Plasma Physics

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Module No. # 01
Lecture No. # 31
Low Frequency EM Waves Magnetized Plasma

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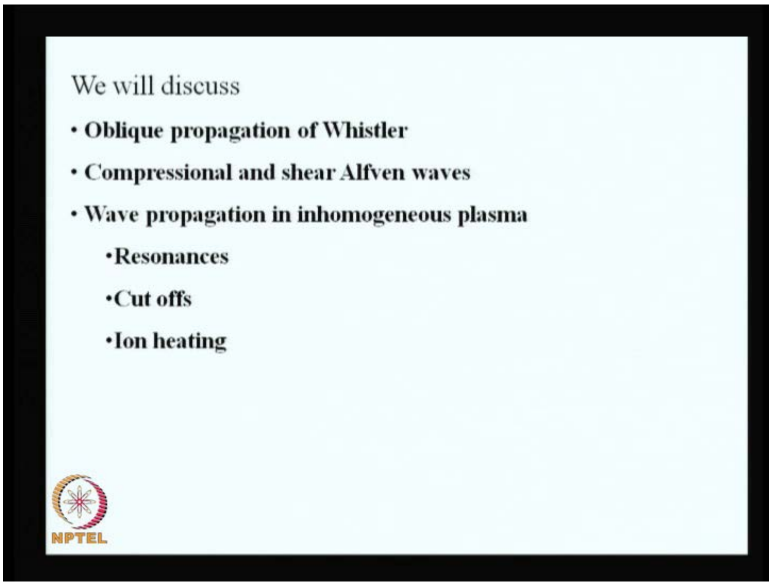

Today, we will discuss low frequency electromagnetic waves in magnetized plasma. We shall talk about whistlers, which propagate at an angle to esthetic magnetic field, then we shall discuss compression and shear Alfven waves propagating at an angle to magnetic field and we shall discuss some aspects of wave propagation in inhomogeneous plasmas. We shall refer to resonances cutoffs and ion heating. The references are prime roughly the same as for last three lectures. Let me remind you that, we are talking about a plasma, in which magnetic field exists in a particular direction say, I will call this as the direction z axis. So, this is parallel to the esthetic magnetic field and the plasma balls are too far away from the main body of the plasma. So, we are going to ignore any sort of reflections and we are essentially considering the propagation of a wave at an angle to magnetic field.

So, this is the k vector this is the direction of ambient magnetic field, and we would like to examine the nature of waves of frequencies which are significantly below electron cyclotron frequency and in these plasmas omega c often is of the order of omega p. So, omega p is something like this so, this condition is certainly satisfied.

Now, we have already talked about wave propagation in the direction of magnetic field in this frequency range and we have already introduced two important waves, one is called whistler wave, the other one is called Alfven wave. However, today we shall begin afresh because wave propagation at an angle to magnetic field has different character. First of all, the polarization no longer remains circular. Secondly, the direction of phase velocity and group velocity they become different and there are some sort of resonances that appear and hence we should relook into this issue.

Now, we have we are what we are considering, we are considering an electromagnetic wave, whose electric field in general is of this form E is equal to A some complex amplitude exponential minus i omega t minus k dot r and I will choose my k vector to lie in the x z plane. So, k vector I will consider to have a x component, which I will call as k X plus have a z component, which I will call as k z. The angle between k and magnetic field so, we are considering magnetic field to be parallel to z axis and whenever need arises, I will refer to the angle between k vector in B field as theta. So, this is the angle at which, wave number exists not necessary with that the wave travels in that direction group velocity is only referred to the direction of a propagation, which would be different than theta. So we shall refer to return to the discussion little later.

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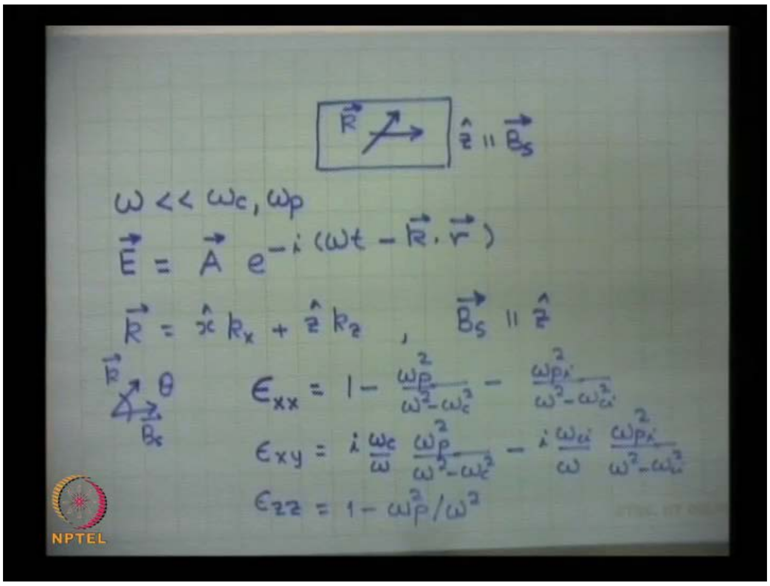

Before, I do some analysis. Let me remind you that, we have talking about a plasma, in this such a plasma and if I treat this plasma to be cold, when the thermal effects are not important. In that case, this permittivity tensors to the tensor has finite components which are expressible in terms of three quantities epsilon x x epsilon x y and epsilon z z. Epsilon x x is 1 minus omega p square upon omega square minus omega c square and similarly, there is a ion term, which is omega p i square over omega square minus omega c i square. Now, what happens.

When omega is much less than omega c, omega you can ignore here and this term becomes positive and this is usually smaller than the second term. But, we should keep it general, this second term can be negligible as well depending on, how of a small omega is. So, I will ignore this term here and similarly, epsilon x y if I write down the electron contribution, this is omega c upon omega into omega p square upon omega square minus omega c square and similarly, ion contribution which is ion cyclotron frequency omega c i upon wave frequency into ion plasma frequency square upon omega square minus omega c i square and epsilon z z is 1 minus omega p square upon omega square.

