

**Plasma Physics**  
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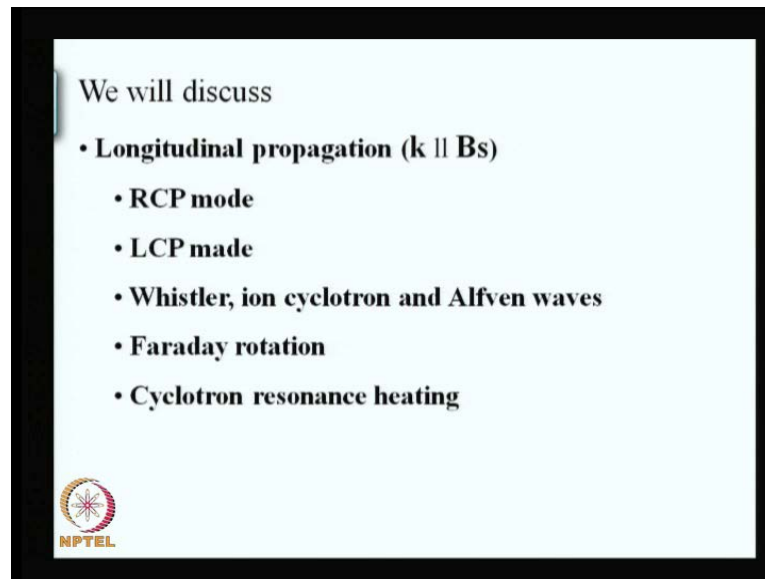
**Module No. # 01**

**Lecture No. # 29**

**Longitudinal Electromagnetic Wave Propagation**  
**Cutoffs, Resonances and Faraday Rotation**

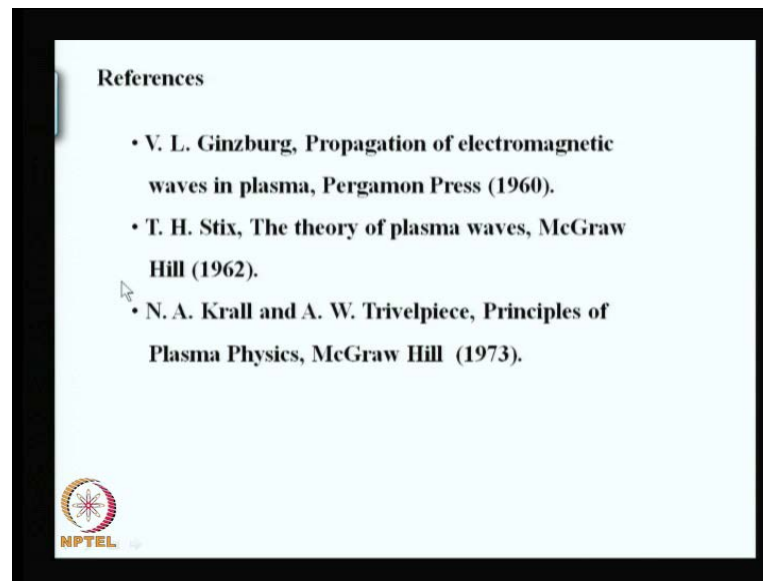
Today, we will continue discussing the propagation of electromagnetic waves in a magnetized plasma. We will discuss longitudinal electromagnetic wave propagation in the direction of magnetic field. We will discuss cutoffs, resonances and Faraday rotation.

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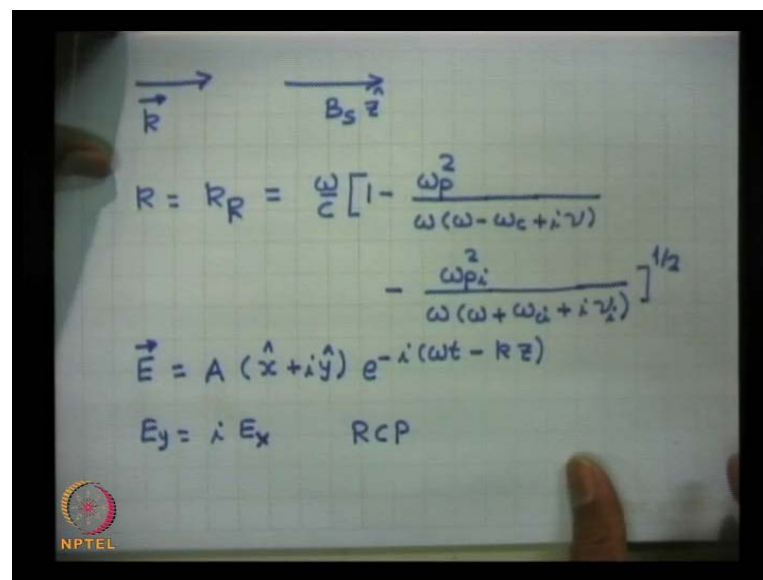
Well, with respect to right circularly polarized mode and left circularly polarized mode that we introduced yesterday, we will discuss these resonances and cutoffs. Then, we will discuss the lower frequency waves called whistler waves and cyclotron waves and Alfvén waves. We will discuss the phenomenon of Faraday rotation and cyclotron resonance heating of electrons and ions.

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The references for today's presentation would be three books by Ginzburg, Stix and Krall and Trivelpiece.

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Yesterday we were talking about the waves travelling in the direction of dc magnetic field. So, this was the direction of dc or **esthetic** magnetic field  $B_s$  which we took as z axis and we considered the propagation of wave with wave vector  $k$  parallel to  $B_s$ . And we found that the value of  $k$  depends on the estate of polarization of the wave. If the wave is right circularly polarized, then  $k$  was equal to  $k_r$   $k_r$  which we defined as

$\omega$  by  $c$  and  $1 - \omega_p^2$ ;  $\omega_p$  is the electron plasma frequency divide by  $\omega$  into  $\omega - \omega_c$ , where  $\omega_c$  is the electron cyclotron frequency.

Then, there was a term here; plus  $i$  times the collision frequency  $\nu$ . This is the electron contribution to  $k$  and the ion term was  $\omega_p^2$  which is ion plasma frequency square divided by frequency of the wave  $\omega$  into  $\omega + \omega_{ci}$  plus  $i$  times the collision frequency of ions to the power half. This was the wave value of wave vector when the electric field of the wave was some amplitude  $A$  and  $x$  cap plus  $i$   $y$  cap exponential minus  $i$   $\omega t$  minus  $k z$ . So, this implies that  $E_y$  is equal to  $i$  times  $E_x$ . This is called right circularly polarized wave RCP.

