Plasma Physics

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Module No. # 01

Lecture No. # 33

Ion Acoustic, Ion Cyclotron and Magnetosonic Waves in Magnetized Plasma

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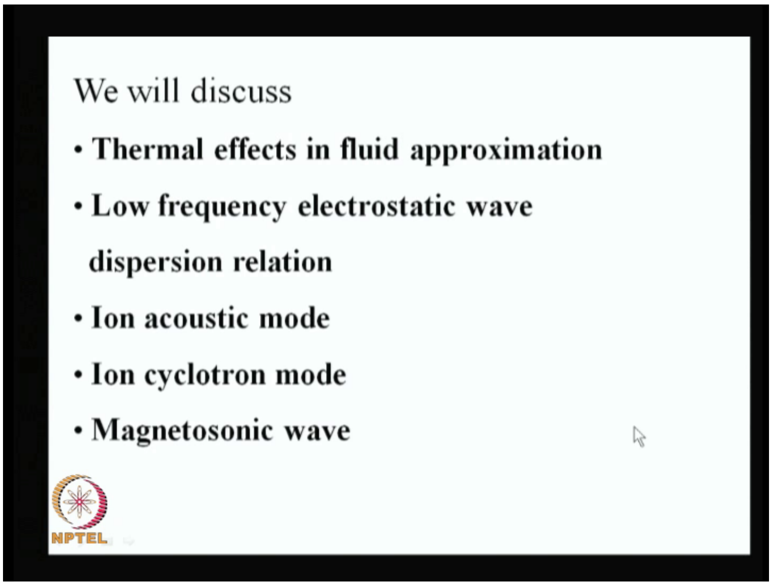

Well in this lecture, we shall discuss electrostatic waves in magnetized plasma including thermal effects and we will consider these waves to be of low frequency like the ion acoustic wave, ion cyclotron wave and magneto sonic wave.

Well, magneto sonic wave is primarily an electromagnetic wave, but it gives rise to charge compression rarefaction and hence, it has a very strong electrostatic character or as well.

We will discuss the validity of fluid approximation when we include thermal effects, we will drive a dispersant relation for low frequency electrostatic wave, then discuss ion acoustic mode, ion cyclotron mode and then, we will start afresh and discuss magneto sonic wave.

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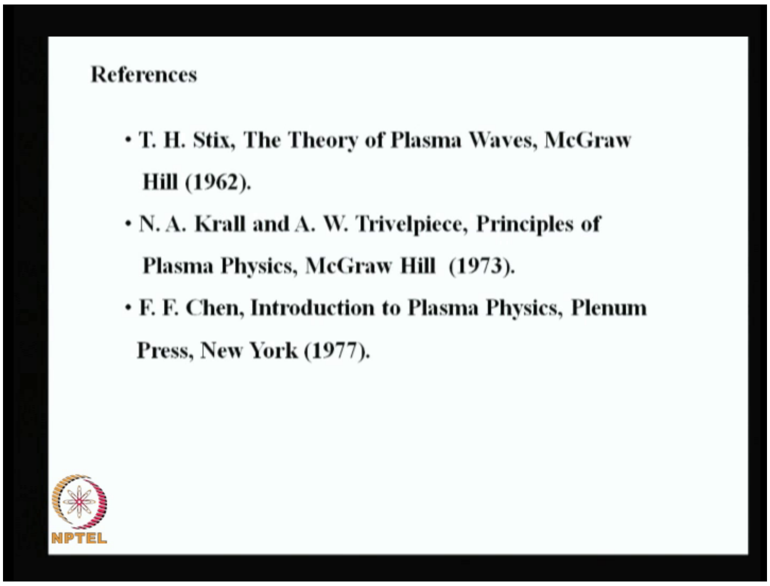

Well, these are the references, three books by Stix, Krall, Trivelpiece and Chen.

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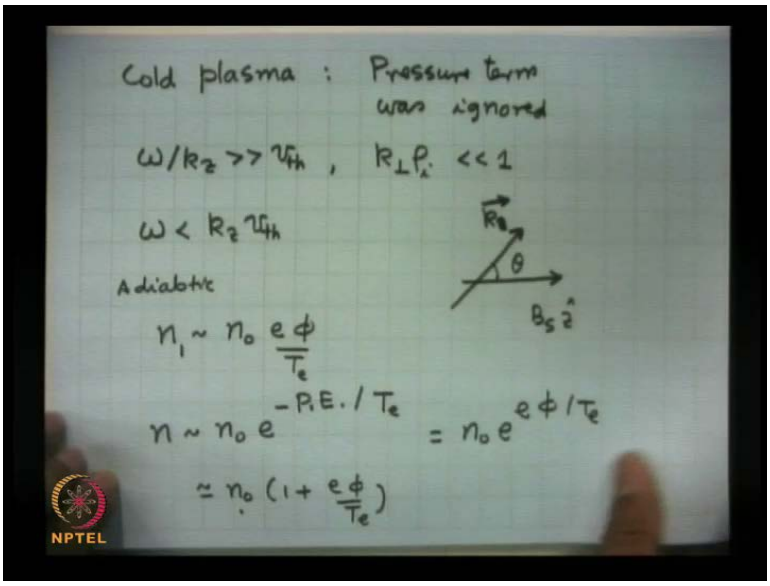

In my last lecture, I was talking to you about electrostatic waves in a cold plasma and in a cold plasma I ignored the pressure term in the equation of motion. So, pressure term was ignored. Well, this is all right as long as the velocity of phase, velocity of waves is much bigger than thermal velocity and I mention to approximations that when omega by k z is much bigger than the electron thermal velocity and larmor radius of the electrons and ions and ions in particular k perp rho i, this was much less than one, then this is could approximation.

However, a great variety of waves in plasmas they may satisfy this condition, but this condition is not satisfied, especially for ion acoustic wave and for ion cyclotron waves we have a situation omega is less than k z v thermal of electrons.

