Plasma Physics

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Lecture No. # 03
DC Conductivity and Negative Differential Conductivity

Well friends, in this lecture, I wish to apply the equation of motion and continuity to understand the phenomenon of electrical conduction in the presence of a dc electric field.

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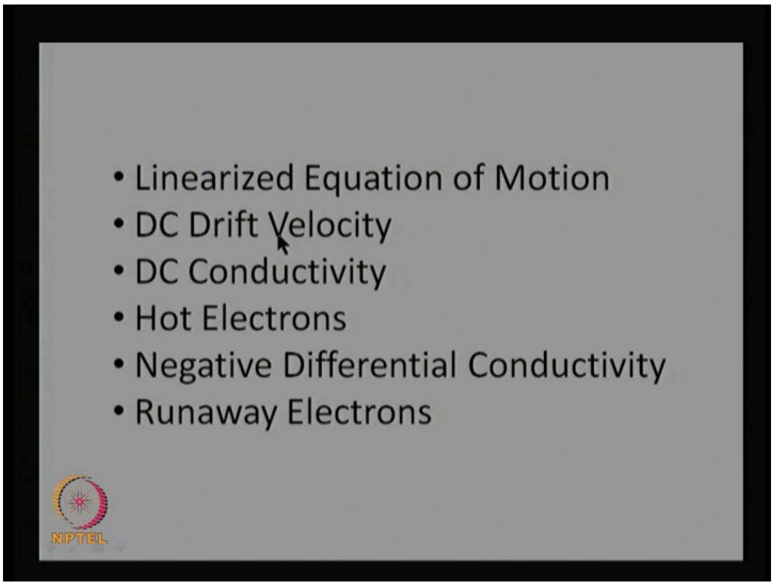

Well, I will discuss the linearization of equation of motion, drift velocity, I will reduce from there, and then I will obtain an expression for dc conductivity. Well, at high electric fields, I will discuss the phenomenon of hot electrons. And in some cases, especially in semiconductors like gallium arsenide, which is called a solid state plasma; the phenomenon of negative differential conductivity, which gives rise to very important gun oscillations. And finally, I will discuss in plasmas, strongly ionized plasmas, the phenomenon of runaway electrons. But I will touch upon these last two topics rather briefly, just to illustrate the strength of fluid equations.

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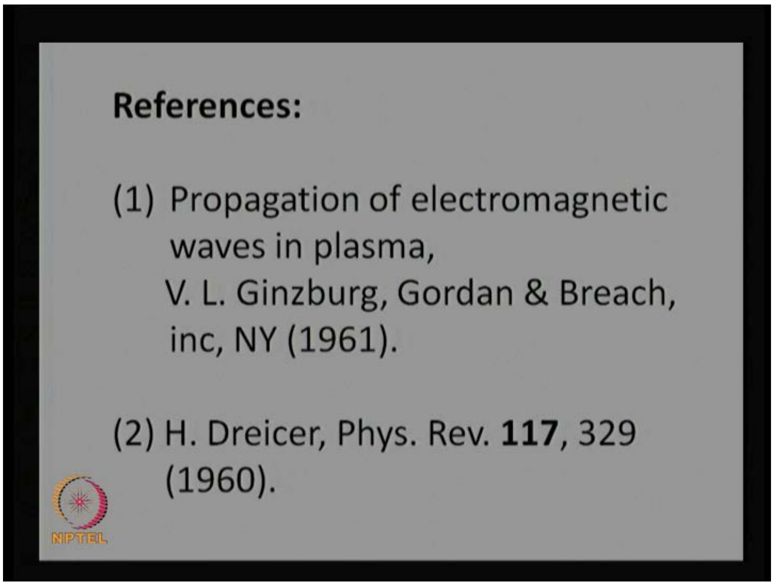

And well, I would like to mention two references with respect to the hot electrons and the runaway electrons; one is a book by V. L. Ginzburg, last chapter of the book talks about non-linear phenomena in the presence of dc and ac fields. And then Dreicer, who discussed the phenomenon of runaway electrons.

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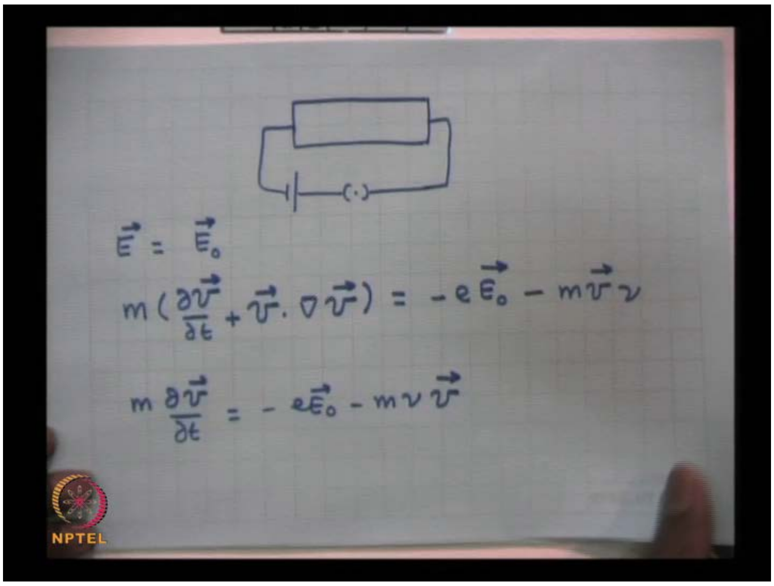

Well, let me begin with a simple geometry; suppose I consider a semiconductor or a plasma, does not matter what it is, apply a dc electric field to it. So, when there is an electric field, if there are free carriers like electrons or holes or in case of plasma, they are free electrons, then they will move. So, if I want to understand the response of electrons to this dc electric field. So, let me consider the electric field of the system to be, say E 0, the dc electric field I will designate as E 0. And the plasma is collisional. So, the equation of motion would be, m delta v by delta t plus v dot dell v is equal to the electrical force, which is minus e E 0, then there is a collisional drag, which drag force, which is minus m v nu. I am presuming that the density and temperatures are uniform and hence, I am not writing the pressure gradient term. I am also presuming that there is no dc magnetic field and hence, I have not written v cross v force.

Well, if I further assume that the source E 0, also called the driver, if it does not depend on position, then the response v should also not depend on position. And hence, this convection term can also be ignored, because v does not depend on position. So, gradient, this term will be 0. So, my equation then becomes, m delta v by delta t is equal to minus e E 0 minus m nu v. Well, one can solve this equation, exactly simple equation, but I am not interested that much at the moment, as a problem, you can discuss that if time T equal to 0, if you switch on the electric field, how much time it will take for the velocity to acquire steady state value. Well, we can do that problem later.

