Plasma Physics

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Module No. # 01
Lecture No. # 26

Tokamak Operation

Today, we will continue our discussion on Tokamak. We shall discuss Tokamak operation; in this, I will discuss basic elements of plasma confinement and Tokamak operation, the parameter regime of Tokamak, inductive current drive and ohmic heating in Tokamak. Well, my presentation will be primarily based on a book by John Wesson Tokamaks and also, I will refer to an article by N Fisch, theory of current drive in plasma published in reviews of modern physics.

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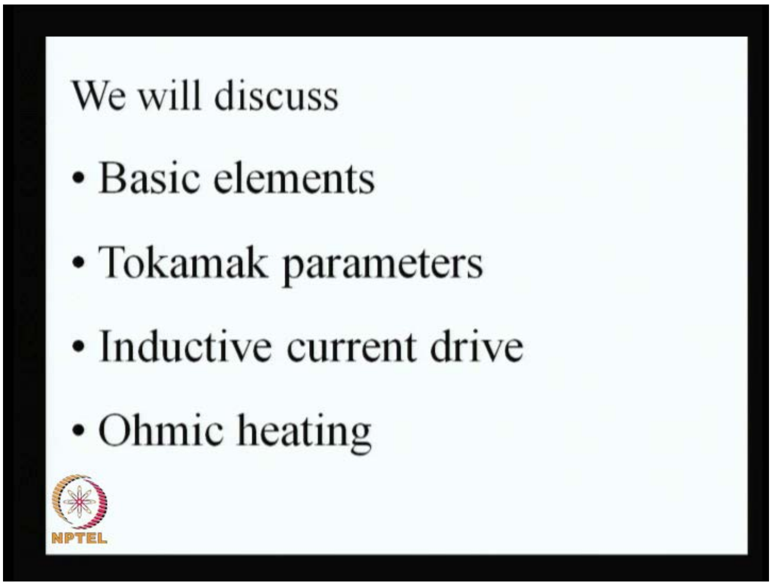

We will discuss

- Basic elements
- Tokamak parameters
- Inductive current drive
- Ohmic heating

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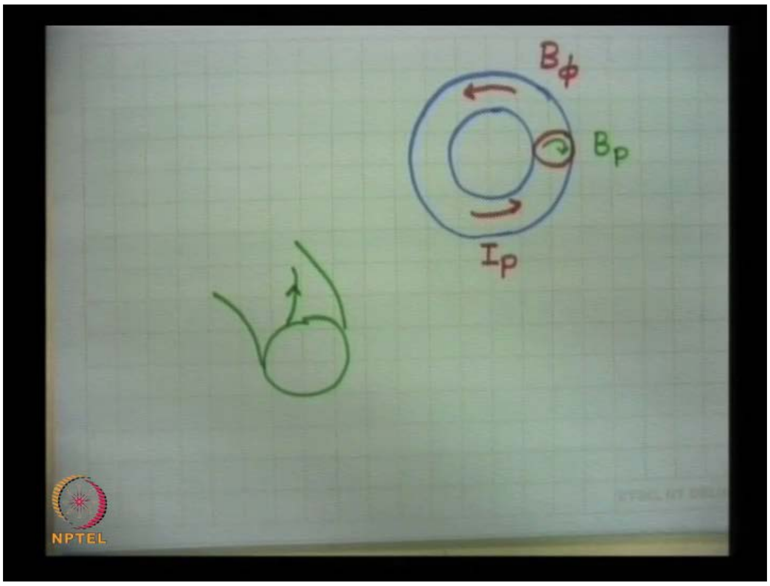

I think, let me begin with the basic configuration of a Tokamak which looks like this. it is a toroid and the toroid has a magnetic field which is in this direction, I will call as the toroidal magnetic field B phi and it also carries a current - plasma current - in the phi direction I called as I p, the plasma current typical value of magnetic field in Tokamak is around 4 tesla to above 100 tesla.

If you examine the cross section of a Tokamak then, there exists a magnetic field produced by the azimuthal current, the toroidal current. The magnetic field will be in this direction; this we call as poloidal magnetic field B p; some people call this as B theta also. So, this is the basic configuration and if you look at the any line of force, we define what we call as the magnetic surface. So, if I plot this cross section of the Tokamak like this; so, the Tokamak goes like this, we will extend it. Then, any line of force will move on a surface; this is a line of force which will move on a surface of constant minor radius; well, not constant minor radius but, it will move on a flux surface. For the sake of simplicity, one can visualize as if, the flux surfaces are circular. Then, this will move on a circle but, on a toroid of constant radius minor radius every line of force.

