**Plasma Physics**
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**Lecture No. # 35**
**Landau Damping and Growth of Waves**

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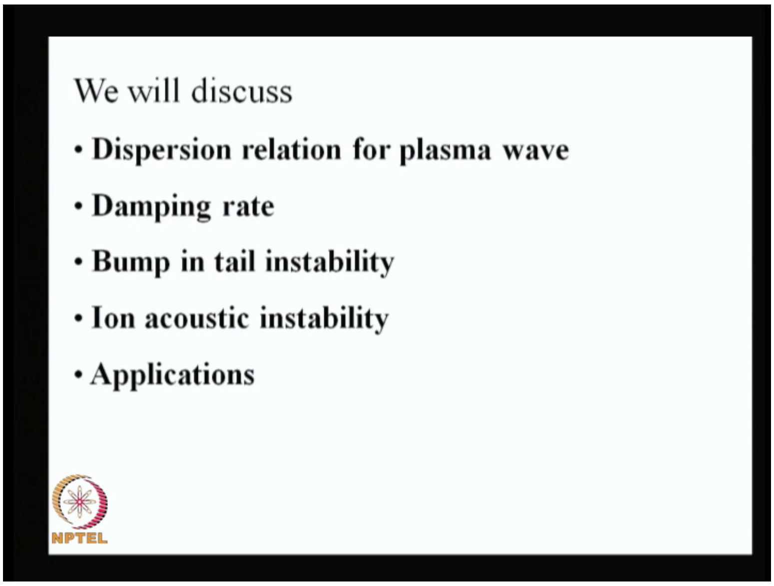


Today, we will continue our discussion of Vlasov theory of plasma wave, and we shall discuss Landau damping and growth of waves. First, we will drive a dispersion relation for plasma wave, then deduce the damping rate, and in case of a drifting Maxwellian distribution function or bump in tail, we will examine the possibility of wave amplification giving rise to instability. We will also use to discuss the growth of ion acoustic wave by a drifting electron distribution function; and we discuss the consequence of such instabilities.

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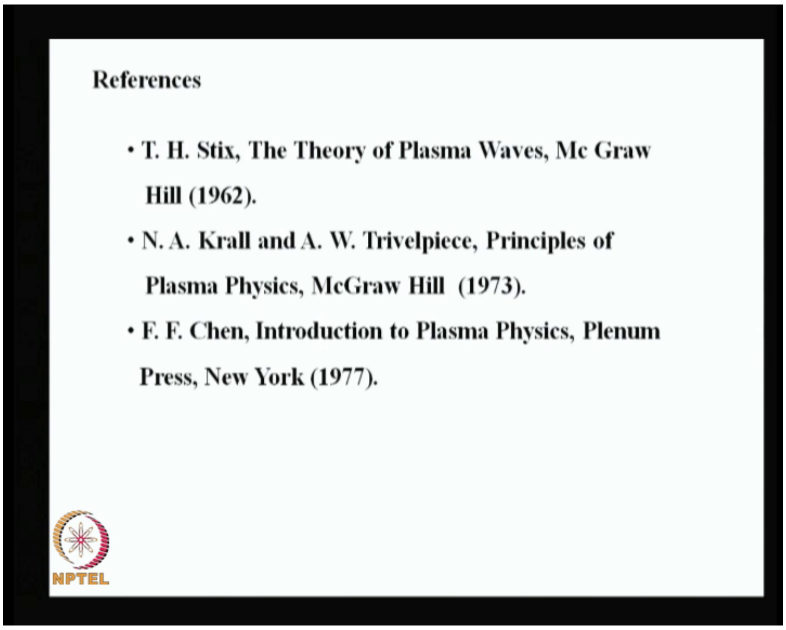


The references for today presentation are the same as before as in the last lecture; three books by Stix, Krall and Trivelpiece and by F. F. Chen. .

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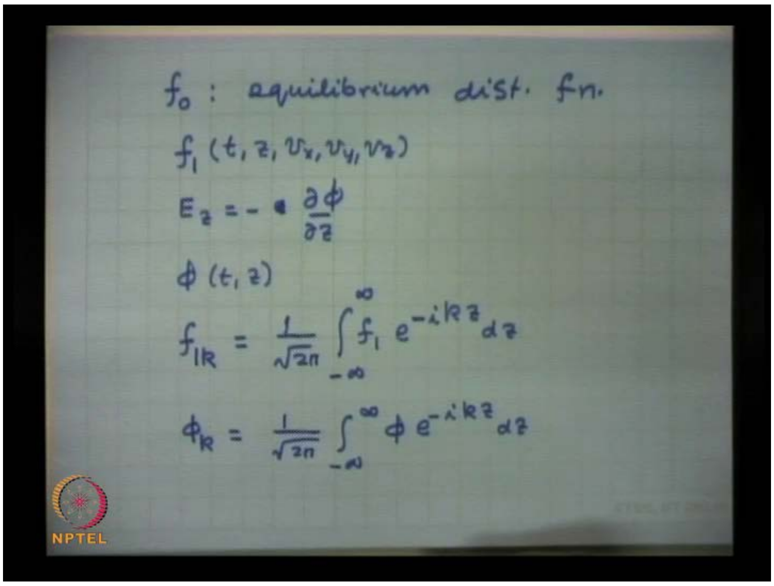


Let me recapitulate what we were doing last time. We were examining the possibility of having a plasma with equilibrium distribution function F 0. This was the equilibrium distribution function, perturbed by a small perturbation f 1 self consistently with the creation of an electric field, which we wrote down as having E z is equal to minus e sorry minus delta phi by delta z, and we were considering that f 1 was uniform in x and y. So, it was a function of t, function of z, and function of v x, v y, v z in general; phi was a function of e and z. So, a small initial perturbation n, the distribution function and self consistent electric field creation in the system; how about this perturbed perturbation will evolve in time? Will it dump out or will it continue to grow? Let there was a issue I would I was think to address.

For this purpose, we had introduced fourier transforms, and we introduced a quantity called fourier transform of f 1 f 1 k that was in z. So, this was I had written as 1 upon under root 2 pi and integral f 1 exponential of minus i k z dz minus infinity to infinity. This was the fourier transform of f 1. And fourier transform of phi, I will call as phi k is equal to 1 upon root 2 pi integral minus infinity to infinity potential multiplied by minus i k z d z.