But let me consider a steady state solution, a steady state is reached, when these two terms balance each other. So, the velocity does not change with time. So, in the steady state, if you equate these terms, sum of these two terms to 0, you will get velocity of the electrons to be equal to minus e E 0 upon m nu. So, the electrons have acquired a drift velocity due to the electric field, which is opposite to the direction of the electric field and it depends on a quantity called the collision frequency. Here, minus e is the electron charge, m is the electron mass and nu is the collision frequency.

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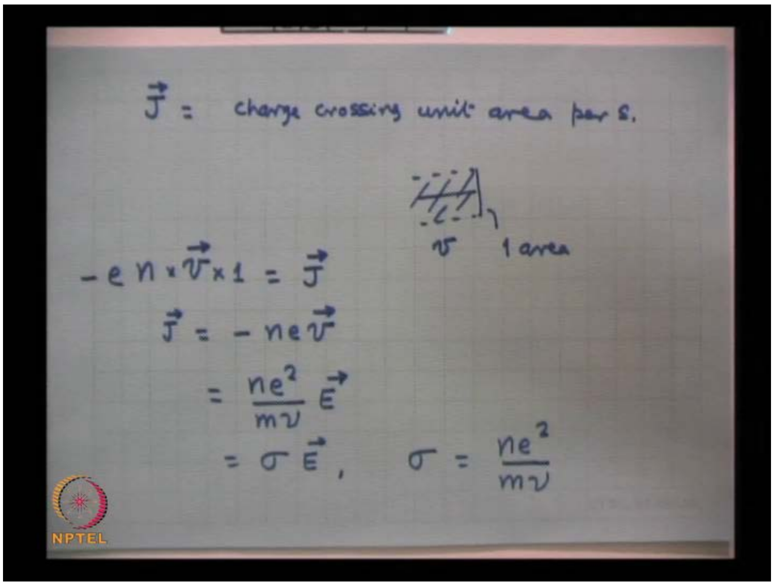

So, if I know the drift velocity, I can quickly write down the current density, because current density, by definition, is the amount of charge flowing, charge crossing unit area per second. So, if my charge is moving with some velocity v and if I consider area perpendicular to it, in one second, all that charge will move, which is at a distance less than or equal to v, this is unit cross section. So, all the electrons that are filled in this region, in this volume, cylinder of unit cross section length v, they will be crossing this. So, the number of particles crossing would be, volume is v into 1 and the density of electrons is n. So, n into d v into 1. This is the total number of electrons that will cross and each electron carries a charge minus e. So, this quantity should be equal to J. And v is a velocity so, put a vector sign there.

So, by definition, current density is equal to minus n e v and the v that we just obtained, if I substitute here, it turns out to be, equal to n e square upon m nu into E. People write, J E relationship in terms of a quantity called sigma E. So, this sigma is called the conductivity, which we just look at this, sigma is equal to n e square upon m nu. So, the conductivity is inversely proportional to collision frequency. In solids, collision frequency is around 10 to the power 12 collisions per second. In plasmas, in the hot plasmas, collision frequency could be like 10 to the power 3.

So, there is a 9 orders of magnitude difference in a hot plasma and the metal. In a plasma, typical electron density is, in case of, for instance, tokomak the fusion machine is like 10 to the power 20 or few times 10 to the power 20 per meter cube. In case of a metal, the collision frequency, sorry the electron density is about 10 to the power 28. So, there is a about 8 orders of magnitude difference in n and 9 orders of magnitude difference in nu. As a result, the conductivity of a plasma could be bigger than that of a metal, just look at this. So, in case of metal, let me just write this. This is an important issue that why plasmas are so important, because their conductivity is very high.

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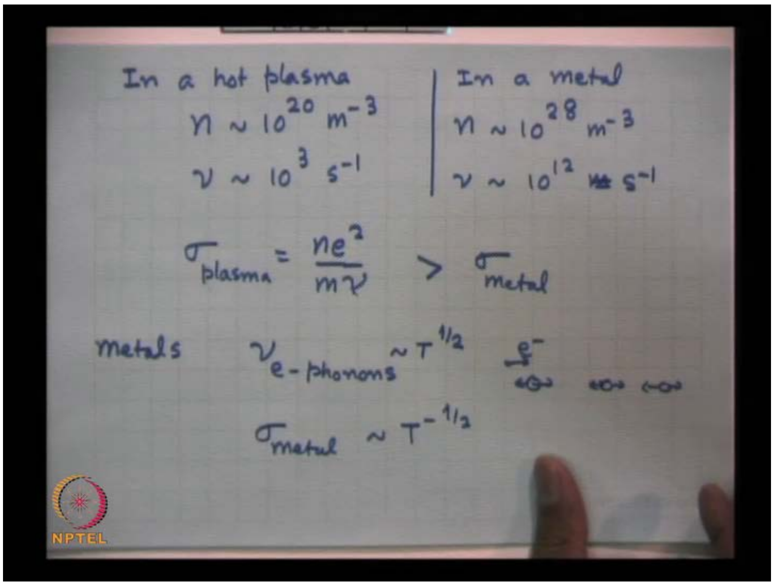

So, in a hot plasma, if I have density of the order of 10 to the power 20 per meter cube, collision frequency of the order of 10 to the power 3 per second. And in metal, like gold or silver, electron density is of the order of 10 to the power 28 per meter cube, collision frequency is ten to the power 12 per second. As a result, if you look at the conductivity of a plasma, which is equal to n e square upon m nu, this could be bigger than the conductivity of metal. It is a very important point, that plasmas have very high conductivity, hot plasmas have very high conductivity. Well, this is something, that you should always remember. Another important issue is, you have heard of (( )) or rather, resistance of metals increases with temperature. Let us see, what happens inside a metal?

The collision frequency, nu has dependence on temperature. In case of metals, the collisions are primarily between electrons and phonons. What does this mean, a lattice, if you examine at any finite temperature, it oscillates. So, when the lattice atoms are oscillating about their main positions, they are oscillating. And when free electrons moves in their vicinity. If these were perfect lattice, these were stationary, they were not static, they were not oscillating, then the electron will not suffer any collision.

In a perfectly periodic potential, well electrons do not suffer collisions. However, when the electrons, these atoms oscillate, the lattice becomes imperfect and there are two modes of vibration of lattice; one is called acoustical mode of vibration, another is called optical mode of vibration. At low temperature, most vibrations are in the acoustical mode and when the electrons moves in the vicinity, it suffers a collision. And this collision frequency increases the temperature, as temperature to the power half.

So, when a temperature rises, what happens when you pass a current through a conductor, the conductor get heated, electrons get heated and temperature rises, so collision frequency rises. When collusion frequency raises, conductivity of the metal, well, because the same expression holds for metal, it will decrease or resistivity will increase. So, this goes like this. So, metal conductivity goes as temperature to the minus half, hence resistivity increases. In the case of plasmas, especially, strongly ionized plasmas, collision frequency decreases with temperature and opposite behavior happens.

