Plasma Physics
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Lecture No. # 32
Electrostatic Waves in Magnetized Plasma

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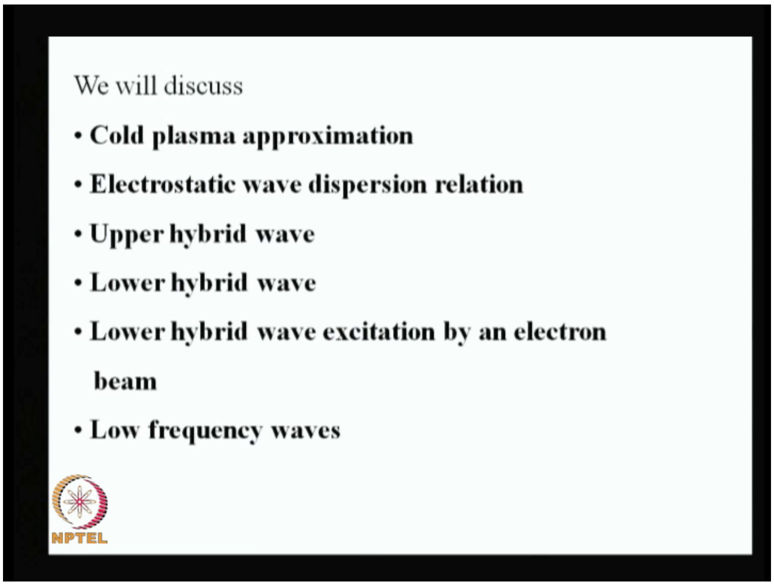

Today, we shall talk about electrostatic waves in magnetized plasma. We will discuss the cold plasma approximation. As today, I will be primarily concerning wave propagation in plasmas, where thermal effects are an important. We will shall discuss rather drive a dispersion relation for electrostatic waves then discuss a private wave, lower hybrid wave, the excitation of a lower hybrid wave by an electron beam, and then we shall talk about low frequency waves in a magnetized plasma.

The references for today's talk are three books one by T H Stix, the other one by Krall and Trivelpiece, and the third one by F F Chen.

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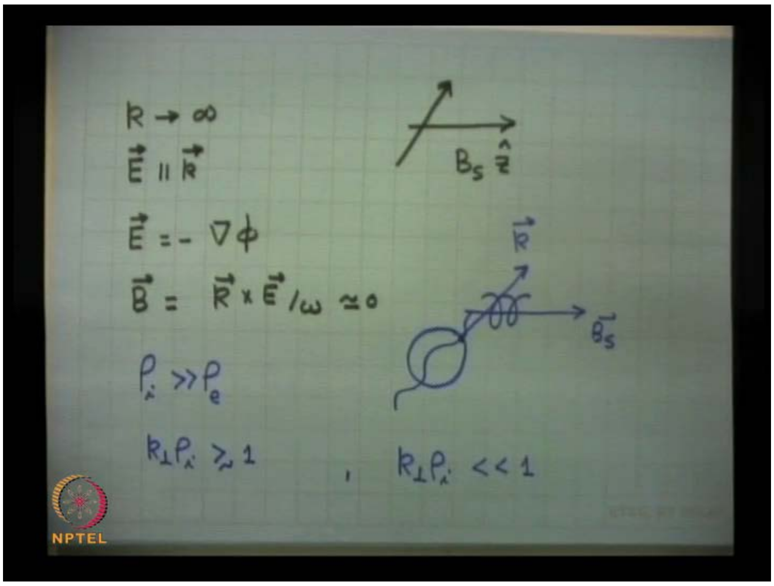

We have already learned about electromagnetic waves, and we found that, if there is a plasma in which there is a magnetic field, say that is in the z direction. And if there are waves travelling at some angle to magnetic field, then there are some resonance is in cutoffs. Resonance is a region, where k of the wave goes to infinity; that is a reason where the electromagnetic wave can get mode convert into electrostatic waves, because electrostatic waves are those waves, for which the electric field of the wave is nearly parallel to the k vector of the wave.

And such waves can be expressed rather in such electric fields can be expressed as gradient of a scalar or potential. So, phi is called the electrostatic potential and E is called the electric field of than an electrostatic wave. Since, E is primarily parallel to k, the magnetic field of this wave which is equal to from third Maxwell equation k cross E upon omega is 0. So, these waves have no magnetic field component, they are primarily electrostatic waves.

And they are very similar to waves in unmagnetized plasmas, the laminar wave or the sound wave. However, the presence of magnetic field introduces a very large variety of modes. And these waves are influenced by the gyration of electrons or gyro motion of the electrons around the lines of force. We know that if there is a plasma at finite temperature then the electrons gyrate about the lines of force like this. Similarly, ions also gyrate and ion larmor radius is bigger than the electro larmor radius much bigger rather. So, what happens that if you are launching a wave into the plasma and this is the static magnetic field and this is the k vector of the electrostatic wave. If k perpendicular rho i is comparable or greater than unity, which means that if the transverse wavelength of the wave is like…

If I draw a picture like this, suppose my wave is going in this way. And if the electro ions gyrate over a very large larmor radius, so that effective wave length is shorter than the larmor radius, in that case non local effects become important. That the electrons, the ion which is at some position during one wave period rotates and moves around and consequently finite larmor radius effects will be important and one cannot use the fluid theory for such waves. It is only when the k perp rho i is less than 1, one can treat all lines like a fluid. Otherwise, finite larmor radius correction effects will be very important. So, we shall discuss the finite larmor radius effects through a kinetic theory at a later stage, but today I will be restricted my discussion to plasmas, where finite larmor radius effects are unimportant.