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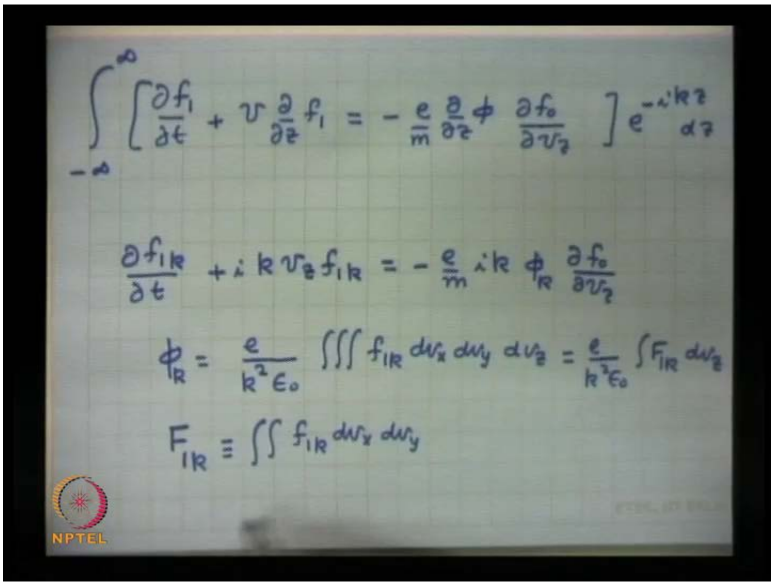


So, what we did was that, we had a linear linearized (( )) equation, which was of this form delta f 1 by delta t plus v delta delta z of f 1 is equal to minus e delta delta z of phi upon electron mass minus e is the electron charge multiplied by delta f 0 by delta v z and when I multiply this equation by on both sides by exponential minus i k z dz and carry out the integration over z from minus infinity to infinity. I got an equation for f 1 k and that equation was, let me write down that equation f 1 k was, delta f 1 k delta t plus i k v z f 1 k is equal to minus e upon m i k phi k delta f 0 by delta v z. So, this is the equation governing the fourier transform of perturbed distribution function. And this phi k is related to f 1 k through the poisons equation, which was phi k is equal to e upon k square epsilon 0, where epsilon 0 is the free space permittivity and triple integral f 1 k dv x dv y dv z.

You may note one thing in here, that I can easily multiply this equation by dv x dv y and integrate over v x and v y. because there is no derivative or no other coefficient having v x and v y so, it is very simple. So, I can easily multiply this equation by dv x and dv y and integrate after all and I can let me define a quantity f 1 k as f 1 k dv x dv y double integral. So, f 1 k dv x dv y then, this whole double integral will reduce to simply f 1 k. So, I can write down this is simply e upon k square epsilon 0 f 1 k into dv z and this f 1 k will depend only on obviously is on time k and v z, that is all dependence. So, in order to obtain an equation for f 1 k, I multiply this equation by dv x dv y and integrate and the f 0 when I integrate over dv x dv y, I define capital F 0 as f 0 dv x dv y. So, this is called one-dimensional distribution function and F 0 now depends only on v z, because v x v y dependence have been integrated over. So, my equation governing F 1 would be F 1 k would be delta delta t of F 1 k plus i k v z F 1 k is equal to minus e upon m i k phi k delta F 0 by delta v z. So, it becomes a one-dimensional equation as for as, f is concerned its velocity phase dependence is concerned.

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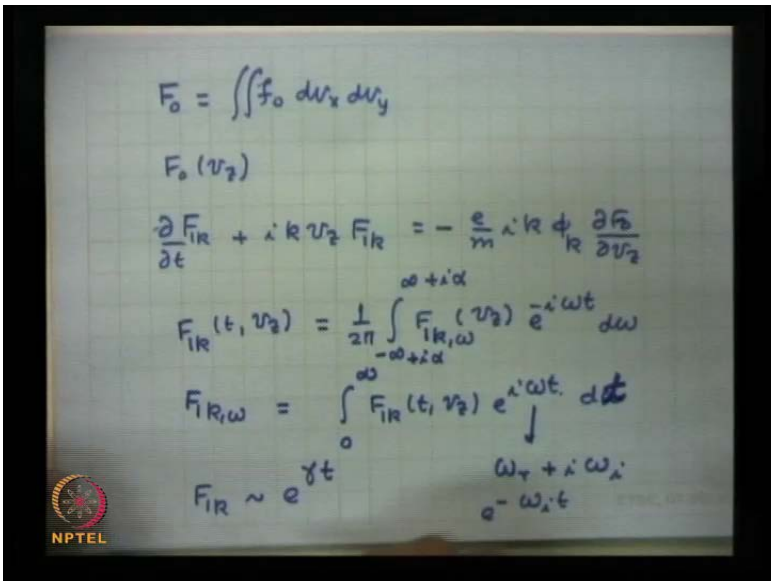


Now, to get read of this time dependence, we carry out a Laplace transform. Laplace transform has to be carefully done. So, I write down F 1 k, which is a function of time, besides other dependence, it depends on v z also. I write down this quantity as F 1 k, let me m omega. Now, this a function of v z of course is there, exponential of minus i omega t d omega, where F 1 k omega is called the fourier transform of F 1 k. So, I will write down F 1 k omega is equal to F 1 k, which is a function of t and v z, exponential of i omega t dt sorry this is d t, this goes from 0 to infinity and this 1 upon 2 pi in here

What I am saying here is, this Fourier transform I define as the time integral of my distribution function multiplied by exponential i omega t. And this F 1 k may have a dependence on time which may go as exponential of say gamma t. Now, if x this as a exponential of I gamma t or gamma t of this form, then the integration over omega obviously, I expect this goes goes from minus infinity to plus infinity but then omega should be allow to have a imaginary part large enough. So that, the exponential decay part of this is more dominant than the exponential growth part of this. Otherwise, this will over flow because in this integral if omega has a large imaginary part, it does not have a large imaginary part but this has then a time goes on this integral will overflow.

So, this integral should not be over flowed and hence this omega should be allowed to have a imaginary part, which is substantially large, because any imaginary part will omega will be minus. So, suppose I write here this omega as omega real plus i times omega imaginary then, this term will goes as exponential minus omega i into t. What I want, that this integral should not diverge. So, whenever this perturbation has a time dependence of this form. The time dependence of this exponential part should be more dominant than this one, so that this integral converges otherwise, this will not converge.

So means, this is defined only for in that for those values of omega for imaginary part is substantially bigger than gamma. That is that is a important. So, when I carry out this limit, I must be careful, this goes from minus infinity to infinity. but it should have a substantially large imaginary part plus i times some imaginary parts suppose alpha plus i times alpha alpha is like give any quantity greater than gamma. That is important.