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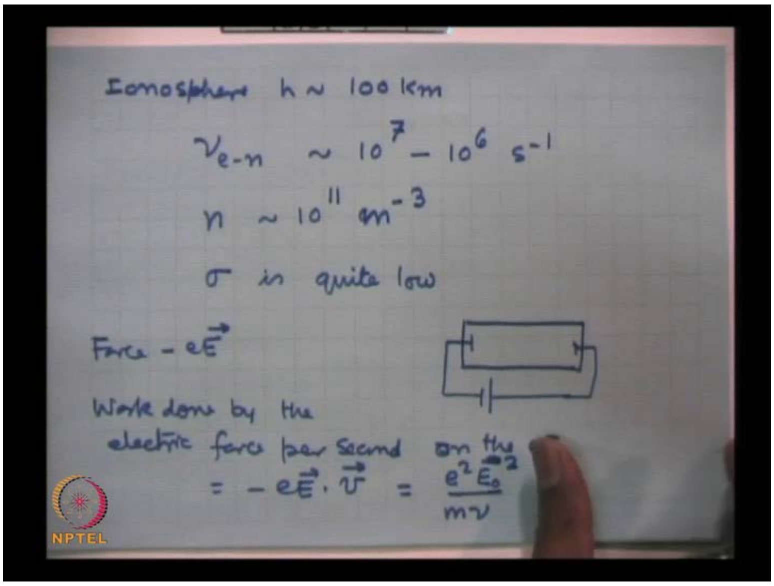

In case of weakly ionized plasmas, like ionosphere, lower ionosphere, at a height of the order of about 100 kilometer above the earth. What happens? The collisions are between electron in neutral atoms and the value of this quantity is about 10 to the power 7, actually decreased the height to 10 to the power 6, this is the range, collisions per second. This is a quite high collision frequency and the density of electrons in the ionosphere, lower ionosphere is around 10 to the power 11 per meter cube. As a result, conductivity is quite low, sigma is low, is quite low. However, even in plasmas of low conductivity, something very important happens, in semiconductor also, it happens, that if you increase the value of the electric field, what will happen?

So, consider plasma or a semiconductor, if you apply or apply a electric field to a plasma like this, if the electric field voltage you increase, just upto a few 100 volt in a plasma or just a 100 volt, that is good enough. Let us see, or a semiconductor, if you are apply having here, apply a voltage of the order of, or the electric field in this region of the order of, say, 1000 volt per centimeter. So, choose a sample of, say, 1 millimeter thickness and this is one millimeter, and apply 200 volts, this will become 2000 volt per centimeter. So, this is quite, you know, you can really apply high electric fields in semiconductors, if the length is small or thickness is small.

What is happening there? When you apply large electric field, the electrons get heated. Why are they heated, because the force on the electron is minus e E. So, when an electron experiences a force due to the electric field and it moves in the same direction, then the work done by the force, by the electric force per second on the electron would be equal to, the force into the distance travelled by the particle in the direction of force, this quantity. If we put the value of v, that I have just derived, distance out to be equal to, e square E 0 square, E 0 square rather, upon m nu, this I had written. So, this is the energy, each electrons gains per second from the electric field. And as a result, its temperature will rise.

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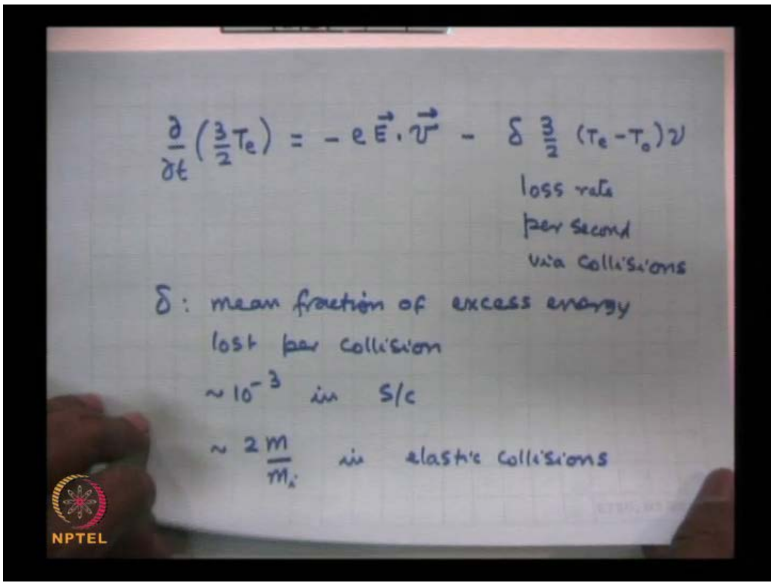

So, let me write down the equation, governing electron temperature. Electron temperature equation would be, delta delta t of average kinetic energy of the electron is, 3 by 2 Kt. But k, the Boltzmann constant, I hide in temperature, so, write this quantity. And this is equal to the electric field work done by the electric field per second, which is this quantity. As a result the temperature will rise, when the electron temperature rises above the temperature of the background ions or neutral particles, the electron loses energy in collisions also.

So suppose, the excess energy of the electron is 3 by 2 T e minus T 0, for instance. T 0 is the temperature of the heavy particle then, ion or they scatter. If these were equal, then the energy lost in the collision is very tiny, negligible. But, when this difference increases, this is called the excess energy of the electron, over the equilibrium value and in each collision, delta fraction of energy is lost.

If there are nu collisions per second, then nu times this quantity is the energy lost per second. So, electrons are gaining energy from the electric field and this is called the energy loss rate per second, via collisions. Delta is the mean fraction of energy lost, mean fraction of excess energy lost per collision. In case of semiconductors, the energy lost per collision is typically, 10 to minus 3, in solids, in semiconductors. In case of plasmas, if the collisions are elastic collisions, this is of the order of 2 m upon m I, in elastic collisions. Let me define, what is elastic collisions? When an electron collides with an atom, it can cause two things, that it can cause the excitation of electrons of the atom to high energy states. Such collisions are called inelastic collisions, because the electron energy is transferred to internal energy of the atom.

On the other hand, if electron suffers a collision and internal structure of atoms is not modified, the electrons does not go to higher energy state in the atom, then the collision is called elastic collision. In elastic collision, this is the fraction of energy lost. Similarly, with a collision with ions. So, this could be of the order of 10 to minus 3, 10 to minus 4.So, in collisions, this small energy is lost. And in the steady state, what will happen? The energy gain will balance with the energy loss. If you equate these two quantities, what you get?