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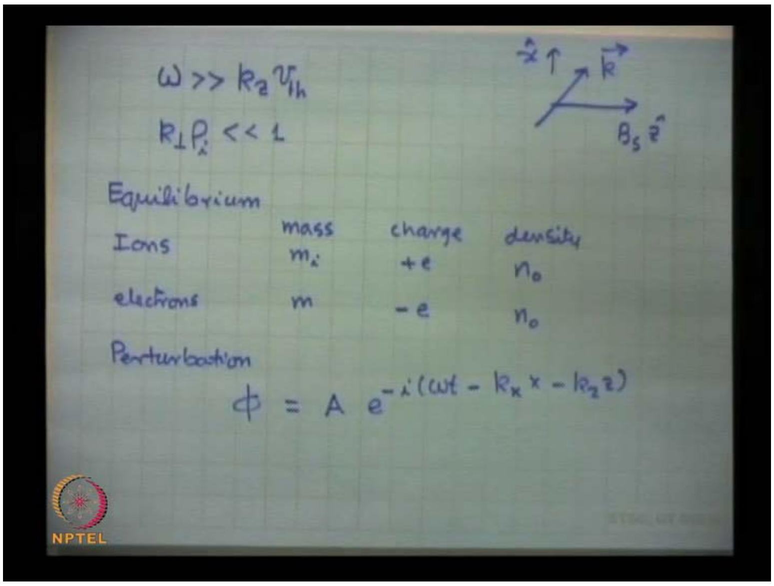

And secondly, second important consideration is, if the wave is travelling then you talk about a quantity omega. If it is bigger than k z v thermal, much bigger than k z v thermal means the phase velocity of the wave in the direction of magnetic field is much bigger than thermal velocity of electrons then we can ignore the thermal effects. This is a important consideration. Otherwise, if omega is less than or comparable to k z v thermal where k z is the z is the direction of magnetic field, so if my wave is going static magnetic field and my wave is going at some angle then, I can always resolve a component of k in the direction of magnetic field and then compare whether the wave frequency is less than k z v thermal or bigger than k z v thermal. If omega is much bigger than k z v thermal then thermal effects are unimportant.

Because electrons cannot really follow electrons move with a random velocity of the order of thermal velocity. And during one wave period, they do not travel a significant distance as compare to wavelength and hence, effects of thermal motion are unimportant. So, today we are talking about two things that omega is much bigger than k z v thermal and also k perp rho I, where rho i is the ion larmor radius and k perp refers to the component of k vector perpendicular to d c magnetic field much less than 1.So, we will consider the plasma to have no thermal effects or no pressure term in the equation of motion. Now, I consider a equilibrium for the plasma. The plasma comprises electrons and ions ions and electrons. To begin, we will consider only 1 spaces of ions, but later on we will generalize our result to two spaces, two ion spaces plasma.

Ions will be characterized by mass m I, electrons by mass m, charge of the ion will be taken as plus e, ion charge electron charge will be minus e and I will be taking the densities to be equal the plasmas quasi, was a neutral. So, the density of ions is n zero in equilibrium and electron consideration is also n zero in equilibrium. Temperature is zero for electrons as well as ions. Now, we perturb this by an electrostatic wave of potential phi, which I will take as some amplitude A exponential minus i omega t minus k x x minus k z z. I'm defining my x axis in such a way that k vector lies in the x z plane. So, this is my x axis and this is my x z plane and k vector lies in there. I would like to find out in terms of k x and k z, the value of omega and we will find that there are many roots of many values of omega possible for 1 pair of k x k z and that gives rise to a number of modes in the system .

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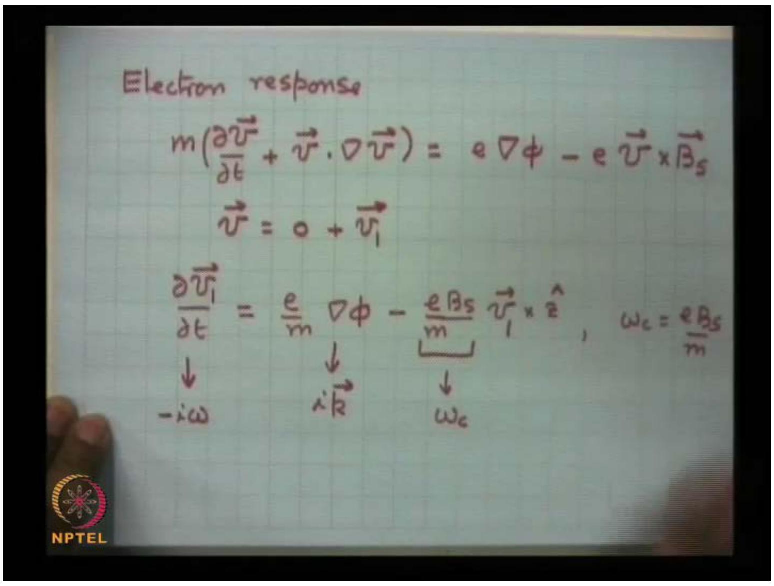

Now, let me look for the response of electrons to such a potential. The equation of motion for electrons can be written as follows, m delta v by delta t plus v dot del v is equal to e grade phi minus e v cross B s the static magnetic field. This is the equation of motion for the electrons. I will write v is equal to the equilibrium velocity, drift velocity which is zero plus some perturbation v 1 and linearize this equation, means I will ignore the products of v 1 with v v 1 and then this equation takes the following form. On dividing this equation by m this takes the form delta v 1 by delta t is equal to e upon m del phi grad phi minus e upon m e B s upon m rather and v cross z cap.

This quantity e B s by m i will call as omega c, electron cyclotron frequency. Since, phi varies in exponential wave manner with respect to t and x y z so this del operator I will replace by i k vector and because v v is the response to phi. So, in the quasi steady state the response should have same t x and z dependence as the source with a source term, then delta delta t I will simply replace by minus i omega and this term by omega c.