You must note that because, we are talking about omega much less than omega c. So, in this case epsilon z z is much bigger than epsilon x x or epsilon x y one thing. Secondly, if your frequency is higher than omega p i obviously, much higher than omega c i. In that case, ion contribution can be dropped and electron contribution will be, if you would look at the electron contributions to epsilon x x and epsilon x y you will find that epsilon x x is a smaller this is larger because, omega c is larger as compare to omega.

So, in a frequency domain where omega is much less than omega c but, much bigger than omega p i. You can ignore the ion motion and epsilon x x is much less than epsilon x y and obviously, both of these are much less than epsilon z z. This is scaling we have to keep in view to simplify our analysis. Now, let me go over to the basic equation, that we the deduce from the wave equation.

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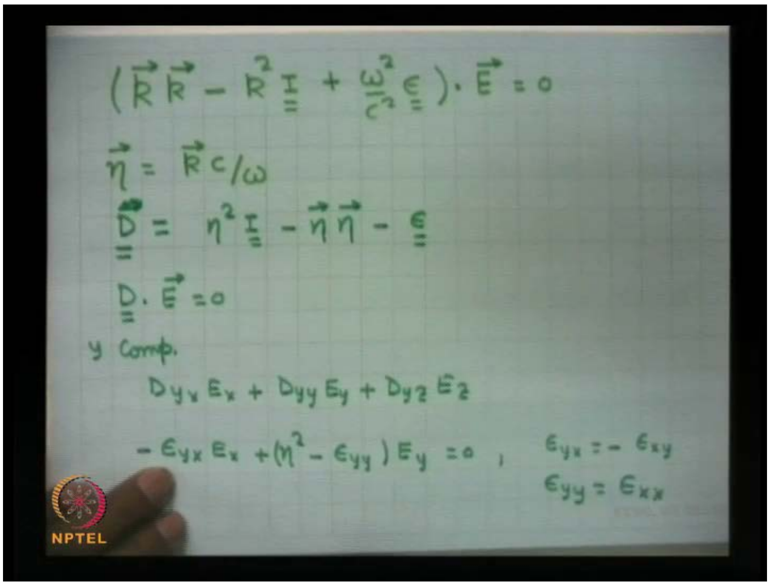

Our wave equation, let to this result that k vector k vector minus k square I vector minus plus I is the unit dyadic omega square by c square epsilon, this quantity dot e is equal to 0. This is the basic result that we deduce from the wave equation by replacing del operator by I k vector delta delta t by minus i omega and just from third and fourth Maxwell equation you deduce this equation. This is very general equation. First of all, I would like to see the character of this equation. If I write because, this is a vector equation this should be true for all components and let me introduce, if you interesting quantities first of all I will call a vector eta, which is normalized k, k vector normalized by c upon omega

So, let me call this as effective well, if it very scalar quantity, I will call this as the refractive index of the medium but, it is tense it situation anisotropic medium so, well it is just treat this as a normalized propagation vector eta. And in terms of this, well, let me call define a quantity D vector sorry D not vector D tensor, D tensor essentially this quantity when put in terms of eta, this will be essentially equal let me define this as eta square unity dyadic, I will write this term first and other terms later. So, this can be written as minus n n eta rather eta eta minus epsilon.

So, let me define this quantity after taking a negative sign common and normalizing k by c by omega like this. Then, this equation becomes D dot e equal to 0. One important thing that you may note here, if I wrote down the y component of this equation recognizing that, when I considering a geometry in which k y is 0. So, write down the y components of this tensor, in that case, this equation will give me write down y component so of this quantity, when I have to write, then I have to write essentially. D y x E x plus D y y E y plus D y z E z and if you recognize that epsilon y z is 0, eta y eta z is 0 because, eta y is 0. So, this term is basically 0, then this equation gives and if I put the values of D from here, this turns out to be primarily D y x is, it has no y x component it has no y x component so, this will give you minus epsilon y x E x and D y y if I calculates this gives me plus eta square I y y which is unity this is 0 and minus epsilon y y E y is equal to 0. So, this quickly gives me a relationship between various components of electric field amplitude or this gives the polarization of the wave.

And similarly, if I write down the, well first of all, I would like to remind you that epsilon y x is the same thing as minus epsilon x y in a magnetize plasma and epsilon y y is the same thing as epsilon x x. So, if I use these two, then I can write down E y in terms of E X in a simple way.

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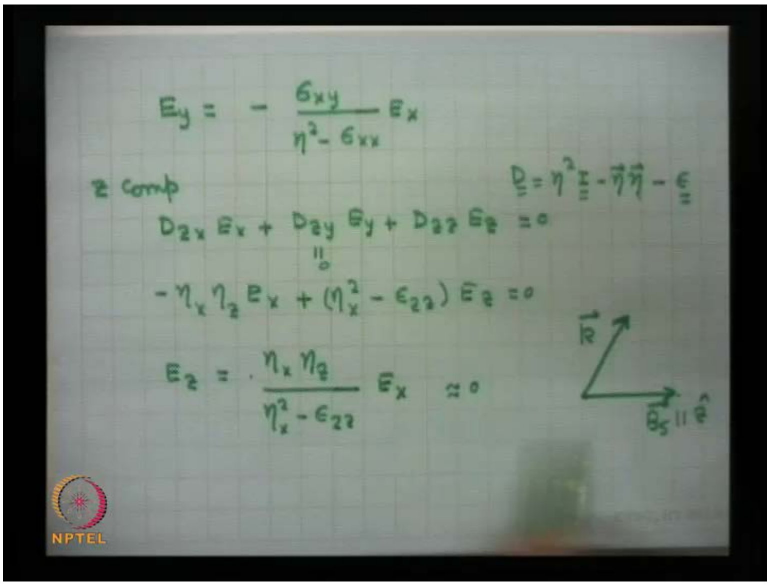

So, E y turns out to be in a magnetize plasma, for a obliquely propagating wave is equal to, this will give me minus epsilon x y upon eta square minus epsilon x x E x and how about the z component of electric field, I can express that also in terms of E x and probably I will do that little later.

Well, why do not I do it now so, z component of this of the vector equation if I had written. I will get D z x E x plus D z y E y plus D z z E z equal to 0, and if I substitute the values of D z x, D z x turns out to be from that expression for D, if you recall this D as eta square I minus eta eta vector vector minus epsilon tensor.

