Plasma Physics

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Module No. # 01
Lecture No. # 30

Electromagnetic Propagation at Oblique Angles to Magnetic Field in a Plasma

Well, today we will continue talking about electromagnetic wave propagation in plasmas in the presence of a magnetic field. Well, we will primarily consider propagation at an angle to magnetic field.

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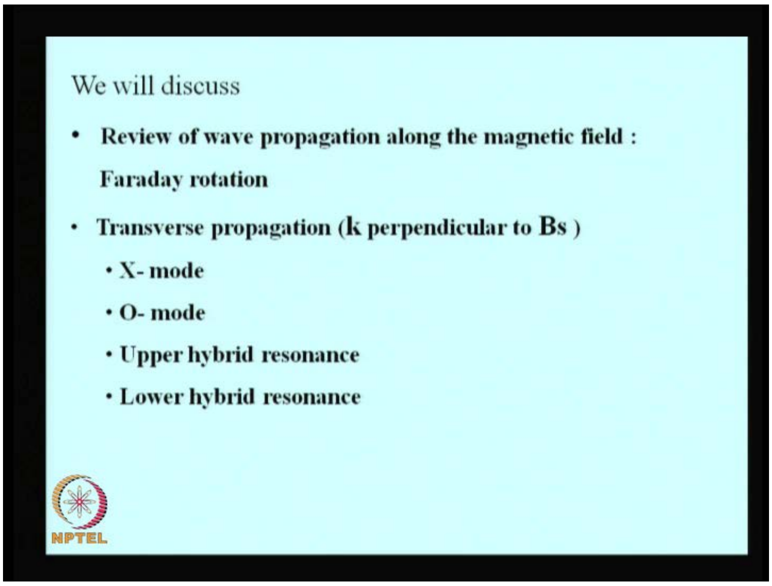

However, prior to that we will review wave propagation when the direction of propagation is along the magnetic field. And we shall discuss the phenomenon of Faraday rotation. Then, we will go over to a special case of wave propagation at right angles to magnetic field called transverse propagation. And there, we will discuss two modes of propagation, the extraordinary mode and the ordinary mode. And we shall discuss two resonances, upper hybrid resonance and lower hybrid resonance.

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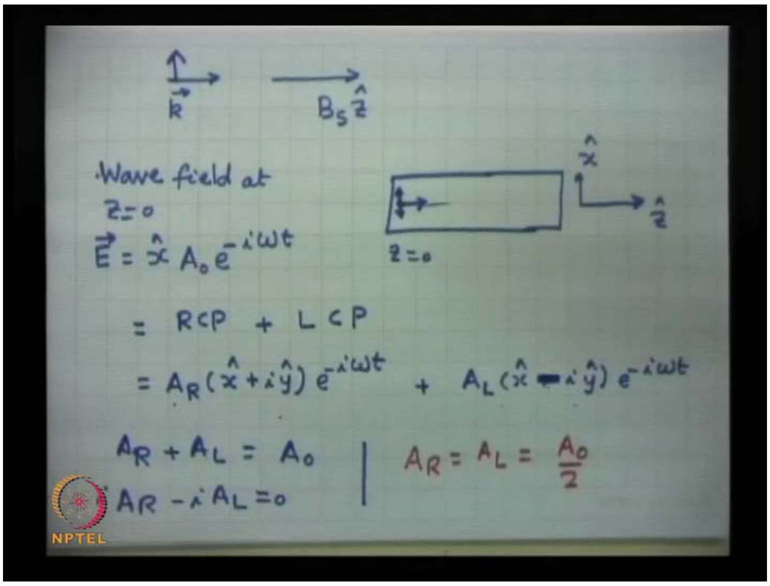

Let me begin with the propagation of a wave, along the magnetic field. This is the magnetic field in the system and we are launching a wave from here. We have learned that, if the wave is right circularly polarized, then it travels with one phase velocity and when the wave is left circularly polarized, it travels in the different phase velocity. What will happen if the wave is linearly polarized. So, suppose my wave is linearly polarized like this, then what k vector this will choose. Obviously, you cannot characterize a single wave vector for a linearly polarized wave. Rather, you should express your wave as a sum of two oppositely circularly polarized waves.

And that is possible for any polarized wave whether, linearly polarized or elliptically polarized to be expressed as a sum of two right circularly polarized waves. So, suppose my incident wave or the actual wave that we are launching into the plasma just inside the plasma. Suppose, I have a plasma here and just inside the plasma I call this as z equal to 0 position I have some electric field of the wave in this plane and this is the direction of propagation. I am given so, wave field inside the plasma at z is equal to zero. Suppose is given to me as E is equal to x cap A zero exponential minus i omega t. So, the wave is linearly polarized, it is a direction of electric field is always along the x direction. In my coordinate system, this is my z axis and x axis suppose a normal to the this one.

So, suppose I choose a linearly polarized wave, I must express this wave as a sum of two waves, one I will call as the R C P wave, plus L C P wave. R C P wave suppose has a amplitude A R but, then it has x cap if the x component of electric field and i y cap is the y component of the electric field exponential minus i omega t. This entire thing is represented as R C P. And L C P is the wave, which is amplitude suppose A L x plus rather x minus i y and exponential minus i omega t. So, what you should do, whenever you are given an electric field of a wave in a plasma at some point, then express this as sum of two fields. Now, the initial wave for instance in the simple case, had no y component. So, while you all see that the x component of this expression should be equal to x component of this plus x component of this, this is no y component. So y component of this plus y component of this must vanish.