Ion thermal velocity may be ignored because that is too small, but the waves may have parallel phase velocity less than thermal velocity of electrons and what is the consequence. The consequence is that if there is a magnetic field in the system and the wave is going at some angle, this is k vector of the wave, this angle is theta. If this theta, well if k z is significant and significantly large, it means that the wave phase varies of the speed of phase propagation along z direction is less than v thermal, then what can happen, that the electrons can quickly follow the variations in potential of the wave and the response becomes Adiabatic. Adiabatic response means what, this is called Maxwell Boltzmann distribution, which says that if density perturbation will be of the order of equilibrium density into e phi upon t. How do we deduce this.

Because we know that if by wave as a potential phi charge of the electron is minus e, then n anywhere should vary as this is perturbed density, it is actual density should be like n 0 exponential minus potential energy P E upon temperature of the electron T e, this is called Maxwell Boltzmann law. That if you have a potential potential energy distribution in the system, the electrons will have a tendency to to those reasons where potential energy is minimum and if I put the value of potential energy as minus e phi, this becomes n 0 exponential of e phi upon T e, if this quantity is less than one, this can be approximated as n 0 into 1 plus e phi upon T e. Hence, the perturbed this is called the equilibrium density, this is the perturbed density, this is so much.

So, electrons can follow the variations in phase of the wave just like an ion acoustic wave, when we talked about this wave in unmagnified plasma. So, this is the important condition and consequently the ion response becomes totally different as compare to the electron ion response, they are totally different ions and electron response are very different because of their thermal velocities.

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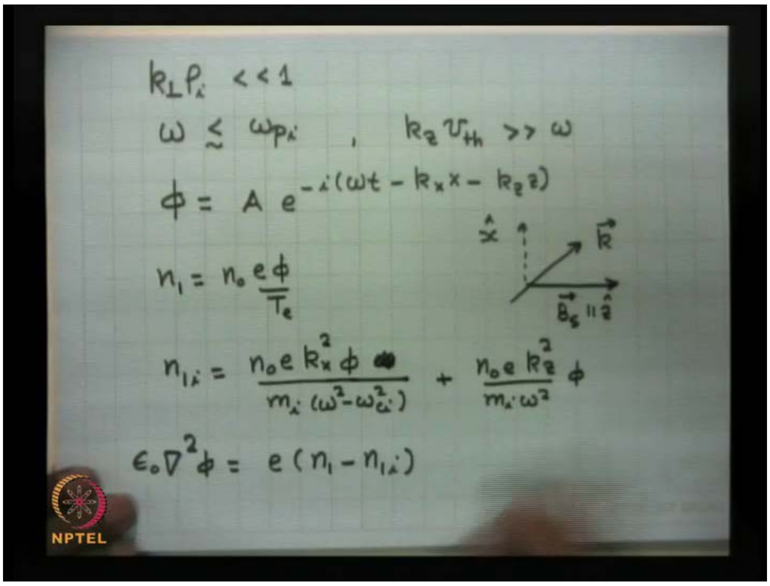

So, let me examine the effect of temperature on waves, in the limit that larmor radius is still satisfy this condition that k perp rho i is less than one. In this limit, I would like to evaluate this and we will be inter essential waves of frequencies less than or comparable to omega p i and I will be considering the case where k z v thermal of electrons is much bigger than omega. Let us examine, I will consider the electron response. So, let me write down first the potential of an a electrostatic wave which is phi is equal to A exponential minus i omega t minus k x x minus k z z. I have chosen k y equal to 0 without any loss of generality because I can choose my x and z axis according to my choice. So, z axis I am choosing along the magnetic field B s is parallel to z axis and this is my x axis and k vector of the wave is some angle here.

The electron response I will straight away take as n 1 is equal to n 0 e phi upon T e, where t is the electron temperature. As for as the ion response is concerned, I will revert back to the old derivation. In my last lecture, I have obtained that if I consider ions to be cold, in that case ion density perturbation due to this wave turns out to be equal to n 0 e k x square upon into phi upon m i omega square minus omega c i square and there was a omega there and then, there was a term because of this n 0 e k z square upon m i omega square and phi. I think this omega is not there. So, this was the ion density perturbation, this is the electron density perturbation, use them in the poison equation which is Del square phi is equal to into epsilon 0 is equal to e times n 1 minus n 1 i.

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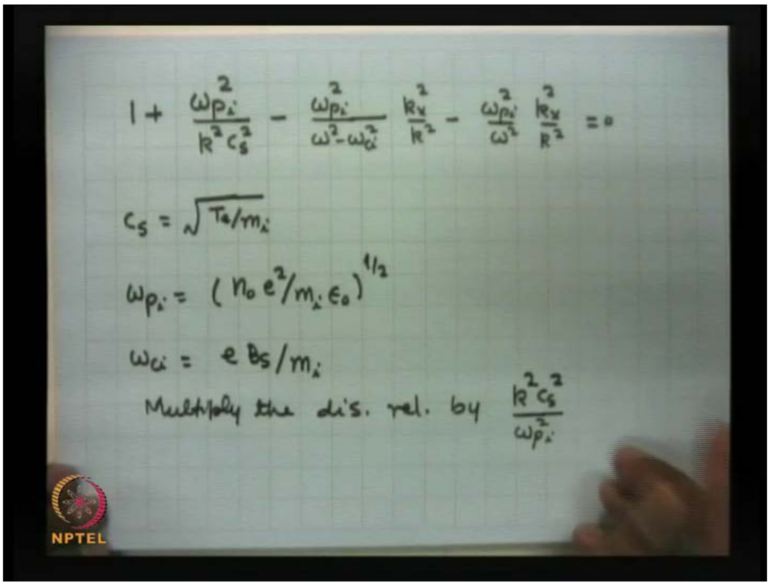

.It can be caused in a simpler form because when we are looking for low frequency waves like ion acoustic wave, then we are looking for omega of the order of k c s, which is significantly smaller than omega p i. So, it is better to multiply this equation by k square c s square by omega p i square. So, multiply the dispersant relation by k square c s square upon omega p i square, then that equation takes the following form. Just substitute it there and you get A, dispersant relation which is 1 plus omega p i square upon k square c s square, I will define c s in a little while, this the electron contribution or electron susceptibility, minus you will get omega p i square upon omega square minus omega c i square k x square by k square minus omega p i square upon omega square k x square by k square equal to 0, where c s I have defined as under root of T e upon ion mass electron temperature upon ion mass, omega p i is the ion plasma frequency which is n 0 e square upon m i epsilon 0 to the power half, omega c i is the ion cyclotron frequency, which is e B s upon m i. This dispersant relation obviously, will have two roots let say by quadratic omega square. It will have two roots

