Plasma Physics
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Lecture No. # 36
Landau Damping and Growth of Waves (contd.)

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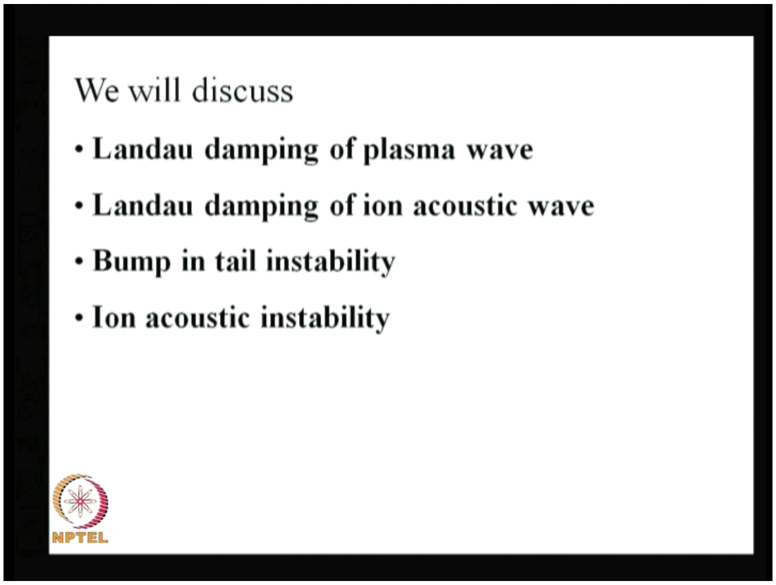

Today, we will continue talking about landau damping and growth of waves in plasmas. We shall discuss landau damping of a plasma wave, then landau damping of ion acoustic wave, bump in tail instability and ion acoustic instability.

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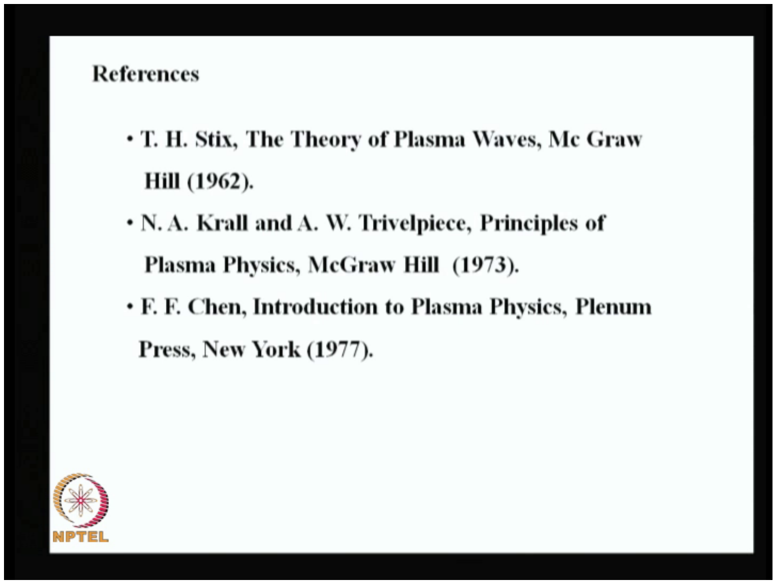

The references for today's talk would be three books; one by Stix, another one by Krall and Trivelpiece, and third one by F.F.Chen.

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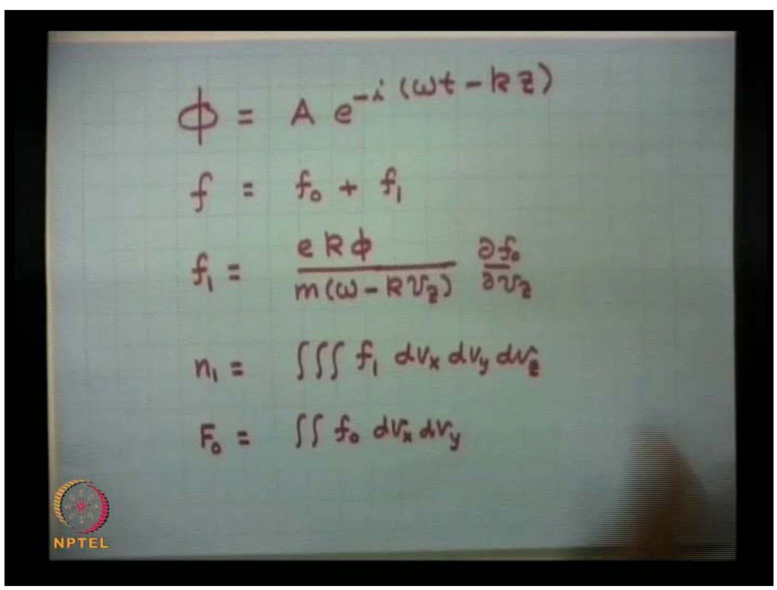

Let me recapitulate, what we were doing last time. We were considering the response of a plasma to a wave of potential phi is equal to A exponential minus i omega t minus k z. And what we have done was that we said that in the presence of this, the distribution function of electrons can be written as f 0 plus f 1; f 0 is the equilibrium distribution function and f 1 is the modification caused by the plasma wave of potential phi. Then, we solve the Vlasov equation for f 1 and we obtained an expression for the perturbed distribution function, which was is equal to e k phi divided by m omega minus k v z multiplied by delta f 0 by delta v z, here minus e is the electron charge and m is the electron mass. One may note here that those particles for which v z is equal to omega upon k, the denominator will go to 0 and they are called the resonant particles. The perturbation and f 1 will be very large and these are the particles which give rise to resonant wave particle interaction and are responsible for damping or growth of waves.

Then, we introduced the density perturbation, which essentially is a triple volume integral of f 1, this f 1 into d v x d v y and d v z. At this stage, we introduced a quantity called one dimensional distribution function because when we integrate this f 1 respect to v x and v y, where the derivative of f 0 comes only through v z and there is no v z dependence here. Sorry no, v x and v y dependence here. So, this can be easily integrated and one introduced a quantity called one dimensional distribution function f 0, which was essentially integral double integral of f 0 over d v x and d v y. So, this f 0 is a function of primarily of v z, only of v z rather.

