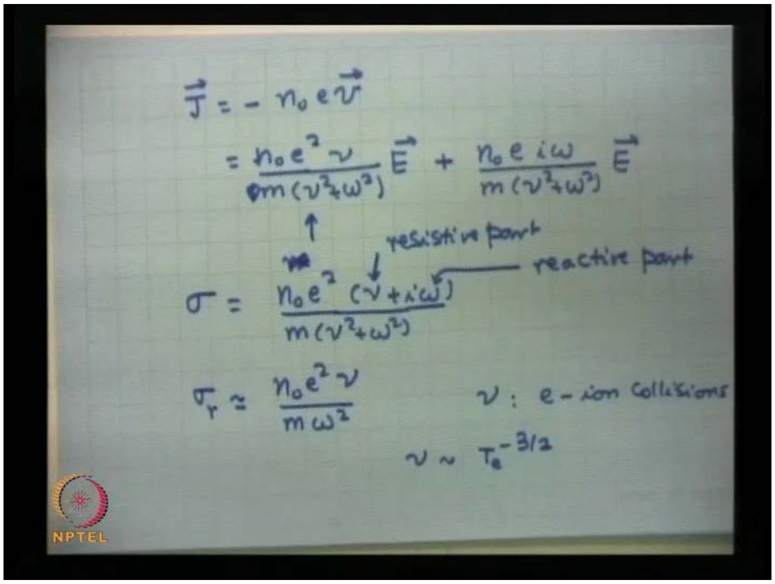

So, if my plasma were of uniform density, I will get J is equal to minus n 0 e v and that will give me two terms here. One will give me n 0 e square upon m nu square plus omega square into nu, sorry this quantity is J into e and another part would be minus, rather plus n 0 e i omega upon m nu square plus omega square into E, this is called the resistive part of conduct, this called whole coefficient, if I take e common, then the total expression is called conductivity. So, let me write conductivity of a plasma n 0 e square upon m nu square plus omega square into nu plus i omega, this part of conductivity real part is called resistive part and this imaginary part is called reactive part or inductive part.

Usually in plasmas, nu is very small as compare to the applied frequency, because you apply frequencies high frequencies and there nu is very small. So, basically resistive part is equal to n 0e square nu upon m omega square. So, rather than calling this entire quantity as the resistive part of conductivity, we call nu a simply resistivity (( )) or rather in general usage. So, nu is a as a manifestation or nu is representative of resistive part. So, that is the quantity effect, a nu have to define. Now, in a collisional plasma electron suffer collisions with ions or neutral atoms and the plasma is strongly ionized and they are nu neutral atoms. So, nu is primarily due to electron ion collisions and as you are familiar that this collision frequency scales as minus 3 half power of electron temperature.

So, in a hot plasma this is a small quantity. So, waves are weakly damped by a collisions in a hot plasma. However, experiments have found that there are waves are strongly observed. So, what is going on? It has been found then the presence of a wave gives rise to density fluctuations and whenever there is a density fluctuation in a plasma, it gives rise to a strong absorption of waves. Now, let us try to understand what is the mechanism of collision less absorption of waves.

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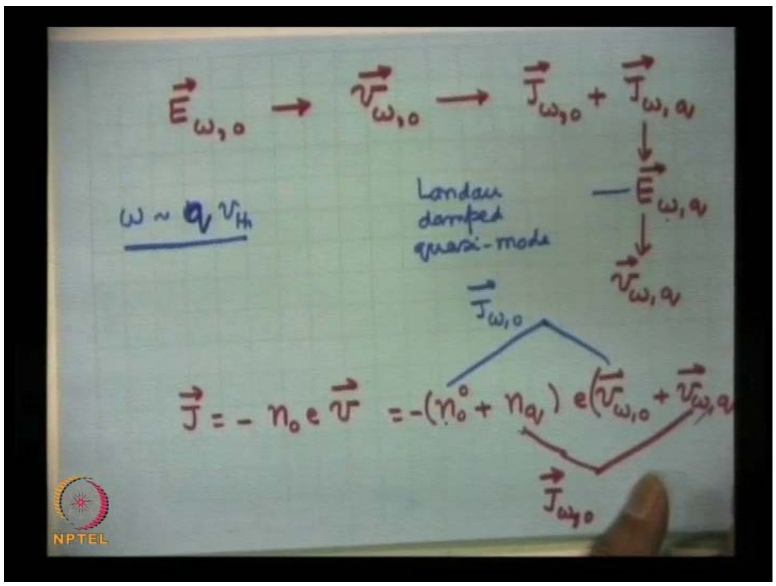

First, let me give rise to a physical picture, what is really happening here? Suppose, I have an electric field that I apply, this electric field has a omega frequency and wave number nearly 0 or tiny. Then, this wave will immediately produce oscillatory velocity of electrons at frequency omega and wave number nearly 0. As soon as, this velocity is produced this will give rise to at a current, which will be at frequency omega wave number 0, plus a additional current will be produced because of density fluctuation, that will be because J is equal to minus n 0 e v, n 0 is two parts, n 00 part plus n q part, e v and I am taking omega 0. So, whenever you multiply this n q with this, you will produce a current at frequency omega in wave number q. This part of current will produce in electric field or potential at omega and q and this will produce an oscillatory velocity of electrons at omega and q.

