Plasma Physics

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Module No. # 01

Lecture No. # 28

Electromagnetic Waves Propagation in Magnetise Plasma

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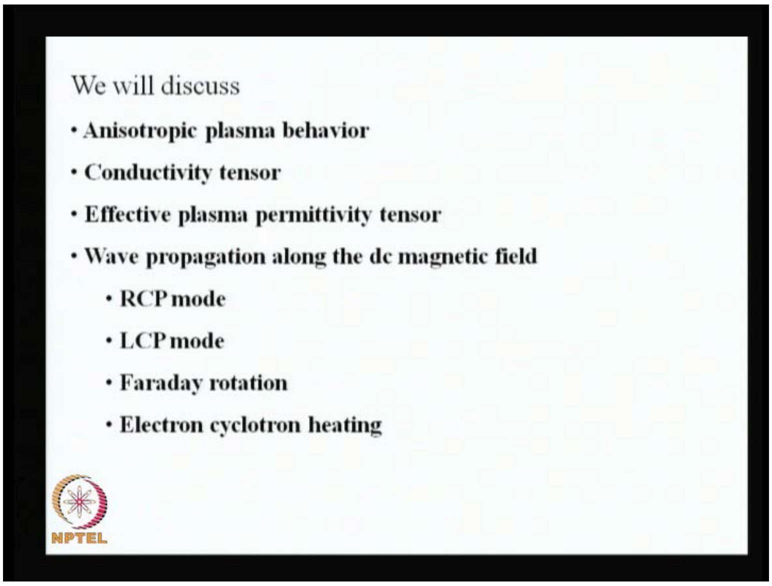

Today, we will discuss electromagnetic wave propagation in magnetize plasmas. Here, we shall discuss the anisotropic nature of a plasma response to an electromagnetic field, and we shall talk about conductivity tenser, effective plasma permittivity tensor, and then we shall discuss a special case of electromagnetic wave propagation along the dc magnetic field. We shall discuss the propagation of the right circularly polarized wave, then left circularly polarized wave, then the phenomenon of Faraday rotation, and we shall also talk about electron cyclotron heating.

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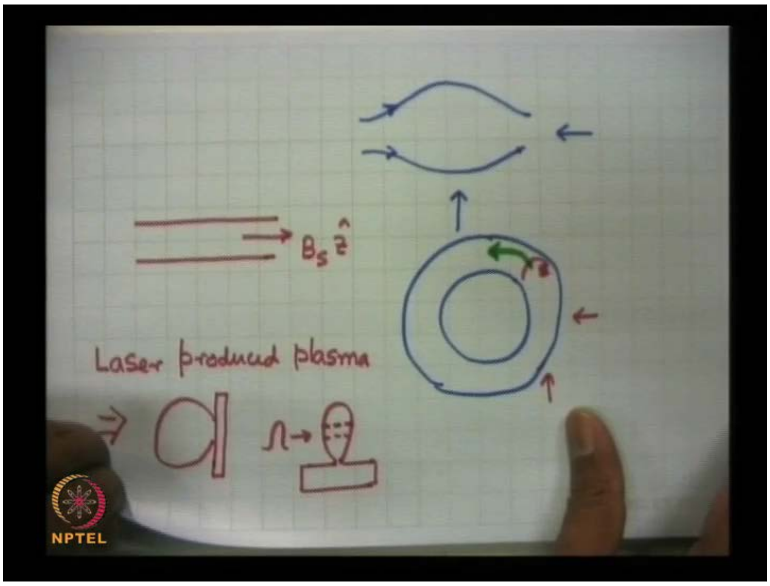

We have already seen the need of wave propagation in magnetize plasmas when we were talking about tokomak heating. Then there are other devices also, in a mirror machine also where the magnetic field lines of force are like this; these are the lines of force in a magnetic mirror, then either you launch wave from here or you may launch from here. In either case, the wave is propagating in magnetized plasma either perpendicular to the line of force or along the line of force or opposite to the line of force.

In general, wave propagation in any arbitrary direction to magnetic field is important. In tokomak of course, we have seen that the lines of force is my tokomak for instance, then there are two kinds of magnetic field; one of them is a toroidal magnetic field and then there was a poloidal magnetic field like this. Due to the toroidal current, and if you want to heat the tokomak, then you need to launch waves from either here or from here. So, they are either coming perpendicular to the line of force or along the line of force or some angle.

Then, there are other devices smaller devices like q machine or a beam plasma system, where you have a cylindrical plasma column with an axial magnetic field, this kind of situation. So, if you want to heat it or you want to study the interaction of a large plasma chamber with an electromagnetic wave, then obviously, we should take into consideration the presence of magnetic field.

Even in plasmas where you do not apply externally any magnetic field, just like laser produced plasma, laser produced plasma, there also you observe a strong magnetic fields. For instance, you can choose a thin foil, metal foil on which you shine laser, then it forms a large plasma plume and in the plasma, people have observed strong magnetic fields, multi mega gauss magnetic fields have been observed.

These days there is lot of work going on plasmas; if you call as gas jet target plasmas? So, this is a gas chamber, gas cylinder or gas container and the gas is released through a small nozzle and forms a plum here. You shine a laser pulse on the plasma. So, a plasma is created here and people have observed mega gauss, hundreds of mega gauss magnetic fields in such a situation when the laser intensities are around 10 to 18 watt per centimeter square or more. So, here again, the problem becomes important that wave has to travel either along the magnetic field or at an angle to magnetic field and magnetic field can have very significant influence on wave propagation.

So, today we shall look into the effect of magnetic field on wave propagation. A few things I would like to emphasize right in the beginning; the major effects that we anticipate from the presence of magnetic field is as follows.