And that immediately leads to x component, when I equate this plus this which is A R plus A L, then this should be equal to A zero and y component of this should be zero, this is A R into i and this is minus i into A L. So, then you solve these two equations, then they reveal that A R is equal to A L and each of them is equal to A 0 by 2. So, this is simple way of expressing any linearly polarized wave as a sum of two circularly polarized waves, one is right circularly polarized the other is left circularly polarized.

Now, the issue is this we have done at z equal to 0. Now, I know that, because this is a right circularly polarized wave, this will travel with a wave vector k R and this is a L C P this will travel with a wave vector k L. Now, at any position z, I can write down these two fields. So, E at any position z at time t would be equal to A R x cap plus i y cap exponential minus i omega t minus k R z and the left polarized wave would be A L x cap minus i y cap exponential minus i omega t minus k L z.

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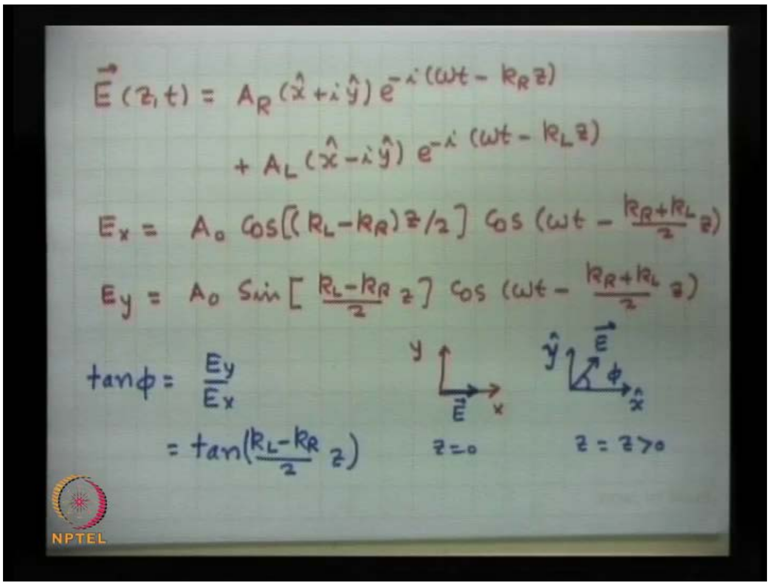

If, I assume collisions to be unimportant, so that the damping is not significant. Then, I can choose k R and k L to be real. In that particular case, it is easier to explicitly write E x and E y. And if you simply take E x as the real part of this, E x is simply A R into exponential plus A L into exponential, and take the real part it would be and remembering that A zero sorry A R is equal to A zero by two and A L is also A zero by two. It turns out to be simply A zero and Cos when you simplify this k L minus k R into z by two into Cos omega t minus k R plus k L by two into z.

So, what I have done, I have used A R is equal A zero by two, A L is equal to A zero by two and have taken the real parts of E x and use Cos A plus Cos B formula, I get this result. Similarly, if I evaluate E y take the real part of, first write down E y in complex rotation, then take the real part of the right hand side and use sign A minus sign B formula, you will get this is equal to A zero sign of k L minus k R by two z into Cos omega t minus k R plus k L by two z. And if, I divide the two so what is happening that. Suppose, this is my x direction, this is my y direction if, I am plotting the electric field initially my at z equal to zero, my electric field was like this but, after a while the electric field has gone into this direction, let this direction be phi. This is the electric field of the wave at z is equal to zero and this is a position suppose z is equal to position z which is different than zero. So suppose the angle these electric field makes with x axis is suppose phi, then from here you will see that ten phi is equal to E y upon E x and that gives you this Cos, when you divide this by this this time dependent factor goes away, and what you get is ten of this factor k L minus k R upon two into z.

So, this factor within the bracket is the same thing as phi and phi is called faraday rotation. That, the polarization of the electric field which was in this direction earlier has become inclined, the electric field is inclined at an angle phi to the x axis. The wave is going perpendicular to this plane z direction. So, the the electric wave is still remains the electric field when you view at any position z, it will still remain to be linearly polarized but, the direction of or orientation of the electric field is different than the orientation of the field in the beginning.

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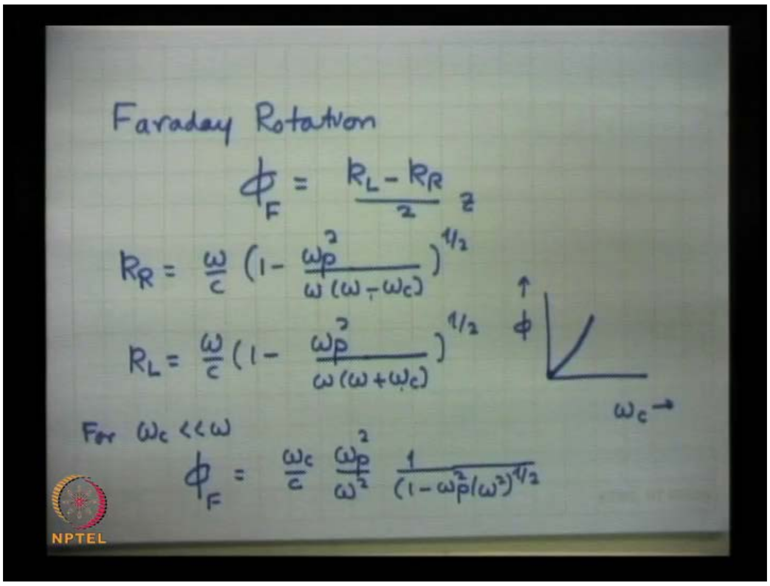

So, this angle phi is called Faraday rotation. And it turns out to be faraday, phi is equal to k L minus k R by two z. Well, what is the significance of this quantity. In a plasma, in distant plasma for instance, I would like to find out in a distant region of space magnetic field. what would I do or suppose I take a laser produce plasma and some magnetic field is generated there. I want to magnetic mirror the magnetic field, what I can do. You pass the electromagnetic wave linearly polarized electro wave through the plasma. And then, find out the polarization of the emerging wave and from there you deduce the magnetic field. So, faraday rotation is a very important technique to determine the magnetic field. Let me tell you, how will this depend on magnetic field. You know for high frequency wave k R is equal to omega by c 1 minus omega p square upon omega into omega minus omega c to the half.