So, what will happen? If I substitute this v omega q and multiply this by n q, rather this v omega 0 will be actually now, you have additional velocity v omega q. So, the product of this n q with this v omega q, if you take the complex conjugate, it will give rise to, so this term and this term together will produce current at omega frequency and wave number 0. So, there are two sources to produce current at omega frequency that the frequency of the wave and wave number of the wave omega frequency and 0 wave number. One is the product of n 0 0 and v omega 0, this product will give rise to J omega 0 and then this new product will be there, this term will be there, even if there were no density fluctuation in the system, density oscillation in the system. but this is the new term that will appear only when n q is finite. So, whenever there is a density ripple in the system, your current will have additional term.

And what is can happen, this electric field or potential that you have at omega frequency in q wave vector, if it is such that omega is of the of k v thermal q v thermal, in that case this will be heavily Landau damped and it will have a finite imaginary part in its response. So, v omega q will not be in the same phase as the u omega q but because of the Landau damping term, Landau resonance term or Cerenkov resonance term, it will have a part out of in phase with the electric field, not pi by 2 out of phase and that gives rise to J, which is in phase with the electric field. Otherwise, because of these two terms if there are no collisions, then J and e are out of phase by pi by 2.

But, this product because of this resonance will have a damping term. So, physically what is happening, that wherever you launch an electromagnetic wave of frequency omega into a plasma, it produces in a ripple density plasma a quasi electric field at frequency omega in the q wave vector and that electric field damps the or takes feeds energy into the electrons by Landau damping. So, this is a quasi mode which is Landau damped. So, I will call this as Landau damped quasi mode and that gives rise to effective collisions or effective resistivity in the plasma. So, I would like to make a simple calculation for this and then I will indicate, if you permit finite wave numbers, what will happen? Is it trivial extension beyond that? So, let me precede a step by step.

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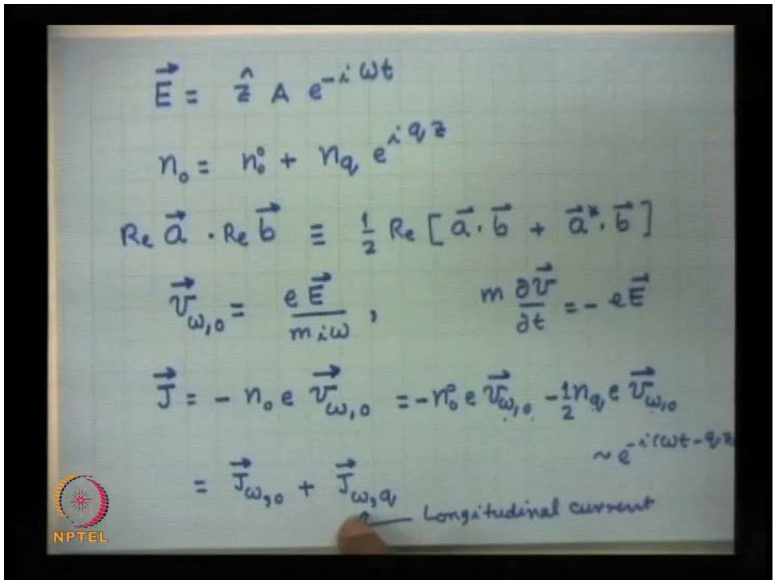

Let me redefine the problem. I am considering a plasma subjected to an electric field E is equal to say z cap A exponential minus i omega t and the plasma has a density n 0 is equal to n 0 0 plus n q exponential of i q z. Well in proceeding further, I would like to remind you of one complex number identity that whenever you are encountered with numbers like a, which is a complex number and another complex number b, then the real part of a into real part of b is identically equal to half the real part of a dot b plus a star dot b or a dot b star, either of the you can just put a star on either a or b, not on both. You can prove it.

So, what I am expecting now, that as a consequence of this electric field, the electrons acquire a velocity v at omega frequency and that would be equal to e E upon m i omega. This is I am getting from a collision equation of motion and you may just check it, m delta v by delta t is equal to minus e E, this is the equation I am solving by replacing delta delta t by minus i omega, you get this result. So, this electric field quickly gives rise to this velocity. Now, if I substitute this in this expression for current density J, which is equal to minus the electron density into v e v omega. I think it will be better if I put a subscript 0 also here, because wave number I am taking to be 0 here, there is no space dependence.

So, it will produce this product will have two terms, one is n 0 0 e v omega 0 and with a negative sign, another will be minus n q e v omega 0. Now, please remember I have to multiply the real part of this quantity density is a physical observable, it cannot be complex. I have written in the complex notation, implying that real part of this quantity is implied. Similarly, v omega 0 is also a physical quantity, which is the real part of the complex number. So, here what I have to do, real part of this is to be multiplied with the real of this and as using this identity, I have to put a half in there.