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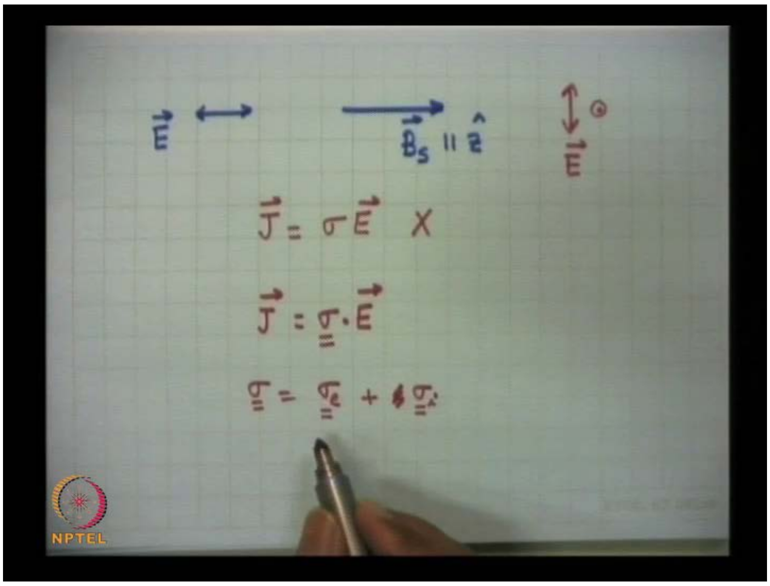

Suppose this is the direction of magnetic field, static magnetic field which we normally take to be the z axis. You may note that if I have a plasma in which magnetic field axis, then if I apply the electric field to a plasma, alternating electric field like this, oscillating electric field in this direction, I do not expect much of influence of magnetic field on the wave on the plasma response because under the influence of the electric field, when the electrons move in the direction of magnetic field, the v cross B or Lorentz force on them due to magnetic field will be 0.

However, if I have a situation where the electric field is acting perpendicular to the line of force like in this direction, it is oscillating like this, the electric field… In this case, when the electrons move in the direction of electric field, the Lorentz force v cross B force will immediately start acting on the particle and that will be perpendicular to the velocity and magnetic field. So, that will be perpendicular to this plane. The force will be in this direction. So, particle will acquire a velocity in that direction.

What a situation that the electric field is applied in one direction, electrons acquire a drift in that direction as well as in a direction perpendicular to electric field. Consequently, the current that you see here is no longer parallel to electric field. So, J normally we write J equal to sigma e which implies that J and e are in the same direction. But as I mentioned to you that if the electric field as a component perpendicular to magnetic field, then J and e will be in different directions and hence this relation is not valid.

What you require? If you want to allow a different direction for J in comparison to e direction, then this coefficient cannot be a scalar quantity because whenever this scalar quantity, it implies that J and e are in the same direction. So, then J becomes can be written as something like a tenser; sigma becomes a tenser. We have talked about this earlier in our early lectures that the plasma response to a magnetic field is such that in general, Ax component of electric field will produce J not only in the x direction, but in y and z direction also. And electric or conversely, current in x direction is not only produced by e in the x direction, but E y and E z as well.

So, this is a conductivity tenser and we have deduced an expression for this and I just want to tell you that if my magnetic field is chosen in the z direction, in that case, conductivity tenser which was sum of two terms; the electron conductivity tenser plus ion conductivity tenser sigma i plus not, this i is not there. This is the electron conductivity tenser plus ion conductivity tenser. They were having similar forms and I like to write down only one; other is very similar.

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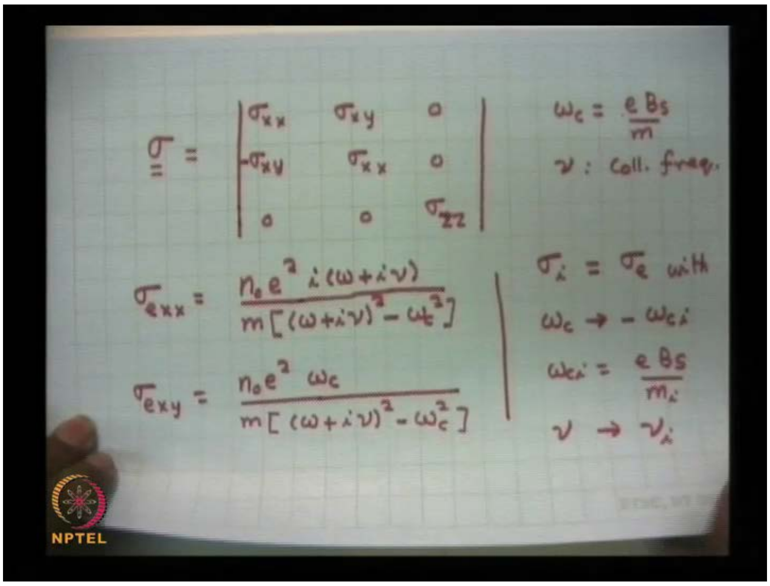

So, it turns out it was something like this which was derived by solving the equation of motion. Sigma was equal to this component of sigma was sigma xx, this was sigma xy and sigma xz was 0. This coefficient was sigma yx which is equal to sigma xy is a negative sign, this was equal to sigma xx, this was 0, and this was 0, this was 0, and this was sigma xx and sigma zz.

So, conductivity tenser is like this; means these two terms are 0, these two terms are 0, these two diagonal terms are equal, and these two are equal in magnitude but opposite in sign. Now for electrons conductivity, the xx component is n 0 e square upon m. This is omega plus i nu square minus omega c square and here you had a term i omega plus i nu. and sigma Exy is n 0 e square upon m omega plus i nu square minus omega c square and here was omega c; where omega c is the electron cyclotron frequency defined as the magnitude of electron charge into magnetic field upon mass of the electron. And nu is the collision frequency of electrons, m is the electron mass, n 0 is the density of plasma electrons, and I think everything else…

You can define similar terms for ions. So, sigma ions is equal to sigma of the electrons with following changes; omega c going to minus omega ci; ion cyclotron frequency which is defined as omega ci is equal to e B s upon mi, where e is the magnitude of ion charge mi is the ion mass. And nu to be replaced by ion collision frequency and the well is probably this is all. So, with these changes, conductivity for ions can be written. Once conductivity is known to us, it is easy for us to study the propagation of electromagnetic waves.