So, write down the z x component, this does not have a z x component this as z x. So, it gives me minus eta x eta z E x D z y it does not have a z y component because, I has only diagonal terms so, it is and this does not have a eta y so, this is also 0 and this gives me simply epsilon z y, which is also 0. So, this becomes to contribute. So, this term is 0. So, this is 0. This give me D z z, if I calculate from here, it gives me eta square minus eta z square, which gives me eta x square so, plus eta x square minus epsilon z z like this into E z is equal to 0. From here, I can write down E z component of the electric field of the wave is equal to eta x eta z upon eta x square minus epsilon z z into E x

My point is that, at low frequencies epsilon z z is very large as compare to other components of epsilon as well as we will learn it is much bigger than epsilon x eta x square also, this is a really tiny quantity. So, I can take z to be nearly 0 quite small as compare to E X at times it may become important its impact on particle dynamics may be important but, usually this is a very small quantity.

So, these waves are, if they are travelling at an angle to magnetic field, this electric field is perpendicular to the direction of magnetic field, this by B s and this is my k vector. So, the wave can have a component of electric field perpendicular to magnetic field means, it can have either a component in the x direction or a y component also but, not essentially z component and E y at times can become bigger than E x or smaller than E x depending on the frequency. So, we shall learn about such things in order to arrive at some estimate of propagation constant of these waves or phase velocity and group velocities of these waves, I need to solve the dispersant relation.

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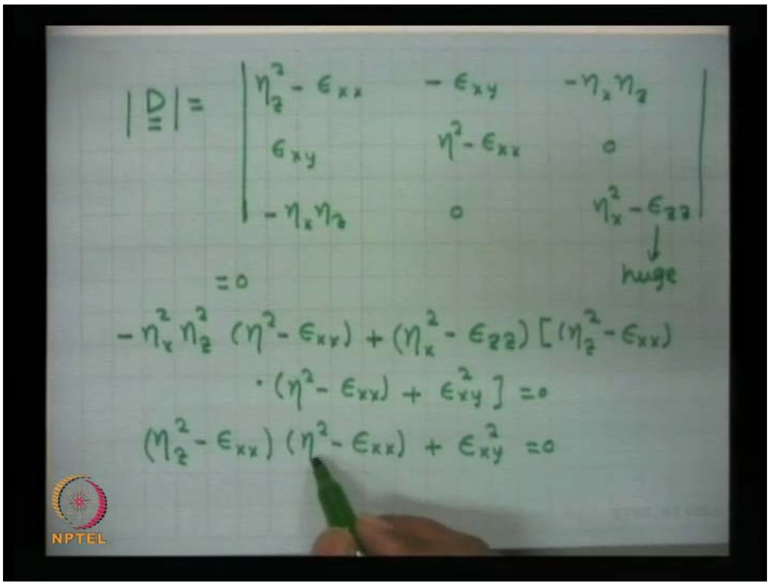

And dispersant relation is D which is the tensor, its determine must vanish and what are the components let me write down. D components, the components are, if you simplify this you will get eta z square minus epsilon x x minus epsilon x y minus eta x eta z, this turns out to be epsilon x y, this turns out to be eta square minus epsilon x x this is 0, this is minus eta x eta z, this is 0, and this is eta x square minus epsilon z z.

Now, if I expand this determinant about this is has to be 0. You can expand any determinant about row or a column. Suppose, I choose the horizontal row the bottom row then, I will get terms in which, this factor has to be multiply with the cofactor its cofactor plus this term multiplied by its cofactor but, because epsilon z z is large huge at low frequencies. So, contribution of this term in its cofactor is a very small as compare to the contribution of this term. So primarily, the determinant will be 0, when the cofactor of this term is 0. So, if I remove the this term may be to have a better appreciation of this quantity, let me do is in 2 steps. First, let me write down the determinant completely which means that, this term minus eta x eta z and cofactor cofactor will be are the 1.

So, when I cancel this column and this row then this determinant is called cofactor, which is this into this minus this term into this term. So obviously, this gives me eta x square eta z square into eta square minus eta x x. Then, I write down this term, which is eta x square minus epsilon z z multiplied by its cofactor, which is means eliminate this column and eliminate this row, then you are left with this 2 by 2 determinant, its value would be this term into this term minus product of these 2 of diagonal terms. So, this gives me, eta z square minus epsilon x x multiplied by eta square minus epsilon x x this term then minus product of these 2 so, gives me plus epsilon x y whole square equal to 0.

Now, what I am saying is, epsilon z z is very large so, if this quantity has to become 0 then, its coefficient really should be 0 because, this is too small letter. So, just put this big bracket equal to 0 and that gives me the dispersant relation. So, the dispersant relation is primarily, let me write on a separate may be write here. Eta z square minus eta x x multiplied by eta square minus epsilon x x plus epsilon x y square is equal to 0 and this is my dispersant relation. So, eta z is simply k z into c by omega eta is k into c by omega so this is the dispersant relation for a wave of low frequency lower than omega c.

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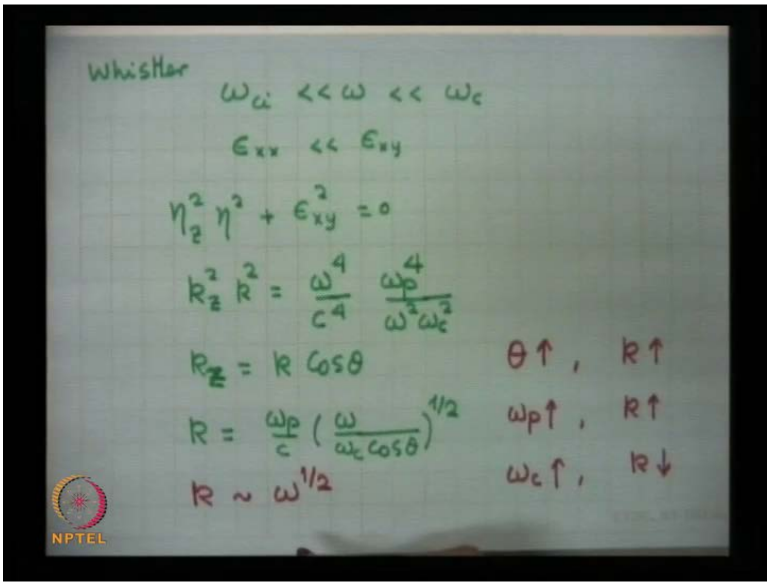

Now, I will specify my frequency to be in this range omega c i ion cyclotron frequency to be much less than omega but, omega is much less than omega c. This band is usually called the whistler wave.