The basic elements of Tokamak operation, we must keep always in view the following elements: number one, we talked about the Lawson criteria which, essentially implies that the plasma, the power output from a fusion device p out must exceed or be equal to power loss.

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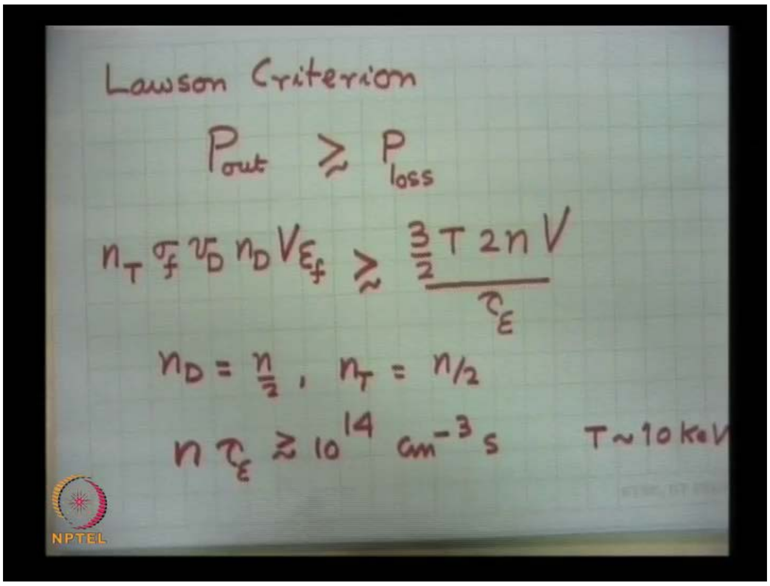

Lawson Criterion

$$P_{out} \gtrsim P_{loss}$$

$$n_T \sigma_f v_D n_D V \varepsilon_f \gtrsim \frac{\frac{3}{2} T \, 2 n V}{\tau_E}$$

$$n_D = \frac{n}{2}, \quad n_T = n/2$$

$$n \tau_E \gtrsim 10^{14} \text{ cm}^{-3} \text{ s} \qquad T \sim 10 \text{ keV}$$

Now, power loss is essentially written as 3 by 2 t; if t is the temperature of electron ion plasma then, this is the average kinetic energy of a particle and there are 2 n particles per unit volume n is the number of electrons per unit volume and same number is the number of ions deuterium, tritium ions. Then, this is the total energy of particles per unit volume; if the volume of the plasma is V then, this is the total energy contained in the plasma and if the energy relaxation time is tau E then, so much energy is lost in so much duration; so this is called the power loss rate; where v is the volume of the plasma. And, as far as the power input is the power output is concerned, well that number of nuclear reactions that take place per unit time per deuterium particle, will be number of tritium atoms per unit volume into fusion cross section into drift velocity of deuterium; not drift rather thermal velocity of deuterium. So, this is effective of the collision frequency or fusion frequency per particle. If there are n deuterium particles per unit volume and V is the plasma volume and the energy produced per nuclear reaction is E f then, this must be equal or of this order this is primarily the Lawson criteria.

This gives you, if I take n D is equal to n by 2, the number of electrons electron density divided by 2 and n T also is equal to electron density by 2. Then, this expression this inequality rather leads to the criteria that the product of density and energy confinement time should exceed. Typically, this should be of the order of 10 to 14 per centimeter cube into second at a temperature T of the order of 10 KeV. So, this is an important consideration we should always keep in view. And, typical operator operation densities of Tokamak are around 10 to 14 per centimeter cube. So, the n is typically 10 to 14 or sometimes 10 to 13; few times 10 to 13 electrons per centimeter cube; so tau must exceed 1 second this is the minimum that is required.

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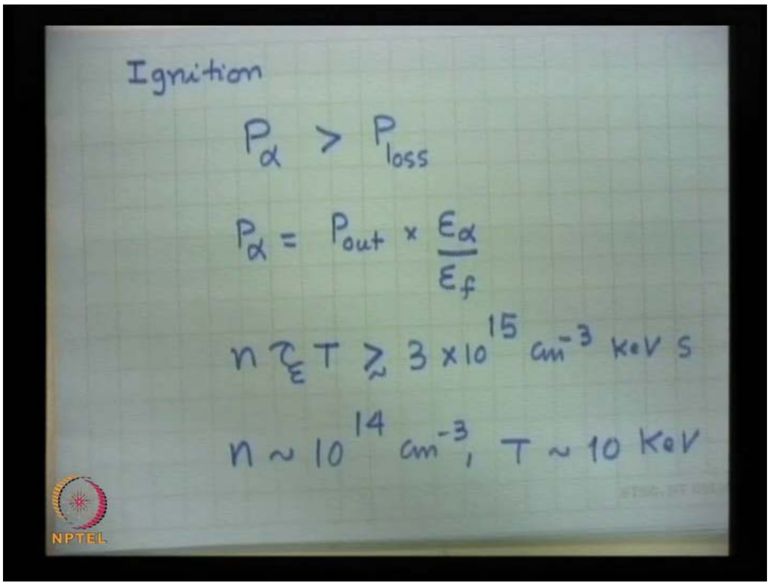