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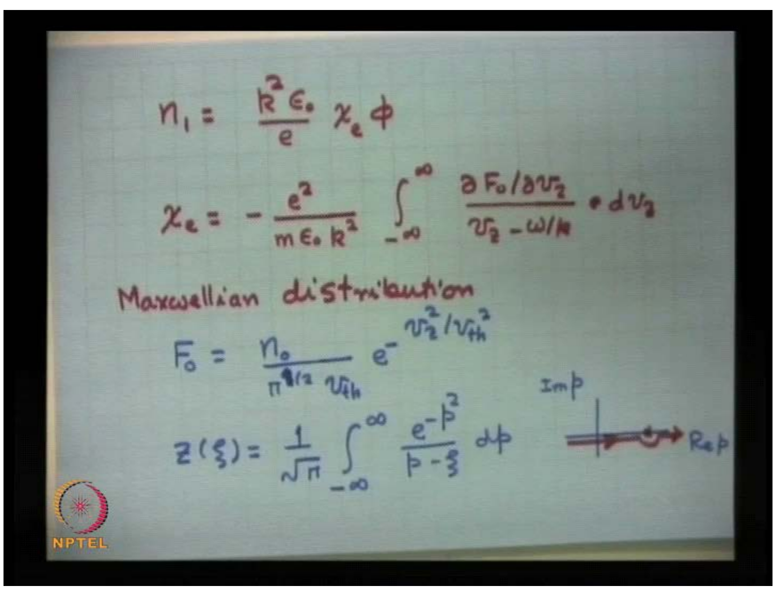

And in terms of this, one can obtain n 1 and we said that I will write n 1 as k square epsilon 0 upon e into chi into phi, epsilon 0 is called free space permittivity and chi is called the susceptibility, let me put a subscript e. I think we put a subscript e because this designates electron susceptibility and the expression for chi e then, was is equal to minus e square upon m epsilon 0 k square integral minus infinity to infinity delta F0 by delta v z divided by v z minus omega by k and this was d v z already, I have written.Sorry, d v z I have to write, d v z.

This is the expression we had written last time and for the case of Maxwellian distribution function, Maxwellian distribution means I choose F 0 as density of electrons per unit volume divided by pi to the power 3 half, sorry 1 half multiplied by v thermal into exponential minus v z square by v thermally square. So, for this case we found that susceptibility becomes simple, is expressible into z functions and z function I defined as z of xi is equal to 1 upon root pi, this goes from minus infinity to infinity exponential minus p square upon p minus xi d p.

And the evolution of this integral has to be done following the Landau's prescription. So, what we do here, if I plot real v z, we treat or p we treat as a complex quantity. So, real part of p be plot here and imaginary part of p, we plot here and then we carry out the integral along the horizontal axis because there is a pole at xi. So, this is the pole at xi, but the integral has to be like this. It has to go round the pole. So, this is the integral line integral. So, following Landau's prescription, one can evaluate this integral and people have indeed evaluated this integral, free run (( )) have carried out this study. z function always has a contribution due to this pole.

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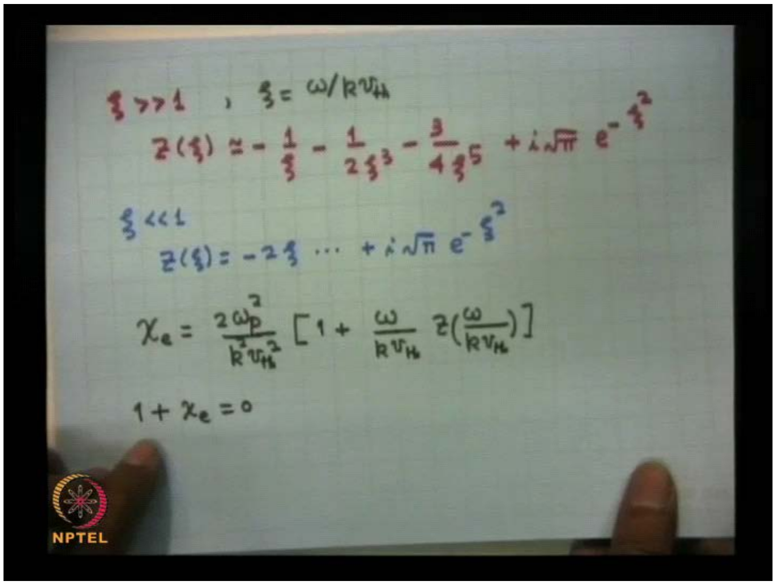

And in the limit, when xi is much bigger than 1, this becomes, this z function of xi is approximately equal to minus 1 upon xi minus 1 upon 2 xi cube minus 3 upon 4 xi to the power 5 plus i root pi into exponential minus xi square. This is the expression for xi. In the limit, when xi is much bigger than 1 and in the other limit, when xi is much less than 1. You can write z approximately equal to minus 2 xi etcetera plus i root pi exponential minus xi square and this factor is nearly 1. So, this like i root pi. Well, in terms of this z function the plasma susceptibility was susceptibility chi e, we had written was, is equal to 2 omega p square upon k square v thermally square 1 plus omega upon k v thermal into z function of omega upon k v thermal and the dispersant relation was for the plasma wave 1 plus chi e is equal to 0.

Now, I would like to simplify this by presuming that the phase velocity of the wave is bigger than v thermal. So, omega by k is v thermal is greater than 1. xi implies here is the argument of z function, which is omega by k v thermal. So, well let me mention here that xi I am talking about is omega upon k v thermal. So, in the limit, when omega by k v thermal is bigger than 1, I can use this expansion and when I carryout this expansion, chi e turns out to be rather simple. Let me write down this expression or rather this expression I will write down.

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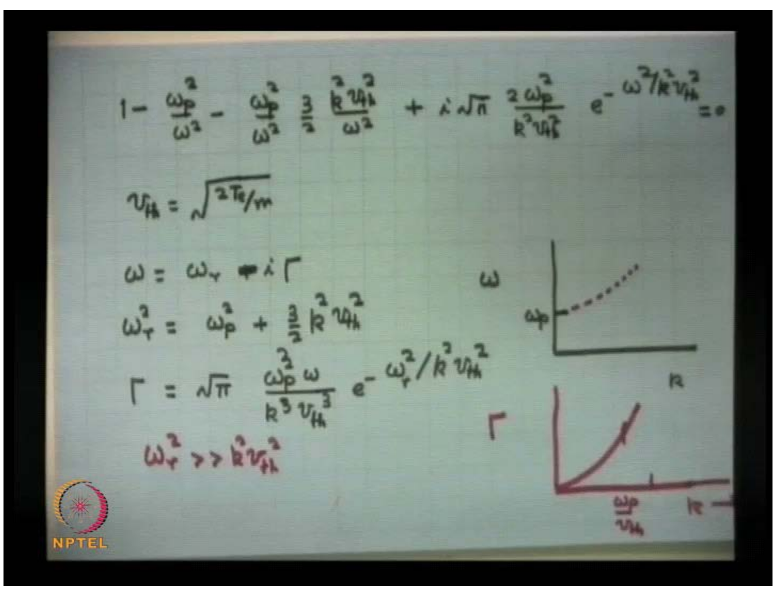

So, if I write this, this takes the following form. 1 minus omega p square by omega square minus omega p square by omega square into 3 by 2 k square v thermally square upon omega square plus the imaginary part i root pi omega 2 omega p square divided by k square v thermally square into exponential minus omega square by k square v thermally square is equal to 0. I forgot to mention that v thermal is related to electron temperature by this relation, 2 T e upon m, where t is the electron temperature, m is the electron mass.