Well similarly, I can write down for k L the only difference is, this is replaced by plus sign. The thing is for any arbitrary omega c you, have to actually obtain the value of k R and then k L and put this here. Obviously, let me just write down this k L is equal to omega by c 1 minus omega p square upon omega into omega plus omega c to the power half. So then, you can plot phi as a function of omega c and this will increase. This is zero when, there is no omega c because if omega c is zero k R and k L are equal. So you will get phi equal to zero and this is linear, and then we may have some character something like this.

So, depending on how far you have gone in omega c. For the case, when omega c is very small as compare to omega, you can simplify this expressions and the result turns out to be rather simple, I will simply write down the result. So, for omega c much less than omega, phi turns out to be faraday rotation turns out to be equal to, I think let me call this as theta F phi F faraday rotation F is equal to omega c by c into omega p square upon omega square into one upon one minus omega p square by omega square to the power half. So faraday rotation is proportional to omega c, which is proportional to magnetic field. So, you can easily deduce from this graph, if this is properly calibrated you can find out the volume of magnetic field by measuring phi. That is why faraday rotation is a useful quantity.

Now, I would like to consider the propagation of waves transverse to magnetic field, which is more relevant to tokomak heating, mirror heating and ionospheric heating in the equatorial region.

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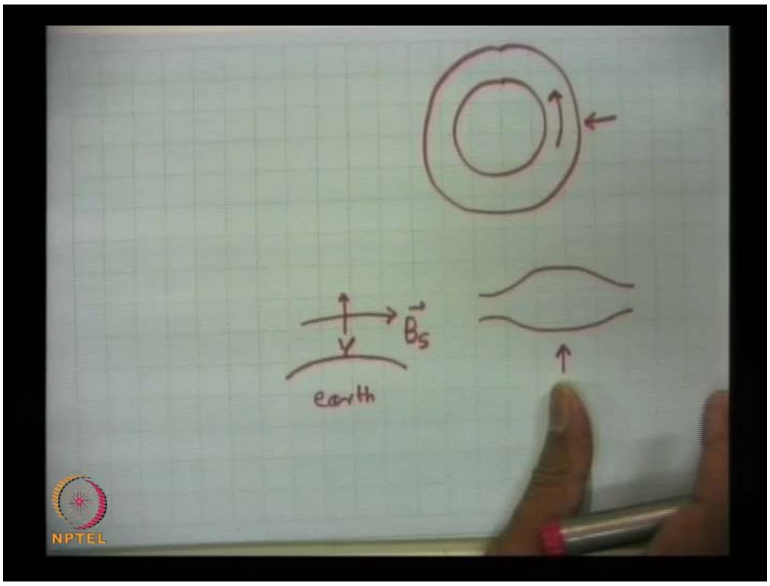

As, I mentioned to you in a tokomak, you have a plasma in this torus and lines of force are something like this primarily, then there is a poloidal magnetic field as well and you are launching wave from here. This is travelling almost perpendicular or may be a some angle.

But, before we go over to discuss the propagation of waves at some arbitrary angle. Let me consider the ninety degree angle because, we can get lot of clue from there. In mirror machine also if I have a lines of force like this and you may be tempted to launch waves from here, which are also normal to the lines of force. In ionosphere, this is my earth the in the equatorial region, the line of force are like this. So, if I am launching a wave from here is going normal to the ionosphere. So, in many situations you are encountered with the propagation of electromagnetic waves perpendicular to magnetic field. And let us see, what kind of physics we can learn about wave propagation in such situations.

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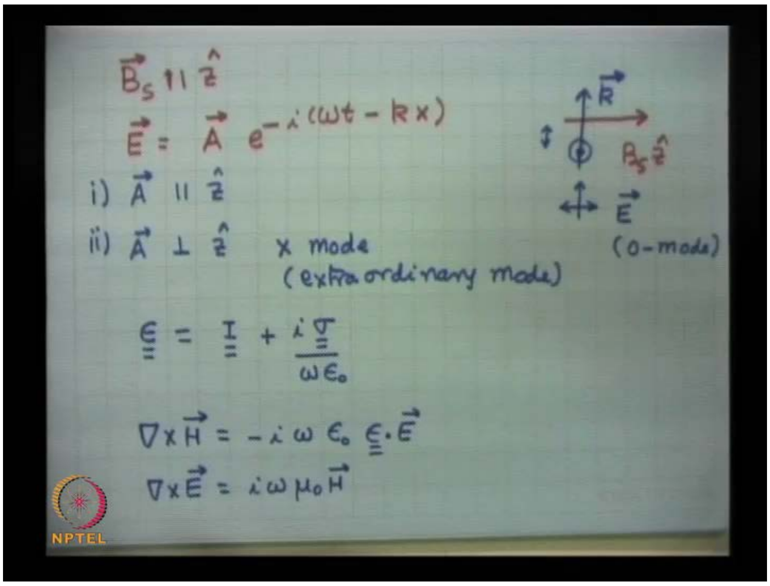

So, I am considering a plasma in which magnetic field is a still parallel to z axis. So, that the expression for effective plasma permittivity, that I deduced is a still valid. But, I am choosing my wave field to be of this form, E is equal to A vector exponential minus i omega t minus say k x. So, I am permitting my wave to travel perpendicular to magnetic field. So, if the magnetic field is here, my wave propagation is normal to this. So, let me write this as there is a at this my k vector of the wave. Now, physically I expect two situations. In a un-magnetized plasma, wave is transverse, the electric field is perpendicular to k. There are two possibilities that my electric field is perpendicular to the plane containing k as well as B like this is one possibility and second possibility is that the electric field of the wave is along the magnetic field.