Ignition

$$P_\alpha > P_{loss}$$

$$P_\alpha = P_{out} \times \frac{\varepsilon_\alpha}{\varepsilon_f}$$

$$n \tau_E T \gtrsim 3 \times 10^{15} \ \mathrm{cm}^{-3} \ \mathrm{KeV} \ \mathrm{s}$$

$$n \sim 10^{14} \ \mathrm{cm}^{-3}, \ T \sim 10 \ \mathrm{KeV}$$

Another important consideration is the ignition criteria. The ignition problem which implies that the alpha particles produced in fusion reaction, so the energy, the power produced by the alpha particle must exceed the power loss. This P alpha is the same as P out with the only difference that rather than using the total fusion energy which is 17.6 MeV, one should have only the fusion energy corresponding to the alpha particle. So, P alpha is the same thing as P out multiplied by energy released in the form of with the alpha particle divided by energy total energy released in the fusion reaction; this is 3.5 MeV this is 17.6 MeV; so this raise you as around one fifth or so. If I put this condition here then, what you get is n tau. Turns out to be, let me just write down this n tau e and temperature when temperature is expressed in (( )) electron volt. This must exceed around 3 into 10 to the power 15 per centimeter cube into KeV into second, where T is in KeV. If I choose this to be 10 KeV temperature then, this turns out to be 3 into 10 to 14 n tau is 10 to 3 into 10 to 14 per centimeter cube into second.

So, these are two important requirements for ignition implies that one does not require them to sustain the fusion reactions. One does not require any external heating source; the plasma can sustain itself. The fusion products like the alpha particle which are released in the fusion reaction are able to sustain the burn; so one must keep this in view. So, one parameter I mentioned to you was the density in Tokamak operations this is around 10 to the power 14 per centimeter cube and temperature we are having of the order of 10 k e v which is about 10 to the power 8 degrees kelvin and confinement time we want to exceed 1 second; well, this is one important consideration.

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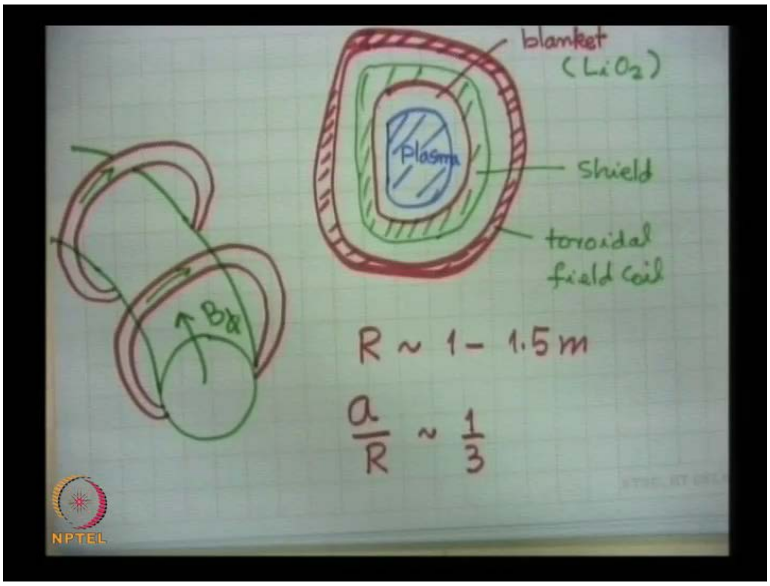

Now, if you look at the reactor, Tokamak as a reactor but, you will and we say is like this; that well, the cross section of Tokamak is really not circular. Close to circle but, not really circular; I can denote like this; this is the plasma in a Tokamak; so this is the cross section of the plasma near the minor at the minor radius.

Surrounding this is a blanket that we would employ to absorb the neutrons. So, let me draw a blanket here. It is a plasma here; this is a blanket here; this blanket is chosen of a compound of lithium because, you would like to use the neutrons for two purposes. One to absorb the neutron and extract energy from the neutron in the form of heat.