Well, if omega is substantially bigger than over k v thermal, this term is quietly small, but it still significant. So, this is a equation, which you can solve for omega and I can write down omega is equal to a real part of omega plus imaginary part of omega, but this turns out to be negative sign. So, I write down simply minus here into gamma and presuming that omega r is bigger than gamma. You can equate the real imaginary parts of this term to 0 separately. So, real part of omega turns out to be omega r turns out to be equal to omega p square, this is equal to this square plus 3 by 2 k square v thermally square, this if you equate this equal to 0, you will get omega r actually. This expression you will get.

And nearly, approximately because I am taking this term to be small as compare to this one and imaginary part turns out to be gamma, which we call the damping rate, it turns out to be root pi omega p square into omega divided by k, this should be k square v thermally square. Actually, this is cube here and this is k cube v thermal cube multiplied by exponential minus omega square upon k square v thermally square. You may note here that the damping is a function of frequency frequency, this omega is omega r actually. But we can suppress the subscript r implying that it is implicit. So, the frequency you may note here, if I plot omega r, omega r as a function of k. Suppose, omega p is here, at k is equal to 0, omega equal to omega p and that is we plot here, this will go something like this.This sort of variation you will get.

Now, if you are too close to omega p, then k will be very small and when k is a small this quantity is very large. So, damping rate will be negligible. So, if I plot here damping rate, damping rate, I will call as gamma as a function of k because when k changes, omega correspondingly change changes. So, use a proper value of omega from this graph and use this here, then plot gamma has a function of k. You will find that when k becomes comparable to omega by v thermal. In that case, this term is nearly unity and this is huge. So, damping rate is very tiny and becomes very large, when k becomes comparable to omega p by v thermal. Suppose, omega p by v thermal is here.

This is omega p upon v thermal. I am varying this quantity k here. So, this is the point where omega k becomes omega p upon v thermal. The damping is very huge here. So, it starts from here and then it increases rapidly. The scale for this is different than the scale for omega, but this could be quite large. Obviously, my approximation fails here; it is not valid up to this point. It is valid only up to somewhere here, because as I mention to you, I have already assumed that omega r is bigger than k square v thermally square. So, this was my assumption. So, I cannot go close to unity here, i should be here may be half or something one-third. So, but it still the damping is strong as you move to shorter wave length or larger values of k and this is a collision less damping. It will be useful to, look at the wave number of phase velocity of the wave over the distribution function. If I plot here f 0 as a function of v z, for Maxwillian plasma this goes like this.

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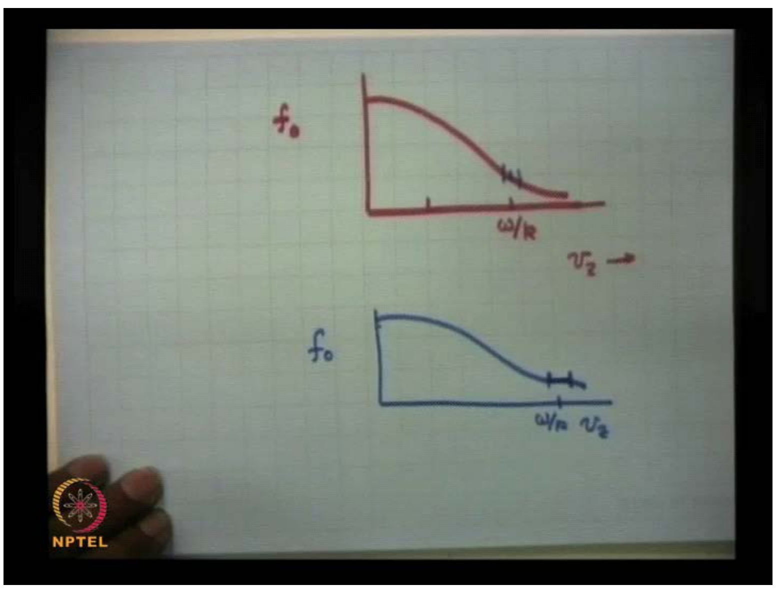

This is my Maxwillian distribution function and my thermal velocity is somewhere here. This is my thermal velocity somewhere here and my phase velocity of wave is somewhere here. So, suppose my omega by k is here, depending on the value of k. This is these are the particles which are called resonant particles. So, in this neighborhood you are expecting resonant interaction of waves with particles, this is the range. What will happen? The particles which are on the left of this omega by k point, these particles are moving slower than the wave. The particles which are ahead of it, they are moving faster initially. But as I mentioned that as the wave grows these particles loose energy. The faster particles loose energy and the slower particles gain energy. Consequently, particles from here to here move and there is a Plato created there.

So, the distribution function appears becomes like this. f 0 if I plot, v z here the distribution function I have plotted here. So, in this a some Plato is created here, this is the range v g. Sorry, omega by k is here. So, a Plato is created horizontal becomes flat. Once this becomes flat, then there is no growth. Because there is no slope in f 0. So, Landau damping will be a transient phenomenon. After a little while, when this flattens, there will be no damping. Obviously, if the electron electron collisions restore the distribution function to be Maxwellian, then this will be ongoing process.

So, laminar waves will damped up to a point and beyond that point the damping will stop. When the particle distribution function has become like this, until or unless there is a restoration of Maxwellian form of the distribution function. It is collision new. So, this is from physics we have learnt. Obviously, the waves of larger k, this omega by k point will be closer here and then, the number of particles in the distribution function are more. So, there are more and more resonant particles and there will be a stronger damping. So, when omega by k is very large. When k is very tiny, then there is hardly any particles and there is very little damping of the waves. So, one can see this.