So, you are creating a wave of frequency omega, current of frequency omega in wave number q because that 2 exponential, 1 exponential of electric field is multiply with the exponential here. So, when you multiply the two, this quantity this product will go as exponential minus i omega t minus q z. So, this will give rise to a current at frequency omega in wave number 0; whereas, this will give rise to a current at frequency omega in wave number q. So, let me call this current. So, I will write down J is equal to J at omega and frequency 0, a wave number 0, because of this term plus J at omega and q and this is the term, which goes as exponential minus i omega t minus q z. So, there are two terms produced.

Now, this is the term that will produce an electric field at omega frequency k wave q wave vector and because this, let us look at the direction of this, J I have taken to be in the direction of v omega 0, which is in the z direction and if q is space variation is also be z direction, this is called longitudinal current longitudinal current and whenever you produce a longitudinal current, it will give rise to density oscillations at omega frequency q wave vector. So, let me calculate the density oscillations in at omega frequency q wave vector.

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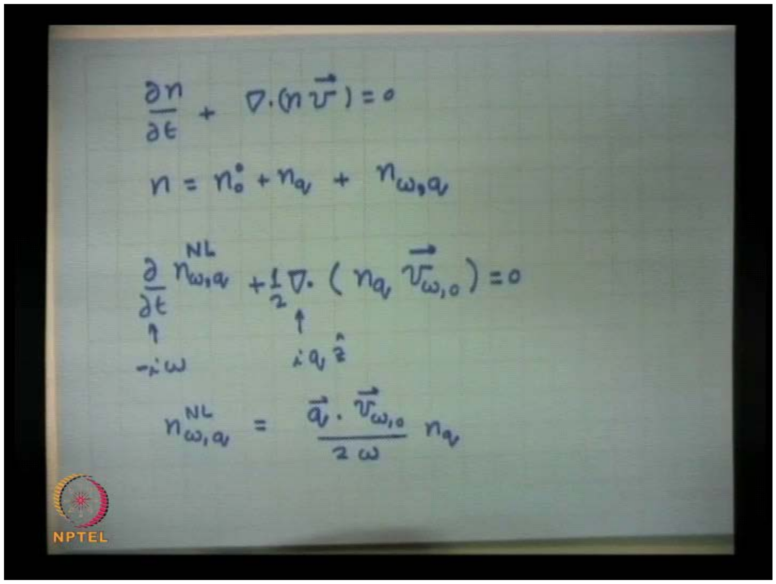

So, I say that the equation of continuity is delta n by delta t plus divergence of n v equal to 0. I say that my n is equal to n 0, that was earlier or n 0 0 plus n q, it was there, plus you have produced an any additional density oscillation at frequency omega and wave number q, because it is current. So, I want to calculate the modified density, this term. So, I put this in here and equate or retain here, also the product that has omega frequency in q wave vector. So, this equation will give me delta delta t of n omega q plus del of n has these many terms and v has omega 0 term. So, I need a n q here and with v omega 0. So, that this product has a frequency omega in wave number q and half will come because this is a product of two complex real parts of two complex quantity. So, I will put half here.

Well, this I call as the non-linear part of omega q. Why non-linear? Because there is no velocity here at omega frequency in q wave number. This omega 0 is produced by the original electric field of omega frequency. but the quasi field that it is produced by this non-linear current is not included in here. So, I will call this non-linear density perturbation. Replace this by minus i omega and this as i q and which is in the z direction. Simplify this, you will get the density perturbation n omega q non-linear is equal to q dot v omega 0 divided by twice omega from here into n q.

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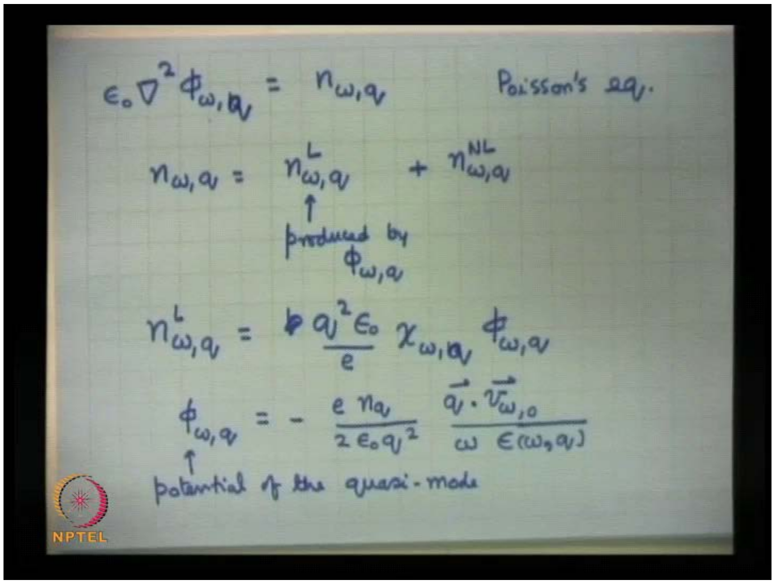

So, it is a simple expression. We got the density perturbation. but as I am saying that density perturbation will also produce a potential perturbation phi at omega k will be produced because del square phi from the Poisson equation is equal to, epsilon 0 if I multiply here, this is equal to density perturbation of electrons at omega k omega q. Now, whenever you are producing a potential density perturbation like at omega frequency q wave vector, this will also produce a density perturbation. Please note the density perturbation n omega q non-linear is caused by the electric field that you have arrive externally. but this is the potential of the quasi field that is generated inside, this will also produce a density perturbation. So, I write n omega q as n omega q linear, which is produce by phi omega q plus n omega q non-linear, which is produced by the pump wave, which is the outside wave.