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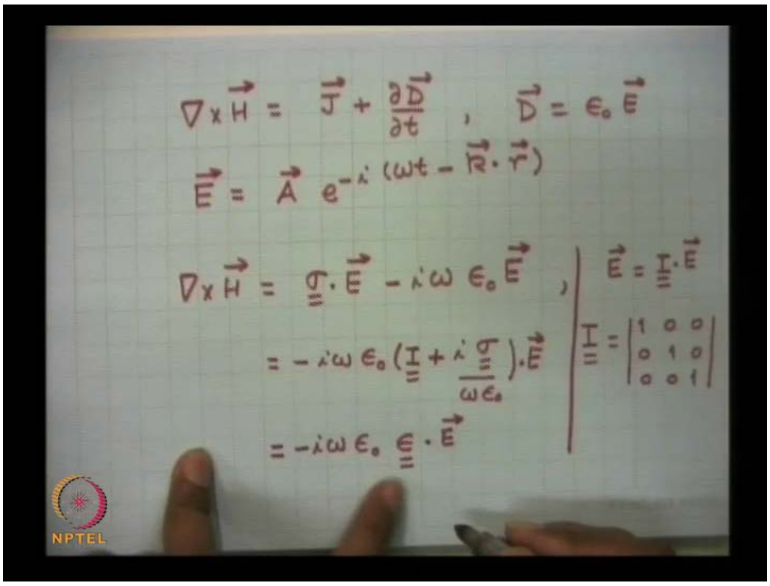

So, let me write down the Maxwell's last Maxwell equation where it is important to combine the conduction current and displacement current. Curl of h we know is equal to J plus delta D by delta t. And in plasmas, D is equal to epsilon 0 into e, where epsilon 0 is free space permittivity. So, if I put this back in here and assume that the fields that I am considering are of this form, A exponential minus i omega t minus k dot R in general. This is called the plane wave solution.

So, if I put this in this equation, what do I get? I can replace delta t by minus i omega and forget the left hand side for a moment, retain it as such. So, curl of h becomes is equal to conductivity tenser dot e minus i omega epsilon 0 into e. You can combine these terms as taking minus i omega epsilon 0 common, and recognizing that e can also be written as a unit matrix dot e, where this unit matrix is a matrix with 1 0 0 terms there, and 0 1 0 there, and 0 0 1 there.

So, if I use this, then this equation becomes i from here, then plus i sigma tenser divided by omega epsilon 0 dot e. This is a nice equation because I can combine the conduction term; this is the current density conduction part, this is the displacement part; together and this becomes a tenser becomes like i omega epsilon 0 and I can call this combination of these two terms as effective relative permittivity tenser of the plasma.

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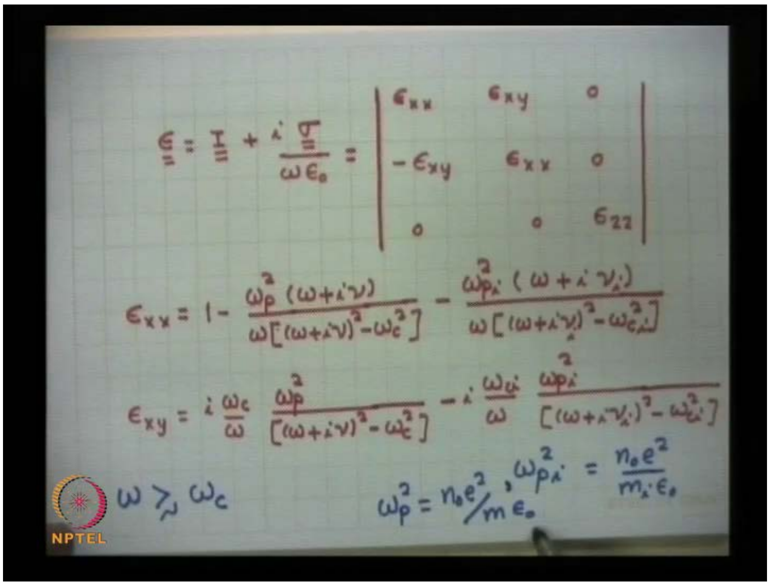

So, this is related to conductivity through this relation and let me write this. So, I am defining effective plasma permittivity is equal to I unit tenser plus i sigma upon omega epsilon 0. Please remember that in an unmagnified plasma, permittivity is a scalar quantity and you normally had epsilon is equal to 1 plus i sigma upon omega epsilon 0, but sigma was scalar. So, this is a tenser version of that. And if you put the expressions for sigma and i here, then this takes the following form. This coefficient this first term is epsilon xx, this is epsilon xy, this is 0, this is minus epsilon xy, this is epsilon xx, this is 0, 0, 0, epsilon zz.

So, permittivity tenser has non-equal diagonal terms; two of them are equal, but third one is not equal, and it has half a diagonal terms which are equal to magnitude, but opposite in sign and other diagonal terms are half diagonal terms are 0.

Now, let me explicitly write down this expression for epsilon xx including the motion of ions together, it is equal to 1 minus omega p square multiplied by omega plus i nu divided by omega into omega plus i nu square minus omega c square. This is the electron term and similarly I must write down the contribution for ion term which is ion plasma frequency square omega plus i nu I; the ion collision frequency upon omega into omega plus i nu square minus omega c i square ion cyclotron plasma frequency ion cyclotron frequency square. This is the expression.

And epsilon xy turns out to be… this is i omega c upon omega ion cyclotron electron cyclotron frequency upon omega into omega p square upon the same factor here, omega plus i nu minus omega c square and the ion term which is minus i times omega c of the ion in ion cyclotron frequency upon omega into ion plasma frequency square upon omega plus i times nu i; this is also nu i here square minus omega c i square. This is the ion term, this is the electron term.

Well, I forgot to mention that sigma tenser had a sigma xx term and sigma xx here also. These terms are called Pedersen conductivity, and the half diagonal terms of sigma are called hall conductivity in plasma jargon. Here, one may note that it depends on the frequency whether if omega is small or large that the relative importance of these terms will is realized.

First of all at high frequencies, when omega is comparable or of the order or of larger omega c, what will happen? You may note that the ion term because omegas c i is very small. So, you can ignore this as compared to omega; omega is too large, then this term is smaller as compared to this. Why? Because please recognize that omega p i which is equal to omega p i square rather is n 0 e square upon m i epsilon 0 and omega p square which is equal to n 0 e square upon m epsilon 0.

