Plasma Physics
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Lecture No. # 34
Vlasov Theory of Plasma Waves

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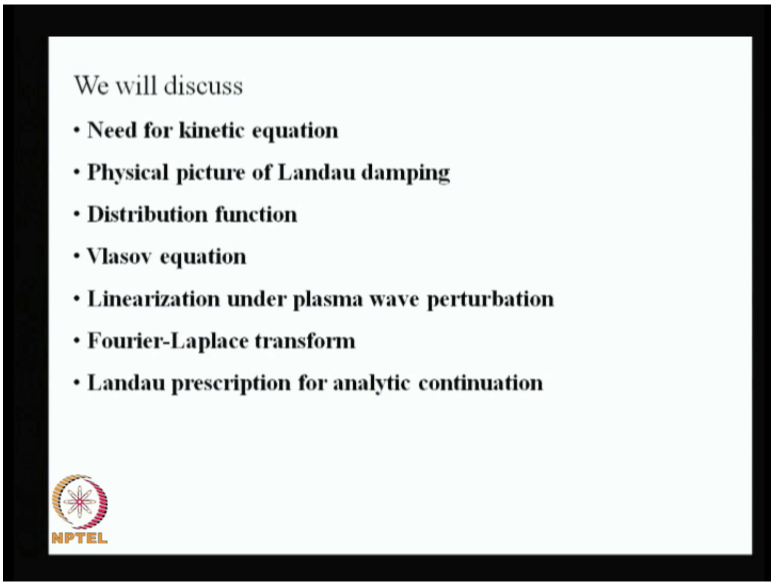

Today, we are going to start kinetic theory of waves in plasmas. Well, today we will discuss, what is the need for kinetic equation or Vlasov equation? We will develop a physical picture of Landau damping, which is a very important effect of kinetic description of plasmas. We will define a distribution function, deduce Vlasov equation, then solve this equation by the process of linearization in the presence of a plasma wave perturbation. We will employ the technique of Fourier Laplace transform to solve this coupled sight of Vlasov equation and Poisson equation. And we will discuss Landau prescription for analytic continuation of the function.

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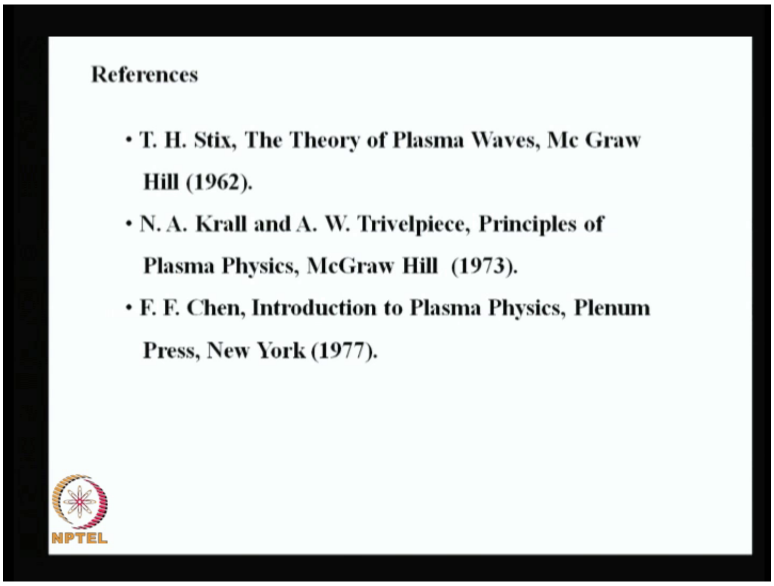

Well, our presentation today will be based on these books; one by Stix, another one by Krall and Trivelpiece and a book by F. F. Chen. Let me begin with the the need for kinetic description of waves and instabilities in plasmas.

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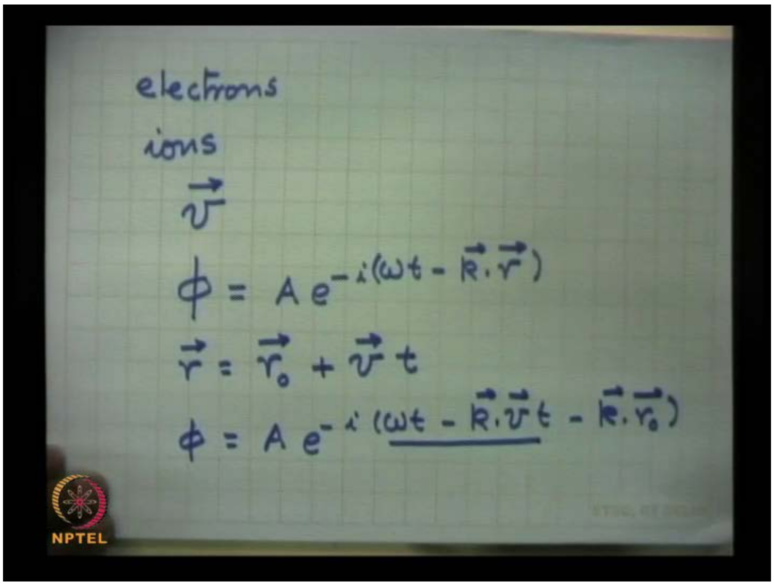

As I mentioned before that plasmas have electrons and ions and both these spaces have random velocities. Especially, the electrons have very large thermal velocities, and often that average velocities or drift velocities are less than the thermal velocities often. And the main consequence of these thermal velocities or random velocities is that if there is a particle moving with velocity v, and if it sees a wave electrostatic or electromagnetic suppose, if this particle sees an electrostatic wave of potential A is equal to A e to the power minus i omega t minus k dot r.

Suppose, this is the potential of a wave as seen by a particle moving with velocity v then, if there were no field, then the position of the particle in terms of time will be given by original position some position at time t equal to 0 plus v into t. So, if I substitute this expression for r in here, my phi appears to be A exponential minus i omega t minus k dot v into t minus k dot r 0. The thing is that, the effective frequency of the wave is not omega, but omega minus k dot v because this is the coefficient of t here.