So, for the whistler wave, if I consider omega c rather omega to be in much bigger than ion cyclotron frequency, what you will find that epsilon x x is too small as compare to epsilon x y and in the dispersant relation, you can ignore the terms or rather epsilon x x term terms and the equation becomes eta z square, eta square plus epsilon x y whole square is equal to 0, this is simple equation If, I put this in terms of k z and k this equation and put the value of epsilon x y, it turns out to be k z square k square is equal to omega four upon c four into omega p four upon omega square omega c square. This is the dispersant relation for a whistler wave what I can do, I can write down k z is equal to sorry k z is equal to k Cos theta, then I can write down the value of k from this expression and it turns out to for the whistler wave k is equal to omega p by c, omega take the fourth root and this becomes omega p by c but, omega square will cancel out so, when you take the and becomes omega upon omega c 1 Cos theta will come from here, Cos theta whole to the power half.

This is the wave number of the whistler wave traveling at an angle theta to magnetic field. You may note few things here, that when theta is 0 Cos theta is maximum equal to unity and k will take a minimum value. So, when theta becomes a small k becomes large.

So, as theta goes up, your k goes up also means the wave will travel with the lower phase velocity. Second thing is that, as plasma becomes denser if you increase the density of the plasma k goes up and if you increase the magnetic field of the plasma, then k will go down and finally, the dependence on omega k is not linearly proportional to omega its under root. So, k is proportional to omega to the power half.

If you have to calculate the group velocity, you should be very careful, never differentiate this expression because, group velocity is a vector quantity. Delta omega by delta k X will be x component of group velocity delta omega by delta k z will be the z component of group velocity. There is no y dependence, k y dependence here so, group velocity y component will be 0. So, in order to obtain the group velocity, what should you do, you should be careful, first you will eliminate theta in terms of k z and then differentiate, one should be very careful

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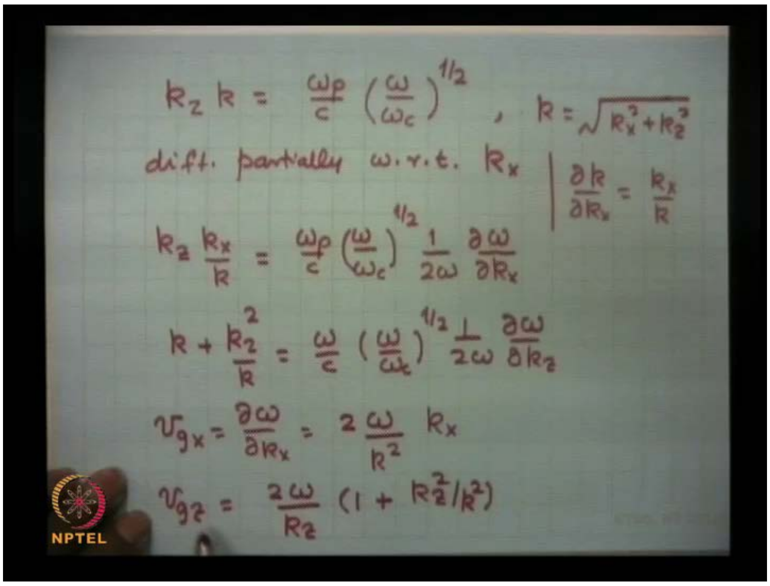

So, let me calculate the group velocity, if I calculate the group velocity, then I should use this relation, k z k is equal to omega p by c omega upon omega c to the power half. Then, I will differentiate this equation partially with respect to say k x. So, differentiate partially with respect to k x and remembering that, k is under root of k square plus k z square. So, when you differentiate delta k by delta k x, it turns out to be delta k upon delta k x turns out to be you can easily see is equal to k x by k.

So, when I differentiate this partially respect to k k x, I get treat k z as a constant and you will get k z into k x upon k for the derivative of this. This is equal to from here you will get omega p by c, differentiate this you will get omega upon omega c to the power half, I can write down like this into actually I should get 1 upon 2 omega into delta omega by delta k x .

Because, when I differentiate with respect to omega, I will get half omega to the power minus half so, omega minus half I have written like this. The advantage in this is that this is the same factor preserved as this quantity and similarly, if I differentiate partially with respect to k z, I will get 2 terms here. One I will get when I differentiate this, I will get k only and when I differentiate this, I will get k z square by k is equal to right hand side will remain the same, except the last term would be let me just write this.

Omega upon omega c to the power half 1 upon 2 omega delta omega by delta k z. So, if you simplify this v g x, which is called delta omega by delta k X, it turns out to be this quantity is the same thing as k z k. So, this becomes 2 omega upon this is 2 omega goes here and this quantity is simply k z k. So, k z will cancel out you will get k X upon k square. And v g z would be delta omega by delta k z this quantity this will give me 2 omega goes up there this becomes, k z into k. So, k z into k well I have to be little careful. So, k I divide so it becomes k z upon k k z comes in there and then you are left with 1 plus k z square by k square.