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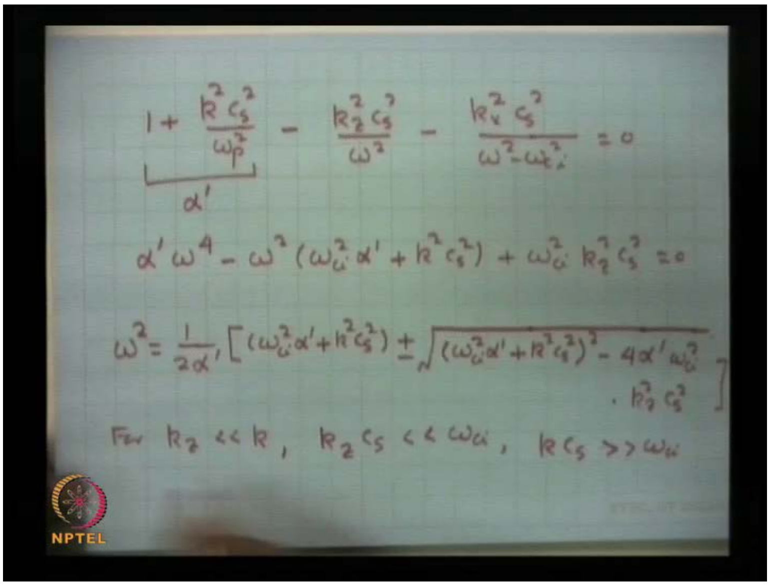

One plus k square c s square by omega p i square, this is the small, minus k z square c s square upon omega square minus k x square c s square upon omega square minus omega c i square is equal to 0 and I may call this quantity as alpha prime, which is close to unity.

In that case, this equation takes this following form, I can cast this into the form of a quadratic equation which gives you alpha prime omega four minus omega square omega c i square alpha prime plus k square c s square plus omega c i square k z square c s square is equal to 0 and this gives me two roots, which is omega square is equal to 1 upon twice alpha prime multiplied by omega c i square alpha prime plus k square c s square plus minus under root square of this omega c i square alpha prime plus k square c s square whole square minus four alpha prime from here and this factor, which is omega c i square into k z square c s square this whole thing, this factor is multiplying here inside the square root.

There are two signs 1 is called the upper sign is called ion acoustic wave, the lower wave I will call simply a low frequency mode whose frequency can be comparable to omega c i as well. If I choose, I think this can one can plot this, but in a special case, when either of this conditions are satisfied that for either k z is much less than k means for propagation perpendicular nearly perpendicular to magnetic field then, this term can be taken to be small.

Or if k z c s is significantly less than omega c i because this is omega c i term or k c s is bigger than omega c i. In either of these cases, this equation simplifies and this is a very. So, if your frequency your are expecting around k c s, if it is much bigger than omega c i, then this expression simplifies to, if k c s is bigger than k c, then this simplifies for the plus sign to. Let me just say, when either of this conditions are satisfied this or this or this, in that case this dispersant relation gives you two roots one is called omega square is equal to k square c s square plus omega c i square. So, when omega is bigger than omega c i, this is like ion acoustic wave.

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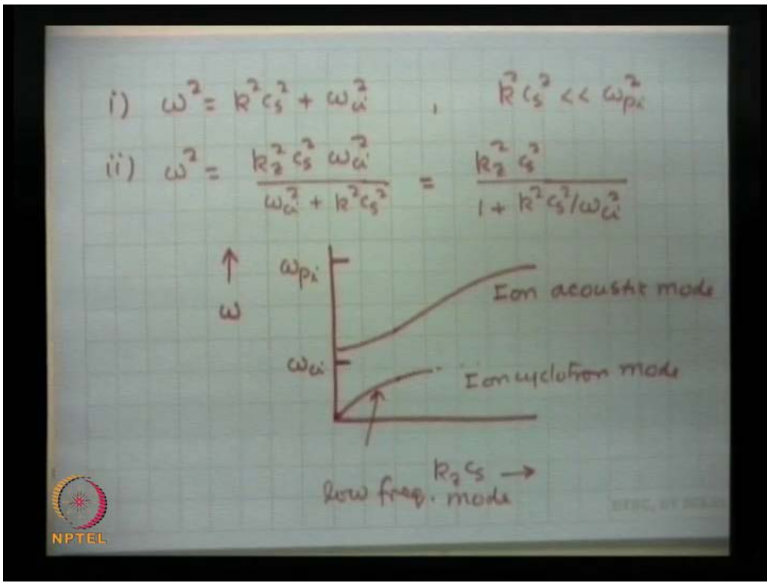

And second case, when omega square of this second root is omega square is equal to k z square c s square into omega c i square divided by omega c i square plus k square c s square. So, what I have done. In order to obtain the second root with negative sign, the second term the under root was taken to be small as compare to the first term and I had expanded the under root using binomial expansion and this can be rewritten as k z square c s square divided by 1 plus k square c s square upon omega c i square.

Now, there are two distinct roots. So, if I plot them omega versus omega here and I let me plot k z c s here. What you get here that suppose there is a frequency, I will call this is omega c i somewhere here and omega p i somewhere here.

Well, this dispersants I have written the limit when I have taken k c s less than omega p i. If I do not take this, then I have already given the general relation omega square is equal to something, you plotted from there and what you get is that the upper root has a frequency higher than this quantity depending on how much k perp you choose, it starts somewhere here and goes to omega p i. Whereas, if I plot the second root it starts from k z equal to 0, so, omega is becomes nearly small and it has a tendency to go towards omega c i.

So, well the approximation that I made is not nearly valid when this becomes larger, but basically there is a, this is a lower frequency root, this is a higher frequency root. I will call this the ion acoustic wave and this is a mode if the frequency is in the vicinity of omega c i, it becomes a ion cyclotron mode. It will smaller than the this mode is called low frequency mode. So, you get different modes; however, we have excluded a very broad category of modes because of this restriction that k perp rho i, it has to be must less than one. If you include the thermal effects in on ions and finite larmor radius effects you get a much richer variety of modes. So, especially I would suggest that when we are talking about the lower frequency modes like these modes, finite larmor radius corrections are very important and the character dramatically changes for these modes. How as for as the ion acoustic mode is concerned, omega is primarily like k c s, this term is usually small and this is a sound wave which certainly exists and well; obviously, I have done this in this limit, but higher frequencies sound wave can take like this omega p i.