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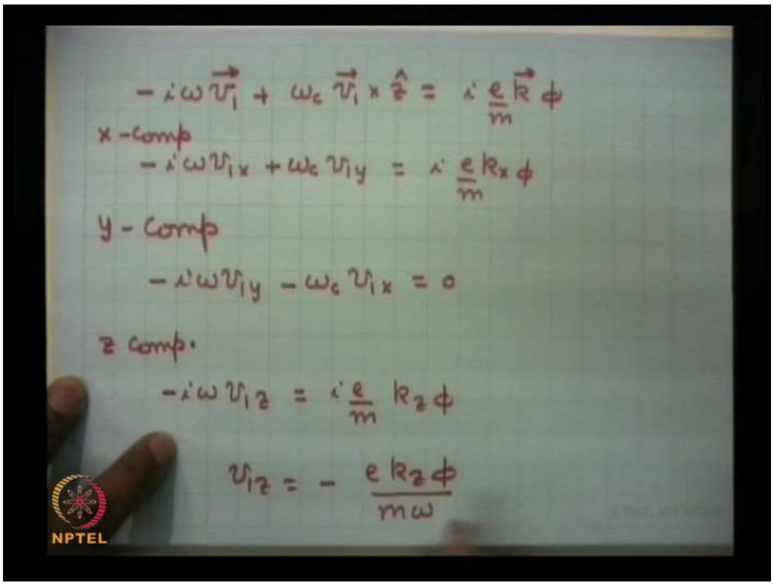

Then, this equation takes the following form, minus I omega v 1 vector plus omega c v 1 cross z cap is equal to i e upon m k vector into phi. This becomes a simple equation and one can write down its x y and z components, and those will be x component will be minus I omega v 1 x plus omega c v 1 y is equal to i e upon m k x into phi. This my x component, y component would be minus I omega v 1 y minus omega c v 1 x is equal to zero, because k does not have a y component. And z component, which is the component along the magnetic field d c magnetic field. This term will not contribute. So, you will get minus i omega v 1 z is equal to i e upon m k z phi this equation gives you straight away v 1 z is equal to minus e k z phi upon m omega, has no influence of magnetic field because the motion is in this z direction.

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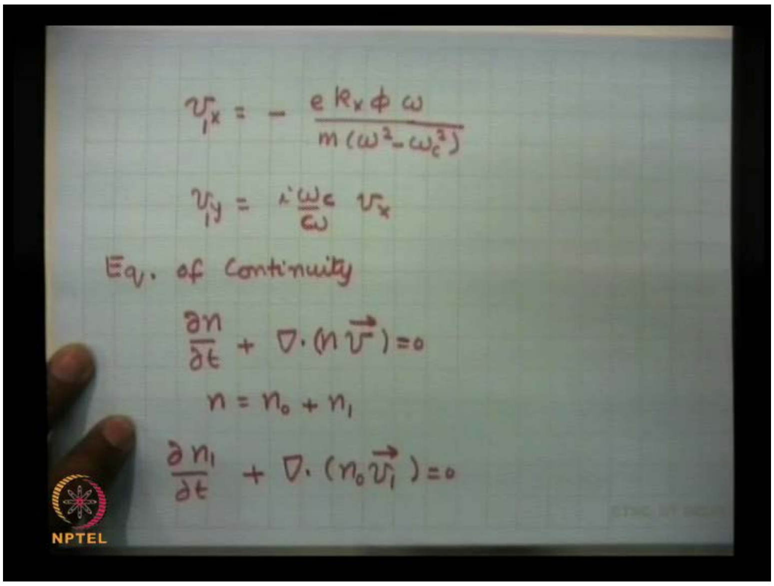

The x and y components of equations simplify to give you v x, which we require equal to minus e k x phi omega upon m omega square minus omega c square. One can see some sort of a cyclotron resonance in this response v x, and v y you can obtain straight away by this. From this value by dividing multiplying by i omega c upon omega into v x, this omega c upon omega into v x. Now, if we go over to the equation of continuity, because in order to obtain the dispersion relation for electro static waves, one requires to solve the Poisson equation, where you require the density perturbation.

So, for that you solve the equation of continuity. And the equation continuity is delta n by delta t plus divergence of n v is equal to 0. We expand n around the equilibrium value n 0 plus n 1. I forgot to write the subscripts one here, they are v 1 x and v 1 y. So, when I substitute this in this equation and linearize, means ignore the product of n 1 with v 1. This equation takes the form delta n 1 by delta t plus divergence of n 0 v 1 is equal to 0, replace delta delta t by minus i omega del by i k.

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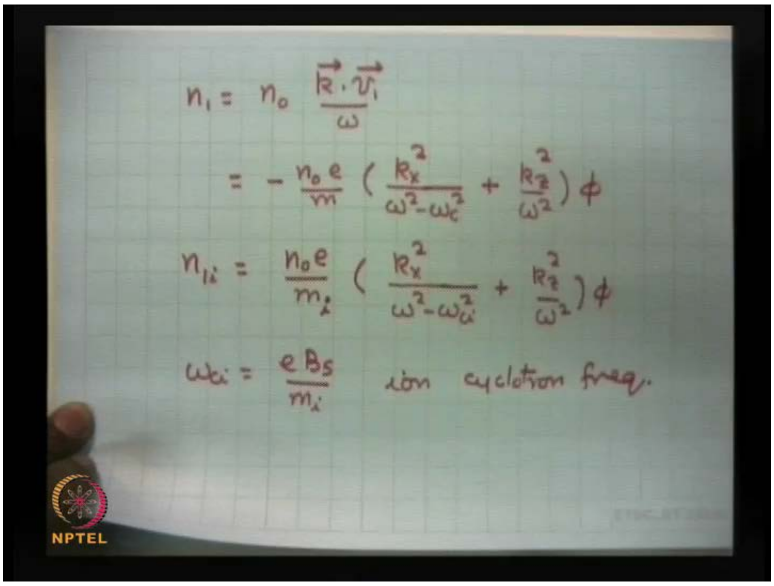

And this equation gives, n 1 is equal to n 0 k dot v 1 upon omega. On using the values of v 1 x and v 1 z, this equation gives you n 0 term as a minus n 0 e upon m outside, inside you will get k x square upon omega square minus omega c square, and the term due to electron motion along the field lines is k z square by omega square into phi. So, this is the electron response. Similarly, the ion response one can obtain n 1 i, the density perturbation of ions due to the wave by replacing the electron parameters by ion parameters.

For instance electron charges minus e replaced by plus e, electron mass is m replaced by m I, omega c should be replaced by minus omega c i, the ion cyclotron frequency. This becomes n zero e upon m k x square upon omega square minus omega c i square plus k z square upon omega square phi, where omega c i is equal to e B s upon m i is called the ion cyclotron frequency. These two density perturbations, this is sorry m i here. These two density perturbations will be comparable, when omega is substantially small as compare to omega c.