If, this is the electric field of the wave, then in the presence of the electric field of this electromagnetic wave the electrons will oscillate in the direction of magnetic field, and I really do not expect any change in the propagation characteristics by the magnetic field. So, I am expecting this wave to be unmodified by the magnetic field and I will call this as O mode ordinary mode, ordinary wave nothing very special about this called ordinary.

But, if the electric field of the wave is polarized perpendicular to the transverse direction, then this wave will as it tries to cause the electron oscillation normal to k vector and normal to magnetic field. The d c magnetic field is starts acting a excreting a Lorentz force on this and the direction of v cross B force will be in the k direction. So, electron velocity will have a component in this direction. And when a electrons oscillate in the direction of propagation then there is a compression rarefaction of electrons and that always produces density oscillations.

Even, if you forget density oscillations for a while any current that is in the direction of propagation will produce an electric field also in the same direction. Or the space charge compression rarefaction that is as I just mentioned that produces a this electric field in the direction. So, this there is always a longitudinal electric field produced. So, I am expecting very different kind of wave propagation, for one case when the amplitude of the wave is parallel to z cap or magnetic field and second one a is perpendicular to z cap. This is a something extraordinary, because it is only in the presence of magnetic field that this sort of wave propagation will occur and we will call this as the x mode or extraordinary mode extraordinary of propagation. Let us examine this way, what we did last time was we defined effective plasma permittivity tenser in terms of which which was actually defined as unit tenser I plus i sigma the conductivity tenser divided by omega epsilon zero.

And in the when I introduce this in the Maxwell equations, the force Maxwell equation was curl of H is equal to minus i omega epsilon zero epsilon tenser dot E and the third Maxwell equation was curl of E is equal to i omega mu zero H. Then, replace this del operated by i k here and here and combine these equations take k cross of this equation and use this one

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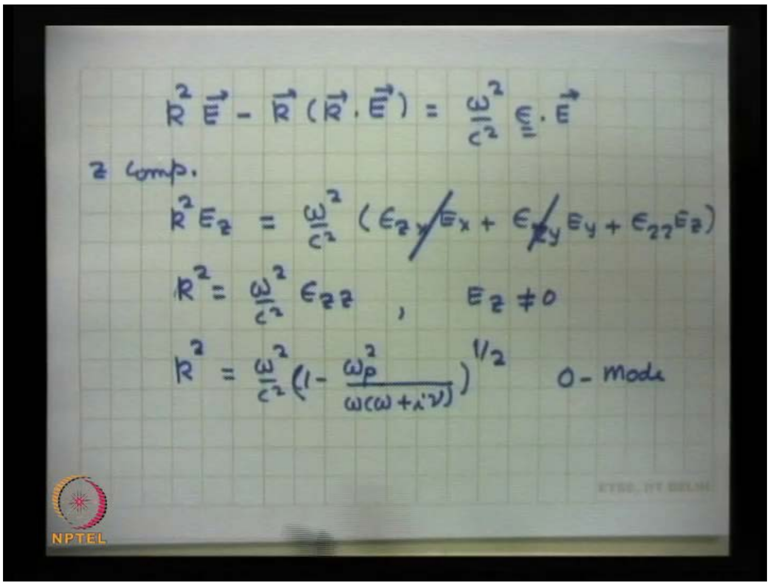

Finally, we got the following we get k square E minus k k dot E is equal to omega square by c square epsilon dot E. This is the algebraic form for wave equation and every information about both the modes is contained in this equation.

Let me write down the x component of this equation z component of this equation first. The first term will give me k square E z, second term does not give me anything because k is in the x direction. So, there is no z component this term is zero is equal to omega square by c square, z component of this product which is epsilon z x E x plus epsilon z z y E y plus epsilon z z E z. If you examine the permittivity tenser, this diagonal this of diagonal term is zero this is also zero this is the only term that survives and k square becomes equal to omega square by c square epsilon z z, so this is zero, this is zero. So, E z equation does not have any E x or E y this is independent of E X and E y and we say that this is a mode for which E z is non zero and k is given by this expression. If you substitute the value of epsilon z z, it turns out to be equal to omega square by c square one minus omega p square upon omega into omega plus i nu, where omega p is the electron plasma frequency, nu is the electron collision frequency to the power half. This has no magnetic field dependence and simply called ordinary mode just ordinary.

So, the thing that we were physically expecting as emerged here, that if the wave is polarized with the electric field in the direction of magnetic field and its traveling perpendicular to magnetic field then, this mode will not be influenced by the magnetic field and actually there is nothing special about this mode, so we shall not discuss much about it. The second case, let me write down the x and y components of this equation of this wave equation, that I have written here. Let me write down the x component and then I will write down the y component.