Secondly, the blanket can be used to produce tritium which is required as a fueling for Tokamak. So, the plasma is here; let me just write this plasma is here; this all plasma here and then outside the blanket is a shield because, some residual neutrons you would like to remove from the system so that, they do not go to the super conducting coils that produce the magnetic field. So, this is I will call as the shield and then there are toroidal field coil.

Let me just note this. This is the toroidal field coil; so, this is a toroidal field coil which is super conducting coil and let me denote this as the shield. This entire thing actually this is let me just mention this is shield and this is toroidal field coil or super conducting coil blanket is contained. Some compound of lithium usually lithium Li O 2 is used; so when neutrons bombard this they produce tritium.

So, this is the entire structure is as you know, this course all around. So, let me draw a picture off this; just a second. Let me draw a simpler picture of this for the toroidal field coils; this is the cross sectional view of a Tokamak. So, if you really go all around then the Tokamak is like this. Cross section is like this and then this goes and these field coils are these. Starting from somewhere here, going like this then there will be a field coil here which goes like this and Tokamak goes all around, there is a current in this field coils like this which produces a magnetic field in this direction b phi. So, Tokamak has this kind of configuration; several field coils along the torus that will produce a B phi typically of the order of 4 to 10 tesla.

Inside the, between the field coils and the plasma lies or rather, you must have inside the vacuum vessel, you must have the blanket shield and the shield to protect the system from or rather to protect the super conducting coils from neutrons which are produced in the system. Neutrons cannot be observed in the plasma on the alpha particles are to be observed in the plasma and consequently, size of the plasma should be substantially large so that the alpha particles before the reach the walls of the vessel or edge of the plasma they are able to deposit their energy and these particles are quite energetic 3.5 MeV is the energy of alpha particles.

So, you require plasma size to be significantly large to be able to slow down those particles and absorb energy from them and typically, measure radius of the plasma the total [mig/big] big radius of the torus is around 1 to 1.5 meter and ratio of minor radius to major radius is called the aspect ratio this could be like one third, one fourth, one third, typically.

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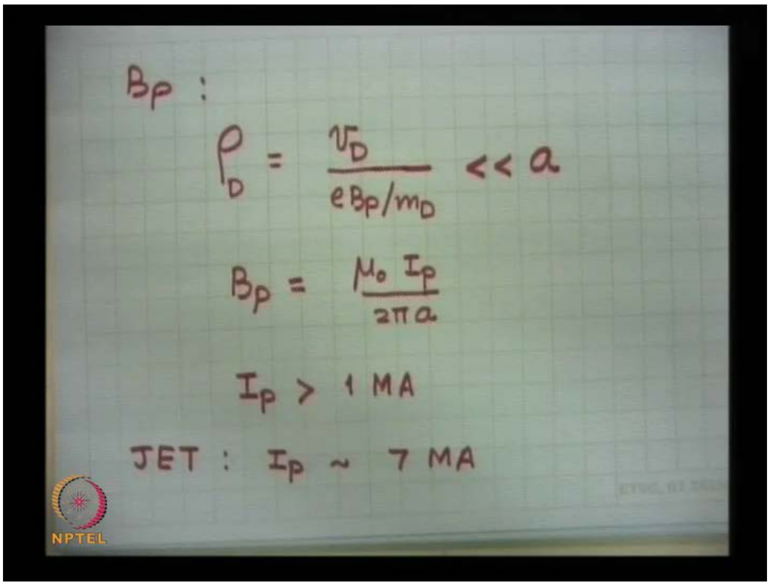

B_p :

$$\rho_D = \frac{v_D}{eB_p/m_D} << a$$

$$B_p = \frac{\mu_0 I_p}{2\pi a}$$

$I_p > 1$ MA

JET : $I_p \sim 7$ MA

Well, an important parameter in this is called poloidal magnetic field and we had talked about this B poloidal is typically has to be so strong that the larmor radius of say, deuterium or tritium due to the poloidal magnetic field which I define as thermal velocity of deuterium divided by e B poloidal upon mass of deuterium. This is called the larmor radius of deuterium. If there were no toroidal magnetic field and the only poloidal magnetic field was present then, for plasma confinement in order to avoid the curvature drift and grad b drifts to cause charge separation, we wanted this to be much less than the minor radius of a Tokamak.

If you work it out and also if you remember that poloidal magnetic field is related to toroidal current I p, typically, as mu 0 magnetic permeability of free space into plasma current divided by 2 pi a where a is the minor digit Tokamak.