Now, this is important because, using these components you can write down the magnitude of group velocity as well as you can write down the direction of group velocity and direction is something important that I would like to write,

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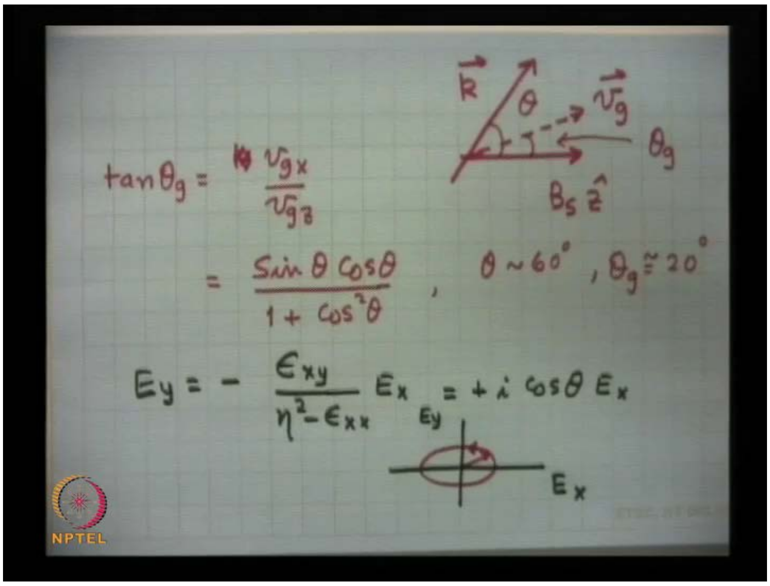

So, suppose your wave is travelling, sorry this is your esthetic magnetic field and this is the direction of your wave travel whistler wave is going here and this is the group velocity will turn out to be in some other direction. Let me call this as v g vector, the angle k makes with magnetic field is theta, let me call the angle group velocity makes with magnetic field as theta g. So, I will call this quantity as this angle as theta g the group velocity angle. Then, tan theta g which will be equal to, sorry v g x upon v g z it will turn out to be equal to let me write down the result because, we have already understood the implication, the result turns out to be equal to sin theta Cos theta upon 1 plus Cos square theta where, I have written k X is equal to k sin theta k z is equal to k Cos theta so you get this result.

Just to have an a idea, if you put say suppose theta is of the order of sixty degrees, if your whistler wave is travelling with k vector at a thirty degree angle to magnetic field then theta just turns out to be, if you calculate put the values of theta here then, take tan inverse theta just turns out to be around 20 20 degree 20 22 degrees. So, the there is tendency in the whistler to travel have a group velocity almost parallel to magnetic field, even if you take theta to be close to say eighty eighty five degrees even then, this as a tendency to esthetic close to a magnetic field, that is a very important characteristics of whistlers that, whenever if the lines of force bend, the whistlers also have a group velocity, it is bending along the lines of force so this is a very interesting characteristic of a whistler wave.

How about the polarization, I had given you the relationship between E x and E y. E y I had written like this, E y was is equal to minus epsilon x y divided by eta square minus epsilon x x into E x and if you simplify this simply just put the values of because this is a small quantity eta square is much bigger than this quantity. So, this is of this order, when you substitute this its turns out to be typically equal to minus, let me just find out this turns out to be equal to i Cos theta into E x. I think, it should be negative sign probably or plus just a second, this is all right the sign is all right this is little gap because, when theta goes to 0 E y should be i E x for right circularly polarized whistler. So, this is this is the kind of sign you get.

So, this wave is not circularly polarized, E y and E x are out of phase by phi by 2 but, the angle the magnitudes are unequals if I plot E x on the axis scale and E y on the y then this will give me a plot of this sort. E x is bigger than E y magnitude wise so, this will be like. This is the control on which the tip of the electric field will rotate in the clockwise sense like this.

So, this is a modification in the polarization with the direction of propagation. So I think, we have learned something quite different, then wave propagation in the direction of magnetic field the polarization of the wave becomes a let me call the group velocity does not have the same direction as the direction of phase velocity or rather k vector so, this is significant difference. Now, I would like to take you to even lower frequencies very low frequencies rather.

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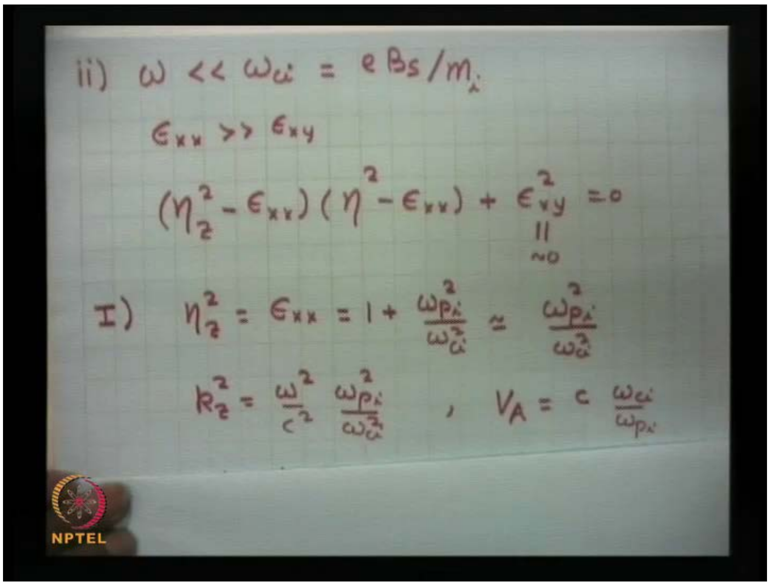

Second case, I would like to consider is, when omega is much less than ion cyclotron frequency, which is defined as e B s upon m i where, e is the charge of the ion, I am considering to be singly ionized of the (( )) should be charge number also should be there magnitude of magnetic field and mass of the ion. In this case, what happens is that epsilon x x turns out to be much bigger than epsilon x y, if you just look at the components. This is the second limit and in this limit, your dispersant relation that we had just written turns out to be like this, your dispersant relation was, let me just recollect eta z square minus epsilon x into eta square minus epsilon x x plus epsilon x y square is equal to 0, this term is nearly 0.

So, there are two roots one when this factor is 0, the other one when this factor is 0. See, if I take the first factor equal to 0, when you put this equal to 0, it means you are talking about eta z square is equal to epsilon x x and epsilon x x value, if you really examine which is equal to 1 turns out to be 1 plus omega p i square upon omega c i square and this is negligible as compare to this. So, this is typically of the order of ion plasma frequency square upon ion cyclotron frequency square, which is a huge number. And if I put k is in terms of eta z in terms of k z, I get k z is equal to omega square by c square into omega p i square upon omega c i square.

Since, we have been calling Alfven velocity is equal to velocity of light in free space into omega c i upon omega p i, this result well you can take the square root and this can be simply written as… so why if I introduce the Alfven velocity then, I can write down the dispersant relation in a simpler way.