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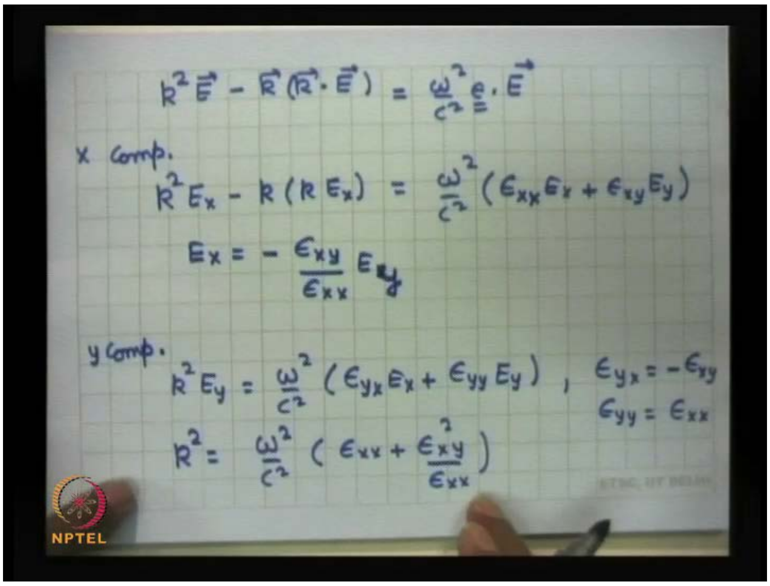

So, let me rewrite this k square E minus k k dot E is equal to omega square by c square epsilon dot E. And let me write the x component, what do I get, the first term gives me k square E x minus this is k x, x direction so simply k and k dot E, remembering that k is in the x direction. So, this becomes k into E x is equal to omega square by c square and write down the x component of this product.

The first index of epsilon has to be x, other should be running with e, it becomes x x E x plus epsilon x y E y the there is another term epsilon x z E z but, epsilon x z is zero so, I do not write that. You may note here these two terms exactly cancel. So, for this mode this factor should also be zero, which gives me a relationship between E x and E y. So, from that you will get E x turns out to be equal to minus epsilon x y upon epsilon x x E x. Now, let me go over to the y component of this equation. Please remember k does not have a y component. So, this term is not contributing, this is the only term that gives me k square E y is equal to omega square by c square epsilon dot E. I should write y component, so first index of epsilon has to be y other should be running with e. So, that is gives me epsilon y x E x plus epsilon y y E y plus epsilon y z E z but, epsilon y z is zero, so I do not write that. What I can do here, this E X is there, sorry I made a mistake this is when I put this equal to zero, I write E X in terms of E y, this is E y here.

So, use this equation to let us write E x in terms of E y here. So, the everything becomes E y, then E y will cancel with this E y. So, when I substitute this here, I get k square is equal to omega square by c square epsilon, please remember epsilon y x is the same thing as minus epsilon x y and epsilon y y is the same thing as epsilon x x from the expression for permittivity tenser, you can check it.

So, when I substitute this here I get a minus sign here and minus sign there, so it becomes simply epsilon x x plus epsilon x y square upon epsilon x x. This becomes the dispersion relation for the extraordinary mode, x mode or extraordinary mode and how about this ratio. If I well, if I want to write down this for a low frequency case, in that case I must include the ion motion but, if I am considering the propagation of high frequency waves then, this becomes simple. So, I will write down this ratio and simplify this as well.

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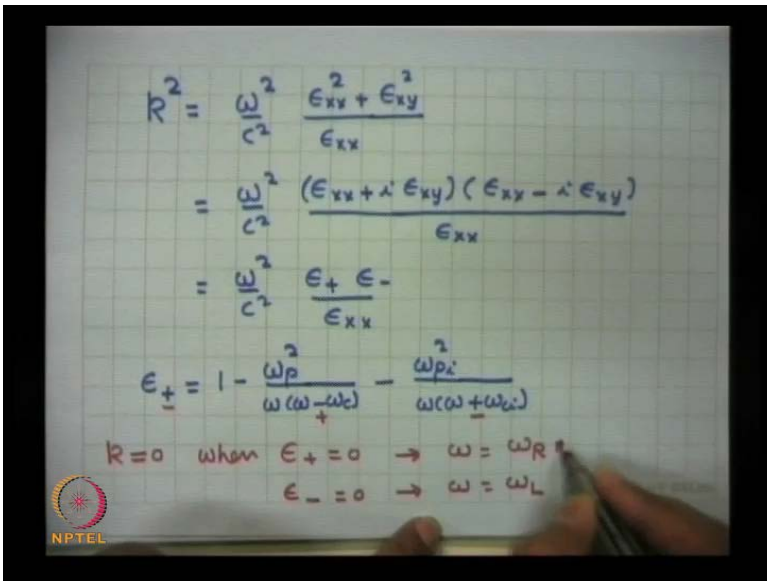

First, let me write down the k square, this is equal to omega square by c square, it can be written as epsilon x x into epsilon x x square plus epsilon x y square. This expression can be written as omega square by c square epsilon x x plus i epsilon x y multiplied by epsilon x x minus i epsilon x y divided by epsilon x x. This quantity is designated as epsilon plus and this is designated epsilon minus upon epsilon x x.

And if, you look at the expressions for epsilon plus and epsilon minus they turn out to be simply the same as appear for the R C P and L C P waves. As this was this can be written as one minus omega p square upon omega into omega minus omega c plus ion contribution, which is omega p i square minus omega p i square upon omega into omega plus omega c i. And if, I have to write down for epsilon minus then change the sign here, so, this is plus here this is minus here. So, if I have to discover the zeroes of k, the cut off when k becomes a zero. Then obviously, when epsilon plus becomes zero or epsilon minus becomes zero.

When epsilon plus becomes zero, the same cut off you got as omega R and when I put epsilon minus equal to zero the same because, in R C P L C P case, your k square will related to this quantities. So, k becomes zero, when epsilon plus is zero which gives you omega equal to omega R the right handed cut off the same frequency as before and or when omega minus epsilon minus is zero which gives me omega equal to omega L, the left handed cut off this I can write down I think I will write down on a separate sheet.