So, you may note here that this electron plasma frequency is much bigger than the ion plasma frequency and hence these terms are unimportant as compared to electron terms. So, ion motion is unimportant. So, for omega bigger than omega c, ion motion is unimportant. Ion motion; ions unimportant. You can forget them contribution. When omega is less than omega c, then also you can ignore the ion motion. Ion motion becomes important when omega approaches omega ci, the ion cyclotron frequency which is a very low frequency as compared to omega c.

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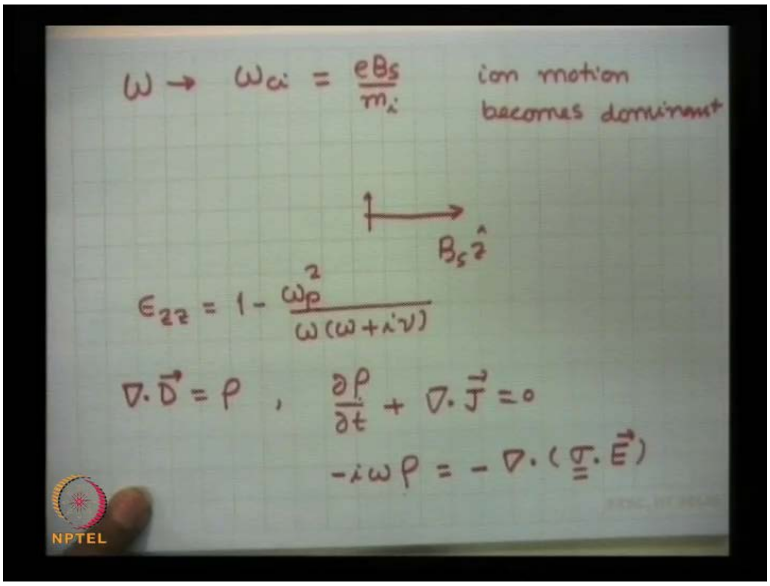

So, when omega approaches omega ci which is defined as e B s upon mi, the ion motion becomes very important. Ion motion becomes very important, becomes dominant. This is a very important characteristic of a magnetize plasma. In an unmagnified plasma, the ion response was unimportant because mass was too heavy, but what really happens that when you apply a magnetic field, the electrons cannot respond to an electric field perpendicular to magnetic field because they gyrate about the line of force. So, if you had a magnetic, if you have an electric field applied, this is my static magnetic field and whenever you apply an electric field in this direction, the electron motion is inhibited by cyclotron motion because the electron will like to go round and round about the magnetic field.

So, magnetic field inhibits the particle moving away from the line of force and hence ion motion and electron motion can be comparable or at times ion motion can dominate over the electron motion. So, that is a very important characteristic of magnetized plasma that the ions start playing a very important role. And we have already seen that magnetized plasma is a medium in which J and e are in different directions. And as a consequence of that, we will learn that in general, the wave propagation depends on the polarization of the wave.

In unmagnetized plasma, whether it is a circularly polarized wave or linearly polarized wave, velocity of propagation is same; k is the same for a given omega, but here we shall learn that it will become polarization dependent. So, there are some interesting features that are we going to notice, and I have written the two components of permittivity tenser; the third in finite component is epsilon zz which remains unmodified by magnetic field and is given by 1 minus omega p square by omega into omega plus I nu. The ion contribution to this is negligible and all other components are 0.

So, once we have constructed the plasma effective permittivity tenser, if you really go over to revisit the Maxwell's equations, you will recognize that even the first Maxwell equation which is divergence of d is equal to rho. If you view this equation in conjunction with the equation of continuity, delta rho by delta t plus divergence of J equal to 0.

For currents that are time dependent, charged density will also be time dependent in general. So, if I replace this by minus i omega delta delta t, then this equation gives me minus i omega rho; the perturbed charged density is equal to from here minus divergence of J. J is equal to sigma e. So, if I substitute the value of rho from here in this equation, and this D is simply epsilon 0 into e in plasma. In that case, you can combine these two terms because rho is in terms of divergence of some quantities, bring this back on the left hand side.

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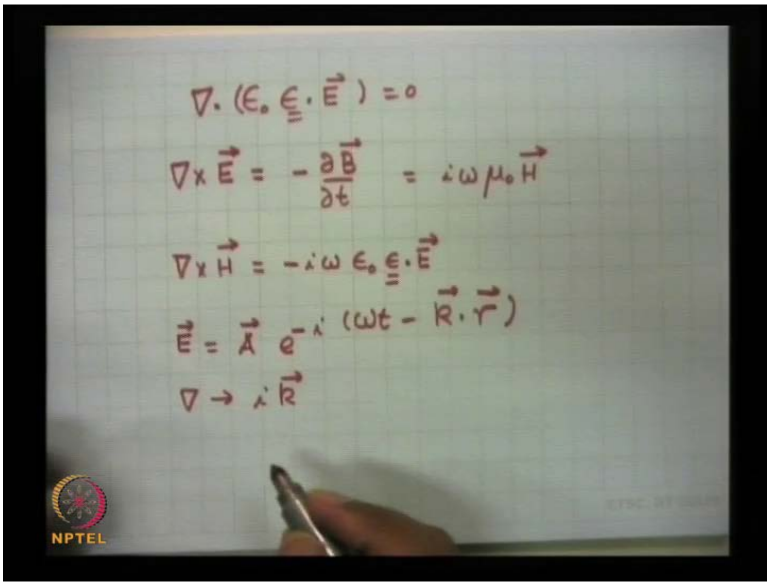

And when you add these two terms, this simply become divergence of epsilon 0 into epsilon dot e equal to 0. So, the first Maxwell equation has become similar to the one that you encounter anisotropic dielectrics of permittivity epsilon. So, the entire information about anisotropy or plasma response is contained in one quantity; that is, plasma permittivity tenser. And the relevant equations for wave propagation are the third and forth Maxwell equations which we write as curl of e is equal to minus delta B by delta t and curl of h; that we have just written, is equal to minus i omega epsilon 0 epsilon tenser dot e and this also becomes equal to I, if I replace delta delta t by minus i omega, so i omega and B as mu 0 h.

Well, to obtain a dispersion relation or understand the propagation of waves, I must replace del operated by i k because I have already presumed for a plane wave solution that my e goes as A; some amplitude vector, exponential minus i omega t minus k dot R in general, if my wave is going in any general direction k.