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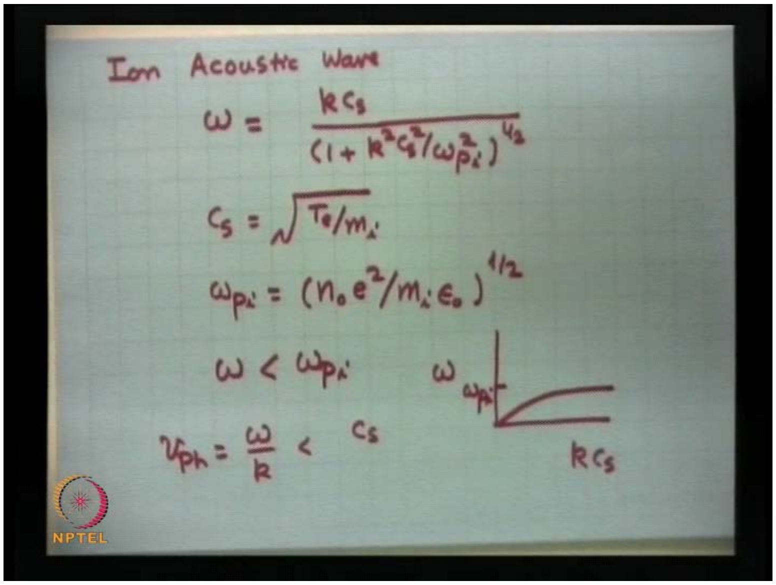

Now, let me examine the ion acoustic wave in the presence, well with kinetic effects. So, ion acoustic wave from fluid theory, we had learnt that this is a wave of frequency omega is equal to k c s upon 1 plus k square c s square upon omega p i square to the power half, where c s is under root of T e upon m i, where m i is the ion mass and omega p i is the ion plasma frequency, which is defined as n 0 e square upon m i epsilon 0 under the root. So, a wave of frequency omega with wave number k will have a phase velocity omega by k, which is less than c s, depends on how close k c s is to omega p i but the highest value of omega is less than omega p i. So, if I plot a graph here omega versus k c s and suppose, omega p i is here, then this goes like this. It saturates. But please remember, in the case of ion acoustic wave the velocity is much less than the electron thermal velocity, 50, 60 times or 100 times a smaller.

However, the ion velocity may be comparable to the velocity of the wave. Especially, if my k is large, then omega by k could be significantly smaller as compare to c s. When k c s becomes comparable to omega p i, there is a significant reduction. So, the phase velocity of this wave v phase, which is defined as omega by k is less than c s, because there is a factor underneath. So, it is possible that larger k waves will have a thermal velocity comparable to ion thermal velocity. Because the velocity is if T e is T i are same, then thermal velocity will comparable to c s and the wave will be strongly damped. So, here the damping is expected to come from the ions. So, what you do? You look at the same expression.

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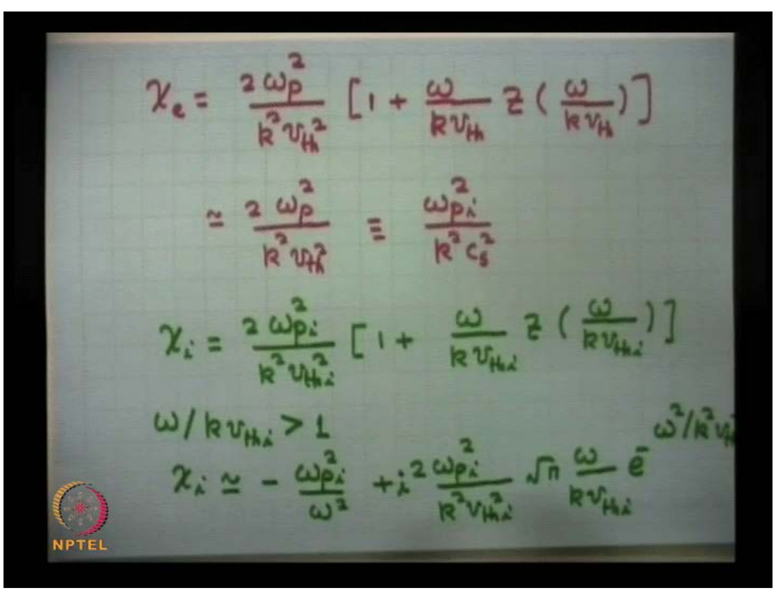

For as we had seen for the electron susceptibility, you can write down the expression for ion susceptibility. Electron susceptibility we had written was 2 omega p square upon k square v thermally square 1 plus omega upon k v thermal of electrons plasma dispersant function of omega upon k v thermal, but we are we are considering the limit when omega is much less than k v thermal, then this quantity is negligible. So, this is primarily equal to 2 omega p square upon k square v thermally square and this you can easily see, is equal to omega p i square over exactly same as k square c s square, because omega p i is square continuous a mass, 1 upon mass, v thermal square also continuous 1 upon mass and it cancel out.

So, whether you put a omega p or p i or c s the same thing happens. So, this is the value of chi e. How about the ion susceptibility? Ion susceptibility chi i is by similarity 2 omega p i square where, omega p i is the ion plasma frequency upon k square v thermal ion square and then this becomes 1 plus omega upon k v thermal of ions z function of omega upon k v thermal of ions. Here, I am assuming the plasma to be Maxwellian.

This term again here, for sound waves we are expecting omega upon k v thermal of ions to be bigger than 1. So, in the large amplitude approximation I can expand this plasma dispersant function z function and in that case, chi i turns out to be approximately equal to minus omega p i square upon omega square that is nearly the real part and imaginary part is simply is equal to plus 2 omega p i square upon k square v thermally square into i root pi. So, I will put here, i root pi omega upon k v thermal of ions and exponential minus omega square by k square v thermal ion square. Well so, chi has this is the ion susceptibility, this not the imaginary part. This is real part and a imaginary part. Real part of this and chi e will determine the real frequency and this will give rise to damping of the wave.