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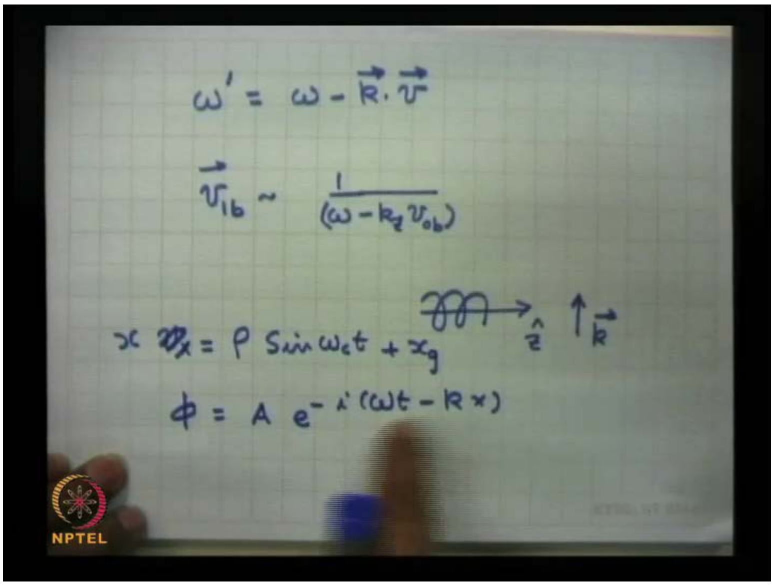

So, the velocity the frequency of the wave as seen by a moving electron is omega dash, which is equal to omega minus k dot v. This is a very important effect, that different particles in a plasma will see the wave of different frequency and as we learnt in the case of two stream instability that the response of a group of electrons which we call beam electrons, the drift velocity that you calculate due to the plasma wave, this scales as one upon omega minus k dot v or k dot v 0, that we call as this is the kind of thing we got. Now, the thing is that, whenever this quantity becomes 0, the response is very huge. So, some particles for some particles this quantity can become 0, those particles are called resonant particles. For other particles, this may not be 0, this will be different.

So, whenever you want to examine the resonant wave particle interaction and that those particles play very dominant role in energy transfer or growth or damping of the waves, you must go to kinetic description, that is one important thing. Second thing, in case of magnetized plasmas, what happens that suppose I have a magnetic field acting along z axis. And we know that, the electrons gyrate about the field lines and their x component of velocity for instance is related to is in terms of time can be written as rho sin omega c t, where rho is called the larmor radius and omega c is the cyclotron frequency. This is the velocity of sorry, I am talking of x. x coordinate of electron can be written like this, plus some constant x 0.

Now, the issue is or I will call this x g actually guiding center. So, the electron will gyrate about the lines of force and x coordinate of the electron suppose, behaves like this. And suppose, I am launching a wave with k vector perpendicular to this this by k vector of a wave. So, if I have a potential phi which goes as A e to the power minus i omega t minus k x for instance. What I should do this, x I should substitute from here. So, there is a time dependence contained in this term. So, the frequency of the wave is seen by the particle, which is gyrating is not omega, it will be omega minus something else, coming from here. And when you substitute this what you get is the following.

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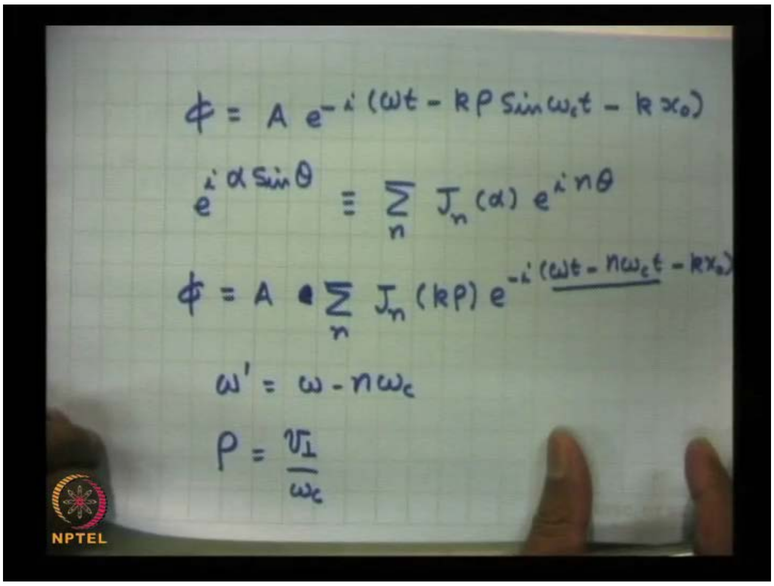

That pi as seen by the moving electron would be A e to the power minus i omega t minus k rho sin omega c t minus k x 0. Now, there is a Bessel function identity, that says that exponential of i alpha sin theta is identically equal to summation over n going from minus infinity to plus infinity integral values, J n of alpha exponential of i and theta. So, what you can do, exponential i k rho sin omega c t can be expanded in this way and phi can be written as A a summation over n J n of k rho exponential minus i omega t minus n omega c into t minus k x 0. So, the effective frequency of the wave is no longer omega, but it is omega prime, which is equal to omega minus n omega c, that is very important.

And, another important aspect is that the Bessel function argument has k rho. So, different effective frequency components will depend on, how much is the larmor radius and how much larmor radius, it depends on the electron velocity perpendicular to magnetic field upon cyclotron frequency. So, it a function of velocity and since different electrons in a plasma move with different random velocities, rho is different for different particles and consequently the response will be the effective field seen by different electrons will be different. That is very important.

So, these two aspects that finite larmor radius effects were important and that different for different electrons. And second, the effective frequency of the wave is dopler shifted and that gives rise to resonant interaction of some particles with waves. From these two aspects, it is necessary that, we go over to kinetic equation or Vlasov theory of waves. Well, before I delve into details of kinetic theory, I would like to mention to you the phenomenon of Landau damping that, so strikingly is described in plasma physics theory as a manifestation of kinetic effects.

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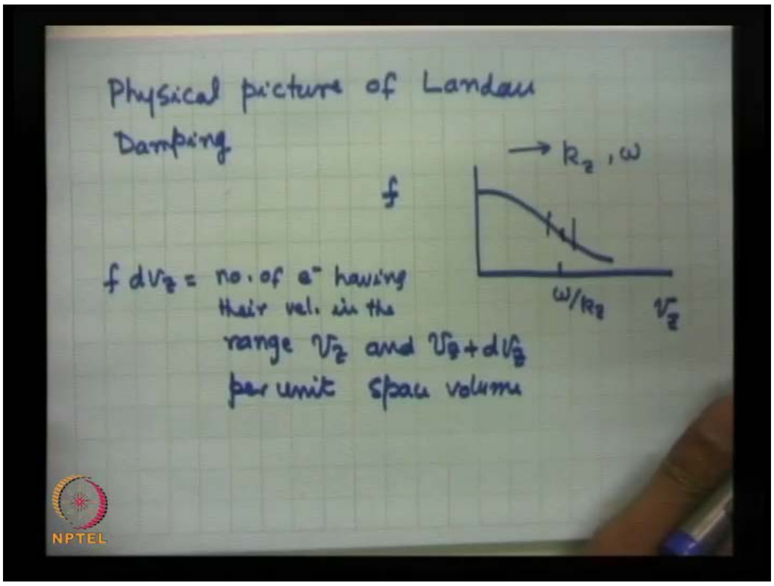

So, let me talk about the physical mechanism or physical picture of Landau damping. We have already learnt the two stream instability, in which there was no collision is still beam energy was transferred to the plasma wave. Now, here we are having a system of electrons many electrons and usually the distribution function in a plasma, if I plot one dimensional distribution function means number of electrons suppose moving with velocity v z and v z plus v per unit range in v z. So, f I will consider to be one dimensional distribution function, which means the number of electrons have a I would say that, f dv z is the number of electrons having their velocities in the range v z and v z plus dv z per unit volume space volume per unit space volume.