We noted that this expression for  $k$  as a resonance at  $\omega$  equal to  $\omega_c$  because usually  $\nu$  is very small as compare to cyclotron frequency. So, this term is very small, an order of magnitude is smaller than this, and hence in the vicinity of  $\omega$  equal to  $\omega_c$ , two things happen: Number 1, this term reverses the sign; for  $\omega$  bigger than  $\omega_c$ , this term is negative of this negative sign, but if an  $\omega$  becomes less than  $\omega_c$  this term becomes positive, if I ignore  $\nu$ . There is no reversal of such a sign in the case of ion response, but the electron response certainly shows a change of sign. And in the vicinity of  $\omega$  equal to  $\omega_c$ , one would expect  $k_r$  to be very large.

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$$\vec{S}_{av} = \frac{|E|^2}{2\mu_0\omega} \vec{k}_r$$
 For a given intensity  

$$|E|^2 \sim \frac{1}{k_r}$$

$$\nu = 0, \quad \omega \gg \omega_{ci} \quad (\text{ion motion is negligible})$$

$$k_r = \frac{\omega}{c} \left[ 1 - \frac{\omega_p^2}{\omega(\omega - \omega_c)} \right]^{1/2}$$

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And we should always keep in view that the pointing flux of a wave or intensity of radiation is equal to modulus of  $e$  square upon twice  $\mu_0 \omega$  and real part of  $k$ , where  $\mu_0$  is the free space magnetic permeability. So, you may note here that if I am sending a wave of a given intensity, then if  $k_r$  increases, then correspondingly amplitude of the wave decrease.

So, I would say that for a given value of intensity; pointing this is magnitude of this quantity is called intensity of the wave. So, for a given intensity, ((No audio from 05:50 to 06:00)) modulus of  $e$  square or wave amplitude square goes as  $1$  upon  $k_r$ ; the real part of  $k$ . For the sake of simplicity, we can easily ignore  $\nu$  to get some insight into the wave propagation. And let us consider the frequency is to be much bigger than ion cyclotron frequency. In that case, the ion motion is neglected. So, ion terms are neglected, negligible and your expression for  $k^2$  becomes or  $k$  becomes  $\omega$  by  $c \sqrt{1 - \frac{\omega_p^2}{\omega^2}}$ .

Now, you may note to the power half that because of this sign reversal, there will be a frequency for which this term is exactly equal to  $1$ . This entire (( )) this negative term besides a negative sign. So, when this quantity is equal to  $1$ , then this  $k$  becomes a  $0$  and on one side,  $k$  will be imaginary because this quantity will be negative. On the other side will be positive or real.

So, let me find out what is the cut off. Cut off means when  $k$  becomes  $0$ , the frequency of the wave for which  $k$  becomes a  $0$  is called a cut off frequency.

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Cutoff (Right handed cutoff)

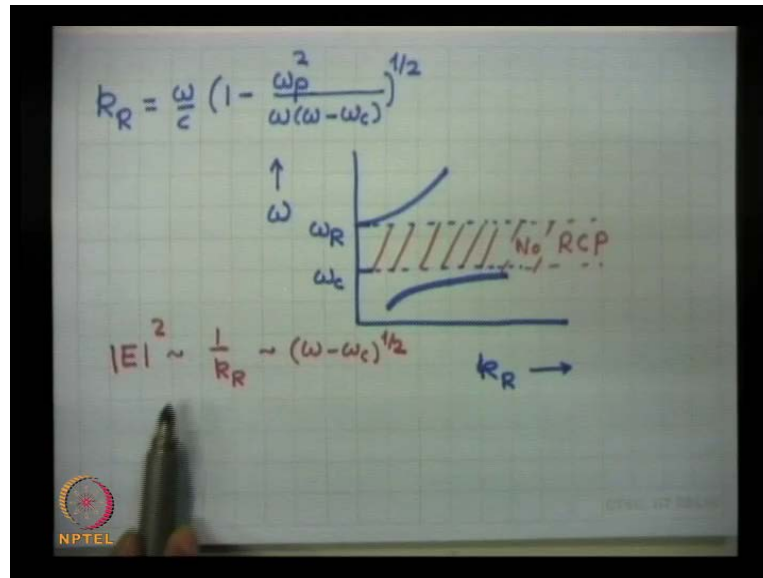
$$k_R = 0$$
$$\omega(\omega - \omega_c) = \omega_p^2$$
$$\omega^2 - \omega\omega_c - \omega_p^2 = 0$$
$$\omega_R = \frac{1}{2} \left[ \omega_c + \sqrt{\omega_c^2 + 4\omega_p^2} \right]$$

Resonance at  $\omega = \omega_c$

So, cut off and because we are talking about the right circularly polarized wave, so, we will call the right handed cut off. ((No audio from 08:02 to 08:14)). So, I am looking for  $k_r$  equal to 0 which implies that  $\omega$  into  $\omega$  minus  $\omega_c$  should be equal to  $\omega_p$  square. And when I am bring this, I can re-write this equation as  $\omega$  square minus  $\omega$   $\omega_c$  minus  $\omega_p$  square is equal to 0, giving two roots and a root the positive this frequency would be  $\omega$  equal to  $\frac{1}{2} \omega_c$  plus under root of  $\omega_c$  square plus 4  $\omega_p$  square. And we call this frequency at which  $k_r$  becomes 0 is a right handed cut off. So, we denote by  $\omega_r$ .

So, this is a important frequency. If you have a wave of frequency higher than  $\omega_r$ , the wave will travel. If the frequency is less than  $\omega_r$ , it will not travel, but this is not true for all low frequency, all the low frequencies because  $k_r$  has a resonance at  $\omega$  equal to  $\omega_c$  at which the plasma term goes to infinity and reverse a sign.

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So, if I plot. So, at this point  $k$  goes to infinity, let us see how it happens. It will be better to plot this expression  $k$  which is  $k_r$  for the right circularly polarized wave as  $\omega$  by  $c \sqrt{1 - \frac{\omega_p^2}{\omega(\omega - \omega_c)}}$ . This is usually plotted as  $\omega$  versus  $k$  here and  $k$  here.