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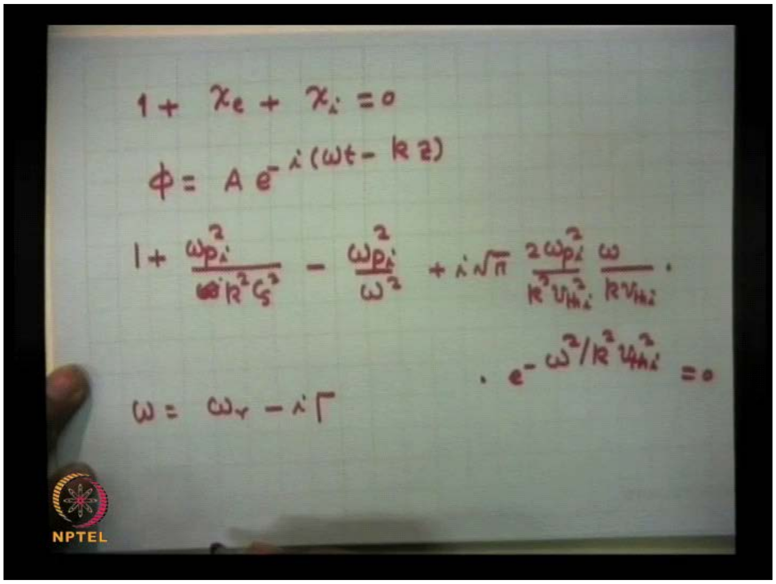

So, let me write down the dispersant relation. The dispersant relation for the sound wave is 1 plus chi e plus chi i equal to 0. chi i essentially is characterizing the perturbation in the electron density, then you multiply this chi by phi and this characterizes the modification in the or perturbation in the ion density by the wave and wave we are considering like this, A exponential minus i omega t minus k z. So, this is the dispersant relation for the sound wave or ion acoustic wave and this turns out to be 1 plus omega p i square upon omega, sorry k square c s square, this is the electron susceptibility and ion susceptibility is omega p i square upon omega square plus i root pi and then, you get 2 omega p i square upon k square v thermal i square omega upon k v thermal i multiplied by exponential minus omega square upon k square v thermal i square.

The real is equal to 0. If I forget this, I get the real part of frequency from here. But when this is included, I can write down omega is equal to omega real minus i times some damping rate, omega r just when obtain from here is the same as I have given before by fluid theory. But this is the additional term that you get because of damping due to ions.

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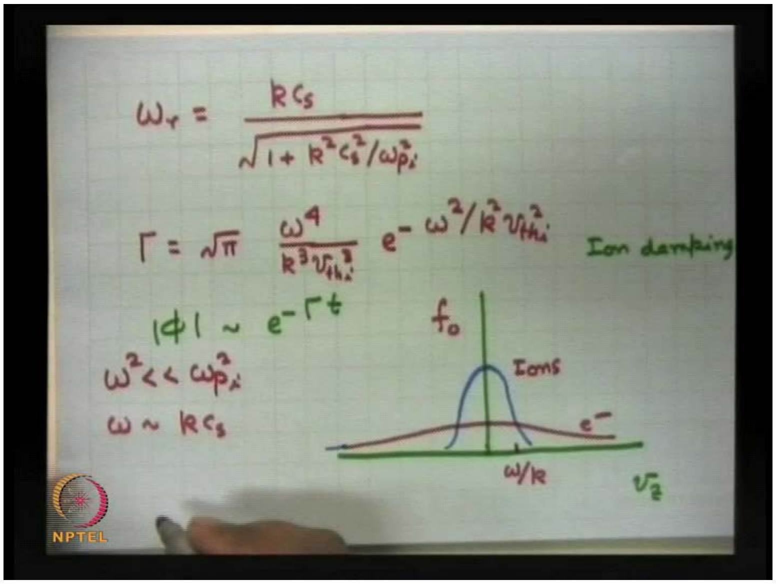

Let me write down the explicit expression for omega r and the damping rate. omega r is k c s upon under root 1 plus k square c s square upon omega p i square and damping rate turns out to be root pi, it turns out to be I guess omega 4 upon k cube v thermal i cube exponential minus omega square by k square v thermal i square. This is called ion damping. So, your phi is the amplitude of the wave, phi modulus if you plot will go as exponential minus gamma t. So, it will damp out. Now, if I plot the distribution function for electrons and ions, what I will get is this. If i plot here v z and put the distribution function for electrons and ions separately, the ion distribution function will be like this. Let me graph by different color. The ion distribution function will be like this and the electron distribution function will be like this. Let me write it by different color and my wave velocity is somewhere here; omega by k is somewhere here.

So, the slope of the electron distribution function is very tiny and that does not give rise to any damping of waves. But if the slope of the ion distribution function, this is ions and this is the electrons, I am plotting here f 0 versus v z. So, these are the ions here, which are responsible for the damping of the wave. It is very important to know that because especially, when you are talking of wave frequency less than omega p i. So, when omega is significantly less than omega p i may be one-third or less, then omega is around k c s and this is the factor here, which is responsible for damping. It can became quite large, the damping rate turns out to be,for low frequency range, gamma is of the order of root pi omega T e upon 2 T i cube exponential minus

T e upon 2 T i, this is very important. This tells that unless T e is much bigger than 2 T i, that will be very strongLandau damping of ion acoustic waves.

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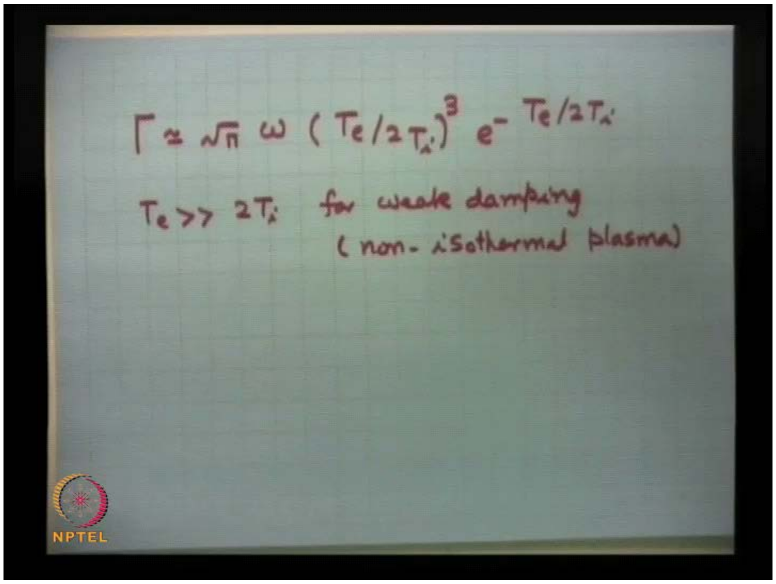

So, T e must be much bigger than 2 T i for weak damping of these waves. A plasma in which temperature of electrons is much bigger than the ion temperature is called non isothermal plasma non isothermal plasma. But certainly there are plasmas in which temperature of ions is much less than the electron temperature and the ion waves are quite weakly damped. Well this argument is true, when we are talking plasmas with singly ionized ions. In plasmas, where ions are doubly ionized or they have charge 2 e or 3 e or 4 e, then this condition changes. But those details are not relevant at the moment. I just wanted to emphasize that thermal effect, because damping of waves. However, with a suitable distribution function, they can give rise to instabilities as well. Physically what would where would you except the instability. A simple example is the case of a bump in tail distribution. I think I will write this on a separate sheet.