And from our theory of Landau damping, we have learnt. Forget all these that if there is a plasma wave in a system of frequency omega in wave number q, then the density perturbation is related to that and that is given by this expression, n omega q is equal to (()) q square epsilon 0 upon charge of the magnitude of electron charge into susceptibility at frequency omega in wave number k into phi at omega, sorry q here, phi at omega q and all the information about kinetic effects is contained in the susceptibility. So, I do not have to solve the vlasov equation. I can simply write down, because I am familiar with my theory of plasma waves, kinetic theory of plasma wave.

So, I can always write down the density perturbation caused by this intermediate or quasi potential, that is created because of this charge variations. So, I substitute this in here, then this equation takes the following form. So, this gives me phi omega k, phi at omega q turns out to be simply equal to, just substitute it here and you will get this is equal to minus e n q divided by 2 epsilon 0 q square into q dot v omega comma 0, I am simply getting this result, divided by omega into epsilon at omega k q, where I will write down the values of epsilon and chi in a minute, this is the total expression I get for the intrinsic of or quasi potential or quasi mode. It is called the potential of the quasi mode or this is the Poisson equation or first Maxwellian equation.

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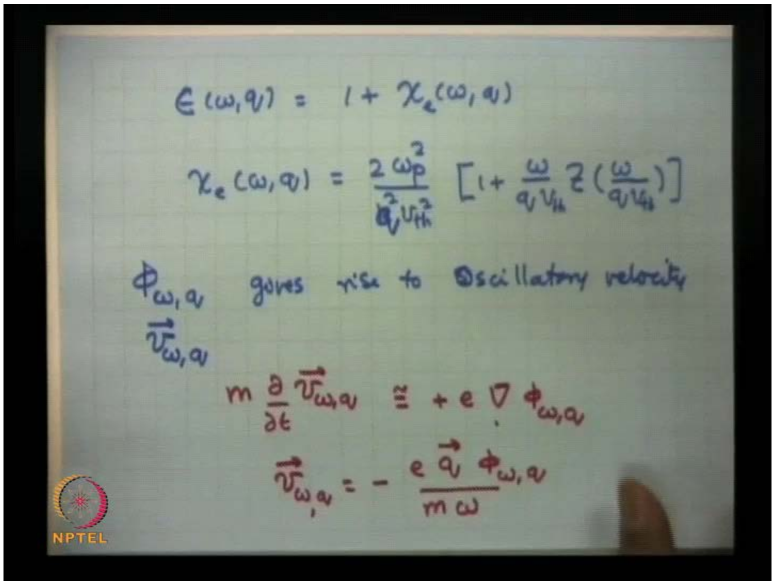

Now, here epsilon is 1 plus chi at omega chi electron susceptibility at omega q and chi e at omega q, I had written in my last lecture as 2 omega p square upon k square v thermally square, k is actually q here into one plus omega upon q v thermal z function of omega upon q v thermal. So, this as a imaginary part and hence, this epsilon will have a imaginary part and consequently phi omega will q will also have a some imaginary part in the denominator. Now, what is the consequence in phi omega q? This self field will give rise to oscillatory electron velocity, phi omega q gives rise to oscillatory velocity v omega q. So, I solve the equation of motion for electrons, which is m delta delta t of v at omega q is equal to the electric force minus e and electric field is grad phi with negative sign. So, grad phi of omega q, just simplify this.

Well, to be regress I should solve the vlasov equation, to obtain v omega q in terms of phi omega q but if omega upon q v thermal is significantly greater than 1, I can still approximately solve the equation of motion. The result is quite accurate. So, in that limit you simplify this by replacing this by i q and this replacing this by minus i omega and you will get v omega q is equal to minus e q phi omega q upon m omega. You may note that these equations are really very simple equations. Though, I am using a number of them. 4, 5 equations I am using and I am also using a, borrowing an expression from kinetic theory for electron susceptibility.

But, then in terms of susceptibility, one can easily write phi omega q, that I did earlier and using phi omega q, I can write down the electron velocity. Now, I am ready to write my current density. My goal was to find current density at omega frequency in wave number 0. So, I can simply write down, J at omega and wave number 0. How much is this, because this is the frequency of the applied electric field and this the wave numbers are applied electric field.