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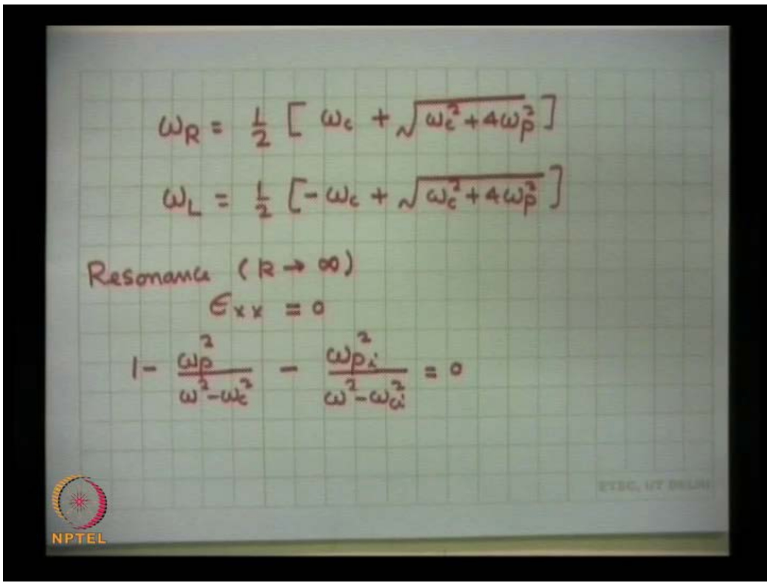

So, omega R turns out to be half omega c plus under root of omega c square plus four omega p square, and omega R omega L the left handed cut off is half minus omega c plus under root omega c square plus four omega p square. So, this is the left handed cut off, how about the resonance resonance you will expect well simplest resonance that is obvious will occur, when epsilon x x equal to zero. Resonance means when k becomes infinity, I will call this as the resonance. Now, what is epsilon x x, if you look at the expression one minus omega p square upon omega square minus omega c square, if collisions are ignored minus omega p i square upon omega square minus omega c i square is equal to zero. This gives me a quadratic equation omega square will two roots. One of them is called upper hybrid resonance the other is called lower hybrid resonance. So, resonance essentially I am saying that, when k goes to infinity, then you get that you will get when epsilon x x is equal to zero or this is zero.

You may also ask a question whether, I not get the resonance via when a cyclotron resonance when epsilon plus becomes infinity certainly, epsilon as when epsilon epsilon plus can become infinity at cyclotron resonance but, the same time epsilon x x also becomes infinity. So, this ratio does not go to a infinity. So, this is the only resonance available for the extraordinary mode and let me evaluate this equation.

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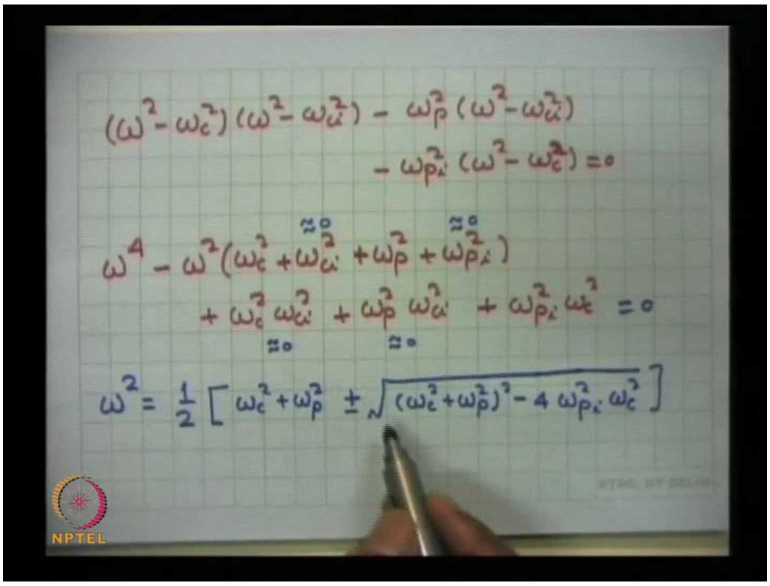

Let me write this equation as such as omega square minus omega c square into omega square minus omega c i square, then you will get minus omega p square into omega square minus omega c i square minus omega p i square into omega square minus omega c square equal to zero.

And I can write this equation as omega to the power four, then I will get omega square coefficients from these products omega c square from here and this will give you plus omega c i square, from here you will get plus omega p square, from here you will get plus omega p i square and then you will get constant terms, from here you will get omega c square omega c i square, from here you will get plus omega p square omega c i square, from here you will get omega p i square omega c square.

Now, recognizing that omega c i is very tiny as compare to omega c. I can ignore this term so, this is nearly negligible as compare to other terms. Similarly, omega p i is much less than omega p so, you can ignore this as well. Out of these terms, omega p square has a dependence on mass is one upon mass and omega c i square has a dependence is one upon mass square. So, the terms that have single power of omega p, they are the dominant ones others are actually out of many terms here, you will note that this is the dominant term others are small as compare to this one these are very small terms. So, this is the term that I written and this is equal to zero.

So, with this ignore this term, ignore this term retain this, you will get two roots and these are omega square is equal to half, like this is like a x square plus b y square plus c equal to zero. So, this gives me omega c square plus omega p square, this one plus minus under the root omega c square plus omega p square whole square minus four a c, a is one c is this expression omega p square i and omega c square. Please note that, this term certainly is much less than this one. So, when I have to consider the plus sign, the contribution this is negligible, you can ignore.