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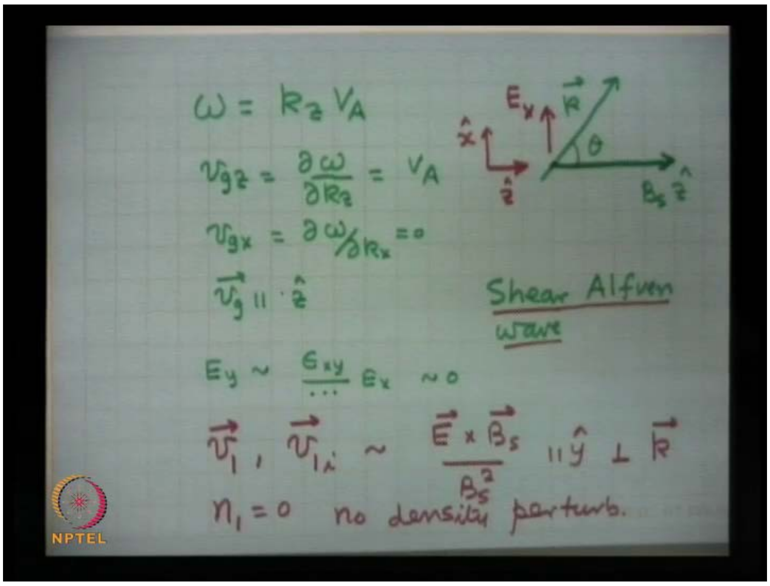

And I should do that on a different sheet. Omega is equal to k z V A. This is important, you may note that if I differentiate this equation with respect to k z, I will get delta omega by delta k z equal to Alfven velocity. But, delta omega by delta k x will be 0. So, the group velocity v g z will be equal to delta omega by delta k z, which will be equal to v a but, v g x because there is no k X dependence here. So, delta omega by delta k x if I calculate this is equal to 0. So, this is the wave, which is traveling at an which has a k vector at an angle theta to magnetic field B s esthetic magnetic field but, the group velocity is always directly along the magnetic field. This is important so, v g is parallel to z axis, another important characteristic of this mode is the electric field, this is actually this mode is called shear Alfven wave.

Why shear, we shall learn in a minute. First of all, about the electric field of this wave a structure of electric field E y, if you calculate is proportional to epsilon x y upon some quantity here into E X and this epsilon x y is very tiny so, this is nearly 0, means the electric field of this wave is primarily in the x direction, what is x direction let me specify by coordinate system. I am choosing my k to be in the x z plane so, this is my z axis and this is my x direction. So, the electric field of this wave is primarily in the x direction because, E y is 0. So, this wave has an electric field in this direction, which I call as E electric field is in the E x direction E x is finite

Now, if the electric field is in the x E x direction, you may note one thing. That electron motion, if you calculate electron drift velocity, perturbed velocity v 1 of electrons or v 1 of ions both at this low frequency, their velocity is primarily in the e cross B direction. They go as E cross B s upon B s square. So, this velocity of electrons or ions is perpendicular to E and B both so, E and B both means in the y direction this is parallel to y.

So, electrons in the presence of this Alfven waves shear Alfven wave, they oscillate primarily perpendicular to the plane of this paper. And that is perpendicular to k vector. So, this is perpendicular to k vector and hence there is no compression of electrons or ions or no charge compression rarefaction and hence there is no density perturbation. So, density perturbation is 0, this called no density perturbation. And the magnetic field of this wave is perpendicular to because, that is k cross E so, if k is in the x y x z plane E in the x direction then k cross E will be perpendicular so, magnetic field of this wave will be perpendicular.

So, if you examine the total magnetic field in the system, it is a super position of the esthetic magnetic field in the z direction plus a magnetic field, which is perpendicular to the plane of this board like this but, varies in the direction of k. So, this total magnetic field will be a sheared magnetic field, that is why this wave is called shear Alfven wave. And this has a very important application in plasma physics.

Here, this is a cold plasma theory we are developing and we find that in the cold plasma theory when they thermal effects are ignored in the equation of motion by omega is equal to k z V A. But, when you include thermal effects and if the wave has large k perpendicular large k X in that case, this dispersant is significantly modified due to finite normal radius corrections and I think, we shall discuss that some time later. But, this is a important wave.

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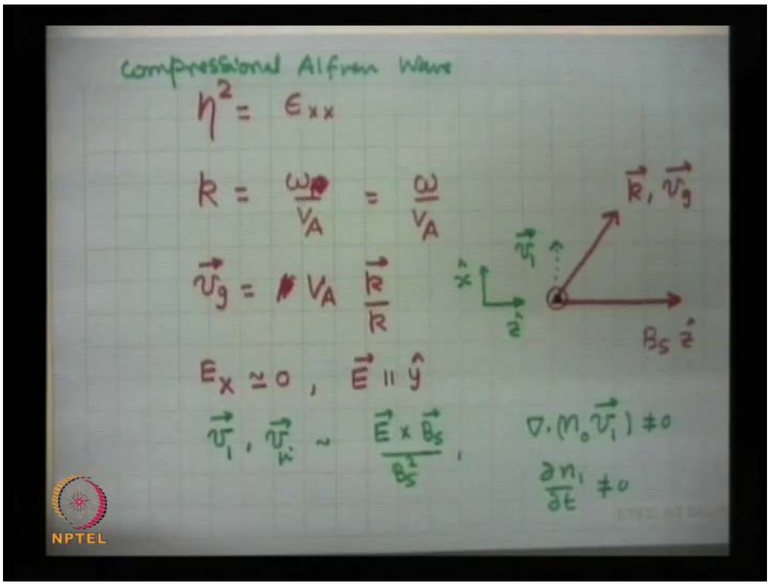

The second possibility is, when the other root of that dispersant relation is 0 and that gives me, eta square is equal to epsilon x x and when I put this in terms of k because, this eta is k c by omega p, then k turns out to be equal to omega p upon v Alfven. This is independent of theta. So, this wave has no anisotropy, the phase velocity of the wave in any direction omega by k will be equal to, sorry this is not a omega p by a this is omega by a omega upon V A Alfven velocity and this is theta independent. So, v g for this we calculate, it turns out to be group velocity turns out to be v Alfven magnitude wise v Alfven but, this is the direction of k vector.