So,  $\omega$  I am having say suppose  $\omega_c$  is somewhere here and right hand cut off is somewhere here. So, what we are doing here? When we have ignored the ion motion, then this equation is true and also when we have ignored the collisions. In that case, at  $\omega$  higher than or rather  $\omega$  less than  $\omega_c$ , certainly all frequencies are possible because this entire quantity becomes positive. So, when  $\omega$  is less than  $\omega_c$ , this is positive and  $k_r$  is certainly is possible for all frequencies.

So, let me this goes like this because as  $\omega$  approaches  $\omega_c$  from below, then  $k_r$  goes to infinity; positive infinity; however, when  $\omega$  becomes slightly more than  $\omega_c$ , then this quantity with this turn without this negative sign is negative is positive, but there is a negative sign.

So, when  $\omega$  is slightly more than  $\omega_c$ , this entire quantity is highly negative and  $k_r$  is imaginary. Wave does not travel. It will travel only when this quantity becomes 1 or less than 1. So, then the mode goes like this. So, there is a gap here between these two frequencies; electron cyclotron frequency and right handed cut off that there is no

propagation. This is the forbidden frequency band. There is no right circularly polarized wave propagating in this frequency band.

It is an important consequence; however, to appreciate the significance of this resonance, let us see what will happen? At the resonance,  $k r$  will increase. So, wave amplitude, as I mention to you, that for a given intensity, wave amplitude goes as or square of this goes as 1 upon  $k r$ . So, around the resonance, you expect that  $k r$  becomes very large. So, this becomes a small. This goes as because  $k r$  goes as, if I ignore this 1, then this goes as  $\omega - \omega_c$  inversely. So, this goes as  $\omega - \omega_c$  to the power half. So,  $e$  square goes like this.

However this resonance can have very important implication for the heating of electrons; obviously, this equation is not valid exactly at  $\omega$  equal to  $\omega_c$  because when this becomes 0, the collision term that we had we had here, you can ignore it. So,  $\nu$  has to be retained. So, this is cannot be exactly zero.

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$$m \frac{d\vec{v}}{dt} = -e\vec{E} - e\vec{v} \times \vec{B}_s - m\nu\vec{v}$$

$$-i(\omega + i\nu)\vec{v} + \vec{v} \times \omega_c = -\frac{e\vec{E}}{m}$$

X Comp

$$-i(\omega + i\nu)v_x + \omega_c v_y = -\frac{eE_x}{m}$$

Y Comp

$$-i(\omega + i\nu)v_y - \omega_c v_x = -\frac{eE_y}{m} = -\frac{i eE_x}{m}$$

$$v_y = i v_x \quad \vec{v} = \frac{e\vec{E}}{m i (\omega - \omega_c + i\nu)}$$

Now, let us examine the electron velocity around a resonance. The electron velocity is governed by the equation of motion which is  $m \frac{d\vec{v}}{dt} = -e\vec{E} - e\vec{v} \times \vec{B}_s - m\nu\vec{v}$ . I will ignore the  $\vec{v} \cdot \nabla \vec{v}$  term because it is due to linearization, and this is equal to  $-e\vec{E} - e\vec{v} \times \vec{B}_s - m\nu\vec{v}$ . This is the equation of motion and  $\frac{d\vec{v}}{dt}$  if I because my electric field is the source and  $\vec{v}$  everywhere is the  $\cos$ ; is the perturbed

electron velocity. So, if I replace this by  $\Delta \Delta t$  by  $-i\omega$ , this equation takes the following form.  $-i\omega + i\nu$ ; when I combine this term with this after dividing by  $m$ , you get  $v + v \times \omega c$  is equal to  $-eE$  upon  $m$ .

It will be useful to write down the  $x$  and  $y$  components of this equation because electric field has  $x$  and  $y$  components.  $x$  component would be  $-i\omega + i\nu v_x + \omega c v_y$  is equal to  $-eE_x$  upon  $m$ . And the  $y$  component would be  $-i\omega + i\nu v_y - \omega c v_x$  is equal to  $-eE_y$  upon  $m$ , but  $E_y$  is equal to  $iE_x$ . So, I can write down this  $-i\omega + i\nu v_x + \omega c v_y$  is equal to  $-eE_x$  upon  $m$ . So, what I am saying is that if I multiply this equation by  $i$ , this should be equal to this equation. So, multiply this by  $i$  and equate to this. You will immediately get that  $v_y$  turns out to be equal to  $i$  times  $v_x$ .

These two equations; the first one when you multiply by  $i$ , the right hand side becomes equal in the two equations. So, multiply the first equation by  $i$  and subtract the second. You will immediately see  $v_y$  is equal to  $i v_x$  and the interesting consequences that once you get this if I put  $v_y$  is equal to  $i v_x$  here, then  $v_x$  can be explicitly written in terms of  $E_x$  and the result is that velocity turns out to be simply equal to  $eE$  vector upon  $m$  into  $i\omega - \omega c + i\nu$ .

So, for a right circularly polarized wave, the electron velocity has a resonance denominator because  $\nu$  if  $\nu$  I ignore, then there is a resonance in the velocity at  $\omega$  equal to  $\omega c$ . That is a very interesting thing. So, the total response of electrons to the right circularly polarized electromagnetic field is to acquire a right circularly polarized drift velocity with resonance at  $\omega$  equal to  $\omega c$ .



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Handwritten derivation on a whiteboard:

$$\begin{aligned} \text{Heating rate} \\ \vec{H} &= \text{Re}(-e\vec{E}) \cdot \text{Re}(\vec{v}) \\ H &= \frac{1}{2} \text{Re}[-e\vec{E}^* \cdot \vec{v}] \quad \text{Time av.} \\ &= \frac{e^2 \nu \vec{E} \cdot \vec{E}^*}{2m(\omega - \omega_c)^2} \quad , |\vec{E}|^2 \sim (\omega - \omega_c)^{1/2} \\ &\sim (\omega - \omega_c)^{-3/2} \\ \text{Heating rate} &= \text{heat loss rate} \\ H &= \nu \frac{2m}{m_i} \left( \frac{3}{2} T_e - \frac{3}{2} T_i \right) \end{aligned}$$

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Now, let me calculate the heating rate. Heating rate of electrons is **sorry** heating rate is a scalar quantity. This is equal to work done by the electric field which is equal to the force the electric force on the electron is minus  $eE$  into the displacement per unit time which is called velocity. So, when you take the dot product of these two quantities; real part of this electric force with the real part of velocity, you will get the heating rate of electrons by the electric field per second. And this by using the complex number identity becomes this is equal to this into real part of minus  $eE^*$  dot  $v$ . This is called time average.