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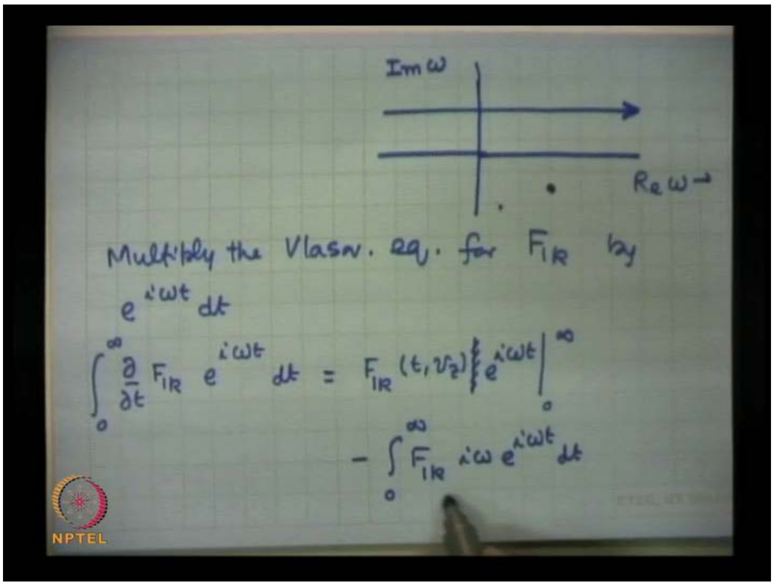

So, if I have to carry out this, if I write down this suppose, I plot real part of omega here and imaginary part of omega here and I find that the Fourier transform has or suppose, this is the value or these are the values any values suppose, this is the value imaginary part omega F 1 k suppose has a dependence on time which is equivalent to imaginary part omega equal to so much like gamma. Then, my line of integration should be here. This is the line of integration of omega integral from minus infinity to plus infinity but at a height alpha. This is the primary the thing means, any singularity in the Laplace transform should be below the line of integration, this what I have to say.

So, when we carry out the inverse Laplace transform, we should be careful about it, but I think, we will refer to this refer this discussion to a little later in stage. Let me first go back to our Fourier transformed (( )) equation. What I do, multiply the (( )) equation (( )) equation for F 1 k by exponential of i omega t dt and carryout the time integral. Let us see, the first term in the equation is delta delta t of F 1 k multiply this by f i omega t dt and integrate. let us see, what do I get, I am doing this from 0 to infinity. This quantity will be first integral of this quantity will be F 1 k and F 1 k as I mentioned depends on time and v z this has to be obtained at 2 limits sorry multiplied by exponential of i omega t. The limits are from 0 to infinity minus differential of this and integral of this. So, it becomes F 1 k and differential of this is i omega exponential of i omega t dt 0 to infinity.

As, I mentioned to you omega has a substantial large imaginary part. So, when t goes to infinity, this vanishes. So, at the upper limit this entire quantity goes to 0. Only at the lower value this will survive and at that this quantity is one and this is F 1 k 0 v z. How about this integral, if I take I omega outside then, this is simply the Laplace transform of F 1 k. So then, this integral is simply sum of two terms. This integral delta delta t of F 1 k exponential of i omega t d t, which was from 0 to infinity is equal to minus F 1 k at 0 v z minus second integral was simply F 1 k omega into i omega. And then the (( )) equation becomes simple, its simply gives you F 1 k omega is equal to simply simplify this, e upon m omega minus k v z k, here phi k there into delta f 0 by delta v z. So, we got a simple expression for f 1 k.

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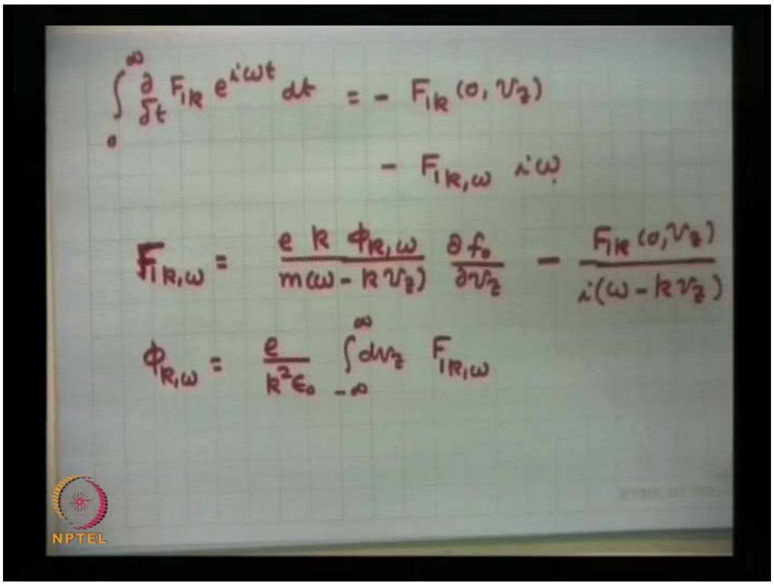


Now, let me go over to the expression for phi k, I have a so, if I multiply phi k equation also by exponential i omega t dt and integrate, I will get phi k omega is equal to e upon k square epsilon 0 multiplied by dv z integral from minus infinity to infinity multiplied by F 1 sorry this is capital F 1 this is capital F 1 capital F 1 k omega multi[ply]- this is all. I think I forgot to write this term, this is also a term there. So, when I multiply this take this on the right hand side it becomes plus actually, when this goes on the right hand side it becomes minus 1 up[on]- yeah this becomes F 1 k 0 v z divided by omega minus k v z into I.

This term, I have taken into account. I had forgotten to take this into consideration so, the I should add. Now, these 2 equations this is phi k omega also here. My suggestion is that or rather what I want to do now. I would like to use this value of F 1 k omega here and I will get two terms, one on the right hand side will contain a phi k omega that I will combine with this term and I will get another term having this integral dv z of this quantity.