However, if I have to consider lower sign negative sign then, the large term will cancel the large term here and hence this is important. But, since this is small, I can carry out a binomial expansion and when you do this,

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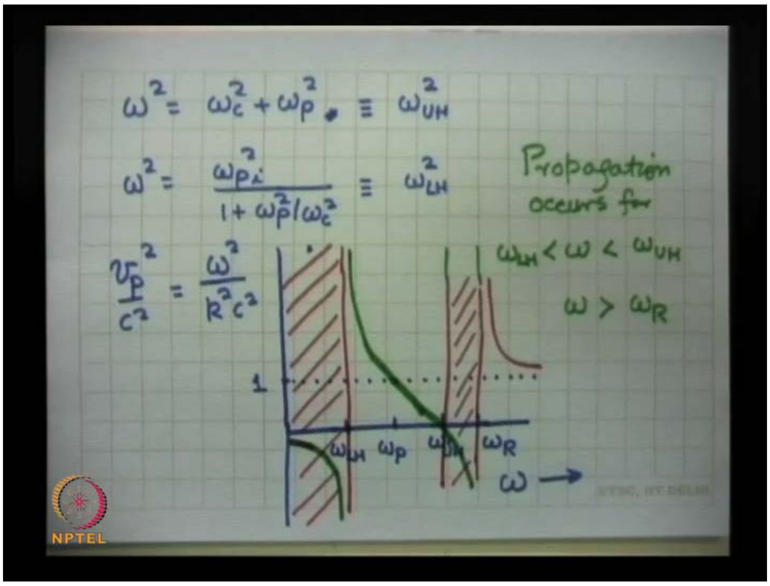

So, the two roots of this turn out to be omega square is equal to omega c square plus omega p square is one root and the other root is omega square is equal to it turns out to be simply omega p i square upon one plus omega p square upon omega c square

There is a special name given for this, this quantity is given a name called omega U H [upper hybrid resonance] upper hybrid frequency square and this is given a name omega L H [lower hybrid frequency square]. So, when your frequency is equal to omega L H or equal to omega U H, you get k very large and even k is very large please remember, B could be large. However, you have to ensure that k cross e is the quantity it is not that when k is large but, the E makes a very small angle with k then it is not important.

So, resonance certainly occurs at these two frequencies, and if I plot the phase velocity of these waves, let me just find the a quantity they called as phase velocity of the wave square by c square phase velocity I am calling as let me call this quantity as omega square upon k square c square, in some books they have plotted this.

So, for the extraordinary mode, if I plot this on the y axis and if I plot here a the frequency, I get some typical characteristics some frequencies one is called the right handed cut off omega R and there is a upper hybrid cut off which is smaller than this somewhere here is the omega U H and then there is a omega p somewhere here and then there is a lower hybrid cut off somewhere here. And, if I plot this quantity phase velocity as a function of square as a function of omega, then I get the plot and suppose this is unity here because, in free space v p is equal to c, phase velocity is equal to c of this ratio is one. So, which is a important guideline to compare the phase velocity of the wave with the phase velocity of the wave when there is no plasma. And it turns out to be that there is a forbidden region here, there is no wave propagation in this region. So, there is no propagation here and the wave goes like this means when omega is much bigger than the right handed cut off, the phase velocity tends to see but, at right handed cut off because k goes to zero. So, this becomes infinity this is k equal to zero basically.

And then, this is the forbidden region again below this below lower hybrid wave frequency, this case which is becomes this is again a forbidden region, in this forbidden region your this ratio is a negative so, the wave they does not travel. Actually, it travels here and passes through a unity at this point and goes to zero here resonance and then it like this. So, wave propagates in those regions which are not shaded like this, this is the region that the wave travels and this is the frequency at which this travels. So, wave travels at frequency higher than omega L H and less than omega U H or. So, propagation bends are, propagation occurs for omega greater than the lower hybrid frequency but less than the upper hybrid frequency and omega bigger than the right handed cutoff. These are two important frequency ranges for propagation of these mode.

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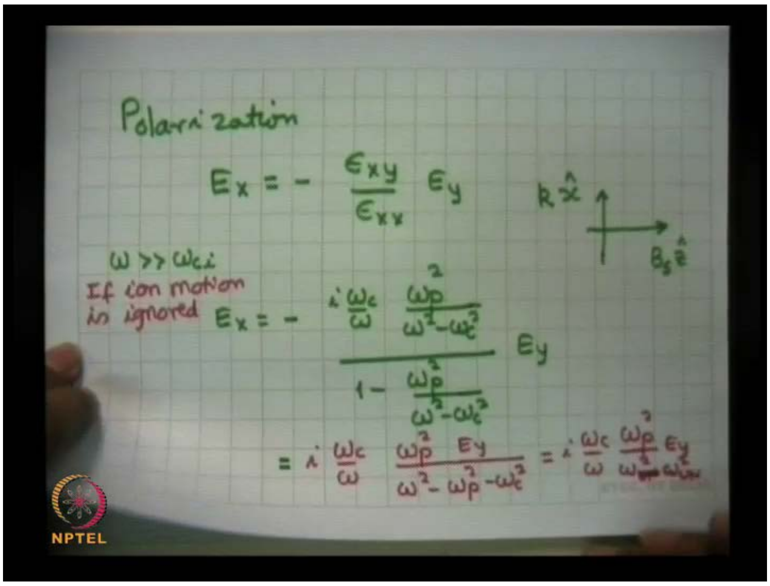

Now, let me talk about the polarization of this mode. We just saw that this wave has E x is equal to minus epsilon x y upon epsilon x x E y. Suppose, I consider the wave frequency higher than the ion cyclotron frequency. So that, the ion motion can be ignored, in that case I can write down these expressions simply as minus this is i omega c upon omega and omega p square upon omega square minus omega c square divided by epsilon x x, which is one minus omega p square upon omega square minus omega c square into E y. Please remember, your d c magnetic field is in the z direction and the wave is going in the x direction.