Well, these modes have been observed in many devises they have driven unstable by a variety of mechanisms, parametric instabilities can drive them, beams can drive them, ion beams can drive them, unstable electron current can drive them unstable and whenever they are produced in a plasma, they give rise to anomalous resistivity because some phonons are produced in the plasma, which can give rise to a stronger collisional momentum loss of charge particles and can cause enhancement in resistivity of the plasma. Now, before I close I would like to go over to a different kind of mode, which is electromagnetic in character, but behaves like an electrostatic wave because it gives rise to a very strong charge compression. It is some sort of a compression Alfven wave.

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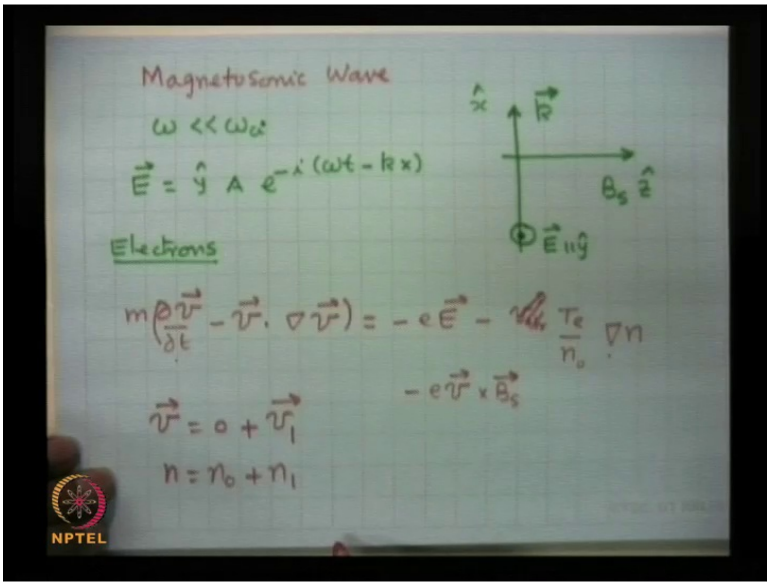

So, I am going to talk about a particular mode, which is called magnetosonic wave. Its dispersant resonance similar to sound wave dispersant relation, but it has a different character.

Consider a plasma with magnetic field along z axis and I am considering propagation of a wave purely in the perpendicular direction to it. Suppose, this is my k vectors of wave and I choose and there has to be self consistently chosen and we will find self consistently later on, that if I choose a wave of electric field, say e parallel to y perpendicular to the plane of this board and this is my x axis. So, if I choose an electromagnetic wave travelling normal to the halo frequency e m wave, whose frequency I will choose to be less than omega c i, may be 1 less than omega c i, ion cyclotron frequency.

And let this wave be polarized perpendicular to the plane of the paper. What will it do. This electrons and ions will experience a e cross B motion and e cross B motion will be in this direction. So, if the electrons and ions move in this direction; obviously, if the velocity of this wave is less than c much much less than c, sound the velocity of light in free space, then that will can give rise to very significant amount of charge compression and rarefaction. So, because of the e cross B motion the when the electrons acquire, a longitudinal velocity longitudinal to k, then there is a charge compression rarefaction and if thermal motions are important, then the electron charge compression may not cancel with the ion charge compression, then balance would be significant. But this wave will also because this as a perpendicular electric field, it will give raise to perpendicular current as well polarization current and that can sustain this this mode. So, this is a mode that gives rise to density compression as well as oscillatory velocity of polarization current in the direction transfers to the propagation.

Let us examine the character of this mode. I choose e is equal to y cap A exponential minus i omega t minus k x, then for the electrons I solve the equation of motion and the equation of motion, if I forget collisions, but include the pressure term, the equation is m delta v by delta t minus v dot del v is equal to minus e E minus v thermally square, sorry not this, t of the electron upon density equilibrium density into gradient of n. This is the equation of motion, but there is a magnetic field term also, which is minus e v cross B s.

There is a wave magnetic field also, but the product of perturbed velocity and perturbed magnetic field, I will ignore. Otherwise, I should include that term also. So, if I presume that my plasma does not have a equilibrium drift velocity, that term is not there and as I mention that, in equilibrium there is no drift velocity. So, I can write down this this v is equal to v 0 plus v one, the perturbed velocity. I substitute it back, ignore the products of perturbed quantities and this equation then takes the simple form. I am going to be delete quick on this, I will substitute this in this, this term will survive, delta delta t I will replace by minus i omega. As I been doing often. This del I will replace by i k, when this operates over n and I will write down as n 0 plus n one, perturbed density, perturbed velocity and then, this equation takes the following form.

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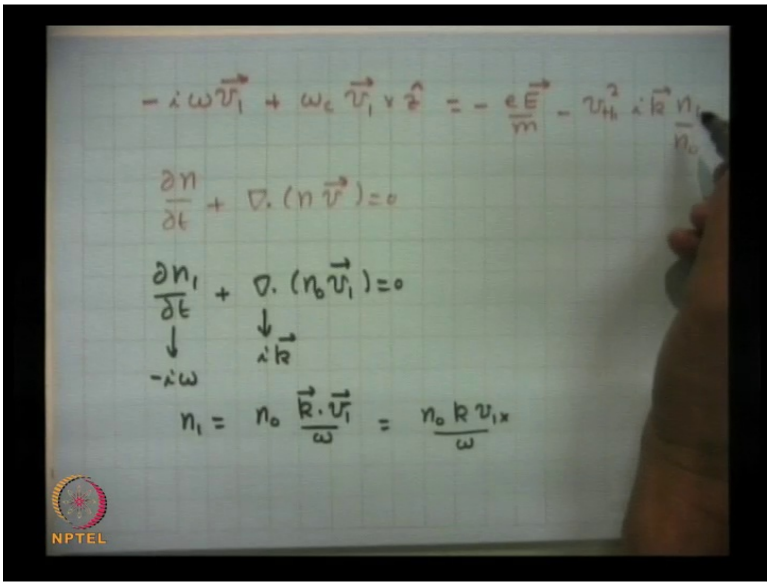

Minus i omega v 1 plus omega c v 1 cross z cap is equal to minus e E upon m minus v thermally square into i k n 1 upon n 0 and the equation of continuity, which is delta n by delta t plus divergence of n v equal to 0. On linearization takes the form, delta n 1 by delta t plus divergence of n 0 v 1 equal to 0 and on replacing this by i k, this by minus i omega, this gives me n 1 is equal to n 0 k dot v 1 upon omega and since, k is in the x direction this simply means n 0 k v 1 x upon omega. Now, it is easy for me to substitute this n 1 in terms of v 1 x. So, that this equation d couples from n equation.