So, if you put this in these numbers in there then, what we really require? You require a current in axis of 1 mega ampere joint European torus Tokamak called jet had a current I p of the order of 7 mega ampere. So, we are really requiring a very huge current in the plasma, several mega ampere current we require to produce, adequate poloidal magnetic field to achieve plasma confinement.

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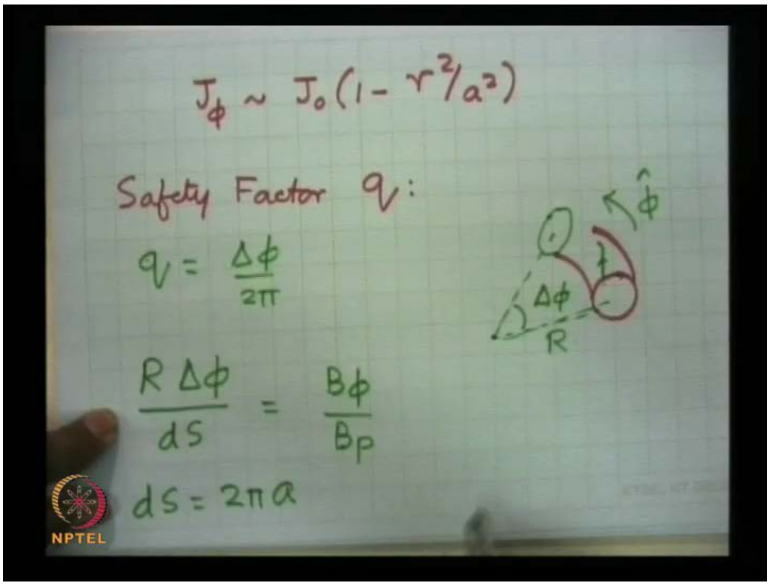

$J_\phi \sim J_0(1 - r^2/a^2)$

Safety Factor q:

$q = \frac{\Delta\phi}{2\pi}$

$\frac{R\,\Delta\phi}{ds} = \frac{B_\phi}{B_p}$

$ds = 2\pi a$

This current is not uniform. This current as I mentioned, when I was discussing the grad shuffle of equation for plasma equilibrium, we found that the current has is typical profile. May be I would say, that J phi can be taken to be some constant into 1 minus r square by a square. This sort of current profile we have different variations but, this is a typical profile. Another important quantity in Tokamak operation is called as safety factor denoted by q and this is an important parameter which is responsible or in terms of which one can talk about the M H D stability of the system. So, one has to be choose a proper q profile and this is defined something like this.

In a Tokamak as I mentioned, that if I choose this as the cross section of a Tokamak and this is the direction toroidal directions phi direction. Then, the lines of force move on the surface; they start from here they will go around it and then they will come back. So, what happens when a line of force moves? Then, after making some displacement in phi direction it will come back on the same point in the poloidal plane.

Suppose this is another poloidal plane here and it is started from some value it arrives, same point here after making a rotation. The angle, this is I measure the angle from here; this is the center and then I go from here. This is the distance I call delta phi; so, delta phi is the displacement in the direction of phi. This direction I called as phi direction for a line of force so that, it arrives in the same location in the poloidal plane; same point in the poloidal plane and I call this q, as this delta phi upon 2 pi, let me define delta phi again the lines of force moves on a flux surface.

So, if you are cutting the Tokamak in the, in a plane perpendicular to phi at many places and find out the location of the line of force. Then, the line of force which had some location in one poloidal plane, will arrive at the same location in the poloidal plane after travelling a distance delta phi. Now, the lines of force depend on the toroidal magnetic field. So, if you are moving a distance delta phi then the toroidal magnetic field displacement in the phi direction is R times delta phi. If the angular displacement is delta phi in this distance is R then, the displacement in the phi direction will be R delta phi.

The displacement in the poloidal direction I call d S or delta S; this will be 2 pi a. so, d S is equal to because it has (( )) circle the line of force has gone around was 2 pi a is the displacement and this should be equal to the ratio of toroidal magnetic field divided by the poloidal magnetic field. Some people write this B theta; so from here delta phi, one can calculate and q turns out to be let me just write down the value of q that you get from here is equal to R B phi upon capital R into b poloidal.