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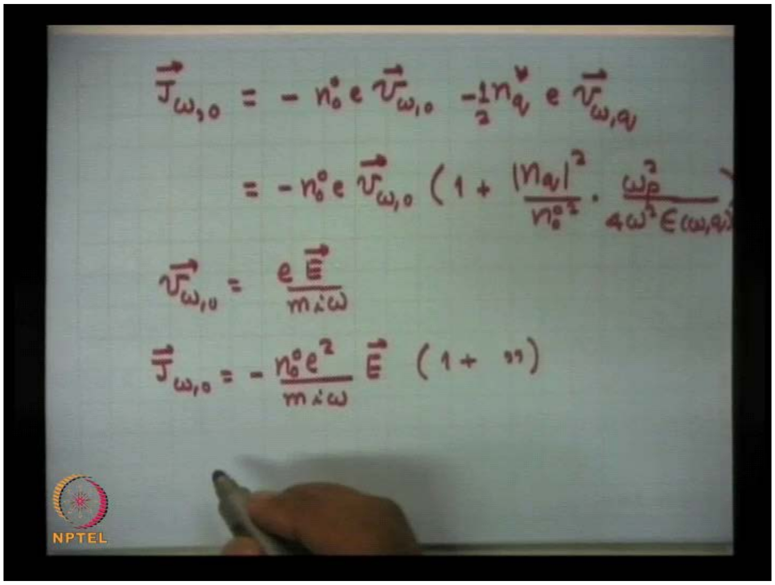

This will be how much, there will be two terms, one is n 0 0 e v at omega 0 and then there will be a term minus n q, star I have to take, because I want to cancel the omega q here, because when I put a complex conjugate here, then exponential i q z from here and exponential minus i q z from here, they will cancel each other. So, z dependence is gone. So, this is the density perturbation present in the system, this is the oscillatory velocity produced by the quasi mode, that is excited by the non-linear interaction of the density perturbation in the oscillatory velocity, due to the incident electric field.

So, now let me substitute the expression for v omega q and when you do this algebra, you will get this expression turns out to be the same as this, n 0 0 e v omega 0 outside, inside you will get a term, one plus n q square upon n 00 square multiplied by omega p square upon 4 omega square and epsilon at omega q. This is the beauty here. The physics has come out with some clarity here, that epsilon which is responsible for resonant wave particle interaction through Landau damping, gives you an imaginary part here. Otherwise, this v omega 0 is out of phase from this with the electric field.

Let me write remind you, v omega 0 we had written, is simply equal to, it was e E upon m i omega. So this is out of phase by pi by 2 from the electric field. Let me substitute this here. So, you will get J at omega 0 is equal to the classical conductivity, which is equal to minus n 0 0 e m i omega and e square upon omega into E. but besides this there is a factor here, one plus this factor and imaginary part of this is equivalent to nu, nu upon omega. So, we have got resistivity here for this epsilon term.

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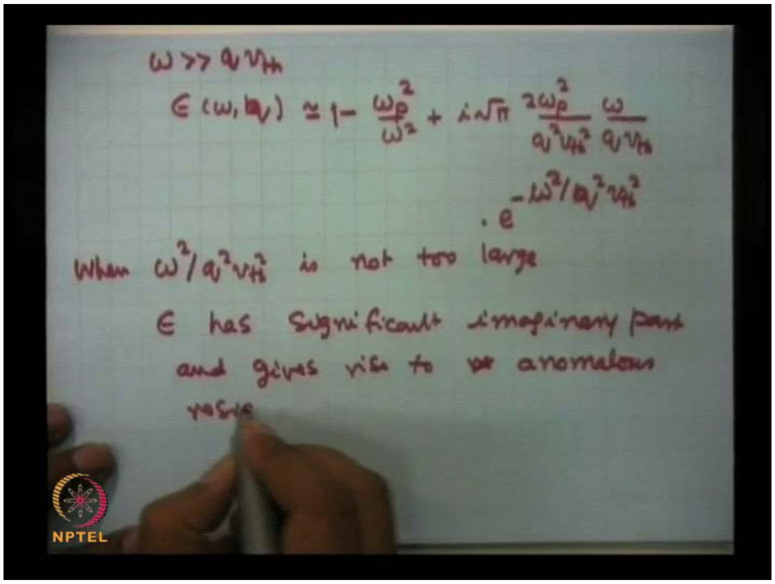

Now, let me talk about this epsilon term, epsilon omega k is called the plasma permittivity, omega q, which is approximately equal to for omega significantly bigger than q into v thermal, this is nearly equal to minus omega,sorry1 minus omega p square by omega square plus i root pi 2 omega p square upon q square v thermally square into omega upon q v thermal into exponential of minus I, sorry minus omega square upon q square v thermally square. So, what we are getting is, 1 upon epsilon the denominator. If omega were very close to omega p, then this is nearly 0 and the second term is purely imaginary. So, there is a huge resistivity.

On the other hand, if omega is half from omega p, suppose omega is much bigger than omega p, this term is simply unity but it is still the imaginary part is significant and it will be important. So, contribution to resistivity will come when omega square upon k square v thermally square is not too large, because if it is too large, then this exponential will be vanish, will be tiny and when omega by q square thermal square, suppose like it is like 10 not 100, then exponential minus 10 is significant is still.

So, when this quantity is not too large, epsilon has significant imaginary part and gives rise to n numbers of resistivity. Let me define effective collision frequency. If you compare this expression for current density from the one that I had derived in a collisonal plasma, then nu upon omega effective collision frequency nu star, I will call it, is equal to n q square upon n 0 0 square into omega p square upon 4 omega square and I should take the imaginary part of 1 upon epsilon. This is what I need.