So, when I take time average of  $h$ , then I get a simple expression, and if I substitute for  $v$  and take the real of this product, it turns out to be  $e^2$  upon  $2m$   $\omega - \omega_c$  whole square then  $\nu eE^*$ . That is a interesting expression which express that it shows you that the heating rate is scales as  $1$  upon  $\omega - \omega_c$  square though as  $\omega$  approaches  $\omega_c$ ,  $e$  decreases; we had just seen, but this  $eE^*$  goes as **...** we have just seen that  $eE^*$  or modulus of  $e^2$  goes as  $\omega - \omega_c$  to the power half. So, when I put this in here, this entire quantity goes as  $\omega - \omega_c$  to the power minus three by two.

So, the heating is very strongly influenced by cyclotron resonance. That is something very important and if you want to find out the rise in a electron temperature because of

this wave which is traveling along the magnetic field, that is not difficult to calculate. It turns out to be heating rate if I have to equate; with the heat loss rate.

Now, in a elastic collisions, suppose like plasma is having only elastic collisions, then in a elastic collision, the fraction of energy that the electron loses per collision is 2 times the electron mass upon ion mass into the average kinetic energy of the electron minus the average kinetic energy of the ion. This is called the axis energy of the electron over the ion energy  $\left(\frac{m_e}{m_i}\right)$  when constant is hidden in temperatures.

So, this is the difference in average kinetic energy of an electron and an ion, and in each collision, this fraction of energy is lost. So, if there are new collisions per second, then the loss rate is so much, heating rate is h. Equate the 2. This nu will cancel out with the nu in h and you will get the rise in electron temperature over the ion temperature in terms of e square and certainly that will show a resonance at omega equal to omega c.

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$$T_e = T_i + \frac{e^2 A^2 m_i}{3 m^2 (\omega - \omega_c)^2}$$

$$\downarrow$$

$$\frac{1}{(\omega - \omega_c)^{3/2}}$$

$\omega = \omega_c$

$\omega < \omega_c + \text{throat}$        $\omega > \omega_c$

$z = -1/2$        $z = 0$

$$n_0 = n_0^0 (1 - z^2/L^2)^{1/4}$$

$$B_2 = B_{min} (1 + (m_R - 1) z^2/L^2)^{1/4}$$

So, the heating rate **one has...** So, the electron temperature rise is equal to the initial temperature or ion temperature plus a quantity which turns out to be equal to e square, which turns out to be actually A square of this wave divided by something like I think 3 into 2; this becomes like I think 3 m square m i divided by omega minus omega c whole square. And this term, as I mentioned to you, because A decreases A square decreases as

$\omega - \omega_c$  to the power  $3/2$ . So, this goes as  $1 / (\omega - \omega_c)^{3/2}$ . That is something beautiful here.

And; obviously, if the electrons are heated by the wave, then the wave must damp out because of collisions. So, if you retain  $\nu$  in the expression for  $k$ , you get an expression for the dumping rate, and let me just mention that expression. Before I actually do that, let me tell you the relevance of this kind of heating scheme.

We have learnt in a mirror machine that the lines of force go like this. The magnetic field is center of the mirror is small and large at the throats. These are the lines of force. And you can typically choose  $B_z$  of a mirror machine; axial magnetic field on the axis on this axis; this is  $z = 0$ , this is  $z = \pm L/2$ ;  $L$  is the length of the machine. So,  $L/2$  is the half length and  $\nu$  I can write down  $B_{\text{minimum}} = B_0 (1 - (z/L)^2)$ . And similarly, density has a maximum here magnetic field has a minimum on the axis; density has this kind of profile. I can write down  $n_0 = n_{00} (1 - (z/L)^2)$ .

So, the density of plasma is maximum here. If I launch an electromagnetic wave that RCP wave right circularly polarized wave here with frequency  $\omega < \omega_c$  at the throat, then this wave will travel and reach a point somewhere  $\omega = \omega_c$  locally. Beyond this point, this wave will not travel. And at this point that the cyclotron resonance condition is met and hence very strong heating will take place.

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$$1 \text{ T} \quad \omega_c/2\pi = 30 \text{ GHz}$$

$$B_s \sim 0.3 \text{ T} \quad \omega_c/2\pi = 9 \text{ GHz}$$

$$\nu \ll \omega - \omega_c$$

$$k_R = k_r + i k_i$$

$$k_r = \frac{\omega}{c} \left(1 - \frac{\omega_p^2}{\omega(\omega - \omega_c)}\right)^{1/2} = \frac{\omega}{c} \eta_R$$

$$k_i = \frac{\nu}{2c} \frac{\omega_p^2 / (\omega - \omega_c)^2}{\left[1 - \frac{\omega_p^2}{\xi \omega(\omega - \omega_c)}\right]^{1/2}}$$

So, the problem of wave propagation along the magnetic field lines is relevant to heating of a mirror machine. And please note that at one tesla magnetic field,  $\omega_c$  upon  $2\pi$  is 30 gigahertz. So, suppose I have a mirror machine that has a magnetic field of the order of say 0.3 tesla, suppose my B field is around 0.3 tesla or 3 kilogauss, then corresponding  $\omega_c$  upon  $2\pi$  would be 9 gigahertz. At lower value of magnetic field, it will be smaller; like suppose I choose a magnetic field of about 0.1 tesla, then this will be like 3 gigahertz, but that is and that is microwave frequency.

So, if I launch a microwave through mirror throat into the mirror machine axially, then electron cyclotron resonance will take place somewhere between the throat and the center. And you can match the frequency of the wave to the cyclotron frequency in the center, and then you can heat the center plasma in the center of the machine. So, you have a great advantage in picking the location where you would like to deposit the energy of your electromagnetic wave and this is an important scheme.