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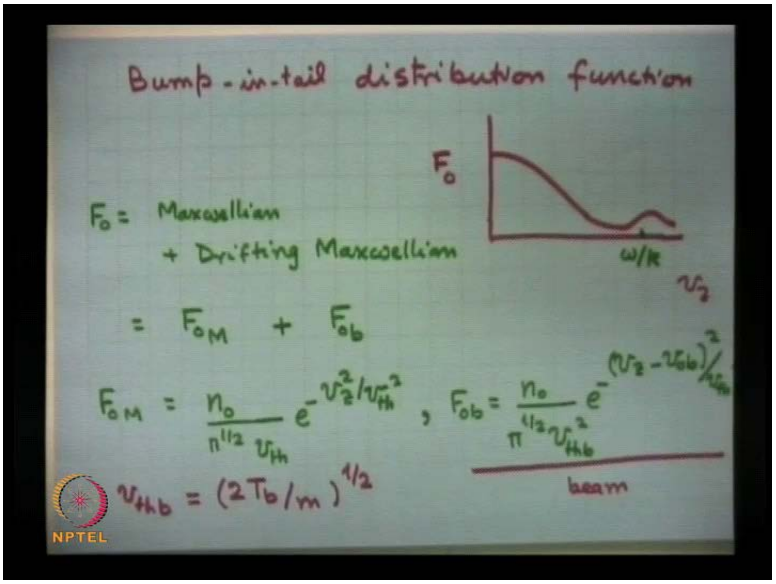

Let me talk about the bump in tail distribution and the instability or the growth of the wave arising because of that is called bump-in-tail instability, bump-in-tail distribution function. So, suppose we have a plasma, which has a distribution like this. I call F 0, suppose is a function of v z if I plot, I get this kind of thing and this goes like this. So, there is a bump in the tail of the distribution function and in case your wave phase velocity were lying somewhere here, the slope is positive here and the wave velocity, the wave that you are considering has a phase velocity somewhere here, then the slope is positive and the imaginary part of omega will be positive and that will give rise to growth of the wave, because here the number of particles moving faster than omega by k are more and the particles moving slower than omega by k are less. So, more particles are moving faster than the wave and they will give energy to the wave. So, wave will grow.

This kind of distribution you can easily write down as a sum of a Maxwellian distribution plus drift in Maxwellian distribution. So, I can write down this, F 0 is equal to Maxwellian distribution plus drifting Maxwellian and so, I can write down Maxwellian. I will write down simply F 0 M plus F 0, the drifting Maxwellian I will call F 0 b, may be something sort of a drifting particles or beam particles moving. Actually, I can call this, actually F 0 either m or F 0 b, the bulk plasma particles.

So, bulk either the plasma electrons are having Maxwellian distribution and some particles are moving with drifting Maxwellian. So, let me write down these expressions. Maxwellian distribution function in one dimension, I can write down as n 0 upon pi to the power half into thermal velocity of the plasma electrons exponential minus v z square upon v thermally square. So, these are my electrons of the bulk bulk electrons of the plasma, but there are some tail electrons, I will call them F 0 b. Suppose, this is equal to n 0 upon pi to the power half, their thermally spread of energy of velocity is suppose less, v thermal b is the thermal spread, exponential minus v z minus v 0 b square divided by v thermal b square.

So, I am writing two distribution functions, sum of 2. This is Maxwellian from here to here. This as a peak at v z equal to 0 but this is has a peak at v z is equal to v 0 b. So, this tense of the peak from this point is v 0 b.And v thermal b is called the, is I can write down v thermal b thermal velocity of beam electrons, or drifting electrons, which I can define as 2 temperature of the beam electrons upon mass of the beam electrons and v thermal is 2 T e upon m but T e and t b could be very different. So, this is the beam part, this is the bulk part, bulk electrons or plasma electrons. If I use this in the original expression for density perturbation, you get the density perturbations.

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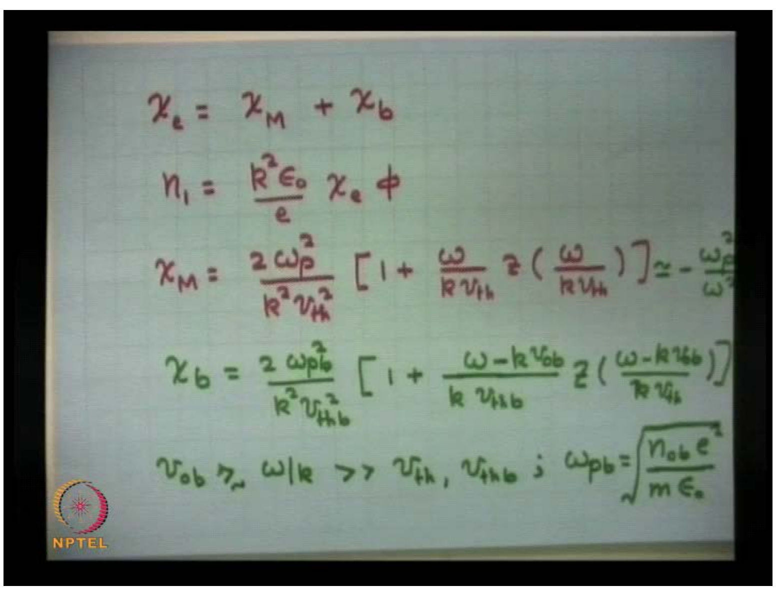

And I can define two quantities. Susceptibility of the electrons turns out to be one comprising the Maxwellian part contribution plus 1 having the beam contribution or drifting Maxwellian contribution. There are two susceptibilities. Obviously, density perturbation of the whole system is k square epsilon 0 upon e into susceptibility of the electrons into phi. So, this chi is a very important quantity. Now, this has a sum of 2 terms. one from Maxwellian electrons and one from drifting Maxwellian electrons called beam electrons and what happens if you work out little bit, just follow the old procedure.