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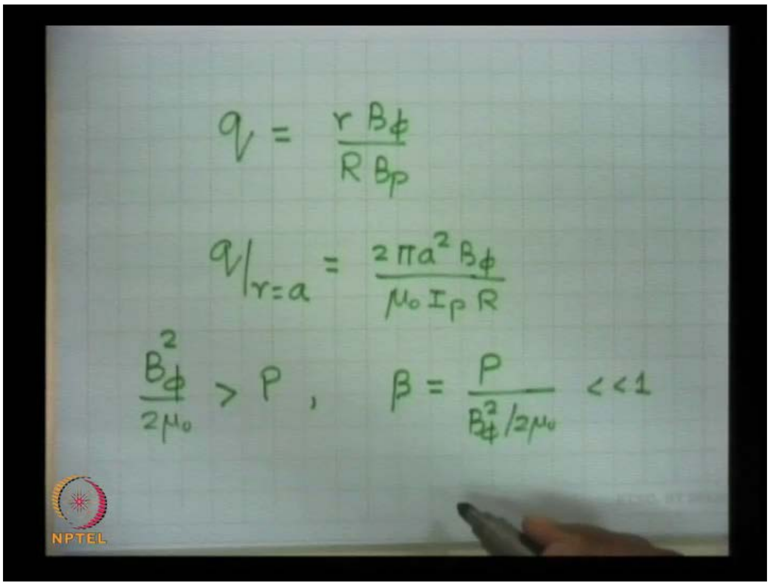

$$q = \frac{r B_\phi}{R B_p}$$

$$q\Big|_{r=a} = \frac{2\pi a^2 B_\phi}{\mu_0 I_p R}$$

$$\frac{B_\phi^2}{2\mu_0} > P \,, \qquad \beta = \frac{P}{B_\phi^2/2\mu_0} << 1$$

Please note that b poloidal is 0 at R equal to 0. So, this has some constant value at R equal to a; this also, this has been constant, a typical value. So, there is some variation of q with R but, less than linear is not like linear variation because, B p changes with R. So, at R is equal to a where a is the edge of the plasma; this is equal to this turns out to be typically 2 pi a square B phi upon mu 0 I p into R where I have put the value of B p in terms of current - plasma current and at small r is equal to a at their point. This is the typical q of the plasma which is around 2 3 or something so one should choose a proper ratio of a by R called aspect ratio in compliance with the plasma current and toroidal magnetic field.

Well, at some general point R q can be expressed in terms of plasma current. I do not think I want to go in those details; so, these are the some important considerations in designing a Tokamak that one should choose B phi such that B square by 2 mu 0 which is called magnetic pressure. This must exceed substantially the plasma pressure; the ratio of plasma pressure to magnetic pressure is known as beta of the plasma and beta has to be much less than 1.

If you want to have to avoid instabilities m h d instabilities, beta of the plasma which is defined as p kinetic pressure of the plasma upon magnetic pressure upon 2 mu 0 this has to be substantially less than 1 and may be 0.2 or something and this puts a condition on B p because, pressure is n T product of density and temperature which we already specified temperature as to be 10 KeV density has to be 10 to 14. So, B phi has to satisfy this condition and that is why I say that at least a few tesla magnetic field is required 4 tesla, 5 tesla, 10 tesla, etcetera. Tokamak for example, I will talk about Tokamak is having about 10 tesla magnetic field and other Tokamak have little less magnetic field. But, typical range would be like between 4 and 10 tesla magnetic field. Well, the issue is twofold; how are you going to provide a current to the plasma? drive a current in the plasma and how are you going to heat the plasma?

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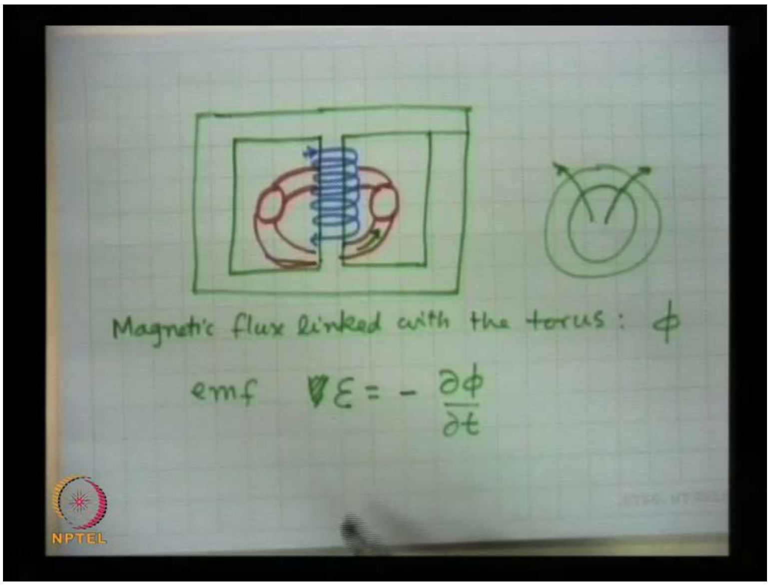