So this wave travels, if this is the direction of esthetic magnetic field, this is the direction of k vector, then the group velocity will be parallel to group to so, this the direction of k and v g both in the same direction, how about the electric field. If you calculate the electric field for this wave, it turns out to be E y is very large. E y E x is really 0, nearly 0 for this mode. I had given you the relationship between E y and E x, this turns out to be large. So, the electric field of the wave is primarily parallel to y means, that is perpendicular to the plane of this board like in this direction, electric field is like this. Now, this as a very important implications. So, let me first represent the coordinate system. This is my z cap, this is the direction of this is my x direction and I am finding my electric field of the wave to be parallel to y axis like this.

So, if you calculate the oscillatory velocity of electrons, due to this wave and oscillatory velocity of ions due to this wave, both will be having their velocity is parallel to e cross B. And this will be upon B s square, this will be in the direction of because, this is y this is so, this will be this direction.

So, the electrons and ions will move in this direction. This is v 1, and this as certainly as a component in the direction of k vector and that gives rise to compression rarefaction of charges. So, this gives rise of compression of electrons and ions because, when the drift velocity is in the direction of k vector. So, divergence of this quantity n 0, the equilibrium electron density into velocity is no longer 0 and that gives rise to delta density perturbation non-zero so, density perturbation is finite. So, this gives rise to compression and that is why this wave is called compression Alfven wave. So, let me write down this is compression Alfven wave. This is also a very important wave, it has been observed in many experiments and it can provide lot of information.

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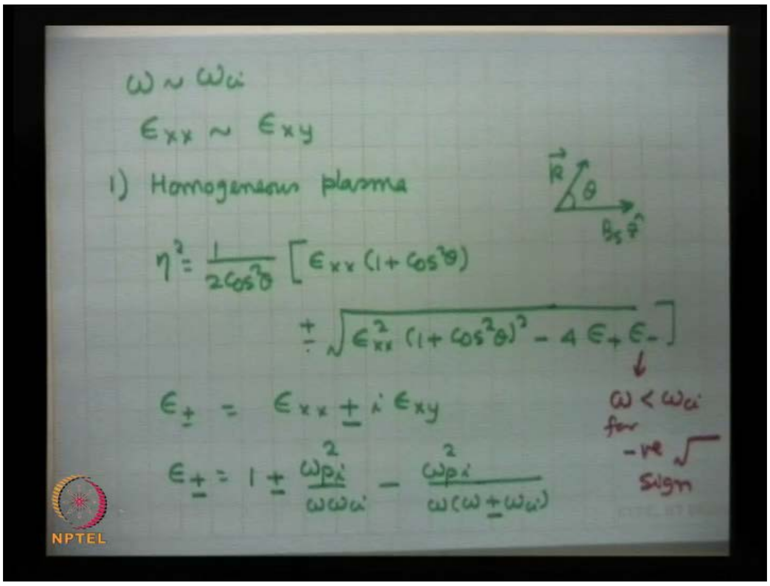

Let me quickly go over to the intermediate frequency range, when omega is comparable to omega c i in that case, ion motion has to be retained and epsilon x x and epsilon x y may be comparable. When you take that into consideration, there are 2 situations one considers. Suppose, the plasma is homogeneous, in that case, you express your k x as k Cos theta suppose, this is the angle between k and magnetic field. So, if theta in terms of theta, your dispersal relation becomes a quadratic equation for eta square and the roots are eta square is equal to 1 upon 2 Cos square theta, multiplied by epsilon x x into 1 plus Cos square theta multiplied plus minus under root of epsilon x x square into 1 plus Cos square theta whole square minus four epsilon plus and epsilon minus.

Where, epsilon plus minus we have defined as epsilon x x plus i times epsilon x y and minus sign is, then you take minus there. Well, important quantity is that at low frequencies, epsilon plus which is equal to 1 plus omega p i square upon omega omega c i minus omega p i square upon omega into omega plus omega c i, this is always a positive quantity but, epsilon minus changes sign. So, if I had a sign here, negative here, this becomes a negative and this becomes negative here.

So, what happens is that, this term can change sign. So, what do you note here that, eta square has to be positive for wave propagation and that will remain positive for both modes, plus and minus modes. For plus modes, certainly, this is always possible. So, if I choose a plus sign, there is no problem wave propagates, that is called extraodinary mode and when you choose a negative sign here, sorry a negative sign then, this term should be smaller than this term. Because, when this is smaller than this term, then only eta square will be positive and that is possible only, when this quantity is epsilon r is positive because, when this becomes a negative, then the sign will be positive and consequently, this term will become bigger than this one and hence the negative root will not be permitted.

So, what you are saying ,what I am saying is that, the lower root corresponds to what we call as the ion cyclotron wave and this occurs, when omega is less than omega c i. For negative under root, if I want this term with the negative sign.

So, the ion cyclotron wave which corresponds to cyclotron resonance here, only for omega less than omega c I, this term is negative is positive and then this is permissible. This is very important thing that, I wanted to mention. And so, out of these 2 roots, only 1 root exists above ion cyclotron frequency and both roots exist below ion cyclotron frequency. No, this is omega c i.