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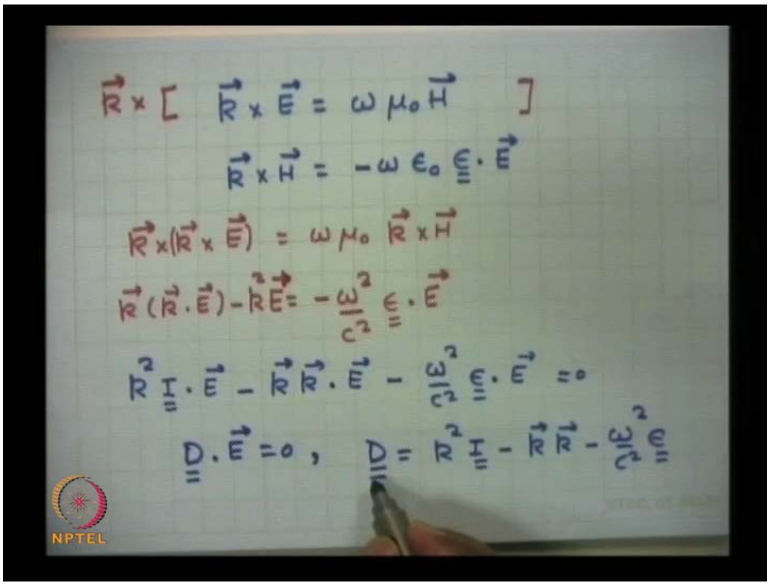

So, just replace del operator by i k vector, you get these equations as follows: the first equation becomes k cross e is equal to omega mu 0 h and the forth Maxwell equation becomes k cross h is equal to minus omega epsilon 0 epsilon tenser dot e.

Now, you multiply this equation by k cross on both sides. Then this equation becomes k cross k cross e is equal to omega mu 0 k cross h. Use the k cross h equation here, and this becomes minus omega becomes omega square, mu 0 epsilon 0 is 1 upon c square, then relative permittivity tenser dot e, and this I can write down as k vector k dot e minus e into k square. This is the equation vector equation which is three components that governs the polarization and propagation characteristics of the electromagnetic wave in a magnetize plasma in general travelling in an arbitrary direction to magnetic field.

And formally, I can write down this equation as this: k square this i term I can write down first and this later, I can write down this k square i dot e this term as such because e i can write down as i dot e, this is unit matrix, this is minus k dot e and this term as minus omega square by c square epsilon dot e is equal to 0.

What I can do? Formally, I can take e outside, and the remaining quantities I can call as some quantity say a tenser new tenser let me call this d dot e equal to 0. Here I am calling d tenser as k square i minus kk and then this is minus omega square by c square epsilon. So, this d is a tenser three by three tenser, and if this equation is to have non trivial solutions, then determinant of d must vanish. Here there are two solutions that one e itself is 0, and then this whole equation is everything is 0, so, but then there is no physics.

But if you want an electromagnetic wave to have finite electric field and it is travel in plasma, then determinant of D must vanish. D may have individual components finite, but the determinant of this is 0. This is an important condition and that is called the dispersion relation. So, dispersion relation of an electromagnetic wave in plasma in general is the determinant of D equal to 0; that is a formal dispersion relation.

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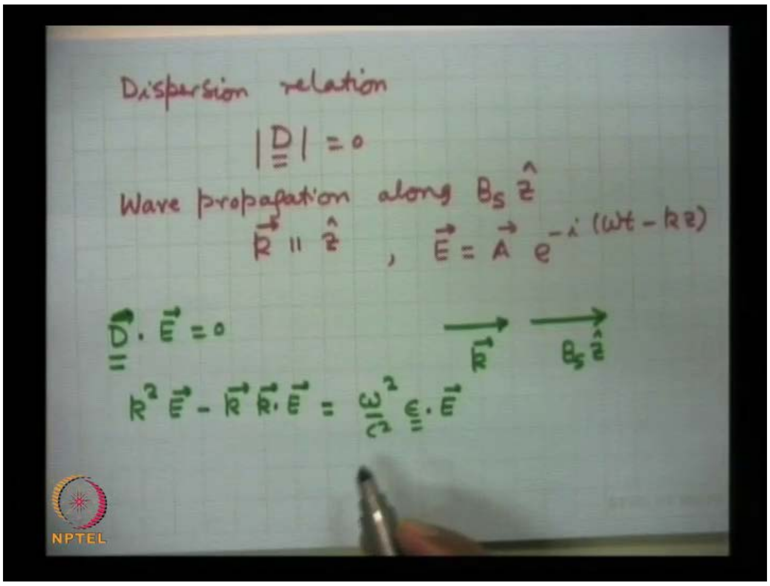

So, let me write down the dispersion relation. ((No audio from 32:03 to 32:11) The determinant of D is equal to 0. Well I think before I delve into the general case of wave propagation at arbitrary angle because this is a little complicated thing, I would like to go step by step. Today I like to discuss the propagation of wave along the direction of magnetic field.

So, suppose I choose my wave propagation along the static magnetic field which is the z direction. So, I am choosing k vector to have only z component; means, I am allowing my wave, electric field to be of this form AE is equal to a vector exponential minus i omega t minus k z. I would like to learn two things; what is the connection between k and omega, for a given omega what is the value of k and what are the relationships what is the relations between different components of A vector. Let us see whether k depends not only on omega, but does it also depend on the relationship A access with ay and so on or is it independent of this relationship. Let us examine this issue.

So, I am going to discuss a particular case when k is parallel to z axis; means I am considering a magnetic field in this direction, and my wave to traveling in the same direction, but I would like to find out whether the wave still maintains its transverse character or not, whether it is a linearly polarized wave or its characters different. Let us examine all that issue.

The entire information is contained in this equation, this simple equation D dot e is equal to 0. Rather than writing the… sorry this D is not here vector is a scale a tenser, this equation was if I write the value of D little more carefully, then this was simply k square e minus k k dot e is equal to omega square by c square epsilon dot e. This was the explicit form of this equation. If I put the value of D properly, then this a this is the form.

I want to write down this in component form recognizing that k has only z component. So, if k has only z component. So, if I write down the x component of this equation or y component of this equation, then this term will not contribute at all. So, I would like to write down the x component of this equation, then y component of this equation and let us see what we get.