So, that this term this because the otherwise, if omega is comparable to omega c or higher, then ion contribution is negligible only at lower frequencies when this term is suppressed because of omega c factor here, and this term is also less important than ion contribution could be significant. So, it is only for low frequency waves that the ion contribution is important.

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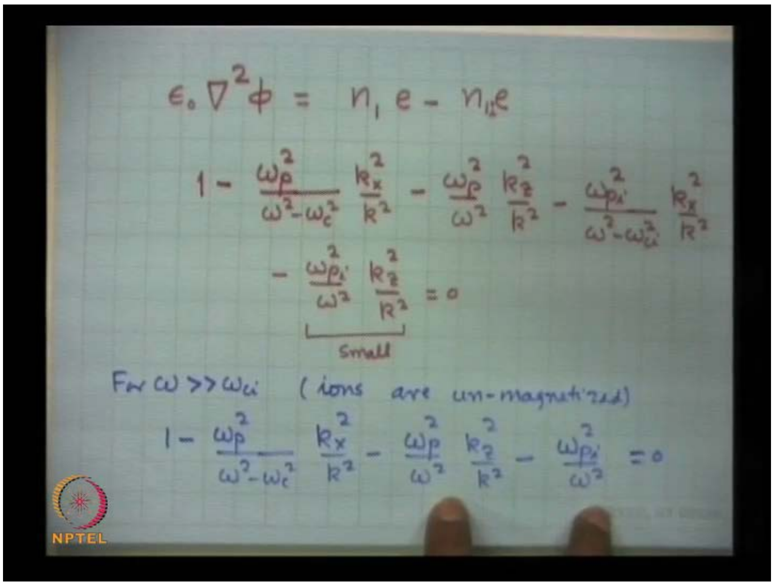

Anyway, let me substitute these in the Poisson equation which is del square phi into epsilon zero is equal to n 1 e minus n 1 i into e, this is ion density perturbation, this is electron density perturbation. This equation we have written a number of times is deduced from the first Maxwell equation. Divergence of d is equal to rho, d I write as epsilon zero into e, e is minus grade phi and rho is the difference of densities of electrons and ions multiplied by the charges.

Now, replace this by k minus k square, put the values of n 1 and n 1 i divided by epsilon zero, this equation takes the following form. One, let me write down this (( )) form of this one minus omega p square over omega square minus omega c square k x square upon k square minus omega p square over omega square k z square by k square minus, this is the electron terms one arises because of this term and ion contribution turns out to be omega p i square upon omega square minus omega c i square k x square by k square minus omega p i square over omega square k z square by k square equal to zero.

Well, you may check this term as k z square by k square and one upon omega square and this is also as the electron term as same term, but there is a large term omega p square multiplying here and this small term omega p i square multiplying here. So, this term you can ignore as compare to this term, tiny small. These are the three terms that must equate to balance with one and that will be the dispersion relation. In order to obtain the dispersion relation, well we go to two different regions. One when omega is much bigger than omega c i and one.

The other one, when omega is comparable to omega c i or less. So, I will consider two cases, one is called the high frequency or medium frequency inter medium frequency waves, when omega is much bigger than omega c i. So, I will consider that case, so for omega substantially bigger than omega c i, I can ignore the ion omega c i here and then, I say that the ion response has become un magnetized ions are un magnetized ions are un magnetized. There is no influence of magnetic field on their response. In that case, well even if you are not ignore this term, because this becomes omega p i square by omega square, same factor. They can combine together to simply give you k x square by k square can, you know just becomes unity.

So in that case, this equation becomes one minus omega p square upon omega square minus omega c square k x square by k square minus omega p square upon omega square k z square upon k square minus omega p i square by omega square, where I have ignored omega c i square and added these two terms, simply this becomes zero. This equation is by quadratic in omega because these two terms can be combined as one upon omega square and this equation can be rewritten in the following form.

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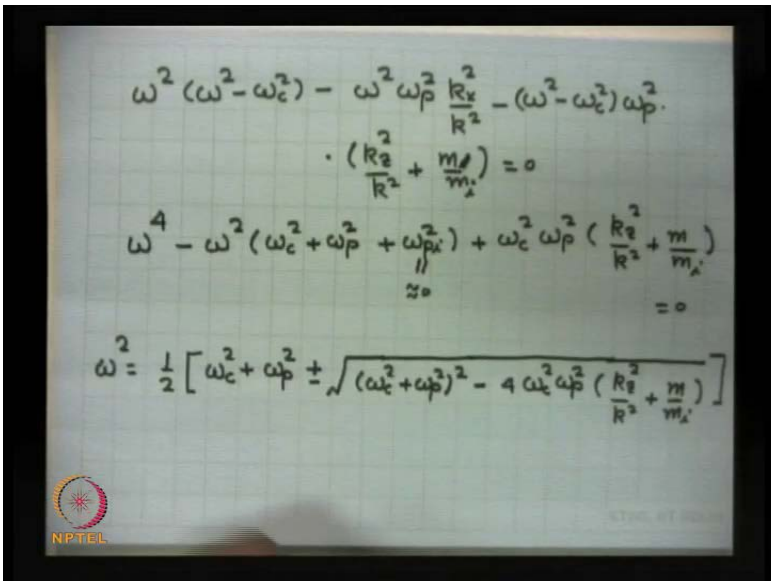

This can be written as omega square into omega square minus omega c square minus omega square omega p square k x square upon k square, then minus omega square minus omega c square into omega p square, then within the bracket there is a term multiplying here as the term k z square upon k square plus m i upon m is equal to zero. We open these brackets, this becomes a quadratic equation omega square. So, omega four here minus omega square and omega c square plus omega p square plus omega p i square minus sorry plus omega c square omega p square into k z square upon k square plus m upon m i m i.

Actually, I made a mistake here. This is not m i upon m, this is m upon m i here. So, this equation this equal to zero is the quadratic equation omega square. It has two roots and I can ignore this omega p i square as compare to omega p square. So, I ignore this, take this to be nearly zero and this equation takes the following form, omega square is equal to half omega c square plus omega p square plus minus under the root omega c square plus omega p square whole square minus four omega c square omega p square into k z square upon k square plus m upon m i. There are two roots.