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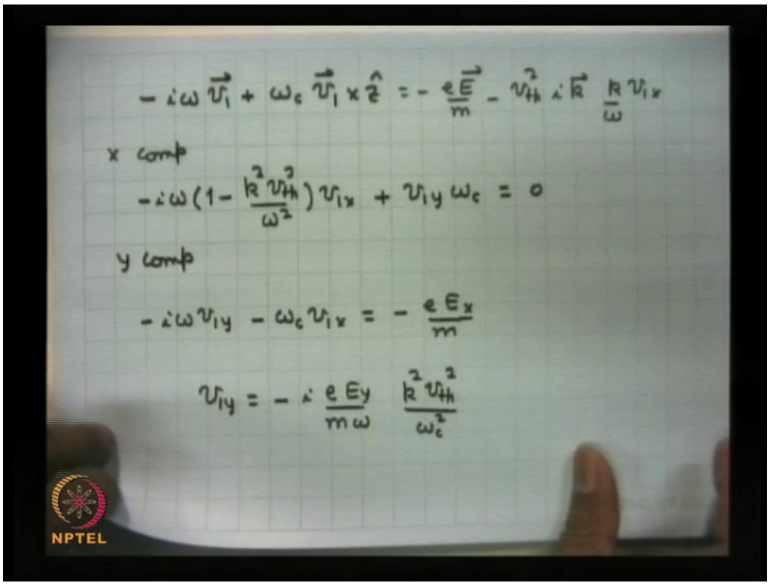

So, let me just substitute it in there and my equation takes the following form. Minus i omega v 1 plus omega c v 1 cross z cap is equal to minus e E upon m and minus v thermally square i k is there n 1 upon n 0 from here would be k v 1 x upon omega. I write down the x components of this equation.

Please remember, e E does not have a x component, I am taking only v y, but this k has x component, so, this will contribute and this gives me minus i omega into v 1 x, bring this also on the left hand side, this will becomes 1 minus k square v thermally square upon omega square multiplied by v 1 x, this gives me plus v 1 y omega c is equal to this term, which has no x components. So, this is 0 and y component gives me, this term will give me, please remember this has no y component, k is in the x direction. So, this does not exist, but this will be finite.

So, for the y component this equation gives me minus i omega v 1 y minus omega c v 1 x is equal to minus e E x upon m. This equation is interesting and what you get is usually for magneto sonic wave k v thermal is much bigger than omega. So, I will ignore this one as compare to this term. You can solve these equations. My goal is to find out the current in the y direction the direction electric field because I want to sustain that. So, I want to solve these equations to, I will eliminate v 1 x and obtain v 1 y. The result is if I obtain v y is equal to is simply this expression, v 1 y turns out to be equal to minus i e E y upon m omega into k square v thermally square upon omega c square. This is the value of electron oscillatory velocity in the y direction.

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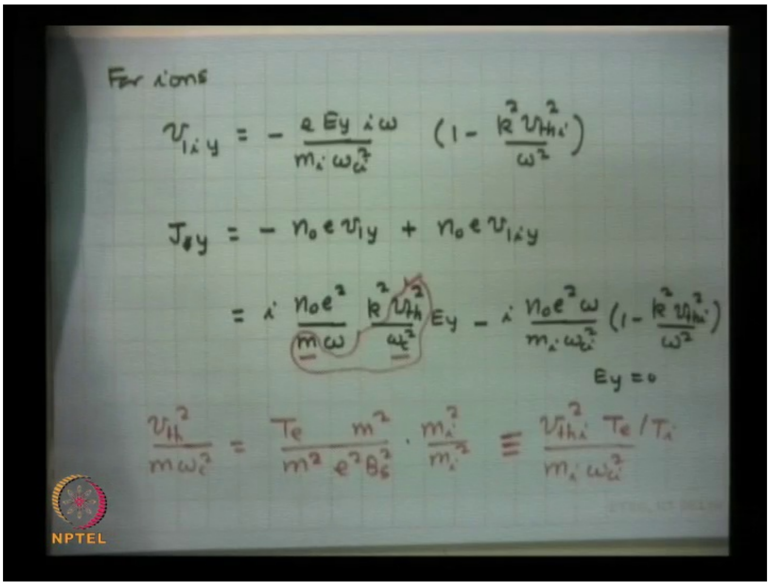

For the ions, I will get the similar procedure, for ions will give me v 1 i, have a y component is equal to minus e E y upon m I omega c i square into i omega here into 1 minus k square v thermal i square upon omega square.

For electrons, I could assume k v thermal much bigger than omega, for ions this is a small. So, I have retain this ratio, this term. Now, I may write down the current density perturbed current density J 1 or simply J, let me call J y. This is equal to minus n 0 e v 1 y, due to the electrons and due to ions will be n 0 e v 1 i y, this is the ion contribution. When I substitute these values of v 1 y and v 1 i y in these expressions, I get perturbed current density turns out to be I times n 0 e square upon m omega k square v thermally square upon omega c square E y, this the electron contribution, the ion term turns out to be minus i n 0 e square upon m i omega c i square and omega up here into 1 minus k square v thermal of ion square upon omega square multiplied by E y equal to 0.

The interesting part is that this m into omega c square and v thermally square, these three combination, this factor and this factor here, v thermal i square upon m i omega c i square. They are roughly same because I can write down v thermally square upon m omega c square is equal to, this is T e upon m, this is how I define thermal velocity square and 1 m is there. So, it becomes m square, this omega c square is e square B s square upon m square.