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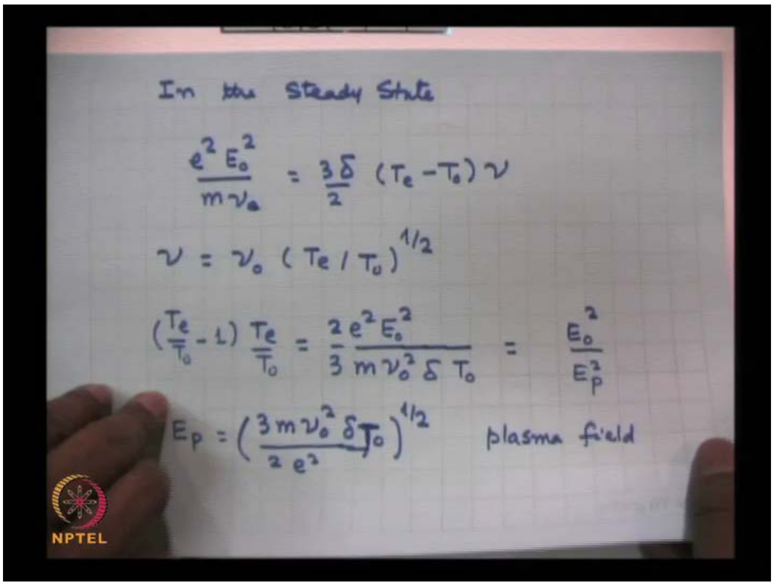

So, in the steady state, when you balance the energy gain from the electric field to the energy lost to the electric field, you get, e square E 0 square upon m nu 0, nu rather, is equal to delta 3 by 2 into T e minus T 0 into nu. I will consider two cases. One is the case, where collision frequency increases the temperature, like some nu 0 into T e upon T 0 to the power half. This is the kind of dependence, you have in intrinsic semiconductors, when the doping level is low or in a weakly ionized plasma. If I take this dependence in here, I can solve the equation for temperature and let me do this.

This equation can be rewritten as, T e upon T 0 minus 1 and this, I can write down, T e upon T 0 outside, is equal to e square E 0 square upon m nu 0 square 2 upon 3 delta into

T 0. This equation can be recast in this form and one can see here, that the left hand side is dimensionless and hence right hand side should also be dimensionless, means the coefficient of E 0 square, should have the dimension of 1 upon e square. So, I call this as, E 0 square upon E P square. Where E P, I am defining as, the quantity 3 m nu 0 square delta upon 2 e square under root into T 0 also is there. So, T 0, let me write this, T 0 there. This is called plasma field. You may note, if E 0 is much less than E P, then you can ignore this term and T e is equal to T 0 is the solution.

So, a significant departure of electron temperature from equilibrium temperature T 0, would be extracted, when E 0 comparable to E P of larger. So, this is really a threshold field for the onset of hot electron effects. So, plasma field is an important parameter in plasma or a semiconductor and this is not very huge quantity.

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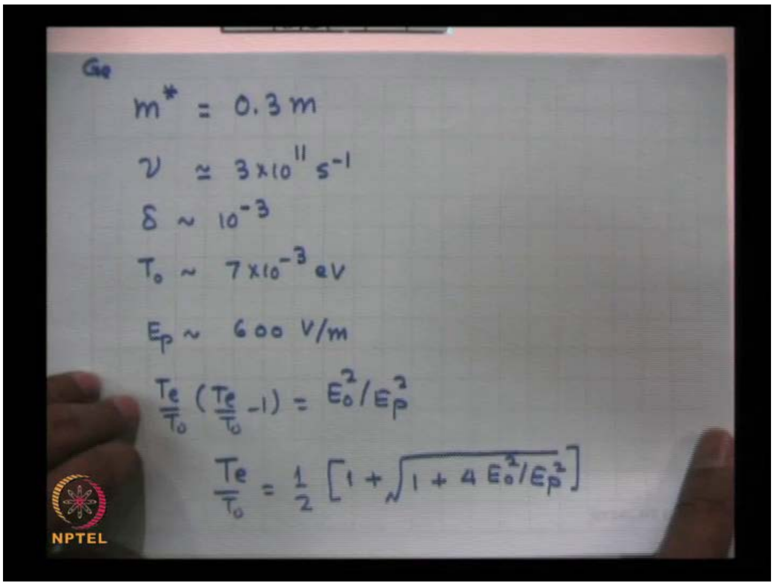

Let me, just give you some typical values, in case of a semiconductor, for instance. You know that, in semiconductors, the electron mass m, let me call this mass is m star called effective mass of the electron in a semiconductor, is equal to, is sometimes less than the free carrier mass. For instance, in germanium, it is like 0.3 into mass m, free. This is a mass of electron in free space, I will choose this. So, in germanium kind of semiconductor, if I choose this, the collision frequency at low temperature, I can choose like 3 into 10 to the 11 per second. And delta, I will choose around, 10 to the power minus 3. And suppose I choose the electron temperature at liquid nitrogen, which is like

7 into 10 to the power minus 3 electron volt. Then, if you calculate E P, it turns out to be of the order of about, 600 volt per meter, which is really quite low field. So, in germanium, this is a very small field, that you have to apply. Just, apply a few (( )) of volts across a semiconductor of a small thickness, you will be able to achieve this condition and hence, the electrons will acquire a temperature higher than the temperature of the lattice.

Now, what is the solution to this equation? The equation was, T e upon T 0 into T e upon T 0 minus 1 is equal to E 0 square upon E P square. If you solve this, it turns out to be, T e upon T 0 is equal to half, this is 1 plus, under root 1 plus 4 E 0 square upon E P square. This is a very important phenomenon in semiconductors. In 1950s, early 1950s, J. B. Gunn observed the characteristics or consequences of such hot electrons in semiconductors.

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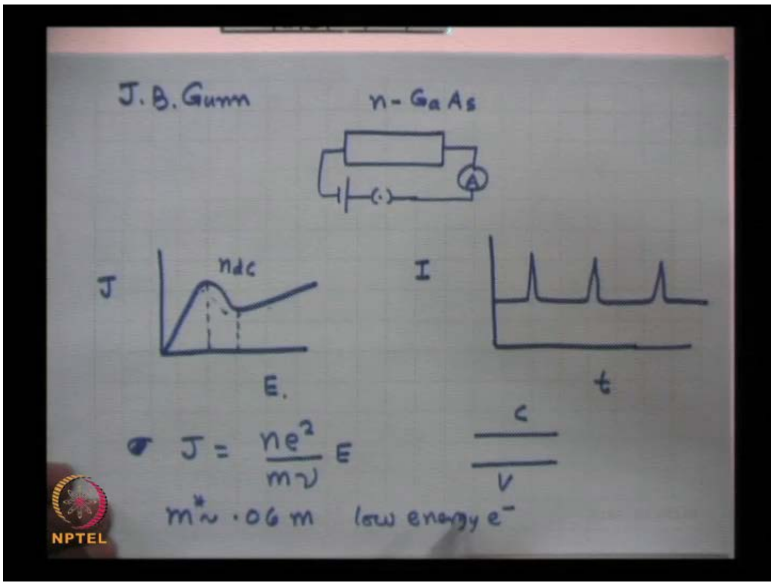

Especially, when he was carrying out experiments on gallium arsenide, he found something very spectacular. What he observed was this, J. B. Gunn, because of this hot electron effect, what did he observe? He considered a n type gallium arsenide sample and he applied a dc voltage here and put an emitter here and observed current in the circuit. When the voltage was low, the current in the circuit was constant. But, when the voltage exceeds certain threshold value, he observed that current here, as a function of time, he observed. So, current had some value and then suddenly, it will show the works like this. It was something very spectacular. He applied typically, 1000 volt per centimeter voltage and he observed this phenomenon. Well, then, and he got noble prize for this. The issue was, what is happening in there? People try to understand, probably, he himself try to understand this.