Magnetic flux linked with the torus : ϕ

emf $\cancel{V}\ \mathcal{E} = -\frac{\partial \phi}{\partial t}$

The primary heating and current drive is accomplished by a transformer action. So, let me schematically demonstrate this. Suppose, I choose a frame iron frame like this; choose an iron frame like this well like this and this is the outer side of the frame. Suppose, I choose an iron frame like this and then I have a primary winding on this like this and pass a current a c current in this coil, this will produce a magnetic flux that will be linked with the entire frame. Now, you bring in a Tokamak whose cross section is here like this and this goes like this, like this. Tokamak becomes secondary of the transformer this is the primary coil of the transformer and plasma having a very high conductivity, higher than that of any solid or any material; any metal it will induce an emf that will induce a current in the system.

So, suppose the magnetic flux linked with well, please examine if you are seeing Tokamak from top. It will look like this and the lines of force that I have produced by you know in the central region in the iron core. Suppose, the lines of force are going like this; so then the lines of force I would say they are so there is a flux linked in the central region here. So, if you examine the entire torus then in the center of the torus there exists a magnetic flux linked to the circuit and whenever this flux changes with time.

So, magnetic flux linked with the torus I will call this as phi when this flux changes with time. Then, emf is induced and emf which I called as v becomes is equal to or I think, I should call something else because v I am calling as volume. Let me remove this symbol here I used emf as E this quantity becomes is equal to minus delta phi by delta t. As long as you can pass a time dependent current in the primary of the transformer the flux linked with the torus will change with time and emf will be produced and this emf will induce say current in the Tokamak in compliance with the following equation.

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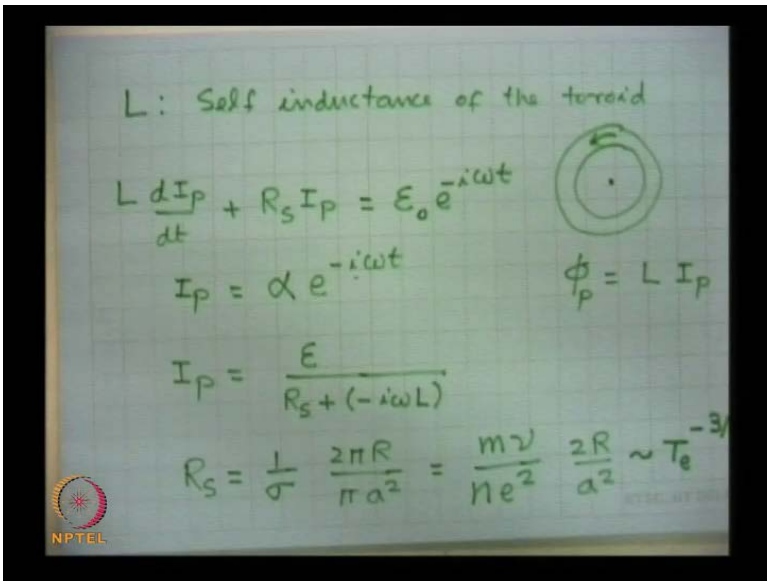

L: Self inductance of the toroid

$$L\frac{dI_p}{dt} + R_s I_p = \mathcal{E}_0 e^{-i\omega t}$$

$$I_p = \alpha e^{-i\omega t}$$

$$\phi_p = L I_p$$

$$I_p = \frac{\mathcal{E}}{R_s + (-i\omega L)}$$

$$R_s = \frac{1}{\sigma}\frac{2\pi R}{\pi a^2} = \frac{m\nu}{ne^2}\frac{2R}{a^2} \sim T_e^{-3/}$$

If I consider Tokamak to have a self inductance L, self inductance of the toroid or Tokamak because, whenever you pass a current in the plasma it is like a coil. So, whenever you pass a current in the plasma there will be some magnetic flux linked by this current plasma. Current phi was not the flux produced by this current phi was the flux produced by the current in the transformer and I am talking of the flux produced by the plasma current if it is induced here. Then, the so I will call as phi p the plasma current produces a flux linked with the circuit and this will be equal to L into I p. This is how you define self inductance of a plasma toroid.

Then, the equation that governs the current would be L d I by d t I p is the plasma current. So, L d I p by d t plus resistance of the plasma R s into I p plus this is equal to emf e. As I mentioned that flux must change with time; only then this e m is produced for the sake of simplicity, we have a better appreciation I will consider this to be like E 0 e to the power minus I omega t in actual operation. This is not a sinusoidal function of time that if continuously increase the current in the primary of a transformers. So, this a linear function of time or something or some function of time.