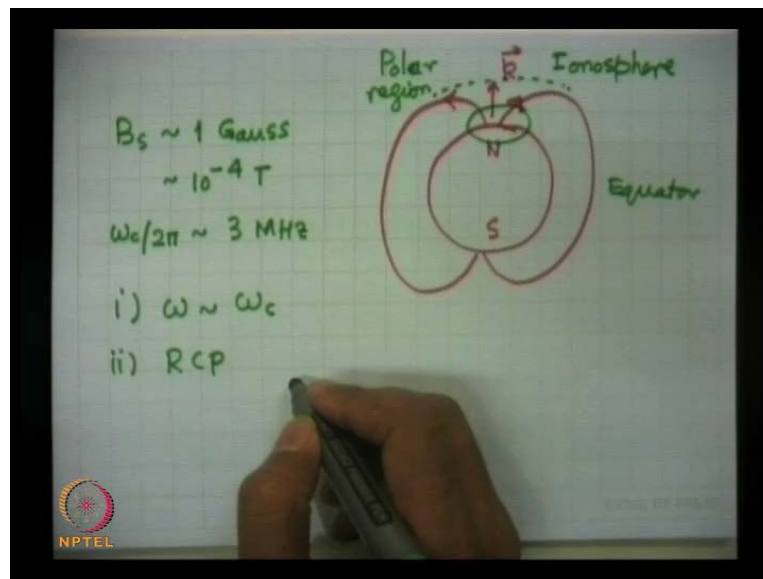
And then, I was talking about the imaginary part of propagation constant and if I include the collision frequency in the expression for  $k$ , but still presume that for  $\nu$  is still less than  $\omega$  minus  $\omega_c$ . In that case,  $k_r$  turns to be  $k_r$  I can write down is equal to real part of  $k$  plus  $i$  times the imaginary part of  $k$ .  $k_r$  real part is roughly the same as before which is  $\omega$  by  $c$   $1 - \omega_p^2$  upon  $\omega$  into  $\omega - \omega_c$  to the power half and  $k_i$  turns out to be equal to  $\nu$  upon  $2c$   $\omega_p^2$

divided by  $\omega - \omega_c$  whole square which is very strong dependence on difference between the  $\omega$  and  $\omega_c$  divided by  $1 - \omega_p^2$  upon  $\omega$  and  $\omega - \omega_c$  to the 1/2. This is very strongly dependent on cyclotron resonance. And this factor within this under root with including the under root sign is called refractive index of the plasma.

So, I can write down this is equal to  $\omega$  by  $c$ ; refractive index for right circularly polarized wave. So, this quantity within the bracket and under root is called refractive index of the plasma for right circularly polarized wave. This is bigger than 1, if  $\omega$  is less than  $\omega_c$ .

So, we have learned something about the relevance of these waves for plasma heating in mirror machine. These waves are also important for plasma heating in the ionosphere.

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Ionosphere is, suppose if this is my earth, and suppose this is North Pole of the earth, this is south pole of the earth. The lines of force go like this. These are the lines of force. So, if you are in the in the polar regions in the pole, this region is called polar region and you launch a wave in this direction or at some angles like a small angles, few degrees, then this is my direction of  $k$  and if I launch a right circularly polarized wave, then obviously, plasma is not there near the surface of the earth, but when you go to a height of about 90 kilometers and above, then, there exists a region here; ionosphere and then your wave

can have cyclotron resonance somewhere there and it can heat the electrons. Now, let me just mention, in the ((C)), this is called the equatorial region and these are the pole; this is called polar region polar poles polar region.

So, in polar regions, electron cyclotron resonance is certainly possible and in the magnetic field of the earth in the equatorial region is around 0.3 gauss and it is higher near the poles. Suppose I choose like 1 gauss; 1 gauss is around  $10^{-4}$  tesla.

So,  $\omega_c$  upon  $2\pi$  will correspond to 30 gigahertz into  $10^{-4}$  which is like 3 megahertz. So, if you are launching a wave of about 3 megahertz, it will meet a cyclotron resonance in the polar region of slightly lower frequency because magnetic field will be may be slightly less. So, if you are in this region, this in this region, if you are having a transmitter somewhere here and launching a wave vertically upwards, then you can heat the plasma via cyclotron resonance heating, ionosphere plasma.

Now, well let me summarize what I have really discussed about heating. I said that if you want electron cyclotron resonance heating, then you should have  $\omega$  close to  $\omega_c$  and the wave should be circularly polarized. So, two conditions are important:  $\omega$  should be around  $\omega_c$  and second, the wave should be RCP. Only then the cyclotron resonance will be there. They are the two conditions for resonance cyclotron heating of electrons.

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$\omega_i \ll \omega \ll \omega_c$  (ion motion can be ignored)

$$k_R = \frac{\omega}{c} \left( 1 + \frac{\omega_p^2}{\omega \omega_c} \right)^{1/2}$$

$$\sim \frac{\omega_p}{c} \left( \frac{\omega}{\omega_c} \right)^{1/2}$$

$$v_{ph} = \frac{\omega}{k_R} = \frac{c \sqrt{\omega \omega_c}}{\omega_p}$$

$$v_g = \partial \omega / \partial k_R = 2 v_{ph}$$

Whistler wave

$\omega \sim k^2$

Now, let me look into the waves of lower frequencies. So far, we have ignored the ion motion. We still can ignore the ion motion if  $\omega$  is much less than  $\omega_c$ , even then you can ignore ion motion, if this is still bigger than  $\omega_c$ ; the ion cyclotron frequency. This is called intermediate frequency range.

So, the waves of frequency bigger than ion cyclotron frequency but less than the electron cyclotron frequency is still ion motion can be ignored. And if I do this, in that case, the propagation constant for the right circularly polarized wave becomes  $\omega$  by  $c$  this becomes  $1 + \omega_p^2 / (\omega \omega_c)$  to the power half. In most plasmas of interest, this quantity is much bigger than 1. You can ignore unity. This becomes equal to  $\omega_p / c$  and  $\omega$  upon  $\omega_c$  to the power half.

This is a wave which has a phase velocity  $v_{ph}$  which is defined as  $\omega / k_r$ , and that would be  $c$ , just divide this, you will get this is equal to  $c$  under root of  $\omega \omega_c$  upon  $\omega_p$ . The phase velocity increases with frequency as under root. It is a dispersive medium and how about the group velocity which is defined as  $\Delta \omega / \Delta k_r$ . It turns out to be 2 times phase velocity. Peculiar case when the phase and group velocities are not equal to magnitude.