So, he says that, if I plot a graph of current as a function of electric field, the graph was like this, at low electric field, when the voltage was low. Then the, it goes like this, but then, it wave like this. The slope of this J versus E curve was different here, than there. This region, between this peak and this minimum is called the region of negative differential conductivity ndc. Because, current decreases with electric field, more the (( )) has the current, that is something very spectacular. What happened?

You just, look at the expression for conductivity or rather J. J equal to sigma e and sigma is n e square upon m into nu into electric field e. The answer to this puzzle lies in this equation, here mass is a quantity, in gallium arsenide, the electrons are have two bands here; one is called valence band and a conduction band. They are separated by forbidden gap. But, all the electrons in the conduction band, do not have same effective mass. The electrons of low energy, they have a smaller mass and the ones of higher energy have higher mass. In semiconductor terminology, we say that, the electrons in the conduction band have many values. The electrons of low energy, their effective mass is, around 0.06, this is effective mass, into free space mass, low energy electrons.

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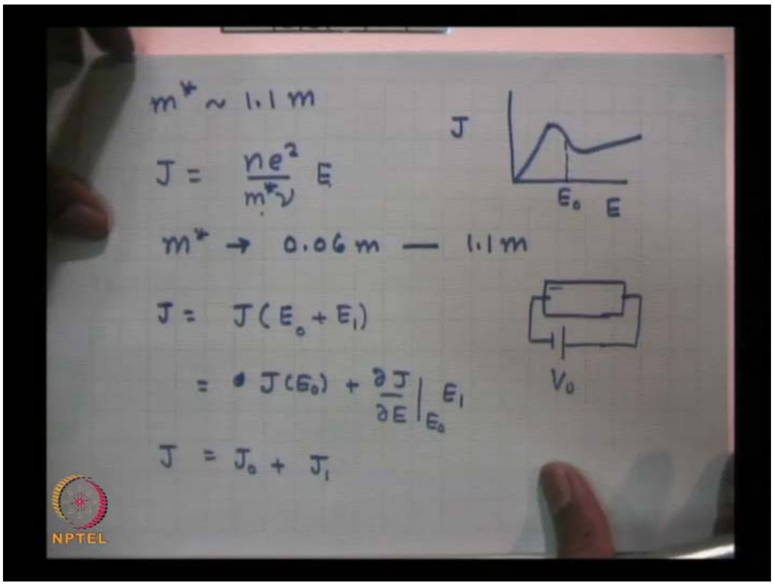

Whereas, high energy electrons in the conduction band have a higher effective mass and that effective mass is, m star of the order of 1.1 into m, where m is the free space electron mass. So, what is happening here? That if you look at the expression for J, which is equal to n e square upon m nu into e, the electrons at low electric fields have low energies and consequently, mass is low. So, conductivity is large, current is large. But, as the electric field is increased, when you vary the electric field in the circuit, then the electrons acquire larger energies and they get heated, they acquire large energies. So, when the kinetic energy becomes large, the effective mass increases. So, m star, this is actually should be m star here, m star goes from 0.06 m to 1.1m.

And that, the transition that takes place here. So, the graph becomes like this. So, when you plot J versus E, the electrons current density increases initially, because m does not change much. Electric field increases, J increases. But later on, when you increase E, this m increases rapidly. So, electrons get transferred to higher energies and then, this falls down and then, you get this kind of behavior. Now, what is the consequence of this, you can very easily see. If I have, J is equal to say, current density, a function of electric field E. Suppose, the electric field that we apply, is biased at this voltage E 0, the external battery that you apply in the circuit, suppose produces an electric field E 0. But suppose, there is a small fluctuation in the electric field, E 1. I would like to find out, what is the consequence of this, enhance the electric field in the semiconductor.

So, your semiconductor, you are biasing with an electric field E 0, voltage v 0, length is L. So, v 0 by L is E 0. And now, let us see, what happens? Suppose, in the semiconductor, there are additional electric fields produced by the accumulation of charges or ratification of charges somehow, what will happen? This, I can expand by (( )) this is small, this will give me, J at E 0 plus delta J upon delta E at e 0 into E 1, this is the Taylor expansion of current density. So, current, this is called equilibrium current. This is called the perturbed current. So, I will call this as, J 0 plus J 1. So, now the current in the semiconductor is, J 0 plus J 1. And if this J 1, could be a function of position and time, let us see, how it will evolve?

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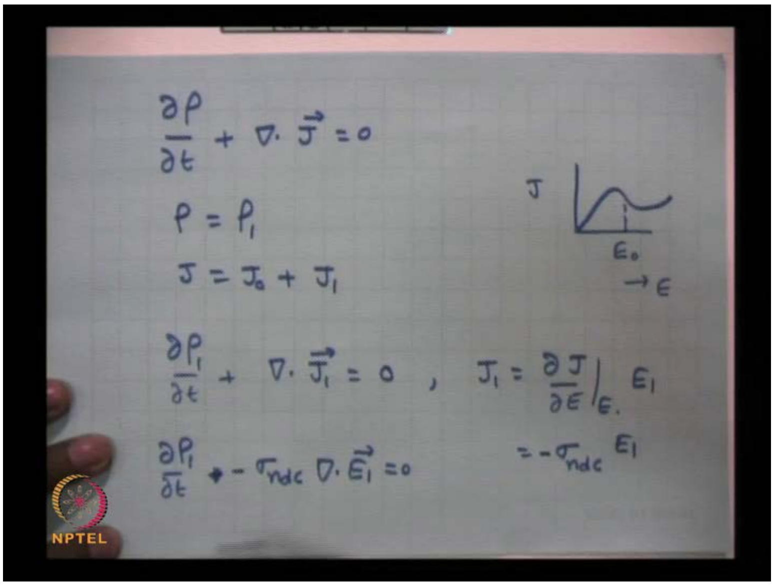

If there is a current in the system, If you look at the equation of continuity, rate of change of charge density plus divergence of J equal to 0. If I put, J equal to J 0 plus J 1; J 0 is the uniform current density, J 1 could be perturbed current density. So, write rho is equal to rho 1, because in equilibrium, there is no net charge. So, there is only perturbation. And J, I am writing as J 0 plus J 1. Put this in there, this becomes delta rho 1 by delta t plus divergence of J 1 is equal to 0. But, J 1 I had written as, delta J upon delta E at E 0, where we have biased the semiconductor, into E 1. This ratio is normally, written as, sigma ndc and the slope was negative, so this can be written like this. So, if I write this ratio as, sigma ndc with a negative sign, because this slope was negative. Just like this, even if it is, remember, I am talking of, this is my E 0 here, I am plotting E here and J there, so slope is negative, into E 1. So, if I put this in here, I get delta rho 1 by delta t plus, rather not plus, minus sigma ndc into divergence of E 1. But, we know divergence of E from first Maxwell equation is related to charged density.