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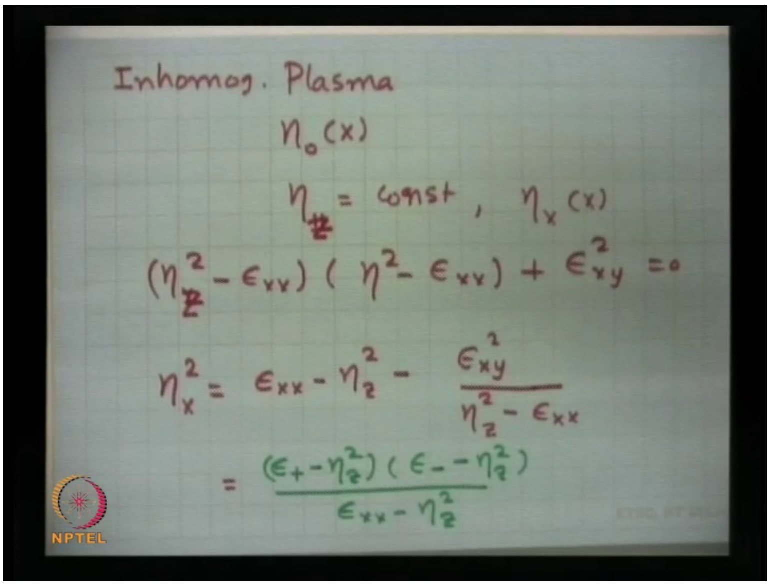

In a inhomogeneous plasma, something very is important happens. In a inhomogeneous plasma, what you have is suppose, I have a density of the plasma n 0, is a function of x or magnetic field is a function of x. In that case, your refractive index eta parallel will remain constant or k z will remain constant, only k x will be a function of x. So, eta x is a function of x and in that case your dispersant relation, which was eta z square rather eta z square minus epsilon x x multiplied by eta square minus epsilon x x plus epsilon x y square is equal to 0. You should not solve it for eta, you should solve it for eta x because, that is the quantity, that is changing and we would like to have it, eta parallel means eta z, this is eta z.

So, if I solve this equation for the eta x square, I get eta x square turns out to be simply epsilon x x minus eta z square minus epsilon x y square upon eta z square minus epsilon x x. And you can simplify this, rearranged terms in a little way little bit and this can be written as epsilon plus minus eta z square into epsilon minus minus eta z square, divided by epsilon x x minus eta z square. Now, this relation is very interesting because, epsilon x x depends on plasma density through omega p and omega p i.

You may note that, this quantity even put equal to 0, this gives you a resonance because, eta x goes to infinity. Whereas, when this is equal to 0 or this is equal to 0 then what we get is called cutoff, when eta x becomes a 0 they are cutoff. So, you always get a cutoff or resonance but, when I put this is equal to 0, that is the whistler mode and when I put this is equal to 0, I get the ion cyclotron mode. But, I am talking of a general mode here, when at this for this particular wave that you are launching from outside into a inhomogeneous plasma suppose, I am considering a plasma like this.

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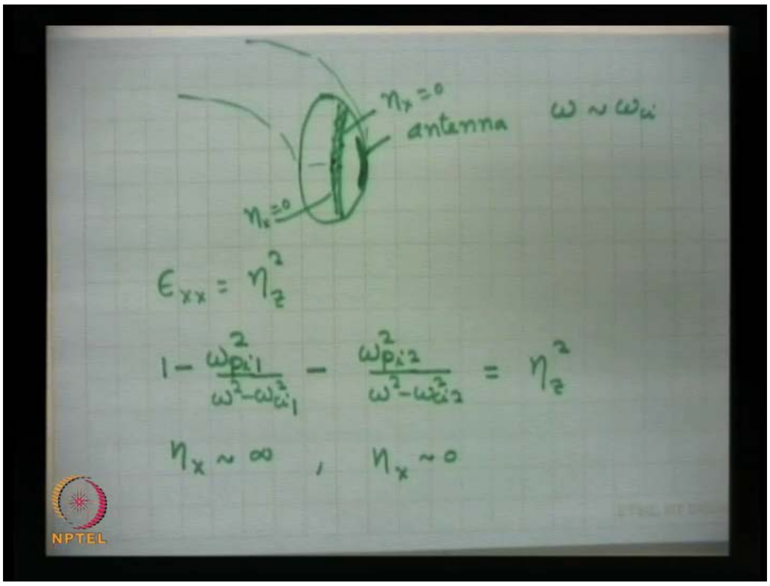

For instance, consider a tokamak. Suppose, this is a cross cross section of a tokamak. Tokamak goes like this, is tokamak. So, what you are getting here is, the density is maximum on the center of the plasma and density is decreasing.

So, if I consider a slag modal of this problem, then I am getting density starting from suppose, a wave is launch from here and you want to go the wave wants to go here and the waves is travelling at some angle, then density is increasing. So, what will happen, that somewhere a resonance will occur and somewhere cutoffs will occur.

So, for a wave of frequency, ion cyclotron frequency launched through an antenna placed here this antenna suppose, I have a wave of frequency around ion cyclotron frequency. I can launched through here, as the wave travels, it will meet some cutoff and some resonance. Resonance will occur at some point for which, epsilon x x is equal to eta z square. Now, this is a important condition because, epsilon x x is equal to, at low frequency is, this is equal to 1 minus omega p i square upon omega square minus omega c i square.

Usually, omega p i is very large so, this cannot be equal to unity. So, if I want this equal to eta z square then I require actually 2 species of ions. So, whenever there are 2 species of ions, one with ion plasma frequency this much and electron cyclotron frequency is so much and another species is omega p i square 2 upon omega square minus omega c i 2 square where, omega p i 1 is the ion plasma frequency of one species like deuterium plasma. This corresponds to tritium plasma, plasma at tritium ions and omega c i to is the cyclotron frequency of tritium. This may correspond to cyclotron frequency of deuterium and then this is possible to have equal to eta z at square. Without this term, this cannot happen because, this term is too big as compare to unity. Because, omega p i is much bigger than omega or omega c i 1 but, with 2 species is possible that one term is positive large the other one is negative large and this condition can be realized. In that case, your eta x will go to infinity. And, in the vicinity of that there will be a region, where eta x will also become 0.

So, whenever this condition is satisfied in that region, this actually happens in the center here, somewhere here so, front layer is the called the cutoff layer in which, eta x is equal to 0 and this is the layer where, eta x becomes infinity called resonant layer. So, the wave can propagate from the antenna to the cutoff but, then there is a small region where, this will penetrate it has to tunnel through and there is a resonance in beyond this also, it can penetrate. But, this is a very important area where, the wave can deposits it energy.

So, Ionian hybrid, this is called Ionian hybrid resonance, which is very important. So, when you are talking about wave of frequencies close to ion cyclotron frequency. You always look for some sort of a resonance and then the wave can be very use very useful for plasma heating. And I think now, with this I close our discussion on wave propagation in magnetize plasmas. Thank you.