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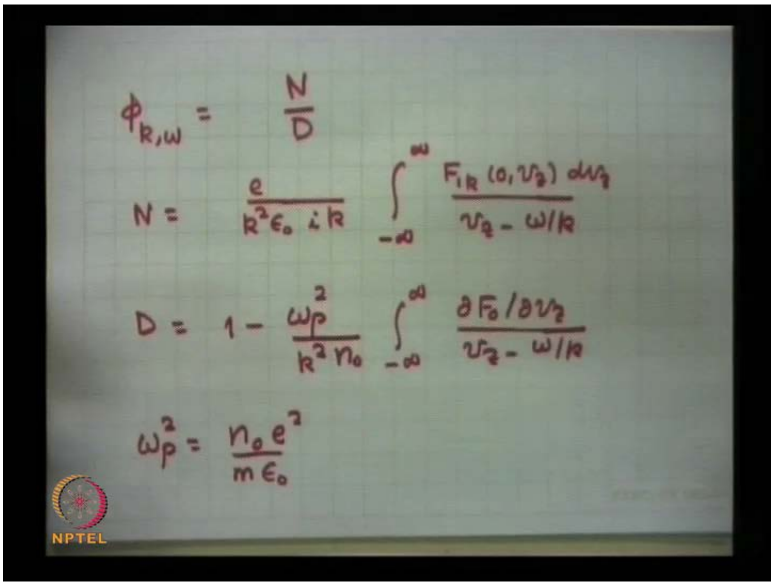


So, let me write down phi k omega I get, is equal to some numerator divided by the denominator. Numerator turns out to be an integral, which is if I simplify this is e upon k square epsilon 0 into i k common and then you will get a integral of this form F 1 k at 0 v z dv z upon v z minus omega by k minus infinity to plus infinity and the denominator of this is it is 1 minus, let me just see the denominator is rather simple. Let me write down this expression. This turns out to be equal to omega p square upon k square into 1 upon n 0 I would say here n 0 the number of electrons per unit volume into minus infinity to infinity delta F 0 upon delta v z upon v z minus omega by k.

So, you get the Fourier Laplace transform of wave potential as a ratio of two quantities N and D and here omega p I have defined as n 0 e square omega p square is n 0 e square upon m epsilon 0. but because there was no n 0 explicitly there. So, I have multiplied denominator numerator by n 0 and this is how I have written. But obviously, 1 n 0 is contained in the distribution function F 0. The important contribution of landau lies in understanding this. The thing is that, whenever you carry out the Laplace transform, you have to integrate to obtain phi from phi k omega, I have to carry out the inverse Laplace transform or integration over D omega from minus infinity to plus infinity above all the poles, and that is a complicated integral. Landau says that, if D equal to 0, hence the singularities. Now obviously, this is integral, this can be 0, if it is 0 for some values of omega after omega is a range and complex frequencies. So, for some values of omega this will be 0.

Those values of omega for which D is 0 are called poles of phi k omega and the maximum contribution to an integral will arrives from those poles. but unfortunately your Laplace transform is not defined for those frequencies of complex omegas at which this becomes 0. So, then you have to carry out what we call as the analytic continuation of the of this integral means, of this Laplace transform has to be this is essentially quantity defined in complex omega plane. So, you want to define Laplace transform in a domain where similarities lie. So, let me draw a graph here. Suppose, I plot here real part of omega and imaginary part of omega here, D the denominator of the Laplace transform suppose becomes 0 at some point here, it may become 0 somewhere here. Then, your integration has to be carried out on this line; this is the contour for omega. Integration has to be carried out here, this is contour one.

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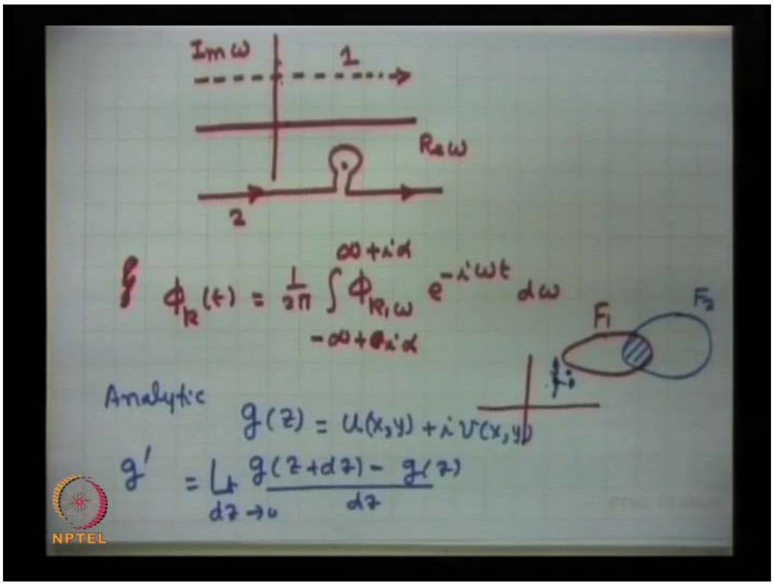


Landau says that, if there are no poles between here and this line. So, if I choose this suppose, this is another path of integration, this is path two because my please always keep this in view. I want to sorry I want phi k at time t that I write in terms of phi k omega exponential minus i omega t d omega ranges minus infinity plus epsilon sorry plus i alpha to infinity plus i alpha this length is i alpha from here to here and this is 1 upon 2 pi. So, in order to carry out this integration over this line in the complex plane like this, landau says that it will be much easier to carry out a integration of this over a another path, which is far below the imaginary omega equal to 0 axis. This is line over a which omega imaginary omega is 0. So, go underneath because imaginary omega will be highly negative.

So, when omega is a large in negative imaginary part, this quantity will vanish. So, only contribution will come from the poles. So, landau suggested that rather than carrying out integration over here, it is more convenient to carryout integration over here because only the contribution of poles will come. but then your function is not defined below the singular line similarity line because as I mention to you otherwise the Laplace integral overflow. So, in order to avoid the over flow of the Laplace integral, you defined your Laplace transform only in upper half upper plane above the similarity. The issue was that, how will you generalize your function.

Now, let me define what is analytic continuation? Suppose, in my complex plane, some function is defined some in some domain, this is a function F 1 is defined and another function is defined F 2 in some other domain, this is the F 2 function is defined here. but if in this overlap region F 1 is equal to F 2, then we say that F 2 is the analytical continuation of F 1. This overlap region may be a simple surface or single line a single curve it mean, it need not be a big finite area region the area of this may be negligibly small, it could be simple joining line. So, continuation of F 1 to F 2, F 2 is called analytic continuation of F 1 if, in the overlap region the two functions are equal or at the boundary joining the two the two functions are equal then this is called analytic continuation. And what is a function, when it is analytic continuation.