Electron velocity in general as a function of electron velocities are in general three components v x v y v z. But forget that suppose, I am counting only the v z of the particle irrespective of how much v x v y they have. If, I consider particles in a unit volume and measure their z component of velocity between v z and v z plus dv z, then their number is found to be proportional to dv z and this quantity, f is then known as particles per unit velocity range. And, if you plot usually this is going like this. So, particles with small velocities are larger in number and large velocities are smaller in number. Usually, this is the kind of behavior you observe.

Now, consider a wave travelling in the system, in the same direction z direction with vector k k z and the frequency of the wave is say omega. So, the phase velocity of the wave would be somewhere here, omega by k z is somewhere here. Now these particles, which are in the vicinity of this omega by k z; they have a v z equal to omega by k z, which is the phase velocity of the wave. These particles will be moving in the same velocity as the velocity of the wave.

And consequently, the frequency of them of the wave seen by them will be 0. However, omega by k z is just a point, what is important that you consider particles in certain range here, slightly slower than omega by k z and slightly faster, what will happen. So, if I consider these particles are called resonant particles or quasi resonant particles. Now, let us see what will happen, how will these electrons, which are moving slightly faster than the wave, how will they exchange energy with the wave, how will they bunch in retarding or a accelerating phases and how the slow moving particles will bunch. So, in what way they are going to have net energy gain from the wave or net energy transfer to the wave, let us examine this is.

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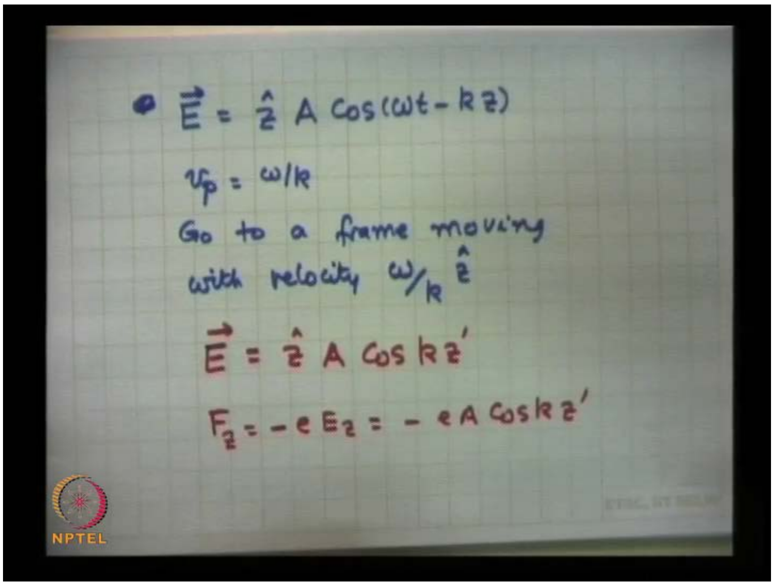

So, if I go to consider a specially a wave of say electric field say, let me write electric field E is equal to. So, z cap A is the amplitude, A cos omega t minus k z. Suppose, I consider a longitudinal wave propagating in the z direction with frequency omega in the laboratory frame and phase velocity of the wave is v p which is equal to omega by k. Now please, in order to understand the interaction of resonant particles with this wave, we go to a frame of reference. So, go to a frame moving with velocity omega by k z cap or v p z cap. In this frame, if I carry on the carry on the Galilean transformation, if I treat the particle moving with non relativistic velocity wave also to be moving with non relativistic velocity. I can carry out the transformation to a moving frame and then the electric filed will be written as, let me write this E would be written as, z cap A cos k z prime, z prime is the coordinate z coordinate in the moving frame. Now, the force on the electron of charge minus e would be minus e into E z, which is minus E a cos k z prime, it is independent of time, but now let us examine, how it looks.

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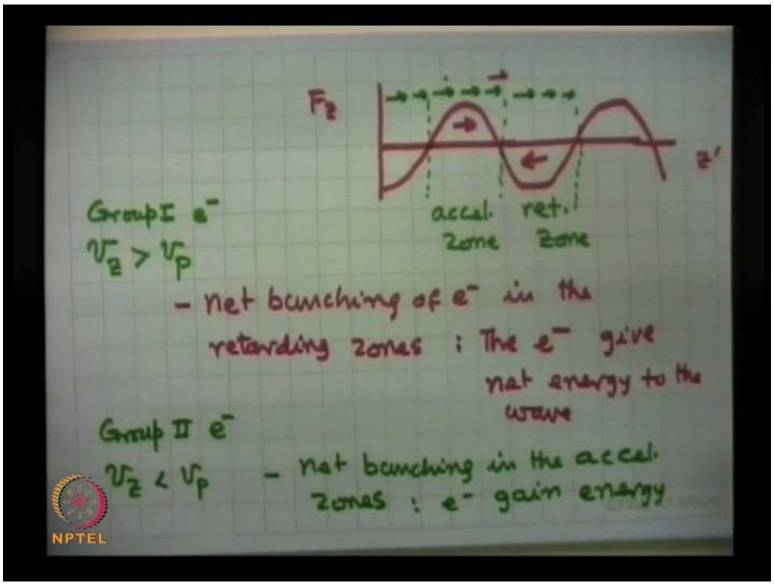

So, the force experienced by an electron in a moving frame will be so much. It starts from here, goes here, comes here. Now, the force F z as a function of z prime I am plotting. I am getting this region, where force is positive in the positive z direction. So, I will call this as the accelerating zone. This force is negative so, I will call this region as the retarding zone, because this tries to track the particles slow down the particles. So, now consider, there are two streams of particles. One moving with v p v z greater than v p, I will call group one particles. In the moving frame, their velocity will be positive because in the left frame their velocity was more than the velocity of the wave. So, these electrons will appear like going this way, like this. Now, what will happen? When the wave is having a positive F force on these electrons, they will move more quickly from here to here. So, they will move rather quickly from here to here, very quickly they will go there.

Whereas, the electrons here they see a retarding force. So, they will slow down and they will spend more time here, then crossing this boundary between retarding and accelerating zone. Consequently, there will be net bunching of electrons in the retarding zones. So, in this case there is a net bunching of electrons in the retarding zones. Let me repeat the argument. Initially, when the wave is not there, all the electrons are moving with a velocity v z minus v p in the forward direction, which is nearly zero, but is still finite.