The root with the upper sign is called upper hybrid wave and the root with lower sign is called lower hybrid wave. Upper hybrid wave certainly has a higher frequency, which is certainly bigger than this quantity. So, when omega is this is large quantity, then what you see here that, this m upon m i term is too small as compare to the bigger terms and you can ignore it. On the other hand, when you consider negative sign because the large part of this term will cancel the large part of this term. So, this ion this is primarily this is ion contribution all others are electron contributions. So, ion contribution can become significant. So, for the upper hybrid wave when I consider positive sign, I not this term is too small as compare to remaining term. So, I can ignore this term and my root becomes.

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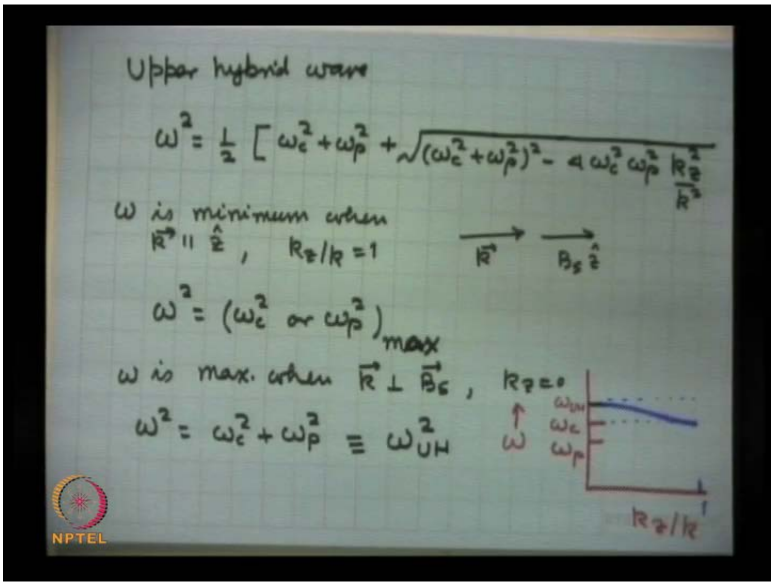

I let me write down this as upper hybrid wave. A root turns out to be omega square is equal to half omega c square plus omega p square plus under root of omega c square plus omega p square whole square minus four omega c square omega p square k z square by k square. This is the quantity that depends on the angle, this is Cos theta. If theta is the angle k makes with the z axis, this this is simply the angle.

You may note, if this angle is zero, then k z by k is unity and this is the has the minimum frequency minimum value. So, omega is minimum when k is parallel to static magnetic field, this is my k, this is my static magnetic field. In that case, k z by k is unity and this becomes a perfect square omega c square minus omega p square whole square minus sign and this gives me omega square is equal to omega c square or omega p square whichever is minimum. So, minimum of the two sorry maximum of the two maximum of the two whichever is larger the value have to consider.

And when omega is when you increase this angle, if k is at an higher and higher angle then, when angle becomes 90 degrees this quantity will have the largest value. So, omega is maximum when k is perpendicular to B s means k z is zero. If I take this to be zero, then these two factors are equal and you get the omega is equal to omega square is equal to omega c square plus omega p square and this is defined as upper hybrid frequency. So, this wave has a frequency starting from omega c or omega p, whichever is larger to omega equal to omega U H. As the angle theta between the k vector and magnetic field varies.

So, if I can plot here a graph, suppose my plasma has omega p somewhere here and omega c somewhere here, then upper hybrid frequency which is the under root of this quantity will be somewhere here. So, if I plot omega here and if I plot k z upon k here, then at k z is equal to zero, the frequency is omega U H and your wave will be localized in this here. it is starts from here, this is unity here. This goes from here to here. So, this upper hybrid wave has a very limited range of frequency between omega c and omega upper hybrid.

If omega p were bigger than omega c, then the frequency domain for this wave is simply bounded between these two values. The wave exists only in a very small frequency range. This is the upper root called upper hybrid wave. Now, let me go and this wave has been found to be a very useful, whenever you launch an electromagnetic wave, then in the vicinity of omega equal to omega U H. The electromagnetic wave gets mode converted into a electrostatic upper hybrid wave and that can give rise to a strong absorption or a heating of particles by the wave. So, this is a very important branch which is relevant to laser produce plasmas, where strong self generated magnetic fields exist and the laser can get mode converted into electrostatic upper hybrid wave.

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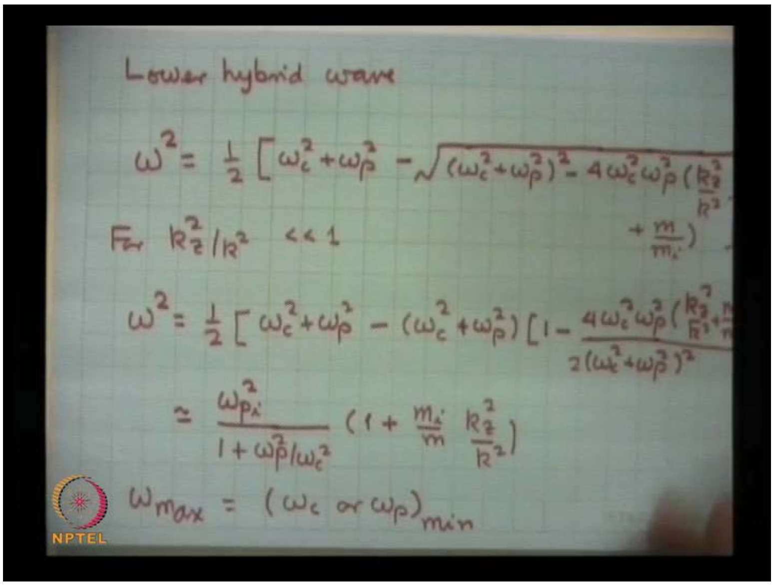

Now, let me talk about the lower hybrid wave. This is a wave with a negative sign in the disperse in that equation. So, omega square is equal to half omega c square plus omega p square minus under root of omega c square plus omega p square minus four omega c square omega p square multiplied by a factor here k z square by k square plus m upon m i. This is here. Now, please remember. If k z by k is comparable to unity or half or one third or one fourth or one tenth even. I can ignore m upon m i because electron to ion mass ratio is a very small quantity of the order ten to minus three, few times ten to minus four or something of that order ten to minus four, ten to minus three.