A function is called analytic, when the derivative of the function you can define suppose, there is a function g a function of z. You call this analytic, when g has a real part function of x and y suppose, I z I write as x plus i y then, this function can be written as u of x y and v of x y. u and v are called the real and imaginary parts of g function and x and y are called the real part imaginary part of z function, of z variable. And there is a condition that, when delta u y some derivative of this with respect to x or y is related to some derivative of y or v with respect to other variable means, the differential coefficient of g is uniquely defined irrespective of suppose, there is a function is defined at this point here and its defined its neighborhood, when you go from 1 point to another point either from here to here, or from here to here, or from here to here, or from here to here it gives the same limit then the function is called analytic

So, when the derivative does not depend on, how z goes to 0, you know we define normally g at z plus dz minus g at z divided by dz and limit dz going to 0, this is called g prime. but dz can go to 0 in a horizontal line or a vertical line but when you give an increment from here to here, or here to here if the rate of increment is same, then only the function is called analytic. Means, it is independent of whether, you make dx equal to 0 or dy equal to 0. so on, which root you move, then the function is called analytic. So, thing is that, if my function is analytic in some domain, the analytic continuation can be achieved by detouring the path of after all the Fourier transform that we obtained here, are in the form of integral over v z. So, Landau suggested that, why do not we treat v z is a complex variable.

Normally, the integral in these n and d the two numerator and denominator of phi k omega. They contain integrals over v z, he suggested that treat v z is a complex variable and presume that, those functions n and d functions, that you have written, if they are analytic and certain region and if you can avoid the singularity, then in all the domains where singularity is do not exists, the function will be analytically continuous. So, what he suggested was this. That if, I have to carry out suppose, I look at this n expression. N is equal to e upon k square epsilon 0 i k minus infinity to infinity F 1 k 0 v z dv z. This is the initial perturbation upon v z minus omega by k. This is some function of omega by k. So, what Landau suggested that, in order to carry out this integration, what you are having is this.

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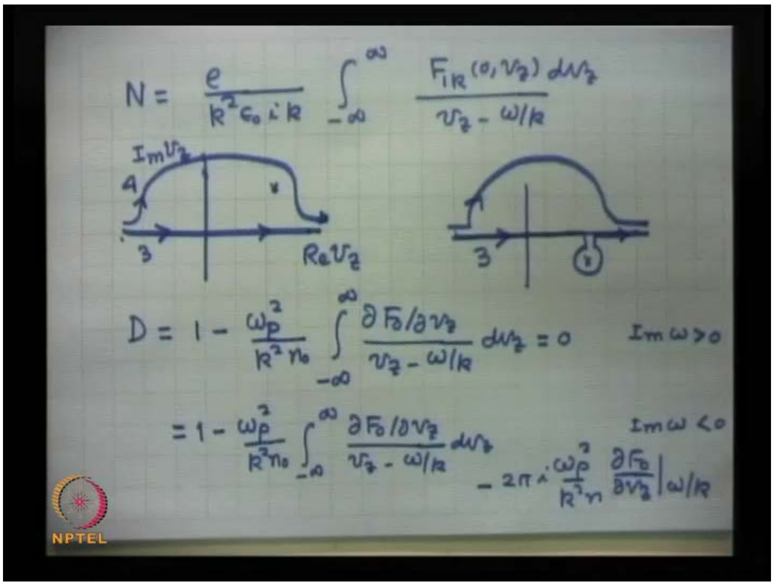


Suppose, I plot real part of v, this integration is over real v z but he says that in order to look for analytical continuation of Laplace transform, I treat v z as a complex variable. So, real part of v z if I plot here, imaginary part of v z I plot here, what do I get. If I move on this horizontal line, my integration has to be turn on the horizontal line, but this omega by k with finite imaginary part omega, I can have a pole here or I can have a pole underneath. Suppose, pole is here then he says that, this integration over this line is the same as over this line, this is suppose, he call as path 3, this is path 4.

The integral along this line and integral along this line, they will differ only by the contribution of the singularity or rigidly of this pole. And, if you have a pole underneath here, Landau suggested, then your path integral should be like, this go over this line go underneath and surround this here and so line integral this is was a suggestion, that the line integral from here to here is the same as line integral over this path plus contribution from the singularity. And you will have the contribution singularity here as well, if the contribution this singularity lies underneath the imaginary v z line, then you detour your contour like this.

Then, n defined by this integral, this the formed contour will be analytically continuous and your Laplace transform is then valid in the domain where imaginary part of omega is negative and far negative. So, just by detouring this, he could show that for imaginary part of omega higher values certainly, this is the same, because if the singularity lies here then and your function is defined there is no problem. The problem arises, when the pole lies here so, he says deform the contour. So, what he wrote was this and the same thing was done for D, D was written as 1 minus omega p square denominator was k square n 0 delta F 0 by delta v z upon v z minus omega by k dv z. This is equal to 0 are the poles, when I put this equal to 0, they are the poles. So, they say that, if imaginary omega is greater than 0 means, the poles lie here then the zeroes of D will be given by this integral.

Then, we are carrying over the integration over the real line. Whereas, if your poles lie imaginary part of omega is less than 0, then you have to detour the contour and this is 1 minus omega p square k square n 0 minus infinity to infinity delta F 0 by delta v z upon v z minus omega by k dv z, a contribution from this singularity has to added. So, he added a term here, this turns out to be equal to, when you add a contribution to singularity, it is 2 pi i times the residue and it turns out to be equal to 2 pi i times omega p square upon k square n into delta F 0 by delta v z at v z is equal to omega by k. This is the additional term, that he added. So, Landau says that, whenever imaginary part of omega is less than 0, in that case because I wanted to really bring my contour to carryout inverse Laplace transform to imaginary omega being very negative in that case, he says that this detour this contour by this and contour defined like this will have will be analytically continuous. And then, he says that these two can be written together in the form of a single, make a single statement by using what we call as the principle value. You define principle value of a integral through an isolated value, let me just define this.