So, when the wave is there, then the electrons which are in the accelerating zones, their velocity will be enhanced. because the force is accelerating in the forward direction. So, they will quickly move from the accelerating zone to retarding zone. Whereas, the electrons which are in the retarding zone, they will be slowed down because the force is in the negative direction, the force is in this direction, the force is positive here, negative here. So, these electrons will be slow down, they will spend more time here plus some addition electrons have arrived from here. So, there is a net bunching of electrons in the retarding zones.

And, as we see the retarding zones are the areas, where the wave retard the particles its takes away energy from the particles. So, there is a net transfer of energy from particles to waves. So, these particles these electrons give net energy to the wave. Now, if you consider second bunch of particles, second group of electrons. Let me just mention here, group two electrons for which, v z is less than v p. These electrons will appear moving in the backward direction and consequently, the ones in the retarding zones will be quickly moving out to the accelerating zone and the ones which are in the accelerating zones, they will be slow down.

So, there is a net bunching in the accelerating zone net bunching in the accelerating zones and these electrons then gain energy from the wave. So, these electrons gain energy from the wave energy. So, there are two distinct behaviors of electrons, which are moving faster and slower. If the number of fast moving electrons was larger, then there is a net energy transfer from particles to wave and the wave will grow. On the other hand ,if the number of slower electrons is larger, then there is a net gain of energy by the electrons from the wave and the wave will damp. So, it depends on which group number is larger and this will depend from the slope of the distribution function.

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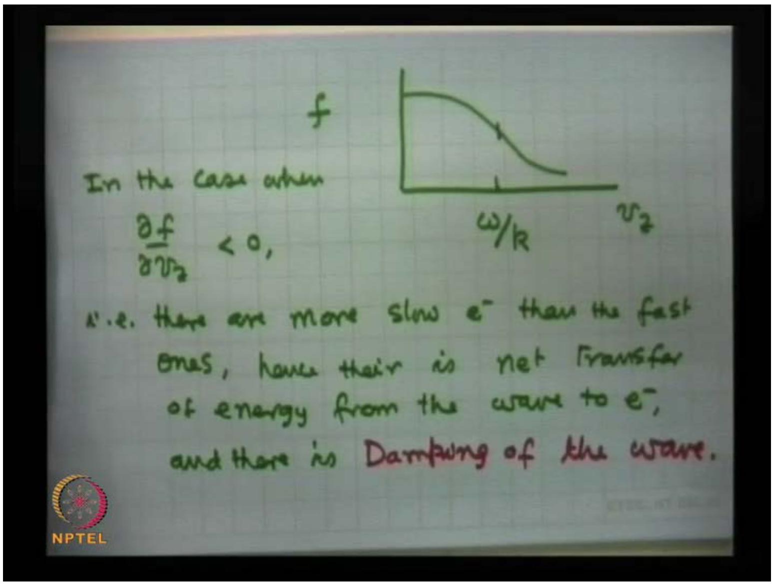

So, if I have a slope of the distribution function like this, f as a function of v z if I plot and suppose the distribution function is like this and this is the point, where phase velocities of wave lies, if the slope of f with v z is negative. It means slower particles are more than the faster particles in that case there is a net transfer of energy. So, in the case when delta f by delta v z is negative .There are more that is, there are more slow electrons than the fast once. Hence, there is a net retardation hence there is sorry , hence there is net transfer of energy from the wave to the particles and there is a damping of the wave and there is damping of the wave, this damping is known as Landau damping. So, it does not really the value of number of particles exactly at omega by k, but in its neighborhood or if the slope of the distribution function that should decide, whether the wave is damped or amplified. Now, all these effects are taken into consideration or through what we call as the (( )) of equation.

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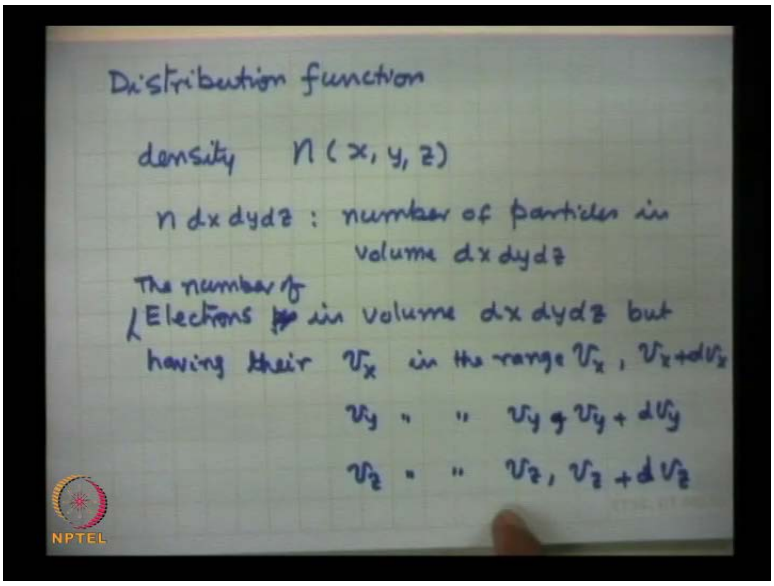

So, before I go over to (( )) equation, let me formally define a distribution function. Well, let me begin with the definition of a density in three dimensional space. Density normally, we use a symbol n. In general, it is a function of x y and z, which means the number of electrons per unit space volume is called a density. If I multiply n by dx dy dz dz then, this gives me the number of particles electrons in volume dx dy and d z. So, if I divided this by dx dy d z, you will get the number of particles per unit volume, that is called density.

But, if I consider unit volume, the electrons that you get n electrons per unit volume, all the electrons are not moving with the same velocity. Some are moving faster, some are moving slower, some are moving upwards, some are moving downwards, some left wards, some right wards and so on. So, if different particles are moving with different velocities. If, I want to study the interaction of individual particles with waves, then I can subdivide these particles into small velocity ranges. So, what we do, we say that out of these n electrons, let the electrons electrons let me define for electrons or the number of electrons rather the number of electrons in volume dx dy d z. Obviously, total number is n dx dy d z.

But, having their v x component of velocity in the range v x comma v x plus dv x, v y in the range v y plus comma v y plus dv y and v z in a small range v z and v z plus dv z. So, let me focus not on all the electrons in this volume element, but out of this, let me find how many particles are there, that have their x component of velocity in the range v x and v x plus dv x, y component of velocity in the range v y and v y plus slightly large velocity dv y and similarly v z. How many particles do I have; Obviously, if the range were not there, then I would expect no particle. So, larger the range larger the number of particles. So, I expect that, this number would be proportional to the products of these velocity ranges. Obviously, it is proportional to dx dy dz dz also as here.