So, as long as k z by k is 0.1 or bigger means as long as the electrons k z by k, I am talking about point 1 means sign, this makes an angle of something like eighty degrees or smaller. So, when k makes an angle of eighty degree with the d c magnetic field or a smaller angle, I can ignore the ion motion and this equation can be simplified can be rather evaluate it. On the other hand, if k z becomes k z by k becomes comparable to m by m i or less.

So, when the wave is travelling nearly at 90 degrees, then this is the term to be retained. However, what I can do in that case because when this is a small this is also a small this is becomes a small term. So, at large angle propagation, so for k z square by k square significantly less than one, this entire term is a small as compare to this one and I can remove this take this out and carry out binomial expansion. When you do binomial expansion this equation takes omega square becomes equal to half, omega c square plus omega p square minus, take this factor common. Sorry, this was a whole square. So, this is omega c square plus omega p square. When this gets out becomes only single power.

And then you are left with one minus four omega c square omega p square k z square upon k square plus m upon m i divided by omega c square plus omega p square whole square and when you take this square it out you can divide by two and remove their square root sign, because this is smaller than one. This can be simplified. This term will cancel with this one and this becomes nearly equal to, this two will cancel with this four and this two.

So, essentially this becomes plus, minus minus becomes plus. You get this is equal to omega p i square. If I take m upon m i outside, it becomes omega p i square, one power will cancel, this becomes equal to one plus omega p square by omega c square and because I have taken this m upon m i outside, this becomes one plus m i upon m k z square by k square. This is known as the lower hybrid wave, a very important wave, very useful for current drive and heating in tokomak. It is a very prominent wave in beam plasma systems. It is also very important wave that has been found to be a very effective in quenching the instabilities in q machine and so on.

So, lower hybrid wave this is a electrostatic mode, where ion motion is un magnetized as I mentioned before because of frequency is bigger than ion cyclotron frequency, but ion plays ions play important role when k z upon k is less than one much less than one. If k z by k were not much less than one, suppose they are comparable to unity in one third, one tenth, one fourth. In that case, ion motion can be dropped and you can find here that the if I take k z tending to one, say k z by k tending to unity then, this gives you the omega frequency. The largest value of omega is from this equation if I ignore the ion motion and take k z upon k equal to unity. In that case, this turns out to be equal to omega c or omega p whichever is smaller. So, this is a wave whose frequency, the minimum value will be when k z equal to zero then, this is the minimum frequency. This frequency under root is known as the lower hybrid frequency.

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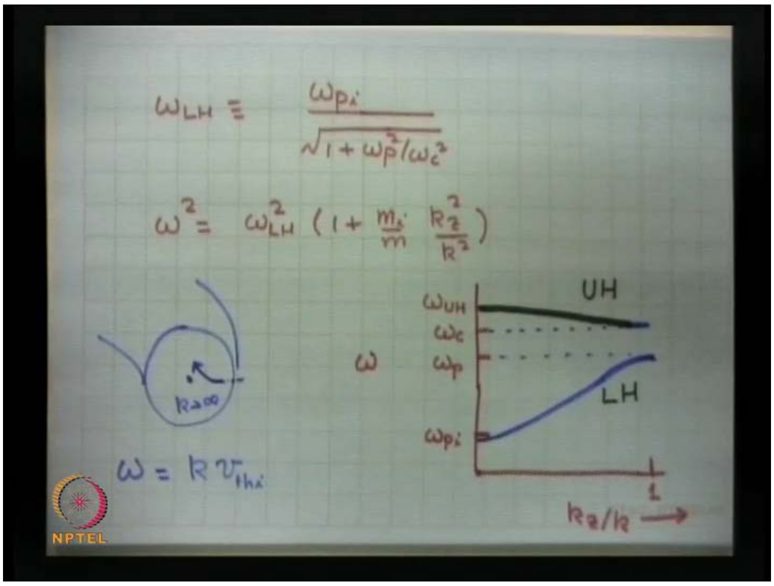

So, let me define a lower hybrid frequency omega L H as omega p i upon under root one plus omega p square upon omega c square and in terms of this omega for this wave turns out to be dispersion relation. For the lower hybrid wave at large angle of k with respect to magnetic field turns out to be equal to omega L H multiplied by one plus m i upon m k z square by k square. And if I plot the dispersion relation, plot omega here and let me plot k z by k here. Let me put this one here and zero here. Lower hybrid frequencies close to omega p i because in most plasmas omega p is less than omega c. So, this is close to ion cyclotron ion plasma frequency. So, the ion plasma frequency somewhere here and ion cyclotron frequency is much less than this somewhere here.

And omega, suppose I choose a plasma with omega p smaller than omega c and upper hybrid frequency is somewhere here. So, these are the various frequencies characteristic of a magnetized plasma. If I plot omega versus omega versus k z by k for this wave, you may note here that it is starts the minimum value starts from omega lower hybrid, which is slightly less than omega p i because it is vector. So, somewhere here, it will starts from somewhere here. This is the lower hybrid frequency and the maximum value it will go to omega p. So, this will be like this. This is the kind of structure, this wave will this is the dispersion curve for this wave. This is the frequency maximum frequency it can take.

It is in the lower frequency side that this ion motion is important and at higher frequencies the wave is primarily governed by the motion of electrons and there is no wave between omega c and omega p. Upper hybrid wave starts from upper hybrid frequency and goes down to the omega c from here, this is the branch. This is the upper hybrid branch; this is called the lower hybrid branch of the wave.

So, linear wave splits into two branches. One is upper hybrid branch, the other one is a lower hybrid branch. This goes up to this point here. This wave is very very important the interesting part of this wave is that a wave launched into a tokomak from outside can get mode converted into this mode and as the wave approaches this lower hybrid resonance because as wave goes it from here. Very interesting thing you can notice here. From this is itself you can notice. Suppose I have a tokomak plasma. This is the cross section of a tokomak. You are launching a wave from here, the wave is opposing towards the center. As the wave comes in here, the density is maximum in the center of the plasma and zero at the edge.