So, what I am saying is that m square cancels out. What I can do. I can multiply this equation by m i square upon m is m i square and this can be written as v thermal of ion square into T e upon T i upon m i omega c i square exactly. So, this is a important comparison of this term with a similar term here and except for this temperature ratio of electron temperature to ion temperature. These two terms are of same value and of same sign minus minus becomes plus the same sign.

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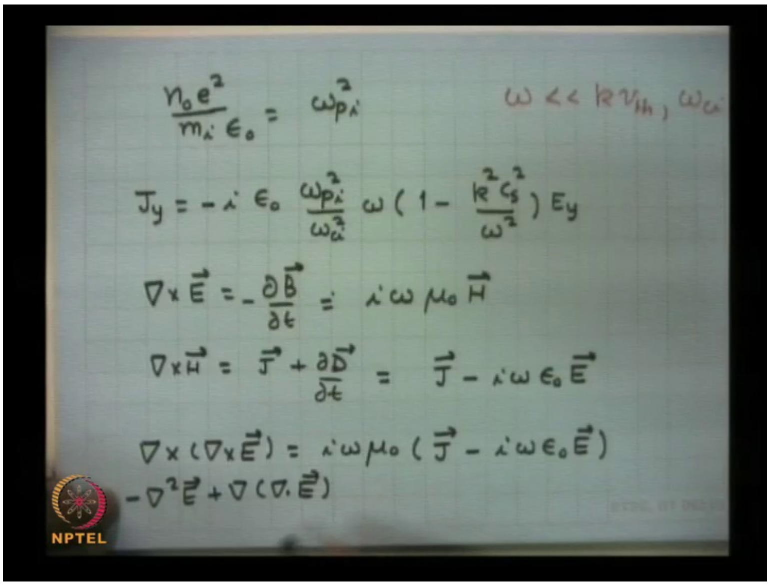

When you do carryout this in here and try to express n 0 e square upon m i as multiply epsilon 0 is equal to omega p i square, then the current density turns out to be simply J y is turns out to be equal to minus i epsilon 0 into omega p i square upon omega c i square omega multiplied by 1 minus k square c s square upon omega square E y. So, this is a very simple expression for current density, thermal effects have been carefully included in here. In the limit that I have taken omega much less than k v thermal and omega also to be less than omega c i, this is what we have assumed. In that limit the perturbed current density so much and let us simplify, rather substitute this in the Maxwell's equations. What do we get. Let me begin with the, let me first deduce the Maxwell the wave equation. You begin with the third Maxwell equation, which is curl of e is equal to delta B by delta t with the negative sign and replace delta delta t by minus i omega. So, this becomes is equal to i omega mu 0 H and now, you take the fourth Maxwell equation, which is curl of h.

Curl of h is equal to J plus delta D by delta t, but D is epsilon 0 e in plasmas and delta delta t is minus i omega. So, it gives me J minus i omega epsilon 0 into E. Take curl of this equation and use this equation, you will get curl of curl of E is equal to i omega mu 0 curl of H which is this expression. So, J minus i omega epsilon 0 e. When I put the and this curl curl of E, I can simplify as minus Del square of E plus gradient divergence of E.

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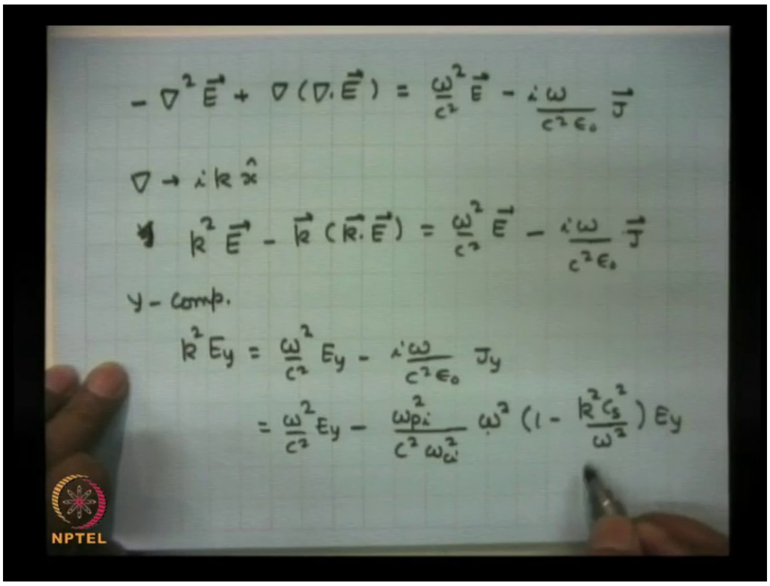

So, this equation can be rewritten as minus del square of E plus gradient divergence of E, this is equal to, if you just put the value of J, then this becomes equal to omega square by c square E due to the last term and this becomes minus, well let me write this as i omega by c square epsilon 0 J. Let me first write this in this form then, value of J I will substitute little later, del square I can write down as minus k square. So, Del I replace by i k x cap in this equation and this gives me, for the y component. First, let me write this. This gives me k square E vector minus k and k dot E is equal to omega square by c square E minus i omega by c square epsilon 0 J. Write down the y component of this equation, this will give me, please remember k does not have a y component. So, this term is not having any y component, this will give me k square E y is equal to omega square by c square E y minus this term i omega by c square epsilon 0 J y. Now, use the value of J y and you will get omega square by c square E y the first term and this gives me minus epsilon 0 cancels out, omega p i square upon c square omega c i square, then you will get omega square into 1 minus k square c square by omega square into E y. E y will cancel out from both sides, we get the dispersant relation.

Please note, when I take this omega square in inside this, will cancel with this omega square. So, what you can do. These two are the omega square dependent terms. The left hand side in the last term are independent omega term, they can combined can be combined together. So, what do you get?