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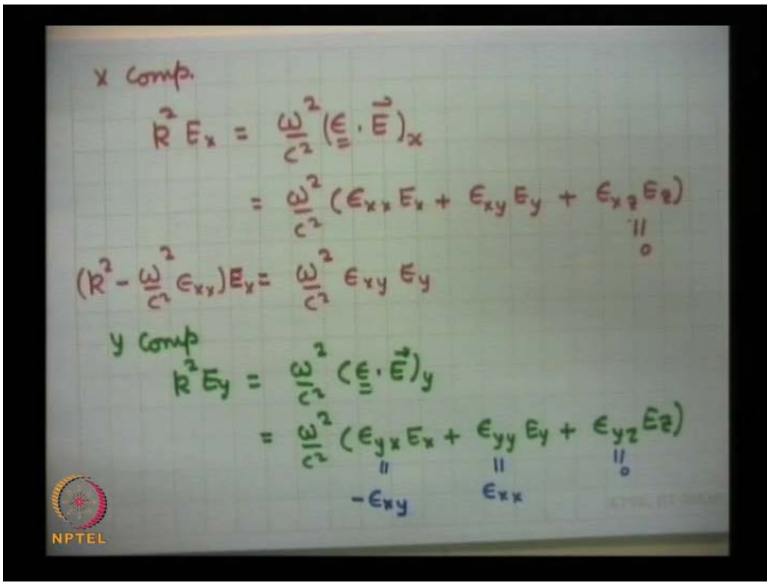

x component’s equation gives k square Ex is the left hand side, right hand side is omega square by c square epsilon dot e. I want the x component of this quantity which implies omega square by c square, I must keep the first index of this product to be x and the second index of epsilon should run and it should be common with the electric field index. So, if it is x here, then should be E x. If it is xy, then the second index E should be having y. If it is xz, then it should be Ez; means, you have to keep the second index of epsilon same as the index on the electric field or the component of the electric field. This is how tensers multiply, but as we have seen that epsilon xz is 0 in plasma where magnetic field is in the z direction, this is 0. So, this gives me… remove this term and you can take this term on the left hand side, So, this equation becomes k square minus omega square by c square epsilon xx into Ex is equal to the remaining term here omega square by c square epsilon xy Ey because this is equal to 0, epsilon xz is 0.

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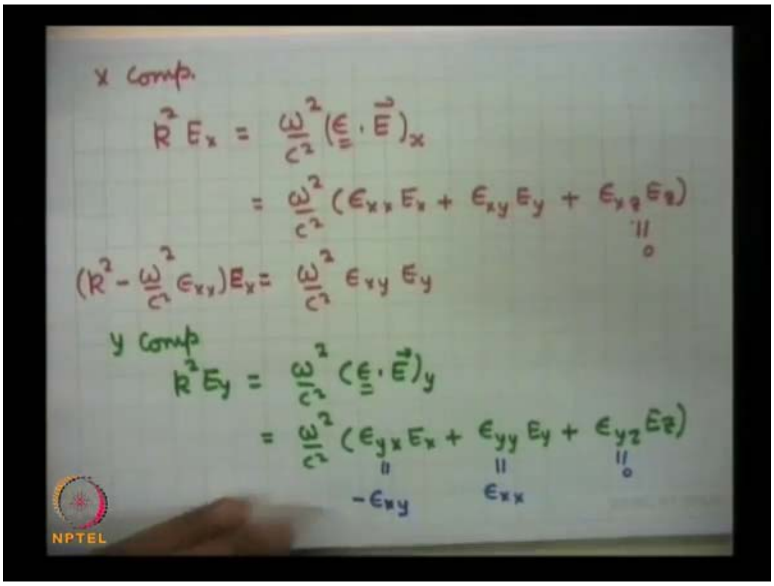

And for the y component, let me write the equation wave equation gives you k square Ey which is equal to omega e square by c square epsilon dot Ey component which turns out to be equal to omega square by c square. This is epsilon, first index is y, second is running. So, yx Ex plus epsilon y Ey plus epsilon yz Ez, but we recognize that this is 0, yz is not there, this is 0. And this is equal to epsilon xx and this is equal to minus epsilon xy. So, when you remove this term, combine this Ey term with the Ey here on the left, and then you get a equation like this.

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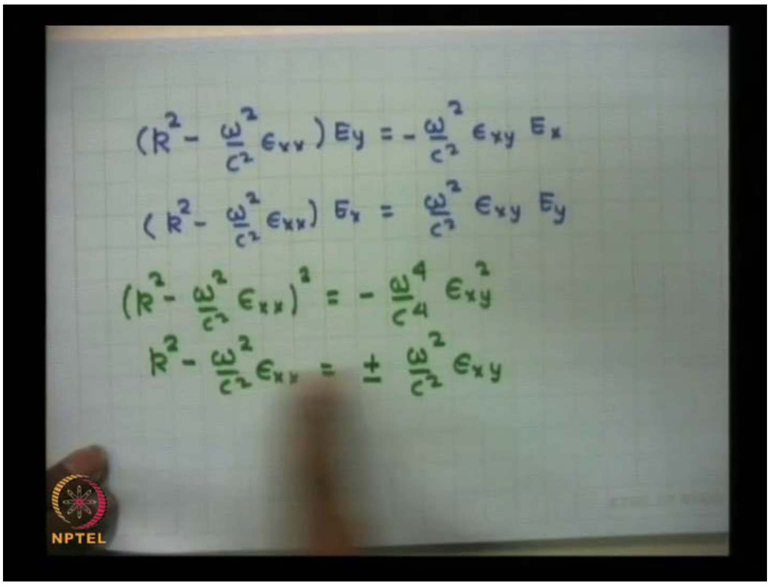

So, this equation becomes k square minus omega square by c square epsilon xx Ey is equal to omega square by c square with the negative sign epsilon xy into Ex. Always also remember the first equation that we have derived for Ex was like this: k square minus omega square by c square epsilon xx Ex was equal to omega square by c square epsilon xy into Ey.