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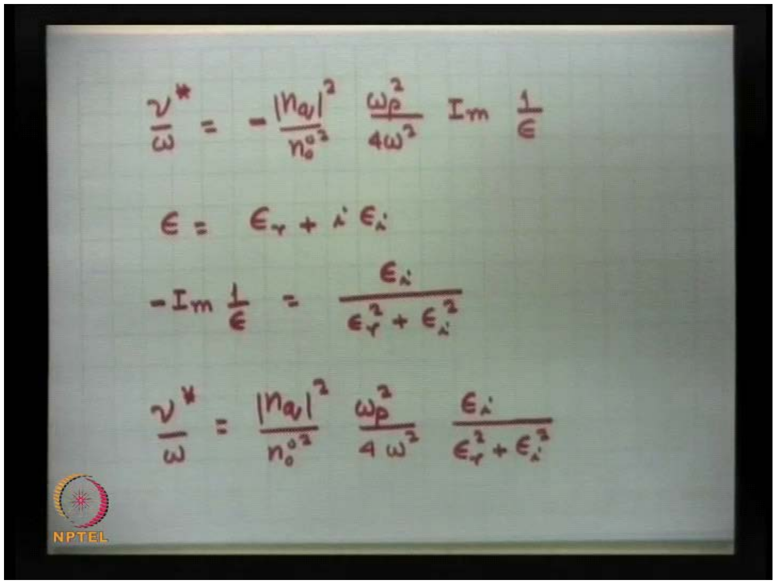

So, if I write down epsilon is equal to epsilon real plus i times epsilon i, i refers here to imaginary part, r refers here to real part of epsilon, then imaginary part of, I think it is minus 1 upon imaginary part. Let me just check, the expression for conductivity that I have derived earlier. It was minus, this is effective, this is minus. So, imaginary part of 1 upon epsilon is, rationalize this epsilon r square plus epsilon i square and epsilon i will be up with a negative sign. So, effective collision frequency that you are getting, this is also called resistivity, upon omega is equal to n q square upon n 00 square into omega p square upon 4 omega square into epsilon i upon epsilon r square plus epsilon i square.

This is a very neat expression and valid quite accurately for resistivity, enormous resistivity of the plasmas. There were no collisions, just because of Landau damping, epsilon i is finite and I have given the expression for epsilon i also, this is a important expression, it is very large when omega is close to omega p. So, when your high frequency wave that you are launching from outside has a frequency close to omega p, then resistivity caused by the density fluctuation could be very large. Now, a word about how are you going to create these density ripples in the system. These density ripples may not be esthetic in time, they may have a slow variation in time.

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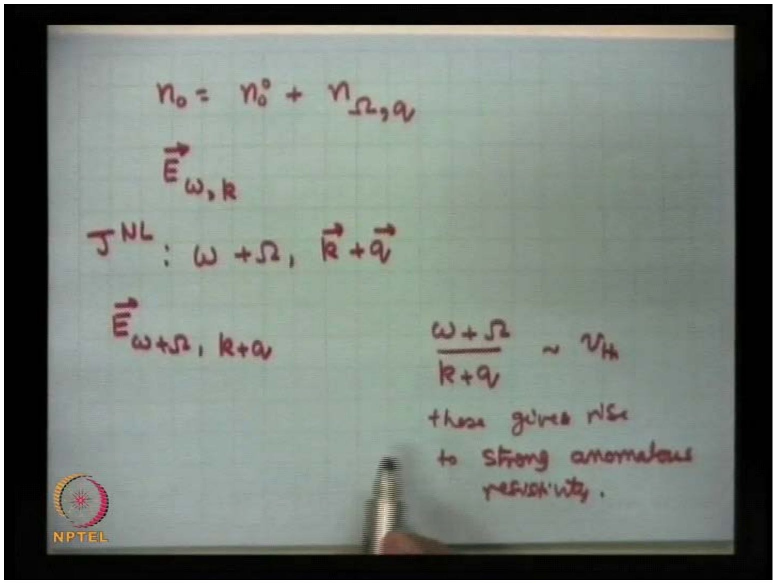

So, rather than having n 0 as n 0 0 plus n cube, you may have n at omega frequency and q wave vector, this may be density fluctuation due to a sound wave or ion cyclotron wave or even plasma wave. So, any wave present in the system will give rise to this and what you require that, when you are launching and the wave that you are having launching from outside, may have a frequency omega and wave number k, may not have k equal to 0. So, in general if your external field is at omega frequency wave number k, you will have a strong damping, whenever omega plus capital omega and k plus q, because the intermediate filed that you will produce by non-linear mixing or a non-linear interaction, the current rather that he will produce, J non-linear will be at this frequency and this wave number. So, the electric field that you will produce in the intermediate case will be at a frequency omega plus capital omega and k plus q.

Whenever they are in Cerenkov resonance, in the vicinity. So, whenever omega plus omega upon k plus q, is of the order of v thermal, not exactly equal to v thermal may be two times v thermal three times v thermal, they will give rise to a strong, these give rise to a strong anomalous resistivity. So, what you require the presence of a low frequency mode in the system and now you may ask a question that the mode may not have a single frequency, it may be many frequency modes or many wave numbers that is certainly possible. So, you have to add up like the expression for (( )) contained n q modulus square, you have to really sum over all such modes and that is not difficult to do and that actually becomes the problem of turbulence theory.