Chi m is the same expression as before, which is 2 omega p square divided by k square v thermally square into 1 plus omega upon k v thermal z function of omega upon k v thermal, whereas, chi m becomes chi b is different, this is twice omega p b square. I may forgot to mention that the density of electrons that I had written earlier. The drifting electrons may have a different density, I will call n 0 b. So, if I take this density n 0 b, then the corresponding plasma frequency will be omega p b upon k square v thermal of the beam electrons square 1 plus omega upon k, this be turns out to be omega minus k v 0 b upon k v thermal of the beam. This is the modification, here is omega only and this is omega minus k v 0 b, which is called Doppler shifted frequency of the wave has seen by the particles and this is omega minus k v 0 b upon k v thermal. So, this total susceptibility of electrons has two terms one has a important term with Doppler shifted frequency and the other one has just a frequency omega.

What you can do? For the instability, you are a or certainly looking for a case, where beam velocity is more than or comparable to omega by k. But this is much bigger than v thermal of electrons or v thermal of the beam electrons that is the case of interest. So, in that limit, this simplifies the great deal. It becomes of the order of minus omega p square by omega square, simply this term and well actually, let me define omega p b here, this is a important quantity, omega p b is I have defined as n 0 b, the drifting electron density, e square upon m epsilon 0 under the root.

Please note that, in this limit when the beam velocity is much bigger than the thermal velocity of electrons or the thermal velocities spread of the beam distribution function, in that case I can simplify these two expressions a great deal. So, chi m turns out to be simply this, chi be will have a real part plus a imaginary part. The real part will be small as compare to the real part of chi m, because if the density of beam electrons is small, then I expect this to be a small modification.

But the major contribution will come in the imaginary part. So, this is what I am going to do here.

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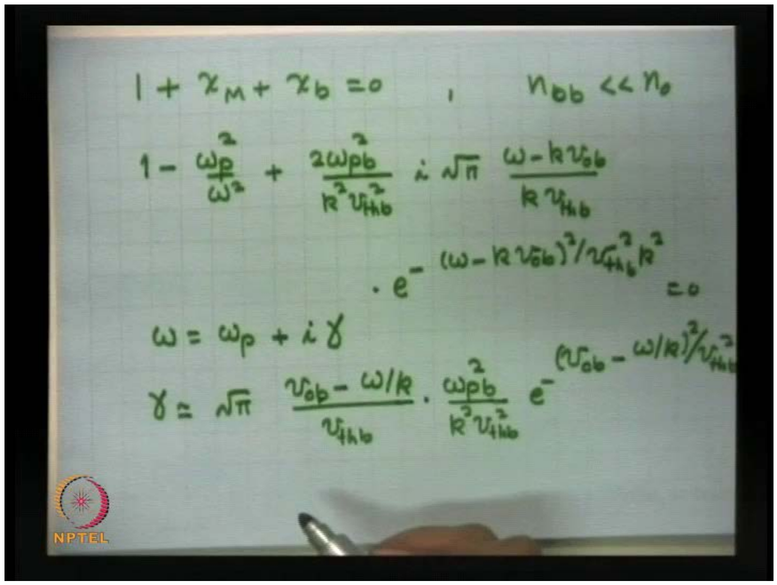

I am going to write this imaginary part of this term and my dispersant relation is 1 plus chi m plus chi b is equal to 0 and when you substitute, this becomes 1 minus omega p square by omega square, due to the second term and chi b I will simply write only primarily imaginary term, which is equal to plus 2 omega p b square upon k square v thermal, beam velocity square into i root pi omega minus k v 0 b upon k v thermal of the beam into exponential minus omega minus k v 0 b square upon v thermal square multiplied by k square, this is b. This is equal to 0. So, this imaginary part because there is a small, well I have written in the limit when beam density n 0 b is significantly less than the plasma density, in that case real part of the beam contribution to susceptibility can be ignored as compare to the real part of the electron plasma electron contribution.

So, if I ignore this, I get omega equal to omega p, but because of this term omega, I can write down as omega p minus or plus rather i times growth rate, just substitute it here and treat equate the real imaginary parts, you will get the growth rate. So, growth rate turns out to be equal to, let me just write down this expression. It turns out to be root pi into v 0 b minus omega by k upon v thermal of the beam, thermal spread in velocity of the electron beam into omega p b square upon k square v thermal beam square exponential minus v 0 beam minus omega by k whole square divided by v thermal beam square. So, whenever the beam velocity is faster than the phase velocity of the wave, more electrons are moving faster than the wave and hence the feed energy into the wave and wave grows. So, this is the growth of the plasma wave, at the expense of energy of the electrons in the tail. So, when there is a whenever there is bump in the tail, whenever there is a positive slope in the distribution function, you will get this instability So, plasma waves are often driven unstable, whenever there is a stream of particles moving faster than the thermal velocity of electrons plasma electrons and they create a bump in tail and you get a instability. This is a very important instability in plasmas.

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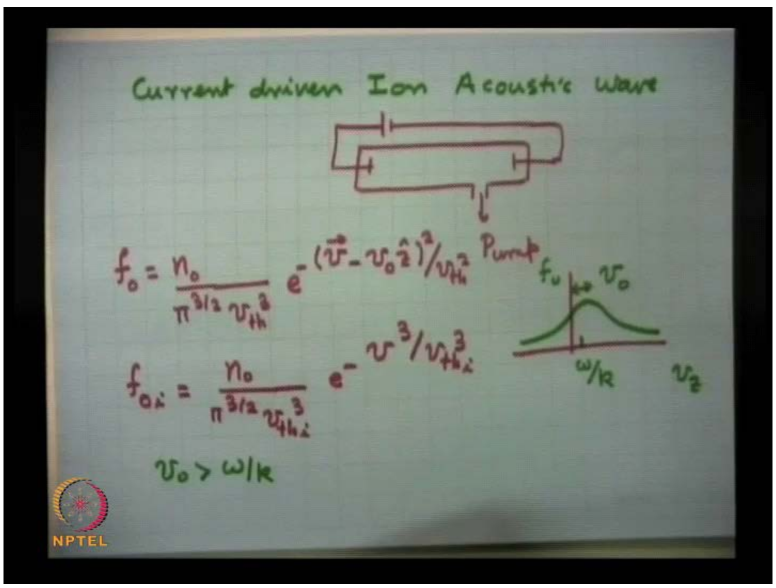

I would also like to talk to you about another instability; you can drive the ion acoustic waves also unstable by an electron current. So, let me just mention finally, current driven ion acoustic wave or simply ion acoustic instability. This system is rather simple. If you take a plasma somewhere and put two electrodes to which we will be certainly apply potential difference, pass a current in the plasma. The distribution function and there is a pump here, so that you can create a vacuum. So, when you have a plasma at low pressure or gas field in at a low pressure and you created discharge, you produce a plasma but the plasma electrons also have a drift, they are drifting towards the cathode anode and the distribution function can be reasonably taken to be Maxwellian, drifting Maxwellian.