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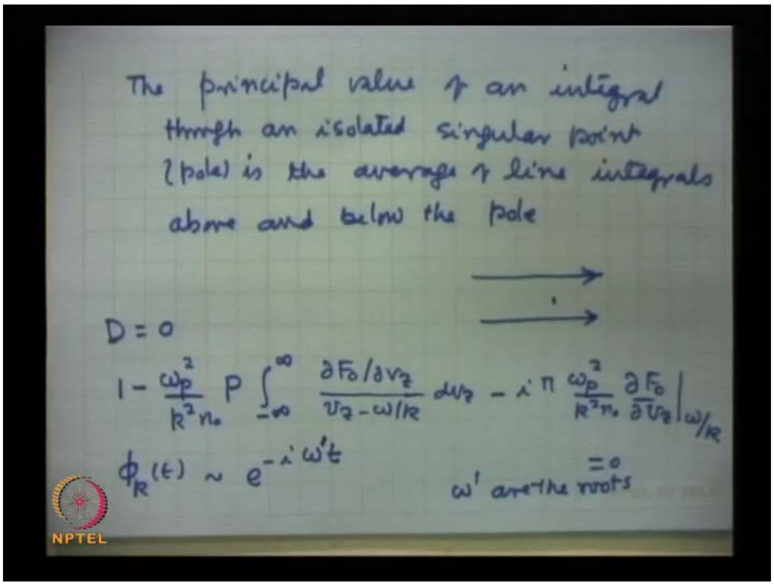


The principle value of an integral is defined as the well suppose, there is a pole somewhere through an isolated singular point called a pole is the average of line integrals above and below the pole. So, you are assume two line integrals, there is a pole somewhere just above, just below carryout the line integrals here and there and average some average of the two will be called the principle value. And if, you write this in the terms of principle value, then denominator of this Fourier Laplace transform will be 0 correspond to 1 minus omega p square upon k square and p means principle value of this integral minus infinity plus infinity, that is along the v z axis real v z axis delta F 0 by delta v z divided by v z minus omega by k dv z minus i pi omega p square by k square n 0, n 0 is here as well, multiplied by delta F 0 delta v z at v z is equal to omega by k and put this is equal to 0. And this is the dispersant relation, because the values of omega which satisfy this equation, in general those values are complex and your phi k at time t will go as exponential of minus i omega t where omega are the values which satisfy this equation, let me call them as omega prime values.

So, omega prime are the roots of this equation. So, the entire problem of examining the growth or decay of perturbations voice down to obtaining the zeroes of this equation. This equation is an integral equation, the first integral or there is only one integral has a velocity space integral from minus infinity to plus infinity but this omega you to allow complex values to omega and principle value, I will just defined as the average of line integrals just above the pole and just below the pole.

And this is the additional contribution that gives rise to damping of waves. If, this were not there as (( )) of did (( )) actually did this theory of plasma waves prior to Landau and he did not get in damping because he did not treat this singularity carefully. So, when you put this is equal to 0, you do not get any damping of wave. but this is the additional term that, you have to keep in view that gives rise to damping of the wave.

Well, I think this entire mathematics can be put in a much simpler way. The entire exercise was, actually I carried out just to show you that, if you consider initial value problem, then you have to carry out the analytic continuation of the function and in that in search of a analytic continuation Landau detour the contour of v z integral treating v z is a complex variable, but once you have recognize that, that whenever you are encountered with the v z integral or integration velocity space, you will follow the Landau prescription then, the rest of the procedures very trivial very simple. And, I would like to rework out the solution of (( )) equation.

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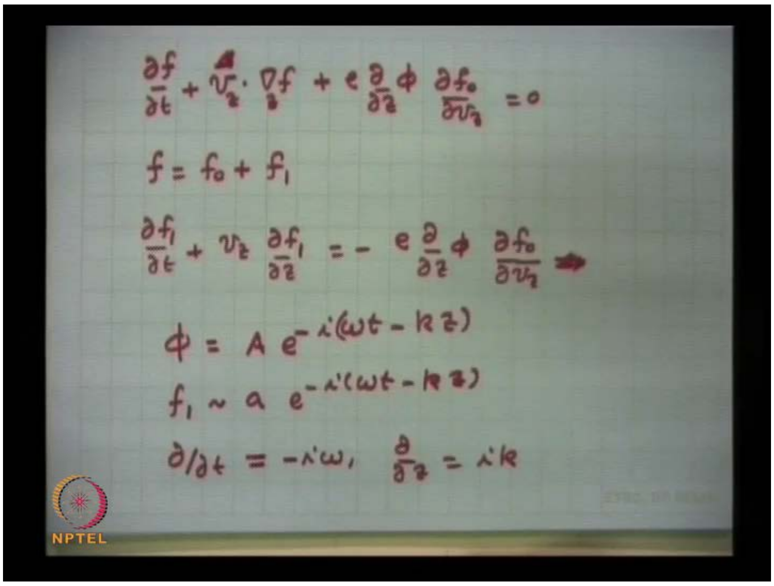


What we were really having is, let me write down the (( )) equation again. Delta f by delta t plus v dot del f minus e rather plus e delta delta z of phi delta f 0 by delta v z equal to 0. This was the one-dimensional, this was the (( )) equation I can write down this z component here and this z here forget this. So, this was my (( )) equation, which I wanted to solve and I can solve this very easily saying that, f is equal to f 0 plus f 1. I linearized this and my equation became delta f 1 by delta t plus v z delta f by delta z, this is the f 1 is equal to minus e delta delta z of phi delta f 0 by delta v z, this is all.

So, rather than doing any Laplace or Fourier transform, I consider, suppose my phi is equal to a exponential minus i omega t minus k z. I presume a Fourier component Fourier Laplace component of phi and I want to find out what is the response of my f 1 in quasi steady state. Then, I will say that my f 1 should also have a similar dependence on t and z. So, I say that f 1 should also have as a exponential minus i omega t minus k z. Substitute this back in here replace delta delta t by minus i omega delta delta z by i k and this equation becomes simple.

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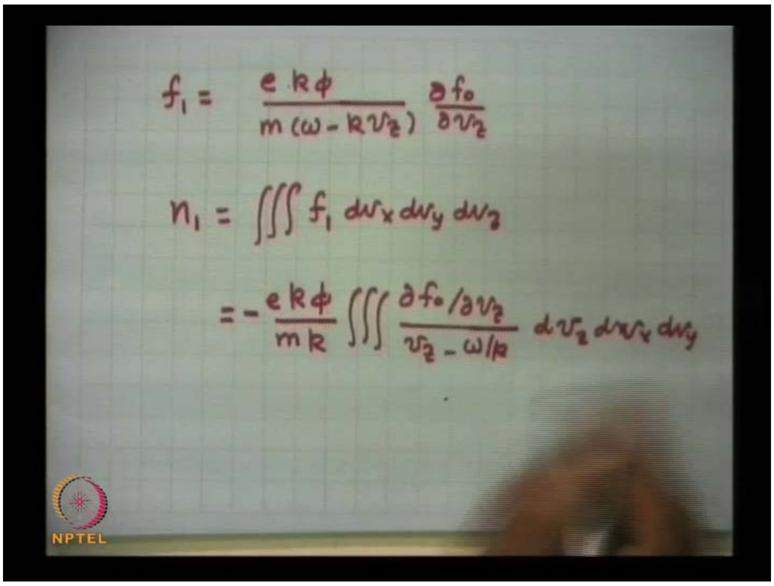