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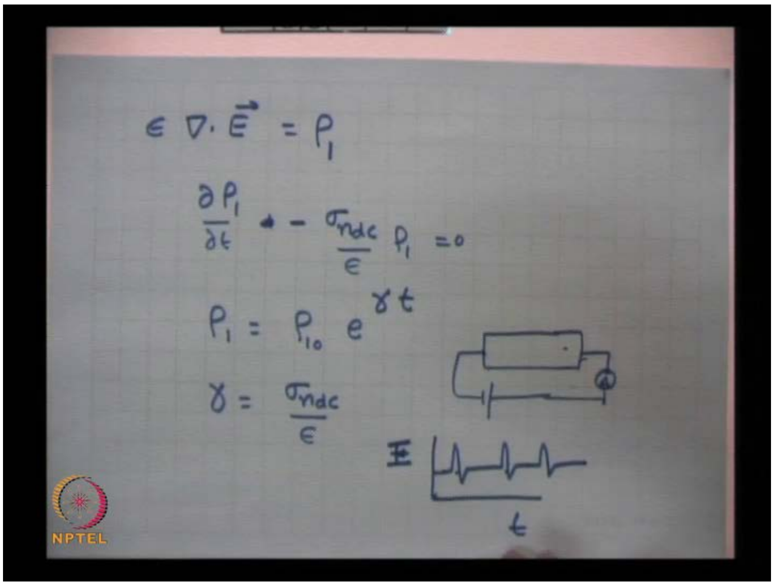

So, just put in there, divergence of E into permittivity is equal to rho, in a semiconductor. So, charged density is only rho 1, because in equilibrium, there was no charged density. It was electrically neutral. So, if I put this expression in this equation of continuity, I get delta rho 1 by delta t, sorry, minus sigma n d c upon epsilon into rho 1 is equal to 0 and this equation gives me a growing solution. This is equal to some constant rho 1 0, exponential of gamma t, where gamma is the called the growth period, which is equal to sigma n d c upon epsilon. What is happening in the semiconductor? Please examine this. You have a semiconductor like this; you are applying a dc voltage and you have an emitter here. So, what is happening? Somewhere, some misplaced charge is accumulating, density is increasing, is a function of space.

So, somewhere, when we apply this voltage in the negative differential conductivity region, then this high density accumulation region will move towards the end, the charge region will move or the electrons will move in this direction. So, when the bunch of electrons comes here, suddenly there is a enhancement in current, sudden burst. So, some sort of electron accumulation, electron domains are created inside the semiconductor. And then, you get this current up, actually current up slightly; I think, I was little casual; the current variation is something like this, J if I plot or current in the circuit, if I plot, as a function of time, this goes like this and then say, something like this. There is always a lower portion’s current. So, there is always a high electron density domain and a low density domain is created there, as a pair. And as it moves out to the wall, you get a sudden enhancement in current. This is a very important behavior, and you can have this frequency of repetition depending on the length of the specimen, and it could be in the microwave range.

Currently, Gunn diode is the cheapest source of microwave production. It does not cost more than few rupees, may be, maximum 100 rupees. It is very cheap. Just like, you get a diode laser, semiconductor diode laser for a very cheap cost; this also very cheap device, for production of microwaves. This is a very important application of hot electrons in semiconductors. So, we have learnt that these effects are, they arise at very modest electric fields and that can be applied very easily.

Now, the next issue is, what will happen in case of a plasma? Similar behavior, we would expect in a plasma, but not negative differential conductivity, because mass does not change that much. Mass can change, only due to relativistic effects, in plasmas. Free electrons do not have mass variation, unless the drift velocities are very high, they approach see, but what can happen in a strongly ionized plasma? Let us see.

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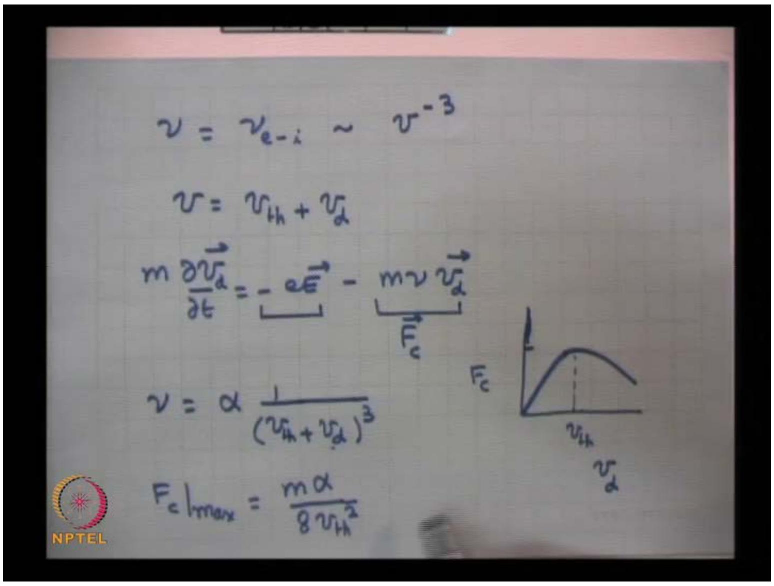

So, let me examine the behavior of a strongly ionized plasma, where the collision frequency is between electrons and ions. As I mentioned to you, collision frequency scales as, v to the power minus 3. This v is the sum of thermal velocity plus drift velocity. Let me call v d, the drift velocity. Actually, I have been calling the electron drift velocity simply as, v. But here, I want to call this total velocity, because in the collision frequency, this is the total velocity that really appears.

Now, please examine the equation of motion, m delta v by delta t is equal to minus e E minus m nu v. This is the equation of motion for an electron, in the presence of a dc electric field, which does not change with position, which is a constant in time, nu is the collision frequency. In the steady state, we want that this electric force should be balanced by the collisional drag force. Let us, plot the collisional drag force, let me call this quantity as F c, collisional drag force, magnitude wise. I will plot magnitude this quantity.