So, if I take the plasma electron distribution function like density of electrons pi to the power 3 by 2 v thermal cube exponential minus v minus, suppose the electrons are drifting with drift velocity v 0 in the z direction upon v thermal square. So, this is the distribution function, then if you work out the and ion distribution function usually in such plasmas can be taken to be f 0 of ions as n 0 upon pi to the power 3 half v thermal of ions cube exponential minus v cube upon v thermal cube of, this is the distribution function of ions, this is i v thermal i.

In that case, the distribution functions will look like this. If I plot here the distribution function for electrons, the electron distribution function will not have a peak here, it has a peak at v equal to. So, if I plot this as a function of v z and f 0 here, the distribution function for electrons will be like this. So, the peak of the distribution function is somewhere here. So, if I have my omega by k just below the peak, here this peak is shifted at distance v 0, the distance from here to here is v 0. So, for v 0 greater than omega by k the slope of the electron distribution function is positive.So, there are more electrons moving faster than the wave and few are moving slower than the wave and there will be a instability. Mathematics is a trivial for this case.

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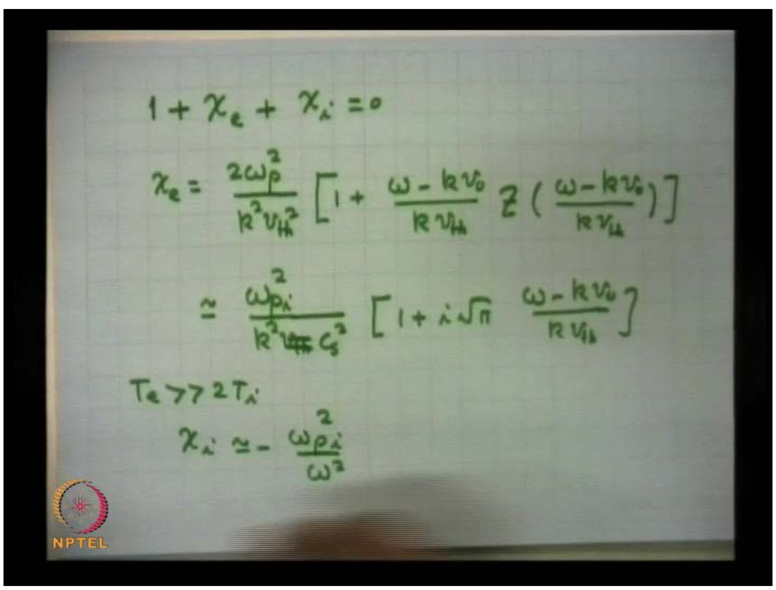

Just, let me write down we are having the dispersant relation, 1 plus electron susceptibility plus ion susceptibility equal to 0 and the electron contribution turns out to be chi e, which is 2 omega p square upon k square v thermal square into 1 plus omega minus k v 0 upon k v thermal of the plasma electrons into z function of omega minus k v 0 upon k v thermal.Since, for sound waves k v thermal is too huge as compare to omega or omega minus k v 0 too huge. So, this quantity, real part of this quantity is negligible imaginary part. So, this becomes approximately equal to, this is becomes equal to omega p i square upon k square v thermal c s square and this becomes 1 plus i root pi into omega minus k v 0 upon k v thermal and if I take ions to be cold, so in the limit when T e is much bigger than 2 T i, ion susceptibility is nearly equal to minus omega p i square by omega square. So, that the ion damping can be ignored. So, in that limit if I just substitute this chi e here and chi i here, you will get instability, whenever v 0 is more than omega by k and it is very trivial to get the dispersant relation.

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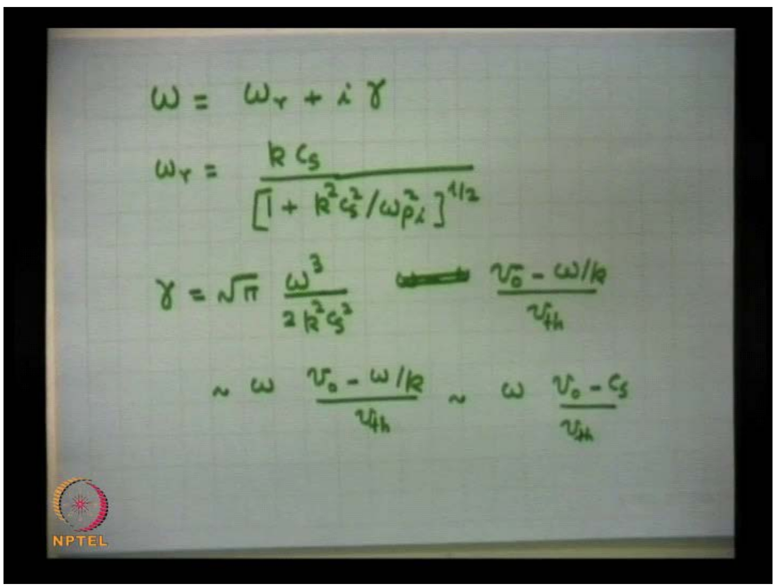

You will get here the dispersant relation will be have omega is equal to omega real plus i gamma, omega real is the same as you obtain from the fluid theory, the real frequency of a sound wave, which is k c s upon 1 plus k square c s square upon omega p i square under root and the growth rate gamma turns out to be equal to root pi omega cube upon 2 k square c square into omega minus, sorry this is v 0 minus omega by k upon thermal velocity of electrons.

As I mentioned omega by k is like c s, v 0 has to be slightly more than c s. So, this quantity is quietly small but significant. So, whenever omega is of the order k c s growth rate is comparable to omega into this ratio v 0 minus omega by k upon v thermal or this like c s, this is maybe I would say one-fiftieth but still it quite significant. So, this is let me write this as omega into v 0 minus sound velocity upon thermal velocity of electrons. So, this is still significant. So, sound waves have been found to be generated in plasmas carrying currents, even simple discharges will produce plasma waves, sound waves.

Sound waves are like acoustical phonons and solids. In a solid, whenever the sound waves travel, we say that acoustical phonons exist. So, whenever you apply a d c electric field to a conductor, the electrons in the drift they suffer collisions with phonons and that gives rise to resistivity. Similarly, in plasmas, if you pass a current in a plasma and sound waves are generated, then these waves can also give rise to plasma resistivity, which could be significantly higher than the resistivity due to particle collisions, electron ion collisions or electron neutral collisions. And I think in our next lecture, we shall talk about the effect of such instabilities on resistivity. I think, I would like to stop at this stage.Thank you.