So, as density increases omega p i goes up, omega p i increases omega L H also goes up. In order to keep omega constant, k z must decrease or k must increase, but k z cannot decrease because the plasma has density variation perpendicular to magnetic field and k z is a component of k vector in the direction of magnetic field. So, k is that does not change and as a result when omega approaches omega L H this term must vanish, means k should go to infinity. That is the advantage. So, when the wave is coming from here, as the wave approaches the center this tokomak center then, k goes to infinity or k perp goes to infinity, k x goes to infinity and the wave becomes very slow. Now, as I mentioned to you that Cerenkov resonance for ions will occur when omega equal to k v thermal of ions. Though, in our formalism we have ignored thermal motions.

But as the wave approaches the lower hybrid region lower hybrid layer in the plasma, when omega p i or omega L H becomes close to omega, k becomes very large and k becomes very large means the wave becomes very slow. So, many ions they can move with the same velocity as the phase velocity of the wave and consequently they can resonantly take energy and momentum from the wave and can get be can be can be heated. So, that is a very interesting scheme. Now, let me mention the relevance of this mode to other devices and one important device is called a beam plasma system

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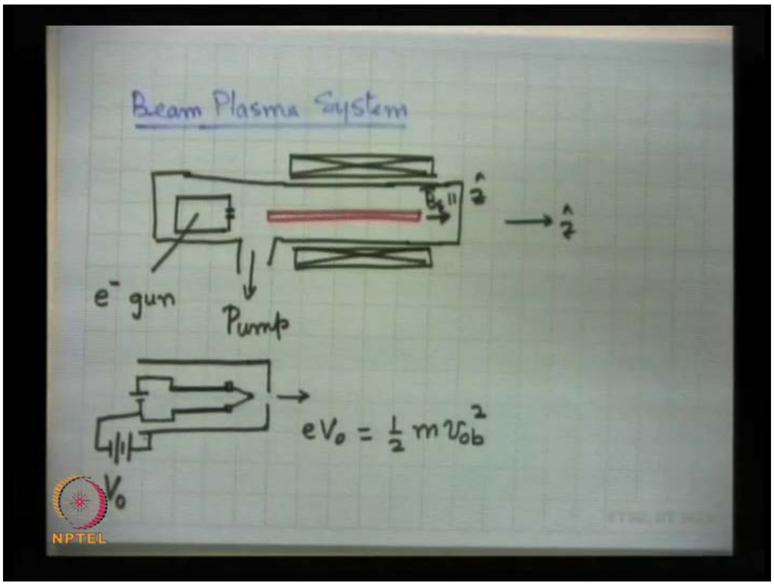

Let me just mention a something about the beam plasma system. Consider a beam plasma system, it is a very important and simple system to a study basic physics of parametric instabilities and other non-linear phenomena. This comprises primarily and electron gun here, that launches an electron beam and then, there is a chamber here which is placed in a magnetic field. So, these are the simply schematic representation of magnetic coils that produces excel magnetic field and what you get here is, well this is vacuum system, this is to vacuum pump and this is closed here.

What you observe here that well, this is the electron gun. Let me simply mention what kind of gun we have. You have two rods to which you connect a tungsten wire and then, you apply a potential difference of nine volt or so, to pass a current through the filament. So, these are two conducting rods, but this is a thin tungsten wire. When it gets heated by passing a current, it emits electrons and then, this is placed inside an enclosure with a hole here. So, actually there is a hole here there is a hole here.

And the gap between this and this is only a few millimeter. We apply a potential difference between these two. Negative and this is positive of about a few hundred volts or may be kilo volt or few kilo volt, then you will get a electron beam. If V zero is the potential difference between the cathode and the anode, then the electron beam energy will be e V zero. So, electron beam will have a velocity given by this relation half m v zero b square. So, as a simple system, it produces an electron beam. Typical values of beam current are in milliamperes and when the beam passes through a low pressure gas, normally people put argon or nitrogen, you can also put helium, then it will produce a plasma on the axis and some something like this you get a plasma here.

Obviously, the radius of the plasma is less much less than the radius of the vessel and much people have observed the generation of waves inside their plasma. Lower hybrid waves in this plasma and those waves gives rise to harmonic generation, they also give rise to modules real instability and so on. So, let us understand how this electron beam drives lower hybrid waves unstable. Right now, we have studied that a plasma in the presence of a magnetic field and in this case the magnetic field is in this direction, B s which is parallel to z axis, this is my axis of a system is z axis. So, in this plasma you are going to have an electron beam and that drives the instability. I can deduce a simple expression for a growth rate, if I ignore the non local effects and taking into a consideration, those effects is also not difficult. One can see this in papers or may be little older papers. On this, let me just simply mention what to happens here.

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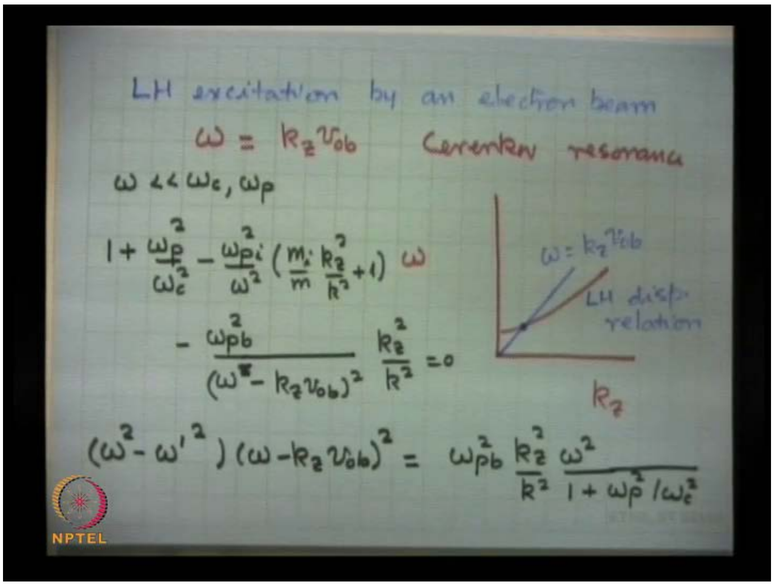

So, I will simply give you a very some scheme of lower hybrid wave excitation by an electron beam. We have already studied two steam instability and what the two steam instability says that, if you have a wave of frequency omega and if you launch an electron beam, then whenever omega is equal to k z v zero beam, if beam is going in the z direction, then this is called Cerenkov resonance and one can have growth of the wave by the at the expense of beam energy. Now let us see, for the lower hybrid wave we have seen that if I plot omega versus k z here, the graph goes something like this and if I draw this equation, then the graph will go something like this, depending on the value of v zero b.