What you can do? Multiply the left side of this equation with the left side here, right we have with the right side. You will get Ey Ex here, Ex Ey there. They will cancel out and you get a dispersion relation. The dispersion relation is k square minus omega square by c square epsilon xx whole square is equal to a negative sign omega 4 by c 4 epsilon xy whole square. I can take the under root on both sides. So, you get k square minus omega square by c square epsilon xx is equal to plus times or minus times omega square by c square epsilon xy.

There are two possibilities. So, k does not have a single value. If there were no magnetic field, epsilon xy is 0 and this term is not there. So, k has simply this value, but now because of the magnetic field, epsilon xy is finite and there are two possibilities; either this equation can have positive sign right side can have a positive sign or a negative sign and corresponding to two signs, we get two roots of this equation, two values of k rather, one of them is called k R, the other is called k L.

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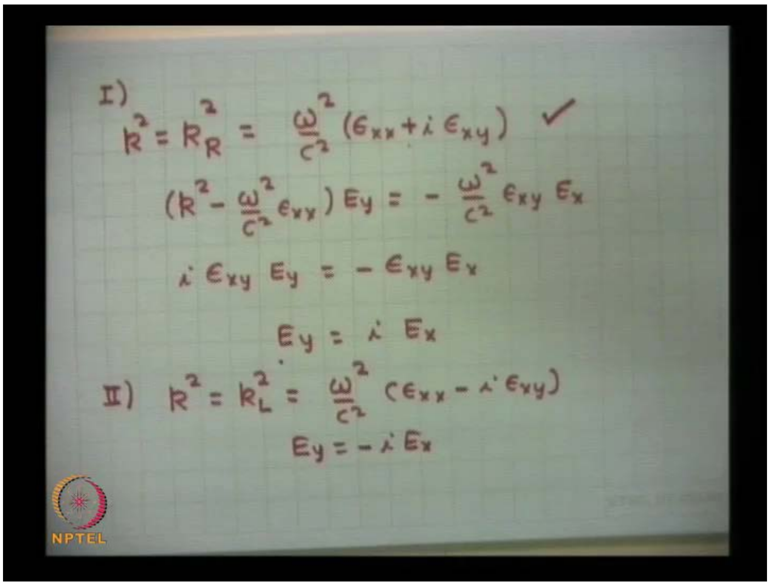

So, let me write down the first case. When I choose k square as k R square which turns out to be equal to omega square by c square epsilon xx plus… actually I forgot, when I took the under root, then minus under root will give you i also here. I forgot to write i. So, I get I times epsilon xy, but if I put this value of k square is equal to k R square, this value in my any of those equations that were relating Ex to Ey, I get an expression for let me write down, for this case, if I choose so much, then the equation which was k square minus omega square by c square epsilon xx Ey which was equal to minus omega square by c square epsilon xy Ex.

Put the value of this expression put the value of k square is equal to this expression. So, when I put this value here, I get I epsilon xy Ey is equal to minus epsilon xy Ex or Ey is equal to I times Ex; means k can take so much value with a positive sign here if E y is equal to iEx.

And this has a very important message in here. We shall discuss this. So, this particular kind of wave with Ey having 90 degree being 90 degree out of phase with the Ex component, this wave will have a propagation vector given by positive sign between these two terms.

On the other hand, if I choose the negative sign there, then this is denoted as kL square; symbol kL square is used for this is omega square by c square epsilon xx minus i epsilon xy and if I use this in this equation, I get Ey is equal to minus iEx. What a dramatic change here? A wave with electric field Ey leading Ex by pi by 2 k value is different and for a wave in which Ey is behind Ex by pi by 2, in that case, the k value is different; means and let me first lets understand the implication of this relation.

What is the meaning of i being here and what is the meaning of minus i being here. We shall just learn that this corresponds to right circular polarized wave and a right circularly polarized wave travels with a different wave number and a left circularly polarized wave travels with the different wave number, but let us appreciate or let us recognize why this wave is a right circularly polarized wave. We shall discuss this.

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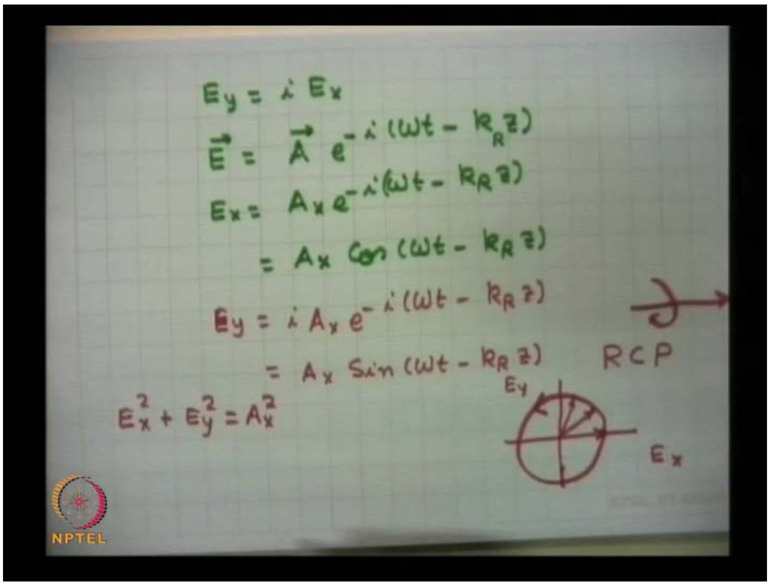

I am saying that Ey is equal to iEx where my electric field I had presumed to be written as A exponential minus i omega t minus k z. And this k, I am calling k R because for this I have considered this or this is my wave field. Ey is equal to iEx implies that if I choose Ex is equal to some suppose Ax, exponential of this quantity kRz and if I count kR to be real, suppose there are no collisions and k R is real, in that case, this equation simply implies is equal to and A x is real too, then this is simply A x cos omega t minus k R z.

Now, let me write down the y component of this equation. The y component of this equation would be Ey or simply Ey I can write down from here, E y is i times E x. So, if I take Ey i times A x exponential minus i omega t minus k R z. Now take the real part of this because whenever I write any field in complex notation, its implied that the real part of the quantity is the physical quantity. So, if I take the real part of this quantity, it turns out to be equal to Ax sign omega t minus k R into z.