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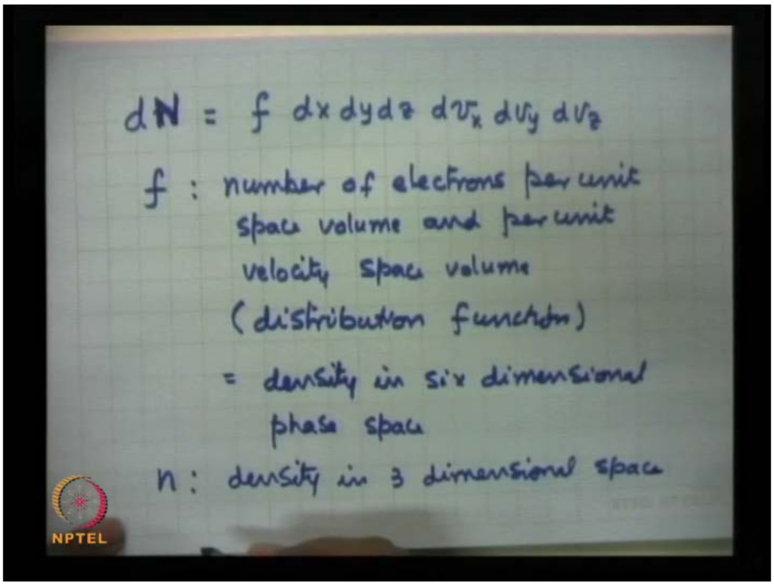

So, I write this number of particles, let me call this d N, total number of particles in the volume velocity range. So, this is f dx dy dz dz d v, it let me call capital n rather not a small n total number of particles dv x dv y dv z. This is the total number of electrons in the volume element space volume element dx dy dz dz and this is called the velocity space volume element dv x dv y dv z. So, f is called the distribution function, which is the number of electrons per unit space volume and per unit velocity space volume. So f, the distribution function is the number of electrons per unit space volume and per unit velocity space volume. This six dimensionally space x y z v x v y v z space is known as phase space.

So, f is known as the distribution function, and which can be viewed as a density and take six dimensional phase space is equal to density in six dimensional phase space. Just as n was the density in three dimensional space, this is the density in six dimensional phase space. So, let me write down for a comparison n was the density of particles or electrons in three dimensional space, x y z space. And just n was a function of three variables x y z f will be a function of six variables x y z v x v y and v z.

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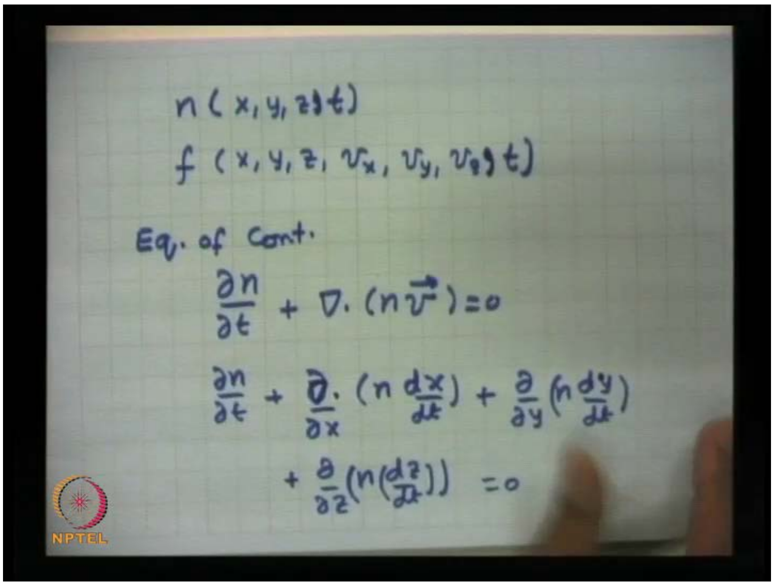

So, let me always keep this comparison in view, n was a function of x, y, z and f in general, is a function of x, y, z, v x, v y and v z. Of course, time dependence is always there like density in any system besides dependent or depending on x, y, z depends on time; f will also depend in general on time. Well, I can write this expression here. Now, we have. So, let me write down a dependence actually time dependence also here and let me introduce a time dependence here as well. Now, in case of three dimensional density, x y z t dependent density, we have a equation of continuity and which says that, delta n by delta t plus divergence of n v is equal to 0, n v is called the flux of particles. So, whenever the inflow of particles is different in the outflow of particles in any small volume element or any volume, there will be accumulation of particles in that volume and this I actually called the equation of continuity.

I would like to write down this equation in slightly different way, which says that delta n by delta t plus divergence of n into, I will write down actually, I can write down this is delta delta x of n v x, which is dx by d t. I think, I should write a plus delta delta y of n dy by d t plus delta delta z of n dz dz by d t, where dz dz by dt is the z component of velocity of a particle. So, this is this is the way, you should you could write this is equal to 0. I would like to, I may write this actually dx by d t in a symbolic form as x dot, dy by d t as y dot.

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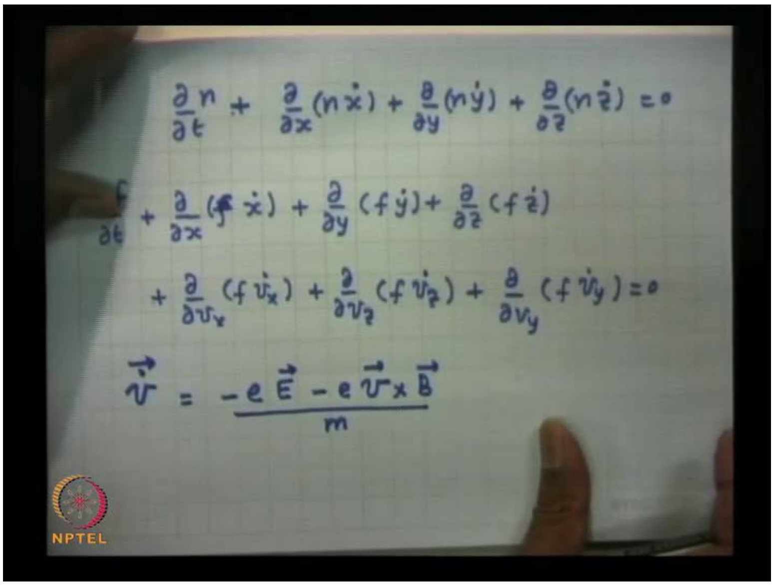

So, rather than calling d d t, I could have written this in this way. Delta delta t of n plus delta delta x of n x dot, plus delta delta y of n y dot, plus delta delta z of n z dot is equal to 0. This is the way, I would like to generalize the equation of continuity to six dimensional space. So, this del operator, I should write down in six dimensional space. By similarity, I can write down delta f by delta t plus delta x delta delta x of n or rather f x dot plus delta detla y of f y dot plus delta delta z of f z dot. These are similar terms here, but now three additional terms will come because of delta delta v x of f v x dot, plus delta delta v z f v z dot, plus delta delta sorry , this is this should have been v y, I will write down v y here v y f v y dot, this should be 0. So, by similarity to equation of continuity, I have written the continuity of equation of continuity for f in six dimensional phase space.