F 1 becomes equal to e k phi upon m omega minus k v z into delta F 0 by delta v z simple. How about the density perturbation, n 1 is equal to f 1 dv x dv y dv z triple integral. Substitute this in here, this becomes e k phi upon m outside, I can take minus k common also, take minus here and k outside here, in the interior you get, delta F 0 upon delta v z upon v z minus omega upon k and this is dv z d x dv x dv y triple integral. The denominator does not depend on v v x v y that derivatives also not respect to v z, it is simply with respect to v z. So, what I can do to be.

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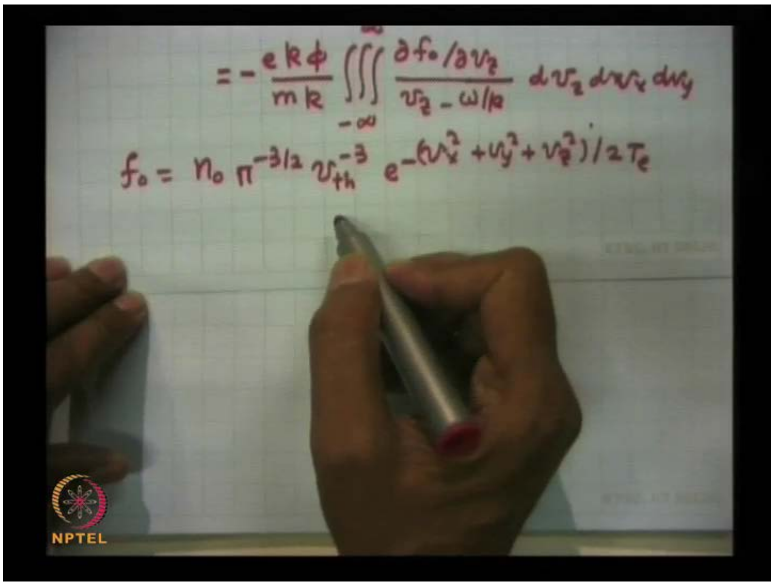


For a special case, when I take f 0 to be maxwellian, which is equal to n 0 pi to the power minus 3 half v thermal to the power minus 3 exponential minus v x square, plus v y square, plus v z square divided by 2 t e. This is the distribution function f 0 for a maxwellian distribution function. My suggestion is that, when you do this, then this when you carryout delta delta v z this factor exponential factor is can be taken out of the differential operator and you can carry out the integration over v x and v y. When you do this, then limits on all components of v x v y v z are from minus infinity to plus infinity. So, on carrying out v x v y integrations you will just turns out to be.

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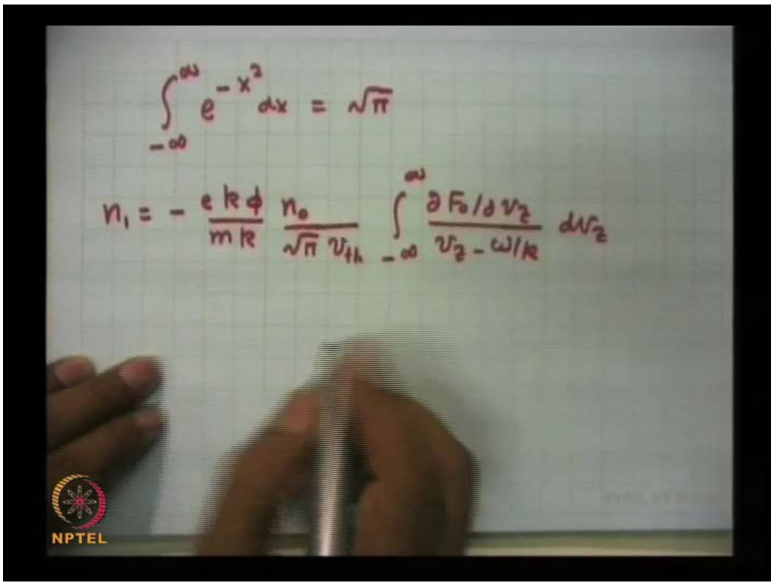

And recognizing that, minus infinity to infinity exponential minus x square dx is equal to root pi. If you recognize this, then this integral turns out to be n 1 is equal to minus e k phi upon m k, you will get n 0 there upon under root pi you, get rest of things cancel out and 1 v thermal you get and you get here delta F 0 by delta v z upon v z minus omega by k and dv z minus infinity to infinity.

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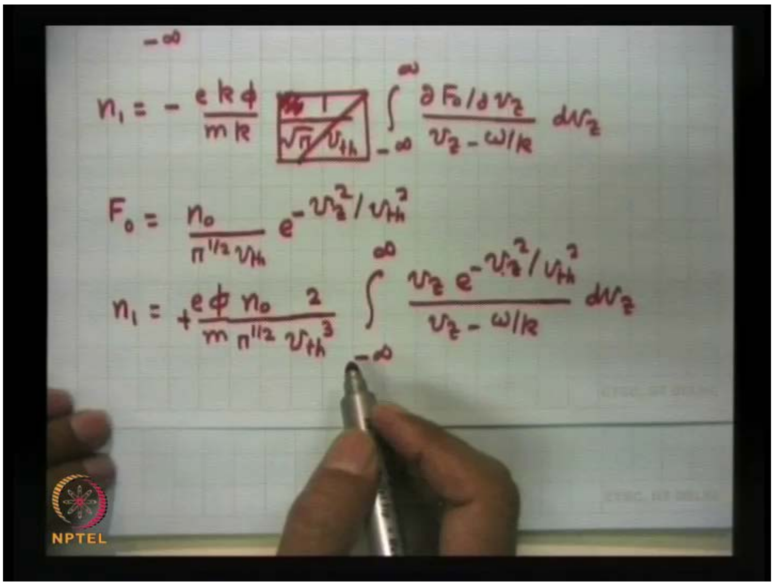


And, if you substitute for, well what is F 0. F 0 is called one-dimensional distribution function. Actually, I made a mistake, this n 0 is not there, that is involved in F 0 this is equal to n 0. Actually, all these factors are not there they have in a part of F 0. So, let me remove them here, they are not there n 0 upon under root pi into v thermal exponential minus v v z square upon v thermally square, this is called one dimensional distribution function. Now, when I substitute this back in here, this integral takes a simple form and let me write down this n 1 is equal to e phi upon m k will cancel out, you will get n 0 outside n 0 negative sign is already there, pi to the power half, then you have v thermal here and if I differentiate this function, you will get twice v z upon v thermally square into exponential.