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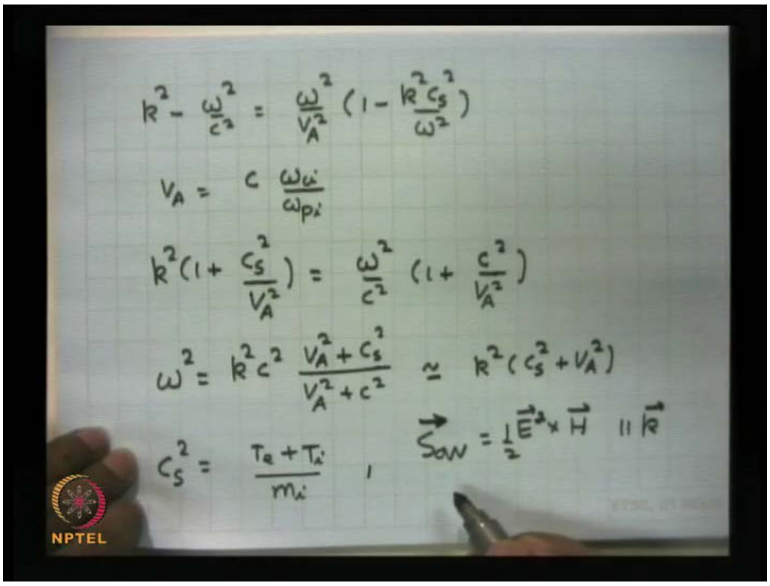

When you combine these, you get k square minus omega square by c square is equal to omega square upon v Alfven square, I will just define this v Alfven, into 1 minus k square c s square by omega square. v Alfven is simply is equal to c omega c i upon omega p i.

So, bring the omega independent from left hand side, you will get k square into 1 plus sound velocity square upon v Alfven velocity square is equal to omega square by c square into 1 plus c square upon v Alfven square and this gives the dispersant relation. Omega square is equal to k square c square multiplied by v Alfven square plus c s square upon Alfven velocity square plus c square. Usually in plasmas, v Alfven is much much less than c, the velocity of light in free space. You can ignore this v a square, then this c square will cancel with this c square and this is approximately equal to k square into c s square plus v Alfven square, this is a interesting wave.

It propagates across the magnetic field, it carries energy and momentum across the magnetic field and the phase velocity of this wave is omega by k of the order of Alfven velocity. Usually, Alfven velocity is bigger than c s. So, this travels across the magnetic field. I would consider these to be a some some some modification of Alfven wave due to thermal effects. c c s contains the effect of, actually I forget to define c s. c s is defined as T e plus T I upon m i. This is the thermal velocity, if under root of this quantity is the actually sound is speed in a plasma.

So, this is a some sort of a modification of Alfven velocity, Alfven wave compression, Alfven wave by the presence of thermal effects. Finite temperature of electrons and ions and phase velocity. Well, if you want to calculate the pointing vector or average energy flow, always you can calculate by using E cross H. Since for this mode, electric field is perpendicular to k. H is always finite and this is always finite half, this is star if I want to calculate this, put a star here. So, the average power flow; obviously, this will be parallel to k perpendicular to magnetic field and these waves can carry momentum and energy across the magnetic field and hence, they are very important waves.

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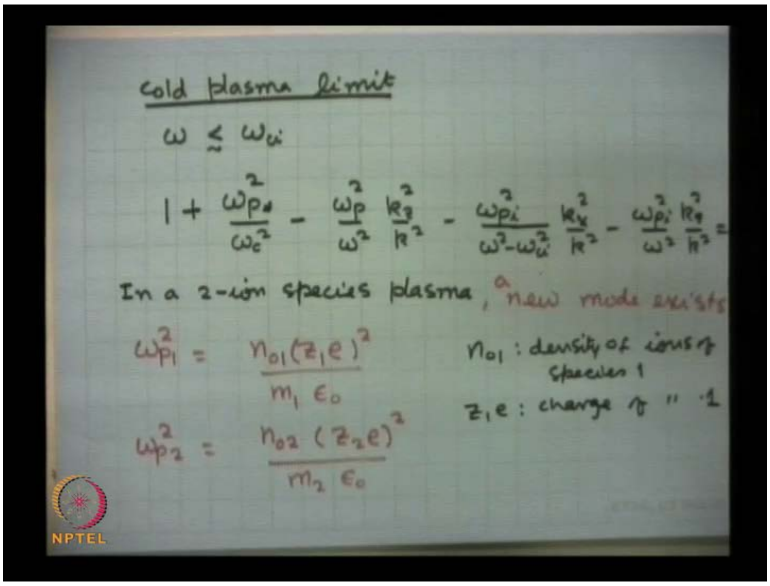

I forgot to mention something about the low frequency electrostatic waves in two species plasma. In my last lecture, I was talking to you about cold plasma limit for the electrostatic waves and in the cold plasma limit, the dispersant relation for the low frequency mode turns out to be, let me just give the, that is a important thing.

I considered the case, when omega is less than or comparable to omega c i and I found a mode actually the dispersant relation was simply equal to 1 plus omega p i omega p square upon omega c square minus omega p square by omega square k z square upon k square minus omega p i square upon omega square minus omega c i square k x square upon k square minus omega p i square k z square upon omega square k square, this is equal to 0. This is the dispersant relation that we derived in the cold plasma limit for waves of frequency much less than electron cyclotron frequency. This is the only term; otherwise, this would be omega square minus omega or rather omega c square minus omega square, this was the relation we had obtained last time.

Here, if you had a plasma like deuterium tritium, in which there are two ion species, then you will get two terms due to ions. The thing is, if you are talking about low frequencies, at low frequencies if k z is very small, you can ignore the electron term here and usually in plasmas omega p is less than omega c. So, this is like this this is of the order unity. So, these terms are small and these terms must balance each other.

In the vicinity of omega equal to omega c i, these terms must balance each other and if k z is tiny, this term is negligible. One term cannot balance itself, but if there are two species, two ion species then, one ion term could be positive, another ion term could be negative and there is possibility of a new mode of wave propagation and. So, in a two species plasma two ion species plasma a new mode exists and let me generalize this dispersant relation to a two species plasma. What I should do. For each species, let me define a plasma frequency. Suppose, the density of one ion species is n 0 1 and charge of this species is z 1 e, this is charge. So, this is density into charge square upon m 1 is the mass of one ion species into epsilon 0.