So, E x is called the longitudinal component of your wave, this is your k vector in this direction and E y is called the transverse component of the electric field. So, I am expressing the longitudinal electric field in terms of the transverse electric field and we find that, this component can become very large, when this quantity becomes a zero or epsilon x x equal to zero is also called the resonance, either upper hybrid resonance or lower hybrid resonance.

In both cases, E x becomes much bigger than E y and the wave becomes almost like a electrostatic wave because, when the electric field is parallel to the direction of propagation the wave is called electrostatic wave. So, at a point at resonance this term becomes zero and hence, the wave becomes like its polarized that becomes like a longitudinal wave. What I can do. I can take LCM of this and rewrite this expression without any approximation. As this i omega c upon omega then, omega p square divided by omega square minus omega p square minus omega c square and this quantity these two terms together are called omega upper hybrid frequency. So, I can write down this is equal to I, this into E y omega c upon omega into omega p square upon omega upper hybrid square into E y sorry.

I made a mistake, this is omega a square minus omega E y H square. I think there is not much room here let me write separately. This I have ignored, this I have written if I ignore the ion motion. If ion motion is ignored, but if ion term is retained then you get low hybrid resonance as well.

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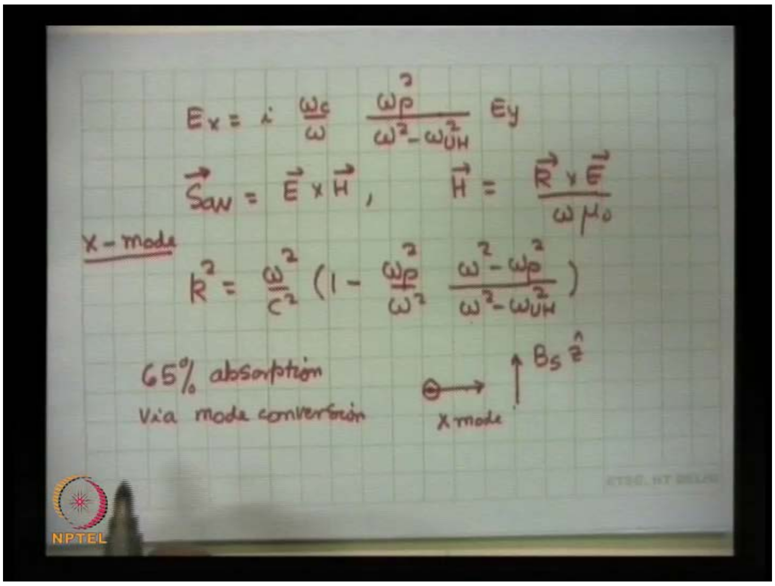

So, at for high frequency response E X is equal to i omega c upon omega into omega p square upon omega square minus omega U H square into E y. Two things are important. At omega equal to omega U H longitudinal field is much bigger than the transverse component of the field.

And secondly the two fields are out of phase by pi by two. This is important and please note in the pointing flux, when you calculate of the wave this is E cross H and because H is k cross E upon omega mu zero. So, longitudinal component will not contribute, because if I want pointing flux in the x direction then, E X will not contribute. So, this really does not play much role in energy propagation, but its coupling to E y has a consequence on k. So, the consequence of longitudinal field appears through the propagation constant k.

Now, let me write down explicitly for a high frequency wave, k square is equal to omega square by c square, it turns out to be one minus omega p square by omega square omega square minus omega p square divided by omega square minus omega U H square. This is the propagation dispersion relation for this x mode extraordinary mode, sometime the wave is also known as upper hybrid electromagnetic wave because, it shows upper hybrid resonance. You may note here, that omega p certainly less than omega U H. So, the wave will travel for omega less than omega U H but, omega bigger than omega p certainly in that region this quantity will be whole second term will be positive so, the wave certainly will travel. And resonance at omega equal to omega, U H k tends to infinity and we have already seen explicitly this behavior. So, this is a very important wave in laser produced plasma what really happens is that often when the laser goes into a plasma, then you are expecting a transverse magnetic field and transverse magnetic field if the laser is polarized perpendicular to the direction of magnetic field, then this will travel as x mode.

So, in laser produced plasma which are inhomogeneous plasmas, there is the densities increasing from like the plasma as the wave or laser travels deeper and deeper into the plasma density increases. So, initially what you can happens is that omega is bigger than omega U H because omega p is low but, as the density increases omega becomes comparable to omega U H. As this wave arrives there, it can get mode converted into a electrostatic wave. Here, I have not considered any thermal effects. So, when omega approaches omega upper hybrid frequency, thermal effects become important because the k becomes large so, phase velocity becomes very small. And when phase velocity is a small then thermal effects become important because, when the wave travels with the velocity close to thermal velocity of particles then, Doppler shifting frequency becomes important and thermal effects become important and there is always a possibility of mode convergent of an electromagnetic wave into electrostatic wave and that gives rise to very strong absorption of these waves.

So, in laser produced plasmas, the existence of transverse magnetic fields by some mechanism built in they are called self generated magnetic fields. If you can have this situation then, very strong absorption occurs and we had studied this problem many years ago and we could show that, it can gives rise to something like sixty five percent absorption of laser energy is possible by mode conversion of the laser into an electrostatic upper hybrid wave. So, electromagnetic upper hybrid wave or electromagnetic wave rather, when approaches the cyclotron resonance layer, then it can generate a plasma wave in an inhomogeneous plasma and it can lead to very strong absorption of laser.

So, I think this is a important a problem. I think, I will like to stop here. Next time, I would like to talk to you about wave propagation at arbitrary angles and then related to plasma heating in many experiments. I think we are gradually building up our understanding of wave propagation of plasmas in the presence of magnetic field thank you very much.