So, this is the point of intersection where the natural frequency of an Eigen mode will be equal to the Cerenkov resonance frequency and this is my equation omega equal to k z v zero b and this is my lower hybrid wave dispersion relation, which I have just written. So, this is L H dispersion relation. Now, if you look the look at the dispersion relation for the lower hybrid wave, which is one minus omega p square one plus rather omega p square by omega c square. If I am talking about omega much less than omega c and omega p, in that limit the dispersion relation that I have just written for the lower hybrid wave becomes like this, minus omega p square rather omega p i square by omega square into m i upon m k z square by k square plus one, this is the electron contribution. The first term here also electron contribution, the other one is the ion contribution to the dispersion relation.

But if there is a beam term here, then there is a term like omega p of the beam square upon omega square multiplied by sorry this is not omega, omega minus k z v zero b whole square and k z square upon k square, this is equal to zero. So, this is the additional term that you get and this equation can be recast in a interesting form because if I take this factor common and take one upon omega square out, then this equation takes this form. Let me just write down, this becomes omega square minus, I will call this is omega prime square and this becomes omega minus k z v zero b whole square is equal to some factor on the right hand side which is omega p b square k z square upon k square then one omega square I have taken common, so this should goes here, omega square goes there and this factor comes in here, one plus omega p square by omega c square.

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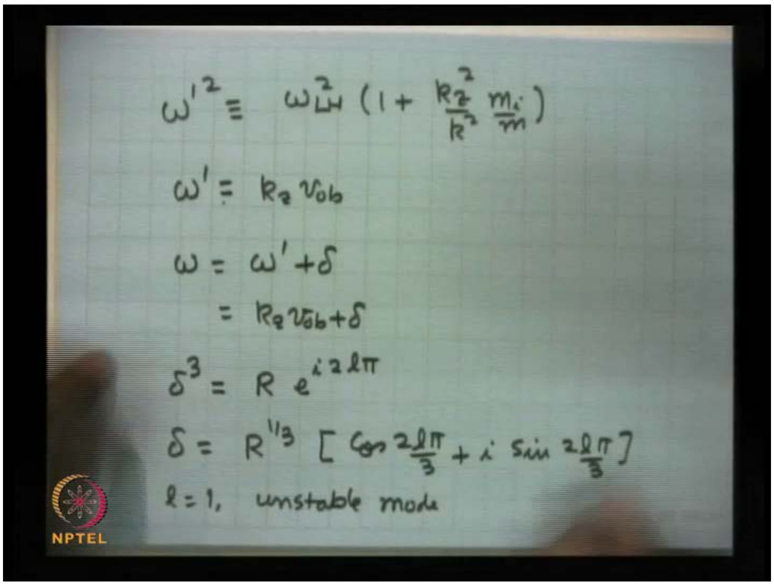

This equation and omega prime is how much, this is the omega prime square is exactly same as omega L H square into one plus k z square by k square m i upon m. This equation, if you examine has the character of the two stream instability, where omega p is replaced by omega prime, rest of the equation is same. So, by same technique you can solve this equation by taking that omega prime, suppose I choose equal to k z v zero b. so, choose a value of k z such that this condition is met, in that case this equation can be solved by expanding omega equal to omega prime plus delta and which is also equal to k z v zero b plus delta and then, you get this equation dispersion relation gets converted into a equation for delta, giving you delta cube is equal to some factor on the right hand side. You multiply this by some i times two L pi and, then gives you delta is equal to r to the power one third Cos of two L pi by three plus i sine two L pi by three and you get a instability for L equal to unity. Unstable mode with growth rate.

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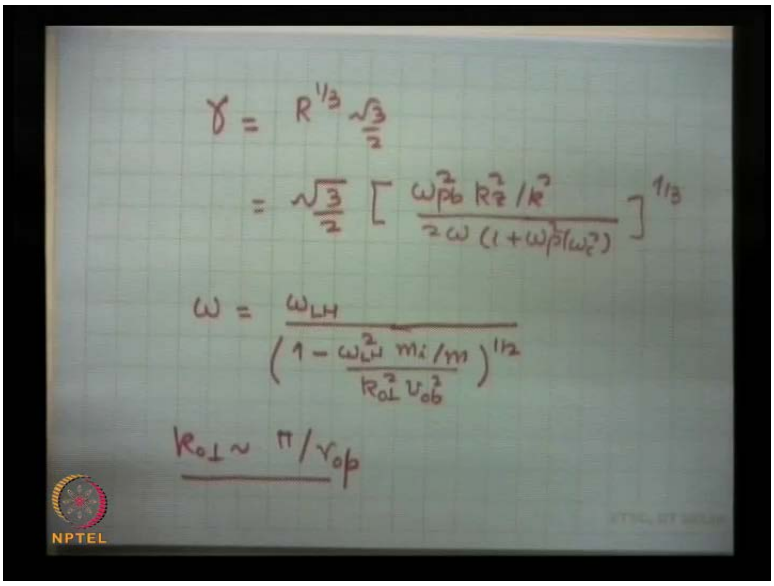

Let me just mention the growth rate, this is a very interesting thing. So, the growth rate turns out to be quite large for this mode and it turns out to be R to the power one third root three by two and if you put the value of R, it turns out to be root three by two into omega p being square k z square upon k square upon two omega into one plus omega p square by omega c square to the power one third and how about omega the frequency of the mode turns out to be equal to omega L H upon one minus omega L H square m i upon m divided by k zero perp square into v zero b square to the power half.

Now in this case, k zero perp is decided by the size of the beam and which is of the which is of size of the plasma, which is of the order of, I would say simply k zero perp is of the order of pi upon R 0 b, R z 0 plasma. So, this is a very interesting way of generating these waves and studying some non-linear phenomena. I think I like to stop at this point. Thank you.