So, here you get, with a negative sign. So, minus is here, 2 will be there and v thermal q will be there and you get here v z exponential minus v z square upon v thermally square divided by v z minus omega by k dv z. Its goes from minus infinity to infinity. People normally define v z by v thermal as a new variable. Let me call this as p so, v z upon v thermal let me call a new variable call p and let me define omega by k v thermal as a quantity called xi. Then, this integral takes the following form, n 1 turns out to be exactly equal to, this is equal to 2 n 0 e phi upon m pi under root into v thermal square and you get this integral of the form p exponential minus p square upon p minus xi d p minus infinity to infinity.

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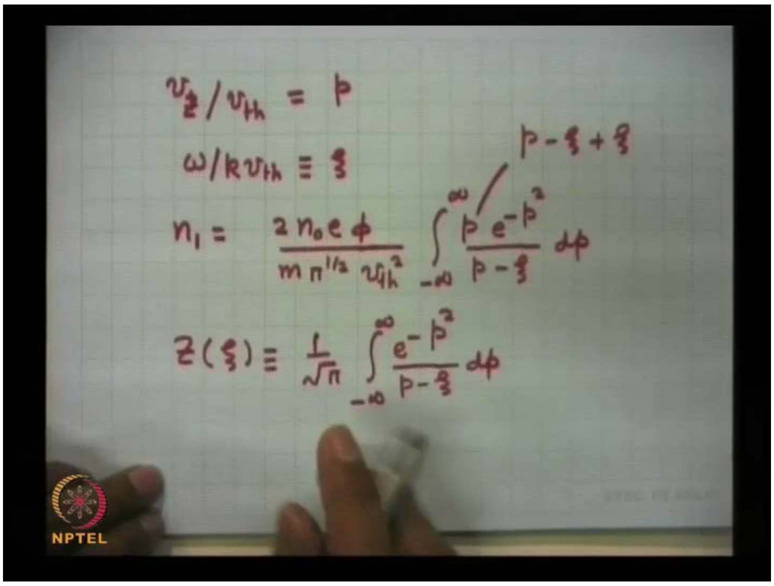

I define a function called z function of xi, identically equal to 1 upon under root pi exponential minus p square upon p minus xi d p. This is called plasma dispersant function. Limits are from minus infinity to infinity and in carrying out this integration one should be careful does this is not only principle value, this has to add a term corresponding to the landau prescription.

So, the entire understanding of or analysis will Landau is contained in the in the interpretation of this integral, not only principle value but the additional imaginary term that I had added because of the going round the singularity. So, now this p can be replaced by p minus xi plus xi and when p minus xi term cancels with this. This gives you root pi so, it becomes a simple expression and this xi can be taken out and you get this entire expression in a simple way, neat way.

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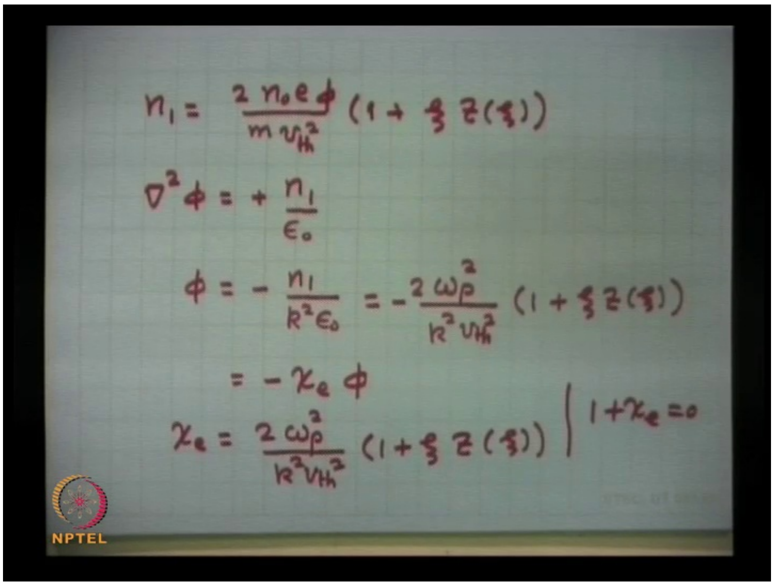


And let me write down this, it becomes n 1 is equal to twice n 0 e upon m v thermally square into phi here, 1 plus xi z function of xi. This is a very important deduction. The perturbed electron density due to the plasma wave is related to this expression. This is the expression for it and I would like to substitute this in the Poisson equation. The Poisson equation is del square phi is equal to minus up rather plus n 1 upon epsilon 0. Replace this by minus k square so, you will get phi is equal to minus n 1 upon k square epsilon 0 and when I substitute this here, you will get this is equal to 2 omega p square upon k square v thermally square with a negative sign into 1 plus xi z of xi. And this entire quantity can be written as minus chi e into phi.

So, chi that is called the electron susceptibility is twice omega p square upon k square v thermally square into 1 plus xi and plasma dispersant function of xi. And this equation gives you, the dispersant relation for plasma waves 1 plus chi equal to 0. So, this is a very important thing that, 1 plus chi e is equal to 0 is the dispersant relation for plasma waves. We do not have time today to discuss these implications. I think we need to need one more lecture to discuss in these implications.

I think, we shall continue next time in discussing the dispersant relation and obtaining the damping etcetera damping rate of the wave. Probably, I like to close at this point but let me remind you that, the entire contribution of Landau really lies reinterpreting the integral over a v dv z in the dispersant relation. And when you properly carryout the integration following Landaus contour Landaus prescription, you always get an additional term that contains the derivative of the distribution functions delta F 0 by delta v z.

As, I mention to you, physically we were expecting that we will get growth or damping of the wave, when the distribution function has a positive or negative slope at the phase velocity of the wave when v z is equal to omega by k and the same thing is contained explicitly in Landaus prescription. I think, we shall discuss this in our next lecture. Thank you.