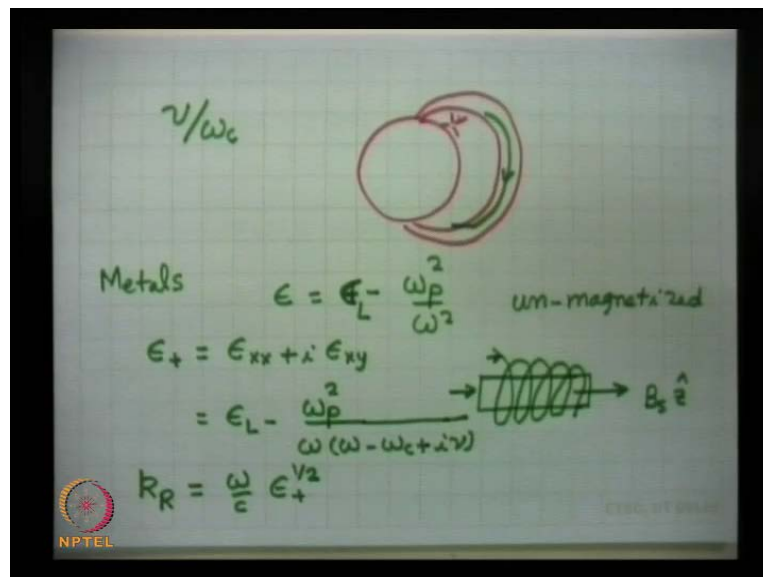
In an unmagnetized plasma, we had seen that group velocity is always less than the phase velocity, but in a magnetized plasma, at low frequencies, in this frequency band,

phase velocity is less than the group velocity. You can just check it. From here you can check that omega is proportional to k square. It is square both sides and we will see that omega goes as k square. Well a special kind of wave where omega goes as k square is called as whistler wave.

In early fifty's, this wave was observed during thunder storms. So, when there is lightning during rainy season, then these waves are produced somewhere; suppose this is my earth, these are kind of lines of force. Suppose I have produced some event has taking place and suppose this is the wave has been produced here by in some thunder storms some wave is generated, this wave has been found to be travelling, entering the ionosphere and travelling along the field lines like this, and can be observed here somewhere here or can leak in any point.

So, what is happening that these waves have a tendency to travel along the field lines. So, the field lines are curve, they also bend to travel along the field lines and can travel almost half the hemisphere from North Pole to South Pole, they can travel half the earth curvature. Very thousands of kilometers they can travel and with very little attenuation.

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The attenuation of these waves is characterized by a parameter called nu upon omega c. In the ionosphere, nu is around 10 to 3 10 to 4 collisions per second. Omega c is around 10 to 7 radian per second. So, this ratio is about 3 or 4 orders of magnitude small as



compare to unity and consequently the imaginary part of  $k$  which is responsible for damping is very weak. And these waves have been used for exploration of the ionosphere; upper regions of ionosphere as well as magnetosphere. They are very useful waves.

Well, these waves have been found to be very useful in case of metals also. We have seen that wave propagation in a unmagnetized plasma is very similar to wave propagation in a metal because effective plasma permittivity that we define in plasma in a unmagnetized plasma was  $1 - \frac{\omega_p^2}{\omega^2}$ , and for a metal this 1 was replaced by what we call as  $\epsilon_L$ ; lattice permittivity unmagnetized.

But if you take a metal rod for instance, and put it inside a solenoid or coil that carries current so that there is a magnetic field produced in this direction. And you can launch a wave through the metal by using another coil, RF coil. So, if you launch a wave here of frequency  $\omega$ , in the presence of magnetic field, the effective plasma permittivity which is relevant is  $\epsilon_{xx} + i\epsilon_{xy}$  and this turns out to be equal to  $\epsilon_L - \frac{\omega_p^2}{\omega^2 - \omega_c^2} + i\nu$ .

So, this is a quantity which is very similar to plasma except that this lattice permittivity appears in place of 1. So, your wave propagation constant for the right circularly polarized wave would be  $\frac{\omega}{c} \sqrt{\epsilon_{xx} + i\epsilon_{xy}}$ . So, when you are talking of frequencies much less than  $\omega_c$  and less than  $\omega_p$ , then this quantity is really unimportant as compared to this entire term. Then the character becomes very similar to in the case of plasma and this wave; this whistler wave in a metal is called helicon wave.

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Helicon wave

$$\omega \ll \omega_c < \omega_p$$

$$\eta_R = \epsilon_+^{1/2} \gg 1$$

$$v_{ph} \ll c$$

$$\omega < \omega_{ci}$$

$$\frac{\omega_p^2}{\omega(\omega - \omega_c)} \approx - \frac{\omega_p^2}{\omega \omega_c}$$

$$= - \frac{\omega_{pi}^2}{\omega \omega_{ci}}$$

So, this has a frequency much less than electron cyclotron frequency and usually in metals, this is less than  $\omega_p$  much less than  $\omega_p$  indeed. So,  $k$  is very large because  $\epsilon_+$  is very large, under root of this quantity which is called the refractive index of the right hand circularly polarized wave; this is much bigger than 1. This can be 10 to the power 4, 10 to the power 5.

So, these waves travel with a very low velocity. You can make the phase velocity of these waves much less than  $c$  and you can employ these waves for diagnostics like measuring the effective mass of electrons inside a metal. In some metals are having anisotropic mass; mass is a tensor like bismuth. You can explore those character is different components of mass by using these waves.

So, helicon waves have been found to be very useful for diagnostics of materials. These waves also adjust in plasmas, in semiconductors, intensive semiconductors as well as doped semiconductors. What then you have to include the effect of holes also, but there this is a fascinating field of study.

Well then, now let me go over to waves of even lower frequencies when I am talking of  $\omega$  comparable or less than  $\omega_{ci}$ . In this case, the electron term in your expression for propagation constant was  $\omega_p^2$  upon  $\omega$  into  $\omega - \omega_c$ .

Now, because your frequency is less than ion cyclotron frequency, it means it is much less than  $\omega_c$ . So, this is of the order of  $\omega_p^2$  upon  $\omega_c \omega_c$ , but  $\omega_p^2$  by  $\omega_c$  is the same thing. As  $\omega_{pi}^2$  upon  $\omega_c^2$  is same. Just substitute for  $\omega_p$  in terms of  $\omega_p^2$  is  $n_0 e E^2$  by  $m \epsilon_0$  and  $\omega_c$  is  $e v$  upon  $m$ . Just you can equate them and you find that mass cancels out, and they are equal exactly equal.