A system in which there are many waves of many wave numbers present simultaneously and you want to examine the response of a another wave or electric field of the another wave, then you have to consider the simultaneous interaction with the oscillatory velocity, due to the incident wave with the density fluctuation, due to all these waves and this has been a major research area over the last several decades and but are the most fascinating manifestation of anomalous resistivity has been infusion devices. It as a plasma interaction, for instance laser produce plasmas. People have seen lot of ion acoustic turbulence and that is responsible for anomalous resistivity.

Even in the ionosphere, in the f region of the ionosphere, I mention to you that we have we see spread up instability, which gives rise to low frequency short wave length density fluctuations. So, that also gives rise to anomalous resistivity but these are important when the wave number of the perturbation has to be such that q has to be such that omega upon q has to be close to v thermal, that is the crucial aspect.

So, all density fluctuations are not going to play a important role in anomalous resistivity. Only those ones that is satisfy this condition, this sort of condition they are going to play a role. This is a very crucial condition and I think plasma is very rich in waves. Especially if there is a magnetic field in the system, then you can get also certain instabilities, drift waves for instance are a major instability in plasmas and they can provide anomalous resistivity, not only resistivity they can also provide anomalous diffusion, anomalous conduction in the plasma.

So, I think before I close today, I would like to mention a word about how would you study the response of a magnetize plasma to electrostatic waves.

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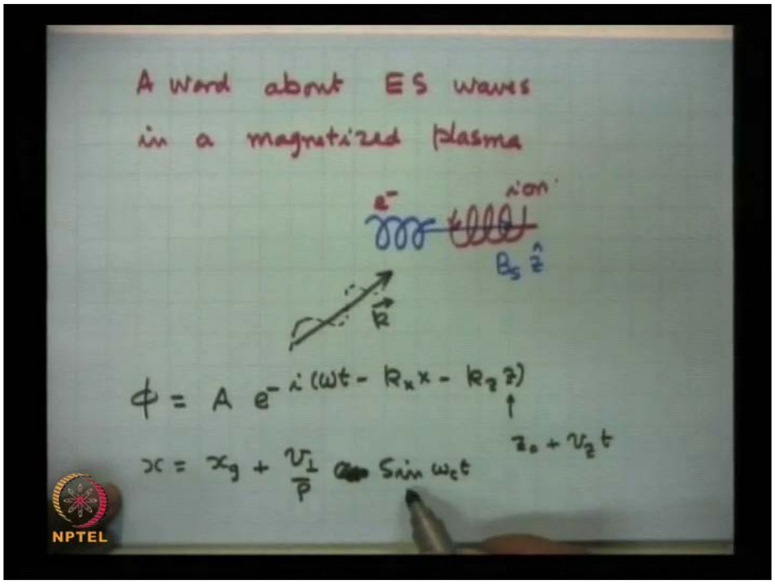

So, a word about electrostatic waves in a magnetize plasma. I am not giving any derivation here. but let me mention two three things. Consider a plasma with magnetic field in the z direction and if you introduce any plasma in this, which is hot into into this magnetic field, then the electrons will gyrate like this and ions will gyrate like this, in larger orbits. This is ion motion, this is electron motion. So, this is electrons and ions. They will gyrate above the lines of force and if you are launching a wave in the system, suppose a wave is going in the system with k vector here, then this wave will have electric fields with these kind of variations.

Now, these gyrating particles when see an electric field as I mentioned earlier, suppose I consider a potential phi, which is equal to A exponential minus i omega t minus k x X minus k z z. Suppose, this is the kind of potential of an electrostatic wave, I consider because this x is not constant, x is particle, x quantity gyrating, it is changing.

So, typically I would write that x is equal to some guiding center plus v perp by rho or cos of, I think sin omega c t. There is a oscillation like this. k z also z also is not constant, this is z of the particle is equal to initial value of z plus v z into t. So, there is a time dependence arising through x and time dependence are arising through z here. So, effective frequency of the potential has seen by the particles is different and one can employ, I think I gave this expression, that whenever you have exponential in the exponential is sin function, you can use a Maxwellian function identity and identity was like this.

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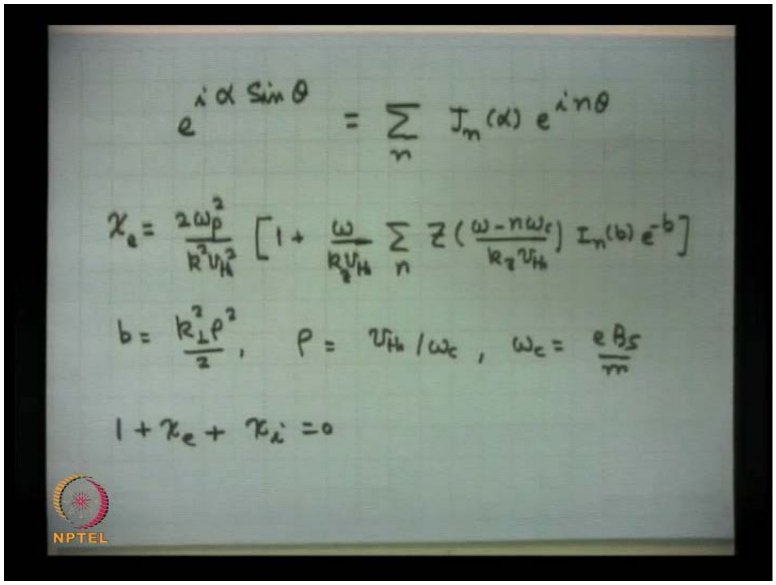