And for second ion species, I can define. Similarly, ion plasma frequency of second species is equal to density of second species ion species, charge of the second ion species upon mass of the second ion species in the epsilon 0. Let me just define what are these quantities. n 0 one, the density of ions of ion species one and z 1 e is the charge of ion species one. Similar quantities are n 0 2 and z 2 for the second species.

And how about this cyclotron frequency, I have to multiply define cyclotron frequency of a species one as, z 1 e B s upon m 1 and similarly, omega c 2 for second species, I will define. Once you define, each of these two terms will split into two terms corresponding two ion species and I am looking for a possibility.

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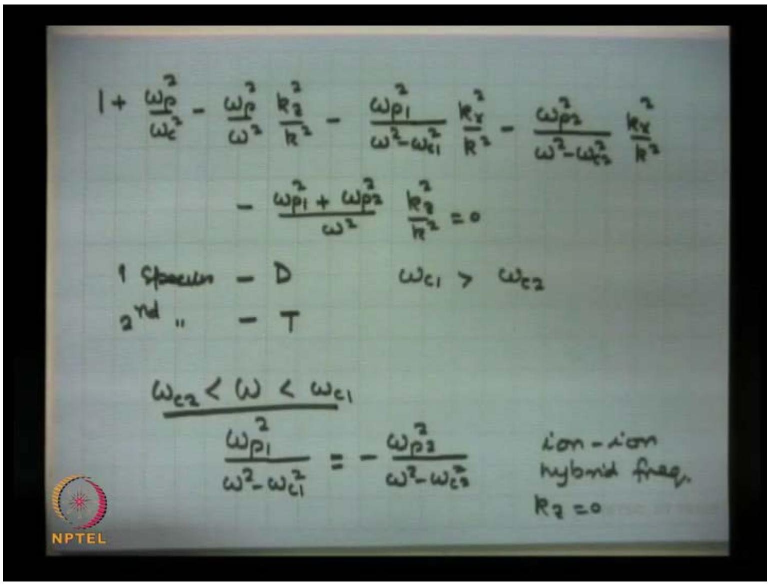

Let me just write these. This modify dispersant relation would be 1 plus omega p square by omega c square minus omega p square upon omega square k z square by k square minus omega p 1 square upon omega square minus omega c 1 square k x square by k square minus omega p 2 square upon omega square minus omega c 2 square k x square by k square minus omega p 1 square plus omega p 2 square upon omega square k z square by k square is equal to 0.

Suppose, my first species is deuterium and second species is, say one species is deuterium and second species of ions is tritium then; obviously, omega c 1 is bigger than omega c 2 because mass of deuterium is less than mass of the tritium. What will happen. Out of these two terms if I choose my omega in between these two frequencies omega c 2 and omega c one, in that case if omega is less than omega c 1 this term will be positive and if omega is bigger than omega c 2 this term is negative and these two terms can balance each other. So, when k z is nearly 0 I can ignore this term this term and this is too small because they are of the order of unity. While these terms could be much bigger.

So, what I am saying is, that there is a possibility of an electrostatic wave, where these two terms can balance each other and when they will balance each other, this factor is common. The frequency you can obtain by saying that omega p 1 square upon omega square minus omega c 1 square is equal to omega p 2 square upon omega square minus omega c 2 square with a negative sign and that gives the frequency of the mode and that is a very interesting thing. This is a very interesting mode and it turns out to have a frequency, something like this, if I simplify this you can obtain the frequency, which is in between this range and this frequency is called upper, this is called ion ion hybrid frequency ion ion hybrid frequency. Obviously, this mode does not have a frequency exactly equal to ion hybrid frequency alone, it can have a frequency because this is dispersant relation, you can obtain omega as a function of k x or k z.

So, for a k z equal to 0, this is the hybrid frequency. Frequency when k z is equal to 0 exactly, but when k z is finite the frequency could differ from ion ion hybrid frequency and I think this is a very important mode which is used for heating ions in tokomak using ion cyclotron waves. So, you choose a frequency of the your signal or R f wave equal to the ion hybrid frequency, let me simplify this expression and give the final expression.

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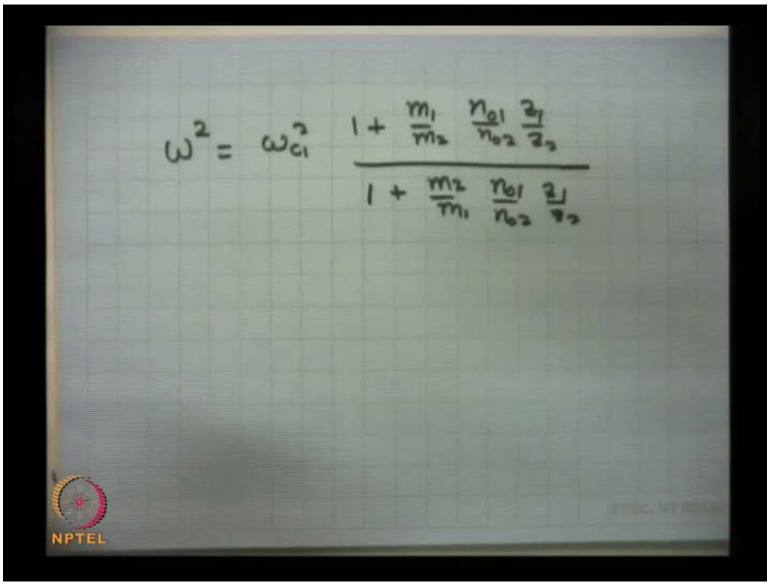

It turns out to be omega square is equal to omega c 1 square into 1 plus m 1 upon m 2 m 0 1 upon m 0 2 z 1 upon z 2 divided by 1 plus m 2 upon m 1 n 0 1 upon n 0 2 z 1 upon z two, the difference in these two is m 1 upon m 2 here and m 2 upon m 2 here. This is the ion ion hybrid frequency and this mode is very interesting mode it has been found to be very useful for heating tokomak.

I think we have talked substantially in detail. The electrostatic waves in the fluid approximation. I think probably now we have to move to kinetic description or kinetic theory to discuss the effects of finite larmor radius and resonant wave particle interactions like the phenomenon like landau damping. Probably, we shall discuss that in our future lectures thank you very much .