So, if I use this approximation for the electron term, then my propagation constant for the right circularly polarized wave takes the following form.

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$$k_R = \frac{\omega}{c} \left( 1 + \frac{\omega_{pi}^2}{\omega \omega_c} - \frac{\omega_{pi}^2}{\omega (\omega_c + \omega_c)} \right)^{1/2}$$

$$\approx \frac{\omega}{c} \frac{\omega_{pi}}{\omega_c} \frac{1}{(1 + \omega/\omega_c)^{1/2}}$$

$$c \frac{\omega_c}{\omega_{pi}} = V_A$$

$$k_R = \frac{\omega}{V_A} \frac{1}{(1 + \omega/\omega_c)^{1/2}}$$

$k_R$  becomes is equal to  $\omega$  by  $c$  into  $1$  plus  $\omega_{pi}^2$  by  $\omega_c \omega_c$ . This is the electron term and ion term is  $\omega_{pi}^2$  upon  $\omega_c$  into  $\omega_c$  plus  $\omega_c$  to the power half. The ion term certainly has a larger denominator than the electron term, and consequently, the positive electron term will be more than the ion term and because the frequency is very low, this electron or ion term both are really much bigger than  $1$ . I can ignore  $1$ . So, then add these terms and this becomes nearly equal to  $\omega$  by  $c$  into  $\omega_{pi}$  by  $\omega_c$  divided by  $1$  upon  $1$  plus  $\omega$  upon  $\omega_c$ .

Well, this quantity  $c \omega_c$  upon  $\omega_{pi}$ , hence the dimension of velocity and is given a name Alfvén velocity and we shall learn its relevance in a moment. So, then my

wave number  $k_r$  can be written as  $\omega$  upon  $v_A$  into  $1$  upon  $1 + \omega/\omega_{ci}$ ; this is the half under root here. So,  $1 + \omega/\omega_{ci}$  to the power half. If  $\omega$  were much less than  $\omega_{ci}$ , then this factor is like unity and these waves travel as if they travel in a medium with phase velocity equal to  $v_A$ . So, then  $v_A$  becomes the velocity of the wave of low frequency and the wave of  $\omega$  much less than  $\omega_{ci}$  is called Alfvén wave.

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$$\omega \ll \omega_{ci}$$

$$k_R = \frac{\omega}{v_A}$$

$$v_A = \frac{\omega}{k_R} = \text{phase velocity}$$

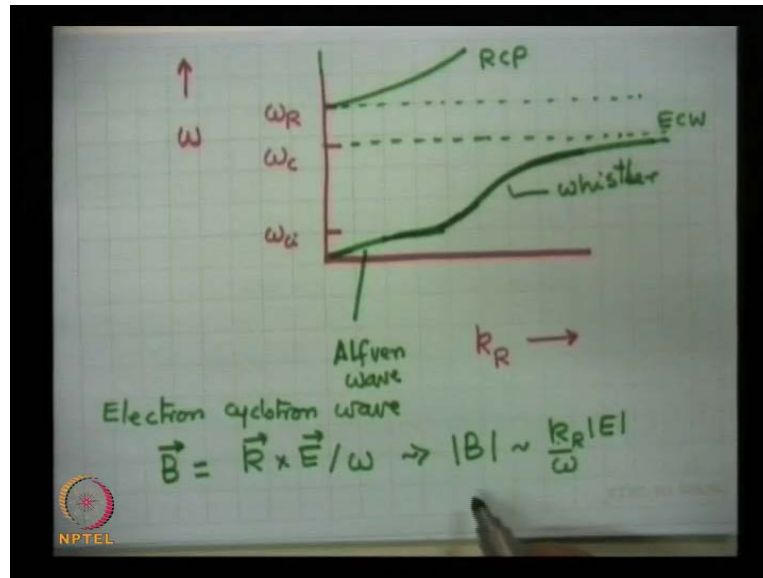
$$\omega \sim \omega_{ci}$$

$$k_R = \frac{\omega}{v_A} \frac{1}{(1 + \omega/\omega_{ci})^{1/2}}$$

So, this is the right circularly polarized Alfvén wave when it travels along the field line and so let me just explicitly write this that for  $\omega$  less than  $\omega_{ci}$ , the propagation constant of the right circularly polarized wave becomes equal to  $\omega$  upon  $v_A$ . So,  $v_A$  is equal to  $\omega$  by  $k_r$  which is called the phase velocity of the wave.

However; if a frequency is not much less than  $\omega_{ci}$ , in that case,  $k_r$  is no longer this much. So, when  $\omega$  is of the order of  $\omega_{ci}$ , in that case,  $k_r$  is equal to  $\omega$  upon  $v_A$  into  $1$  upon  $1 + \omega/\omega_{ci}$  to the power half and the wave is dispersive. This is not a dispersive wave because the phase velocity does not depend on frequency. It is simply  $v_A$ . So, when  $\omega$  is much less than  $\omega_{ci}$ , the wave the medium becomes non-dispersive, but when the frequency is comparable to  $\omega_{ci}$  or higher, in that case,  $k_r$  is this expression.

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So, we have learned different characteristics of electromagnetic wave in various frequency domains. Let me plot a graph of frequency versus  $k$ . I will plot  $\omega$  here and  $k$  for the right circularly polarized wave here. What I am saying that the frequency of interest  $\omega_1$  is called ion cyclotron frequency  $\omega_{ci}$  which is three or orders of magnitude are more smaller than  $\omega_c$ . So,  $\omega_c$  is somewhere here. And then there is a right cut off;  $\omega_r$ .

So, at very low frequency, your wave is an Alfvén wave and its characteristics and its dispersion relation would be something like this. But as it approaches  $\omega_c$ , then its character changes, and then it becomes like a whistler wave at higher frequency than  $\omega_c$ , and this goes increases as square  $\omega$  goes as  $k$  square. So, it goes as it becomes parabolic here, and then this goes to infinity. This is called cyclotron resonance here.

So, this is the branch here called whistler and this is the branch in the vicinity of electron-cyclotron resonance. This is a small bend of spread of frequency is called electron-cyclotron wave, and this low frequency part is called Alfvén wave here.