Now, I would like to remember that, x y z v x v y z v z are all independent variables. So, if x dot is v x, I can take v x out of this differential sign, this v y can be taken to be independent of v y and so on. And similarly, I can write down about these these are of stroll acceleration v x dot is acceleration x component, v y dot is particle acceleration y, z component and so on.

So, normally if we are talking of electromagnetic fields in the system, then v dot vector is equal to the electric force, if the electron is minus e charge and electric field is E and magnetic force is e v cross b, then this is the electric force magnetic force divided by mass, this is the acceleration. So, what I should do, I can replace v dot in these equations by this quantity and these can be written after removing v x ,I can take out of delta delta x operator because v x and x are independent variables.

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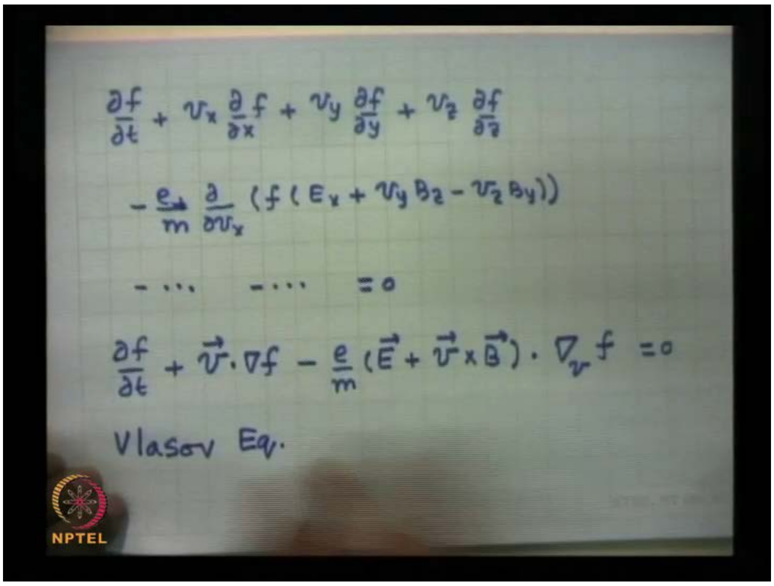

So, when you do this, you get delta f by delta t, plus v x delta delta x of f, plus v y delta f of delta y, plus v z delta f by delta z. These are the three terms, they are having space derivatives. Now, the terms with velocity space derivatives would be, this is plus delta delta v x of f multiplied by the x component of acceleration. So, minus e by m is common. So, I can take this minus e upon m outside. Then, I am having E x plus v cross b x component, which would be v y B z minus v z B y and similarly other terms. I will not write them explicitly, I will just say that similar terms will be for delta delta v y and delta delta v z and this whole sum should be 0. You may note one thing in here, that when delta delta v x operates over this inner bracket, it gives you 0 because there is no v x dependence anywhere, it operates only on f and hence this inner bracket can be taken out because it has no dependence on v x.

Similarly, if I had written delta delta v y term, the term containing e and v that inner bracket will be independent of v y. So, it can be taken out, that is the beauty here. So, what I am saying is that, these acceleration term can be taken out, rather than keeping it inside. And this equation takes the following form delta f by delta t plus these three terms can be cast in this form v dot del f and these can be written as minus e upon m taking these force terms outside, which becomes become E plus v cross B dot delta delta v. So, I will call the grad v operating over f this is equal to 0. This is called the (( )) equation.

In which two things, important things have been ignored. One, no collisions have been taken into consideration and secondly, no ionization or recombination is are allowed. If you have to allow for and collisions then, right hand side will not be 0, it will have some number there. Well, the processes of recombination and ionization and collisions are slower processes in plasmas and consequently, in a studying the behavior on a shorter time scale, then the collision time ecetera. You really have to solve this equation with right hand side equal to 0 and this has very rich physics.

All resonant wave particle interactions are inherent in this equation. Today, I will discuss a very special case, when there is no magnetic field in the system d c or perturbed magnetic field and there is no d c electric field in the system. So first of all, I would like to consider a equilibrium that is called field free equilibrium in which, e is 0 b is 0. So, this term is 0. Obviously, in equilibrium distribution function should not depend on time. So, when this is 0, then this must also be 0. So, for a field free equilibrium, f must be independent of x y and z. So, that gradient of f is 0.

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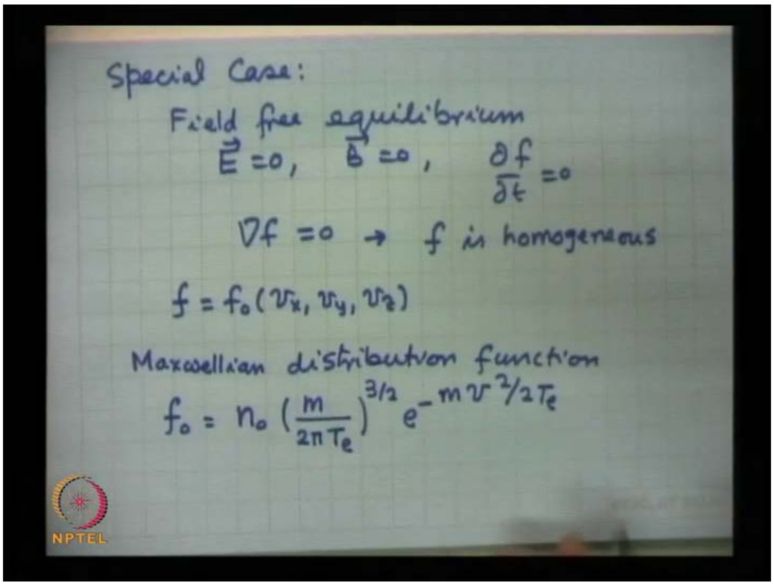

So now, let me consider a special case. Field free equilibrium, means E 0, magnetic field is 0, in the system and delta F by delta t does not dependent on time because we are talking about equilibrium. So, gradient of f must be 0, which implies that f is homogenous independent of x y and z.

We call this distribution function as f 0. So, f is equal to f 0. So, this in general as a function of v x v y v z and many plasmas, this has Maxwellian form. So, I will write down this especially Maxwellian distribution function. It can have different forms also, but for a Maxwellian distribution function, I can write down f 0 as, this is n 0, the number of electrons per unit volume is called density of particles. And then you have m mass of the electron upon 2 pi T e electron temperature in energy units whole when constant is hidden in temperature three half exponential minus m v square upon twice T e. This is called the maxwellian distribution function. And certainly, when you substitute this, in the plasma equation in the absence of any external fields e and b, then this satisfies the Maxwell's equation. For any arbitrary value of n 0 arbitrary value of t e and where, v square is v x square, plus v y square, plus v z square.