That exponential of i alpha sin theta is equal to summation over n J n of alpha exponential of i n theta. So, when you solve the Vlasov equation keeping in view, that your potential has x and z dependence and that has to be taken into consideration and then solve the Vlasov equation for perturbed distribution function. What you get? You get an expression for susceptibility, which is equal to 2 omega p square by omega k square v thermally square 1 plus omega upon k v thermal k z v thermal, this I am writing for a uniform plasma, this multiplied by summation over n arising because of this and there is a term here z function of omega minus n omega c upon k z v thermal of electrons into a modified Bessel function of order n argument b exponential minus b. This is called modified Bessel function, which is stipulated and b is a quantity that is responsible for k perp rho square by 2 and rho is equal to thermal velocity of electrons upon cyclotron frequency of electrons.

Cyclotron frequency is e b s upon m. Thermal velocity is the same as defined before rho rho is the larmor radius. So, this is the kind of expression you get, which is significant modification over the expression for chi that I give you earlier.

Now, you may note here that the argument of this depends on omega minus n omega c [noised] and you can get very strong large imaginary parts of this z function, whenever omega minus n omega c is a small is comparable to k z v thermal or a smaller, because this is same plasma dispersant function as before. So, the argument of depends on omega minus n omega c. So, you get resonances around omega equal to n omega c.

Well your dispersant relation for the electrostatic waves is 1 plus chi e plus similar expression for chi i equal to 0. This is the general dispersant relation. but as you see here, there are many terms in here and consequently this gives rise to many modes and one of the interesting modes is a category of modes is the one whose frequency is around n omega c and b is bigger than 1, k perp rho is bigger than 1. So, this those special kind of modes are called Bernstein waves.

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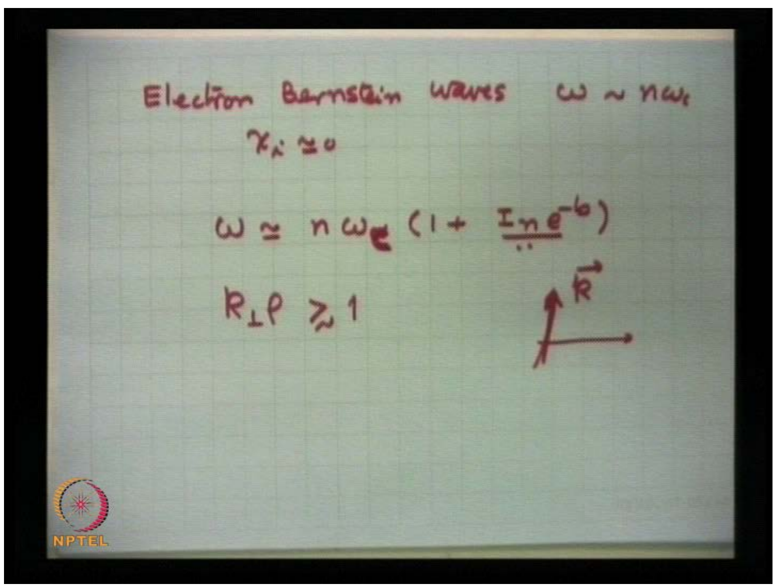

Let me just write this. So, they are called electron Bernstein waves. For these waves, chi i can be taken to be infinite, I am talking of waves of frequencies around n omega c. You can forget this, and when you simplify this expression, omega turns out to be around some integer multiple of omega c i into 1 plus i n exponential minus b divided by some factor here. I forget the factor, but of this or the unity.

So, this is this omega c rather not c i, this omega c. You get this a very special category of modes with k perp rho greater than of the order of unity, very short wave length or large k perp modes. These modes propagate primarily perpendicular to line of force, a magnetic field is here, these waves travel like this, almost perpendicular. And similarly, magnetize plasmas offer ion Bernstein waves of frequencies close to multiples of ion cyclotron frequency; these are these modes can be driven unstable, if there are current in the system and people have observed in beam plasma systems, current driven or electron beam driven Bernstein waves, electron Bernstein waves, and if there are there is lot of turbulence or there are there are some high amplitude perturbation density fluctuation, then they can give rise to anomalous resistivity as well.

So, today we have learnt that presence of fluctuations in the system density oscillation in the system gives rise to absorption of waves of high frequency and I think that is a very front line area of current research as well and lot of new situations are emerging wave waves are unstable and you are having lot of anomalous absorption of waves. And I think, I close at this point. Thank you for your attention.