So, if I plot F c, as a function of, say velocity, v d rather. If this equation is written for drift velocity, so let me call this v d here and v d there. Please understand, nu I can write down, nu is equal to some constant, say alpha and 1 upon v thermal plus v d to the power cube. Let me write like this. When v d, and suppose, when v d is a small, this v d you can ignore and nu is simply alpha upon v thermal cube.

So, if I plot this quantity F c, as a function of v d, this will increase and then, it will acquire some peak value, because when this v d becomes comparable to v thermal or bigger, this will start falling down. This is a important characteristic, what you are expecting that, when we increase the electric field here, the drift velocity will increase and as a result, this quantity will increase and the situation will come, when these two balance each other.

But what is happening here? When you increase the drift velocity, this term does not increase and definitely, it acquires a maximum value, then stops. What is the maximum value? F c maximum, this occurs around v thermal, when drift velocity becomes comparable to v thermal, you get a peak. And F c maximum is equal to m alpha, this becomes like, 2 v thermal cube and this v d is also like, v thermal. So, it becomes, 1 v thermal will cancel out. So, 8 v thermal square. This is the largest value, it can have.

The question is, if you apply the electric field bigger than this quantity, if e E is bigger than this quantity, then there is no equilibrium. So, the electron will experience an electric force, which is never balanced by the collisional drag. And the electron will be continuously accelerated. This field, at which these two balance, this electric field is called Dreicer field.

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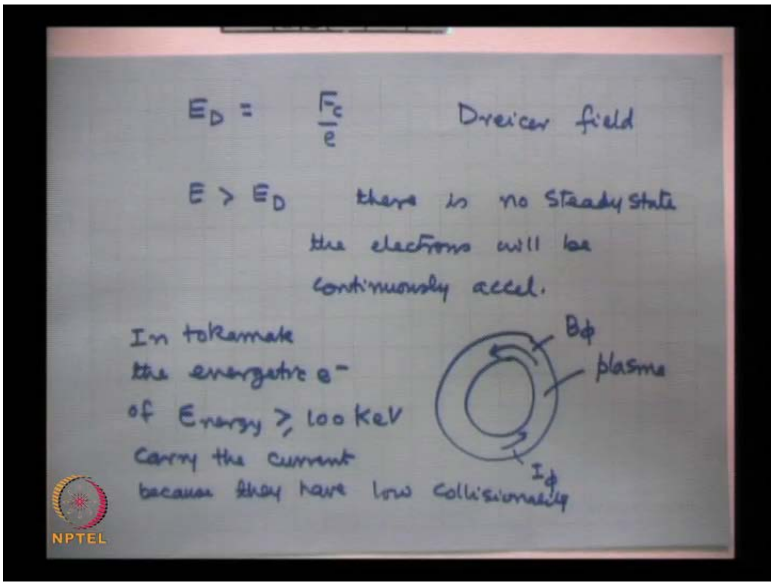

So, let me call this quantity, that Dreicer field E D, I am defining as, F c upon the electron charge called Dreicer field. And when E is bigger than E Dreicer, there is no steady state. The electrons will be continuously accelerated. Well, this problem has been observed. However, such high electric fields are not applied for infinite time or for an infinite length. So, once the electrons are out of this electric field region, their acceleration stops. So, the still, you get an upper limit on the electron velocities, because either the electric field is switched off or its length over, means x is limited. So, this is what you get.

In plasmas like tokamak, this is a very important phenomenon, because it has been recognized that in tokamak. Tokamak is a device like this, as a toroid, which contains a plasma here and this contains a dc magnetic field in this direction produced by coils. So, plasma is filled in these systems like this and it has been recognized that, this magnetic field is not enough to confine the plasma. So, you require a current also, in this direction. So, this is called a, current is required, I phi. If you do not have a current flowing in this direction, the plasma confinement is not possible. The electrons and ions move out to the outer wall and plasma cannot stay there for long.

The issue is who will carry this current? If you want to allow this current, by low velocity electrons, low energy electrons or high energy electrons. If you want to do it this, by low energy electrons, then the collision frequency for them will be very large. So, collisional drag force will be very huge. And hence, that will be very inefficient way of current drive. So, people have recognized that, you must drive a current in the tokamak, the current should be carried by energetic electrons. Though, the average energy of the electrons in the tokamak, is around 10 k e v or less. The current carrying electrons will have energies of the order of, few 100 k e v, because those electrons will have low collision frequency and hence, collisional drag on them will be weak. So, you will require only a small force to maintain them, movement of balance.

So, these electrons are essentially, some sort of a runaway electrons, because bulk of the electrons are 10 k e v or less energy. Some electrons in the system have large velocities and hence, low collisionality and they carry the current. So, in tokamak, the energetic electrons of energy greater than 100 k e v, carry the current, because they have low collisionality, low collision frequency. So, this is a very important phenomenon, which comes to our advantage of driving a current in a tokamak.

I think, these are a few fascinating phenomena, which have been observed in laboratory plasmas, a strongly ionized plasmas, encountered in the laboratory in fusion devices. And dc discharge is also important material processing, there also, the electrons have higher temperature then the background plasma. In fact, in most of the plasma devices, the electron temperature is higher than the ion temperature. Tokamak, for instance, the plasma temperature is about 1.5 times to 2 times higher than the ion temperature. In laser produced plasmas, the temperature difference could be even higher.

But the electron distribution function is not necessarily a maxwellian, in many of these devices, there is a strong high energy component of electrons, we call them as hot electrons or sometimes, we also call them energetic electrons. And they are responsible for lot of wave phenomena, damping of waves or some instabilities in the systems and they are also responsible, as I mentioned, for current driving plasmas.

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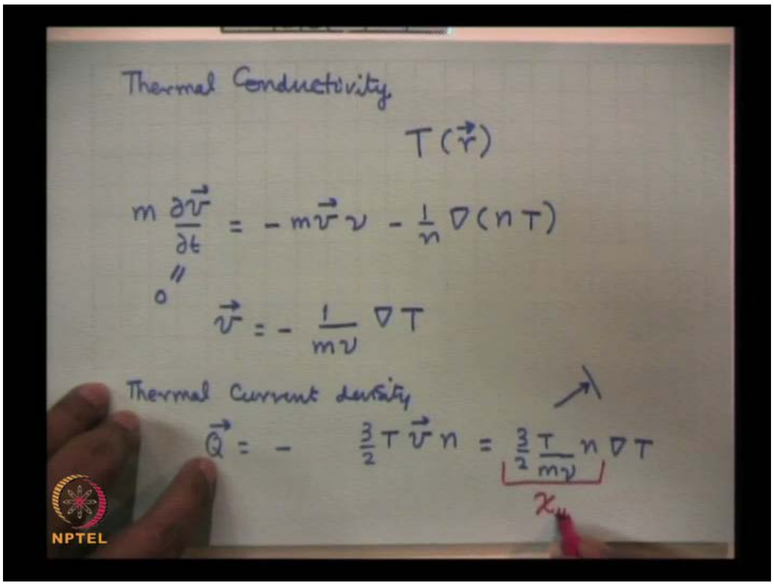

Well, now, I would like to say a few words about the diffusion and electrical thermal conduction in a plasma. The derivation of thermal conductivity and diffusion coefficient is very similar to the derivation of electrical conductivity. And let me, give you a simple derivation of this. Suppose, I have a plasma, whose temperature depends on position, then I would expect that, the plasma would have a high pressure in regions, where temperature is large and low pressure in regions, where pressure is low. So, I will consider the electron motion, that electrons, now, if you write down the equation of motion for the electron drift velocity, m delta v by delta t is equal to, there is no electric field, so I will not write the electric field term, minus m v nu and there is a pressure term, which is 1 upon n gradient of n T.