To appreciate this, let me plot a graph here. If I plot Ex on this axis and Ey on this axis, what you will note? First of all from this itself, E x square plus E y square is A x square and on this plane this for equation of a circle, but what kind of circle? What I say is that suppose I choose without any loss of generality, suppose I choose z equal to 0 or some given some value. So, at an instant, when this quantity is 0, then Ex will be equal to Ax and Ey will be 0 because sign of argument is 0. So, your field will be somewhere here, this is the field.

After a while, when time increases, when this quantity become more than 0, then Ex will have a value less than Ax because cos value will decline whereas, sine value will increase. So, your point will be located somewhere here. At a time later further, this will move to here. So, your tip of the electric vector is going from here to here, then going from here to here; means it is moving on a circle which is moving on a clockwise sense like this on anti clockwise sense from this figure like this.

And if you examine this in terms of a right handed screw, if I take a right handed screw and rotate it like this, then if the direction of rotation of a screw is given by this arrow of the circle and then the direction of advancement of the screw will give you z axis because this x axis y axis here. What I am saying here is that when you view in the direction of magnetic field, suppose this is the direction of magnetic field and if my electric field rotates in the clockwise sense, in this sense in the right handed screw sense where the direction of advancement of this screw gives the direction of wave propagation or magnetic field, then such a wave is called right circularly polarized wave.

So, this represents an RCP; right circularly polarized wave. So, for a right circularly polarized wave, wave number is k R and k R is given by this expression.

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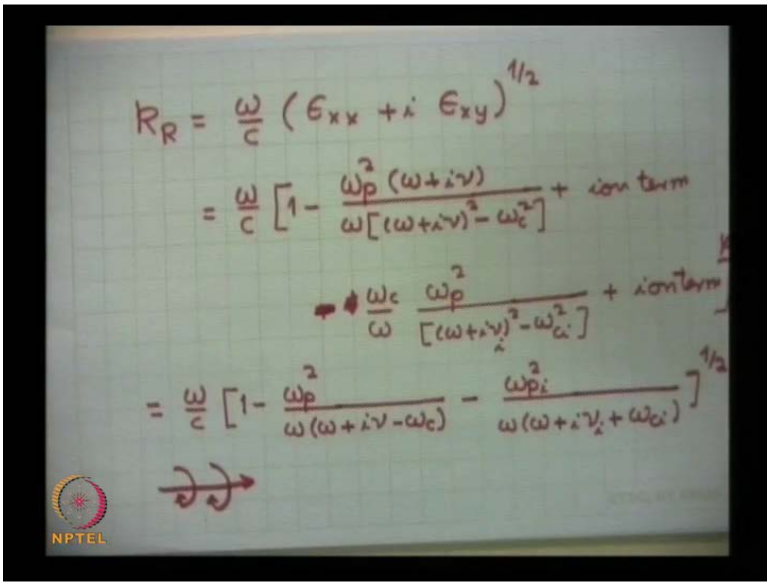

Let me just write down the value of k R. I wrote down this is equal to omega by c epsilon xx plus i times epsilon xy under the root. Put the values of epsilon xx and epsilon xy and this turns out to be omega by c, this one minus the electron term which turns out to be omega p square and it turns out to be omega plus i nu divided by omega into omega plus i nu whole square minus omega c square. This was the value of epsilon xx. Let me write down the terms.

There was similar ion term. For the moment, suppose I ignore the ion term, this is very similar. Let me write down probably plus ion term. Let me just call this ion term and for this, I substitute, I get plus i omega c upon omega omega p square upon omega plus i nu square nu i square omega c i square plus the ion term due to epsilon xy to the power half. What you see here, these two terms have same denominator here and they can be easily combined and this becomes like omega plus i nu minus omega c factor there plus actually i is here, i is there. So, this becomes minus sign actually. this is the minus here and i is gone, there is no i here.

So, this becomes omega plus omega c plus i nu and you can factorize this denominator and one of them will cancel out and it turns out to be simply like this is equal to omega by c, this is 1 minus omega p square upon omega into omega plus i nu minus omega c. This is the electron term. Ion terms also combined, they give you omega p i square upon omega omega plus i times nu I; the ion collision frequency and becomes plus omega c i the ion cyclotron frequency. Usually the collision frequency is much a smaller than omega c in plasmas of interest. nu i is much smaller than omega c i of interest.

So, one may ignore for a while. They are essentially responsible for absorption of waves and suppose I ignore the absorption for a moment, then one may note here that this has a resonance denominator with omega equal to at omega equal to omega c. This will overflow becomes very large and it is changes sign as well. For omega less than omega c, this term is negative so the whole thing becomes positive, whereas; for omega bigger than omega c, this is positive. So, this whole thing becomes negative.

So, there is a change in the character of wave propagation at omega equal to omega c and at omega equal to omega c, around in the vicinity of omega c, k R can take very large value. So, the phase velocity wave will be very tiny. Waves can be made to travel very slowly and electrons can interact with them resonantly at cyclotron resonance.

This is the ion term; the ion term does not show any resonance. The reason is that the electrons in a magnetic field also rotate in the same right handed sense as the electric field is rotating and consequently there is a resonance. For a gyrating electron, if the wave is going in this direction and electron is gyrating like this, this is the direction of electron motion and this is direction of wave electric field.

So, if the electric field rotates in the same sense in which any electron would have a tendency to gyrate, then the magnetic field, then the wave field as seen by the electron will be shifted in frequency and the effective frequency of the wave as seen by the particle will be omega minus omega c and there is a possibility of resonance when they two become equal.

In the case of ion, the ions gyrate in the opposite sense left handed polarized sense left handed sense and consequently for this mode, there is no there is nothing like a resonance. One thing more, we had seen earlier when we were talking about wave propagation in unmagnified plasmas that wave can travel only when omega is bigger than omega p; however, here because of the presence of omega c here, since this term can change sign when omega is less than omega c.

So, irrespective of whatever is the value of omega p, if omega is less then omega c, this term becomes positive and wave can penetrate any dense plasma. What (( )) is the density is and that is a very important characteristic of plasmas that it can of magnetized plasmas that a magnetic field can allow the penetration of a low frequency wave into a plasma of any density. And I think more such fascinating characteristics we shall learn in more detail in our subsequent lectures. I think I will stop at this point. Thank you very much.