So, the whistler frequency starts actually from here to here; almost close to  $\omega_c$ . Just very close to  $\omega_c$ , the wave is given a special name called electron cyclotron wave, electron cyclotron wave. This is called; I can call this ECW and for frequencies higher

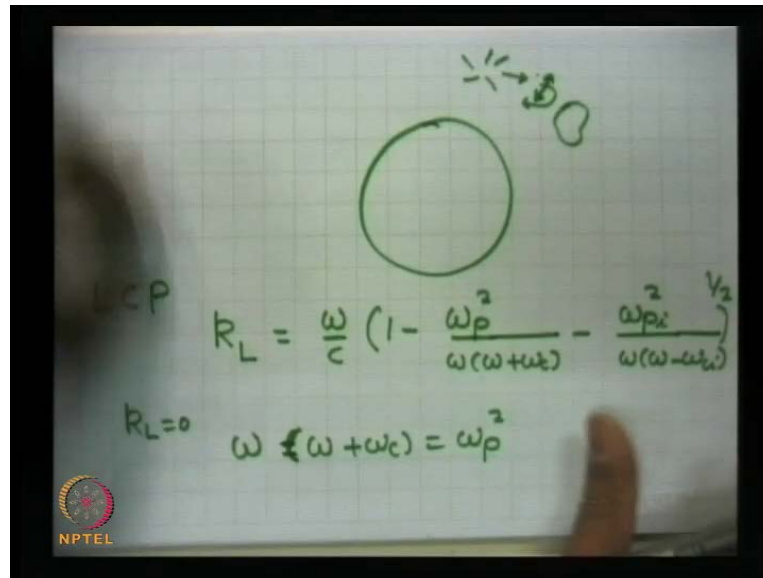
than right handed cut off, the waves goes like this. This is simply called RCP. All these waves are RCP waves when they travel in the direction of magnetic fields, but their characteristics are different, phase velocities are different; obviously, everything is right circularly polarized.

So, we have learned something about this. One thing more before I close my discussion on right circularly polarized waves that the magnetic field of these waves, if we use the third Maxwell's equation is given by  $\mathbf{k} \times \mathbf{E}$  upon  $\omega$  and if this is magnitude wise, so this gives you B magnitude is like k magnitude into magnitude of e upon  $\omega$ .

So, when k is large, this is right circularly polarized. So,  $k \approx r$ . Then b is very large. So, in the vicinity of cyclotron resonance, when k becomes very large, then this magnetic field of the wave is also very large as compared to electric field. And these waves can be employed for deflection of particles. Currently there is lot of efforts to employ large amplitude whistlers because they are primarily oscillatory magnetic fields to deflect particles.

Suppose I have a space craft, and I want to save it from some explorer outside because when some nuclear explosion takes place in outer space or somewhere, then it releases very energetic particles ions. And if I want those ions should not be entering the space craft and should not destroy its electronics, then I would like those particles to be deflected. Then, what you can do?

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If I have some sort of a space craft, suppose this is my earth and suppose this is my space vehicle somewhere here, and I want to protect it from stream of particles that could be produced somewhere in some explosion here. Suppose an explosion takes place somewhere and particles are coming, then I can produce waves with magnetic field perpendicular to this, then the particles which are going this way, these particles will be deflected back.

So, these large amplitude whistlers, because of their large magnetic fields, are very helpful in deflecting particles and currently there is significant amount of efforts being put in to generation of very high frequency waves. And in order to stimulate the conditions in the ionosphere or outer space rather, much above the ionosphere, one what people do? They are doing large scale experiments in laboratories to launch high frequency whistlers and examine the particle dynamics. So, this is about the right circularly polarized wave.

Now, a few similar is the character of LCP; left circularly polarized wave. The only thing is that in this case, the  $k$  vector of this wave is  $\frac{\omega}{c} \left( 1 - \frac{\omega_p^2}{\omega(\omega + \omega_c)} - \frac{\omega_p^2}{\omega(\omega - \omega_c)} \right)^{1/2}$  and the ion term is  $\frac{\omega_p^2}{\omega(\omega + \omega_c)}$  and  $\frac{\omega_p^2}{\omega(\omega - \omega_c)}$  to the power half.

So, rather than having electron cyclotron resonance, you are expecting a ion cyclotron resonance and similar properties occur. If you examine the cutoff of this wave for this wave means when  $k L$  becomes a 0, then you will discover that  $k L$  becomes 0 when  $\omega$  turns out to be... this term is really unimportant when you put  $k L$  equal to 0 because this just dominates over this and you can find this.

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Handwritten notes on a whiteboard:

$$\omega = \omega_L \equiv \frac{1}{2} \left[ -\omega_c + \sqrt{\omega_c^2 + 4\omega_p^2} \right]$$

$\omega < \omega_L$  does not propagate  
 $\omega > \omega_L$  propagates  
 $\omega < \omega_{ci}$  propagates  
 $E_y = -i E_x$  ions can be heated

NPTEL logo is visible in the bottom left corner of the whiteboard image.

So, the high frequency cutoff actually is can have a low frequency cutoff as well. The high frequency cutoff turns out to be is equal to is given by actually  $\omega$  into  $\omega$  plus  $\omega_c$  is equal to  $\omega_p$  square and it turns out to be  $\omega$  is equal to  $\omega_L$  left handed cutoff is equal to  $\frac{1}{2} \left[ -\omega_c + \sqrt{\omega_c^2 + 4\omega_p^2} \right]$ . So, there is a cutoff frequency.

But what is happening here? That a wave of  $\omega$  less than  $\omega_L$  will not travel. So,  $\omega$  less than  $\omega_L$  does not propagate.  $\omega$  bigger than  $\omega_L$  propagates, but if you make  $\omega$  substantially smaller less than  $\omega_{ci}$ , the ion term becomes change a sign and one gets propagation again. So, there is something spectacular happening at  $\omega$  equal to  $\omega_{ci}$ .

So, this wave has a very interesting characteristic. It is a left circularly polarized wave with  $E_y$  is equal to  $-i E_x$  in which ions can be re-cyclotron resonance sensitive and



ions can be heated **can be a heated** via ion cyclotron resonance and that is a very important wave.

In mirror machine, rather than launching a RCP high frequency wave, if I had launched a low frequency LCP wave of frequency less than or slightly less than  $\omega_{ci}$  or comparable to  $\omega_{ci}$ , I can heat preferentially the ions and that is the beauty here. I think other characteristics like Faraday rotation and other phenomena of wave propagation in magnetic plasmas, we shall continue or discuss in our next lecture. I think I will stop at this stage. Thank you very much.