In their steady state, I ignore this term. So, just balance these two, I will get the drift velocity of electrons due to temperature gradient and you will get v is equal to minus 1 upon m nu into gradient of temperature, the sign is negative, which implies that the electrons will drift, this will have a average drift, from temperature, high temperature to low temperature because of negative sign. Now, I can define a quantity called thermal current density. Because, each electron when it travels, it carries an energy of 3 by 2 k T, k hidden in temperature, this is the average kinetic energy of an electron.

So, when electron travels, so suppose, the gradient of temperature is in this direction and the heat is going in this direction, temperature is varying in this direction and the heat is suppose, flowing in this direction. Then, in one second, how much heat is crossing this area, that is called Q and this will be simply, this into velocity into the number of particles it is carrying, this will like, n Q v or n e v, n v in electron charge is not flowing, is the electron energy that is going. So, I just put this quantity there. So, put the value of v, from here, it turns out to be, 3 by 2 T upon m nu into n gradient of T. This coefficient of T, gradient T is called thermal conductivity and is denoted by symbol khi.

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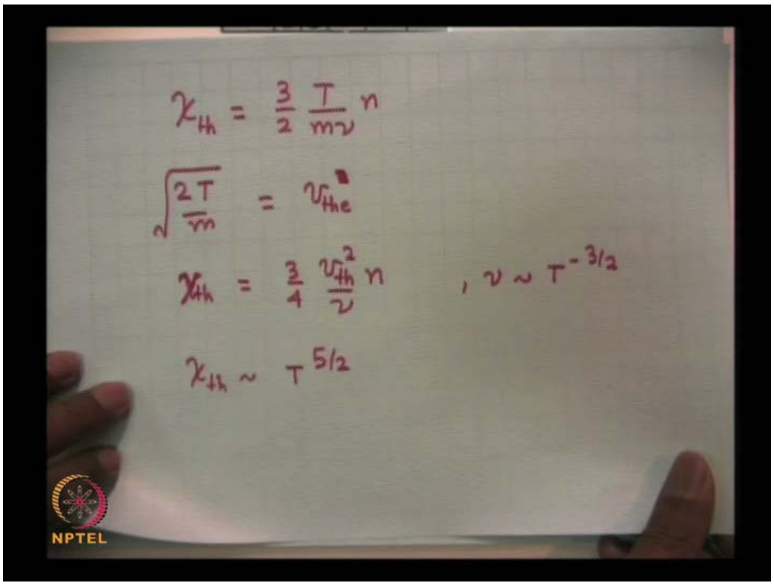

So, I will write down this quantity, thermal conductivity, let me write down, K t h to designate thermal conductivity. And I can write this expression for thermal conductivity, is equal to 3 by 2 T upon m nu into n, sometimes we write 2 T by m under root, is equal to thermal velocity of electrons square. Sorry, this quantity has thermal velocity of electrons. So, thermal velocity, this v thermal, sorry, khi thermal, thermal conductivity can be written as, 3 by 4 v thermal square upon nu into n. Please understand, in a strongly ionized plasma, collision frequency scales as, temperature to the power minus 3 by 2. Hence, thermal conductivity scales as, temperature to the power of 5 by 2 means a hotter plasma has much higher thermal conductivity than a cooler plasma. Similarly, one can have a simple derivation of diffusion coefficient.

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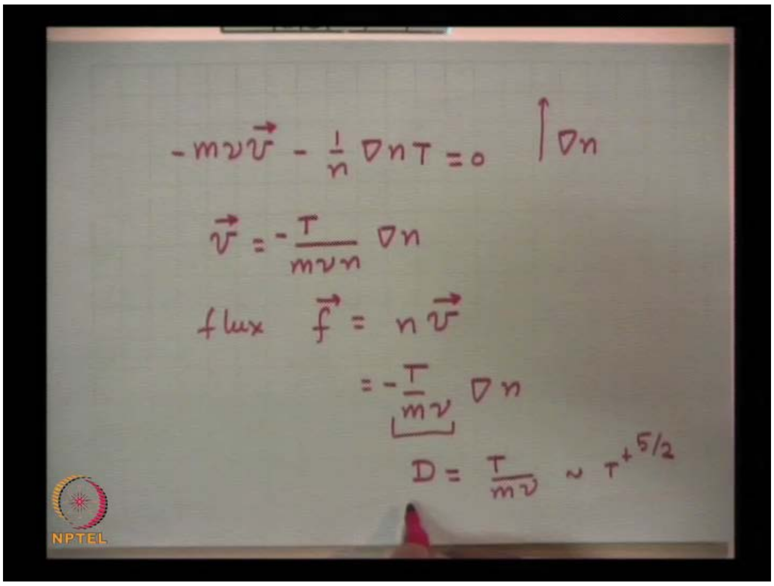

If a plasma has a density gradient for instance, suppose the density changes in some direction. So, density is a function of position. Due to this, the electrons will have a drift. Just similarly, you know, m nu v. In their steady state, this is the collisional drag on the electrons, and the diffusion term or pressure gradient force would be, 1 upon n gradient of n T, but temperature is uniform, density is non-uniform. So, I can take this out and put this nearly, equal to 0 in the steady state.

So, the velocity that you get from here would be equal to, T upon m nu into n into gradient of n, with a negative sign. Here, we define flux of particles. So, flux f would be, n into v. And just put this quantity, you will get, T upon m nu gradient of n, with a negative sign, this coefficient is called diffusion coefficient is denoted by symbol D. So, diffusion coefficient is T upon m nu. This is a very important quantity, is dependence on temperature is very similar to thermal conduction. At high temperature, when collision frequency goes as, temperature minus 3 by 2, this scales as, temperature to the minus 5 by 2, plus 5 by 2. So, these are two important coefficients, thermal conductivity and diffusion coefficient, which in conjunction with electrical conductivity, play a important role in determining the plasma dynamics. I think we close at this point. Thank you.