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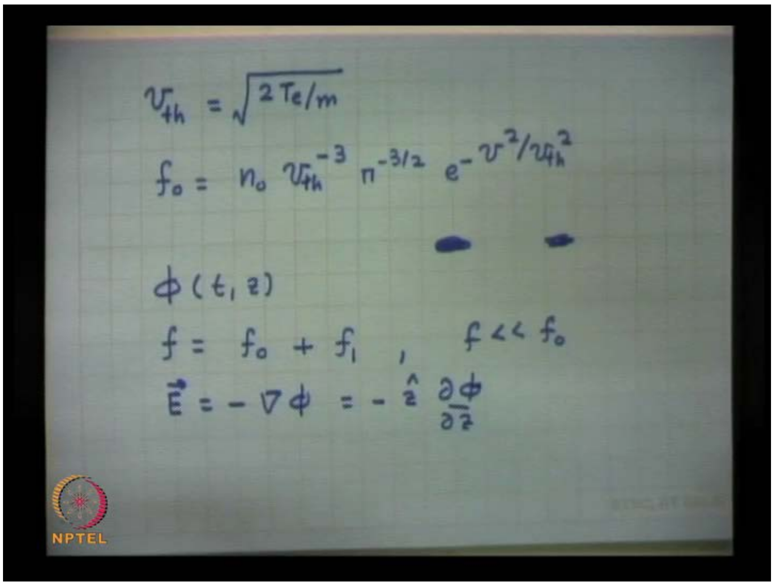

We define a quantity called thermal velocity of electrons as 2 T e upon m under root. Then, f 0 can be written as n 0 into v thermal to the power minus three, then you have a quantity called pi to the power minus three half, exponential minus v square upon v thermal square. So, this is a three dimensional velocity space distribution function, you can call so, called as six dimensional phase space distribution function, which is uniform in x y z, but its depends on values of v x v y v z. I would like to consider now a perturbation, suppose I perturb the system by some compression or rarefaction of charges somewhere. That gives rise to electric field and; obviously, I am expecting that, when you have displaced electrons, which are uniformly distributed somewhere, suppose you have to created more electrons and a depletion of electrons somewhere here. Then, in such a system, you are created electric fields in the system.

And those electric fields will then, move these electrons from high concentration region to low concentration region and that will set an oscillations. So, I would like to self consistently study. If, I have create a some initial perturbation in the distribution function and self consistently perturbation in the electric field has been created, then how this perturbation will grow with time. So, I am trying to consider a simple one dimensional situation. That suppose, I create a perturbation in electric field, which can be expressed as a gradient of scalar potential phi which in general as a function of t and z.

And I say that, my distribution of function, now is equilibrium distribution function plus some perturbed quantity. So, this accumulation of charge is somewhere in a space velocity space or actual space irradiction elsewhere, will gives rise to creation of electric fields or electric potentials. I would like to find out the equations governing f 1 and the equation governing phi. Well, the equation governing f 1, I can deduce from the (( )) of equation treating f 1 to be very small as compare to f 0. And perturbed quantity is to be very tiny.

So, I will consider that the electric field, that is produced by the potential, which is minus grad phi, which is equal to in this case, minus z cap, delta phi by delta z is a small quantity, very negligibly small, very tiny, f 1 is also very tiny. And let us see, how it f 1 evolves with time, that is my fundamentally. So, how the potential evolves with time will damp out or will it grow with time. That is the fundamental problem that, Landau examined in 1946, a very pioneering work he carried out, he addressed this is so.

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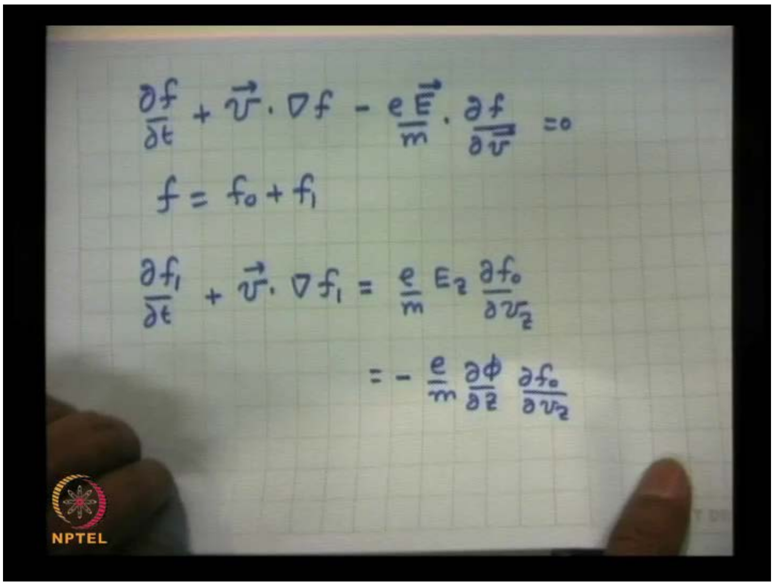

So, I would like to substitute this f equal to f 0 plus f 1 in the (( )) equation. Let me write down the (( )) equation, delta f by delta t plus v dot del f minus e E upon m, dot delta F by delta v, I will write down v vector. I should have written grad v, equal to 0, because there is no magnetic fields, I did not write that. But, I am treating this electric field to be a small perturbation. So, when I put f equal to f 0 plus f 1, I get the and recognizing that f 0 does not depend on time, it does not depend on position, it depends only on velocity. So, this term will be a finite, but other will not be.

But, product of f 1 with e will be ignored. So, I get delta f 1 by delta t, plus v dot grad f 1 is equal to, take this on the right hand side e upon m e z, because I am considering my electric field in the z direction and delta f 0 by delta v z. This is my equation governing f1, but it is in terms of E z, which I can write down in terms of phi as minus e upon m delta phi by delta z into delta f 0 by delta v z. But what is the equation governing phi. The first equation Maxwell’s equation is also called the Poison equation governs phi in terms of density perturbation.

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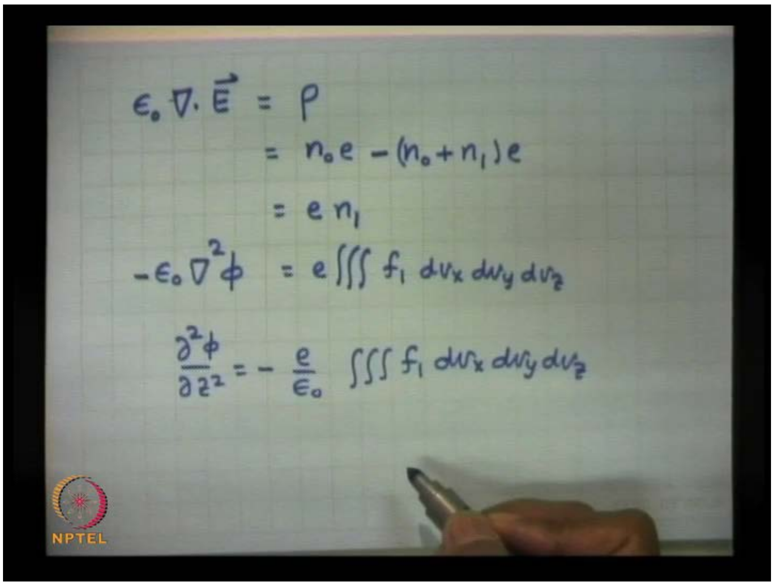

Let me write that equation, the equation is divergence of e multiplied by free space permittivity epsilon 0 is equal to the charge density rho, but this I can write down as electron ion charge density, which was n 0 into e ions are there in the system with density n 0 and I presume that, the density is not modified by this perturbation, its only electron perturbation minus n 0 plus n 1, this is the electron density into charge is minus e. So, n 0 e will cancel out and you will get n 1 e.

But, n 1 e is how much, number of particles you can get from the distribution function e, which is the perturbed distribution function f 1 into dv x dv y dv z. It is a triple integral on v x v y v z ranges are from minus infinity to plus infinity for each. And, e if I write as minus grad phi, this becomes minus epsilon 0 del square phi is this expression. So, its and because I am considering only delta delta z variation, I write down this is equal to d 2 phi, dz dz square is equal to minus e upon epsilon 0, triple integral f 1, dv x dv y dv z. So, these are two coupled equations. One governing phi in terms of f 1 in the integral form and the other one governing f 1 in terms of phi, the perturbed (( )) equation. In order to solve these, Landau carried out Laplace Fourier transform, I will do it in steps. So, consider the Fourier transform first.

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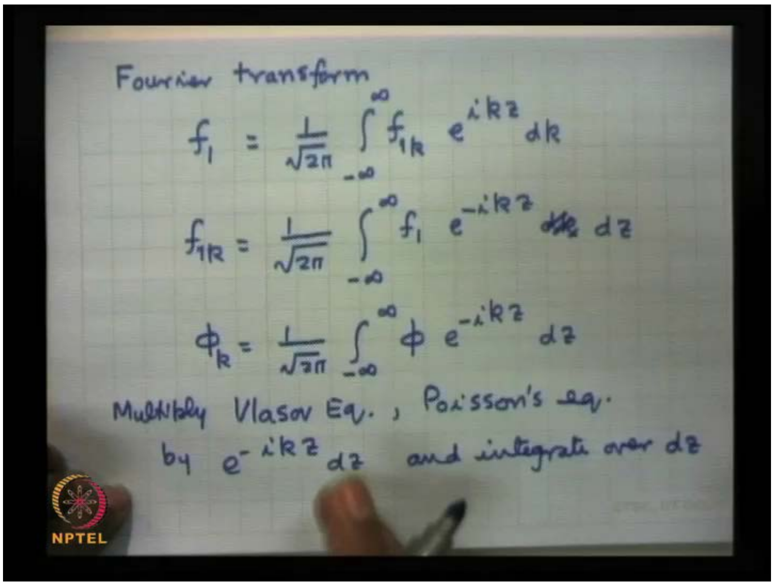

But I am saying here that, if I have a perturbation f 1, which depends on position, then I can write down this space dependence as 1 upon root 2 pi, a quantity f 1 k exponential of i k z dk. Please understand, f 1 in general, has dependence on time z v x v y and v z. So, as far as z dependence is concerned, I have written as a sum of sign k z terms. This is called Fourier transform, f 1 k is called Fourier transform of f 1 and the values of k goes from minus infinity to plus infinity.

And f 1 k can be written as 1 upon root 2 pi, minus infinity to infinity f 1, exponential of minus i k z dk oh dz dz sorry sorry this is dz. So, this becomes my Fourier transform of perturb distribution function. Similarly, I can define a Fourier transform potential phi k as 1 upon root 2 pi, minus infinity to infinity phi, which depends on time and z as I mentioned before exponential minus i k z dz. So, what are my suggestion is that, multiply the Poisson equation and the (( )) equation by exponential factor this minus i k z and integrate over d z. So, multiply (( )) equation and Poisson equation, the phi equation by exponential minus i k z dz dz and carry out the integration and integrate over z d d z, integrate over dz dz over z rather.

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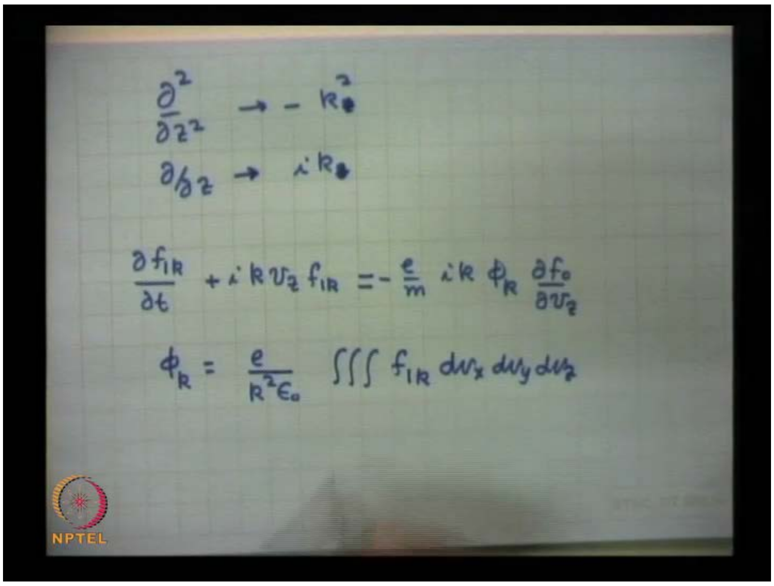

When you carry out the integration, what you get essentially is this is equivalent to when you do this, then you will say that, wherever d 2 dz dz square was there, it is replaced by minus k z square and wherever delta delta z was there its replaced by simply i k z, you can just do it. This integration can be carried out by parts and this can be done.

So, once you do this, the (( )) equation reduces to the following form delta f 1 k by delta t plus i k rather, I do not have to put it is z subscript here, z k is enough. So, i k v z f 1 k and then, there was this term minus e upon m, this is equal to minus e upon m i k phi k delta f 0 by delta v z. And the phi equation became becomes, the phi equation was it is becomes rather phi k is equal to e upon k square, epsilon 0, triple integral f 1 k dv x dv y dv z. These are two couple set still, but 1 derivative is there in time. So, I would like to carry out the Laplace transform, but I think, for today the time is running out.

So, we will continue our discussion in the next lecture. What you want to do, we want to carry out the Laplace transform of these quantities and solve for the Laplace transform and carry out the inverse Laplace transform and when you do this, you have to be careful and we will follow the Landau prescription, we will do it by control integration. I think, this has very rich physics and we shall continue in the next lecture. Thank you.