But, for the sake of mathematical analysis which is very reasonable to reveal physics of this process, let me choose this. Then, the solution turns out to be as if I p is equal to some quantity say alpha exponential of minus I omega t some quasi steady state solution. I can write down and replace d d t by minus I omega so this gives me I p is equal to emf that you are introducing divided by R s plus minus I omega L. This is the impedance due to the inductor inductive part this is the register part total is the impedance of the plasma.

For typical parameter, well let me estimate this R s in terms of plasma conductivity. R s is the plasma resistance which is equal to plasma resistivity which is one upon conductivity multiplied by length of the plasma which is 2 pi R and divided by the area of cross section of the plasma through which the current flows which is pi a square and sigma is equal to electron density into charge square upon mass of the electron into collision frequency.

So, this becomes 2 R upon a square my main point was to tell you that the resistivity of the resistance of the plasma is proportional to collision frequency nu and collision frequency is a function of electron temperature. This quantity goes as temperature of the electron to the power minus 3 by 2 this is a important dependence.

Please note that at low temperature plasma is very resistive but, as the temperature goes up then plasma becomes less in resistive. However, at any temperature up to a few KeV; if you compare these two quantities R s resistive less than omega L and L you can essentially estimate order of magnitude estimation of L you can easily have that. If I have a current I p in the system then the magnetic field at the center you can easily write.

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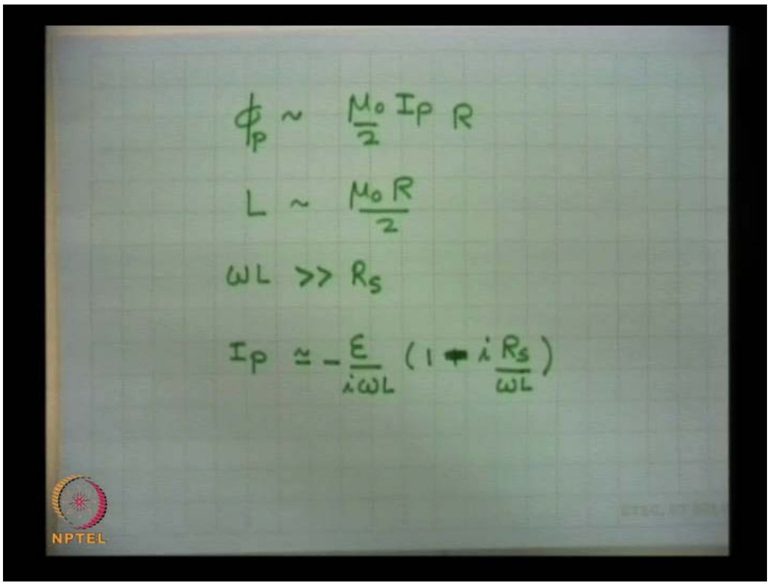

$\phi_p \sim \frac{\mu_0}{2} I_p R$

$L \sim \frac{\mu_0 R}{2}$

$\omega L >> R_s$

$I_p \simeq -\frac{E}{i\omega L}\left(1 + i\frac{R_s}{\omega L}\right)$

If I do this I get phi p typically of this order, let me just find out. So, phi p is typically of the order of mu 0 I p upon 2 into R. So, inductance of the plasma is typically equal to mu 0 into measure radius divided by 2 and then, just you can calculate the value of omega L and you find that omega L, if I choose the time like time of evaluation of emf like a second then, this is like 2 pi upon 1 second, which is like 2 pi by 6 and if I put multiply by L this turns out to be order of the magnitude bigger than resistance. So, this is much bigger than R s as a result the induced plasma current I p is typically equal to E divided by I omega L multiplied by 1. Actually, minus time I let me take common and then this becomes 1 plus I R s upon omega L. I think this is minus sign here.

So, this is the plasma current dependence on this. This is small; so, I p usually as pi by 2 out of phase with the induced e m f however important quantity is the heating so first of all you get the plasma current and you want this current to be typically of the order of a few omega ampere and hence this proper calculation if you reveal will reveal that E does not have to be very large because omega l still is a very large quantity omega L you can just ensure that L has a very small quantity mu 0 which is 4 pi 10 to minus 7 in m k s units R is about the of the order of one in m k s units.

This L being a very tiny quantity of the order of 10 to minus 7 or 10 to minus 6; this becomes very large even if e m f is 10 volt or 100 volt or something, 20 30 volts. So, this

I p. If I want of the order of omega ampere, omega if I choose like 2 pi L is 10 to minus 6 then e m f does not have to be very large; so few terms of volt will do the job.

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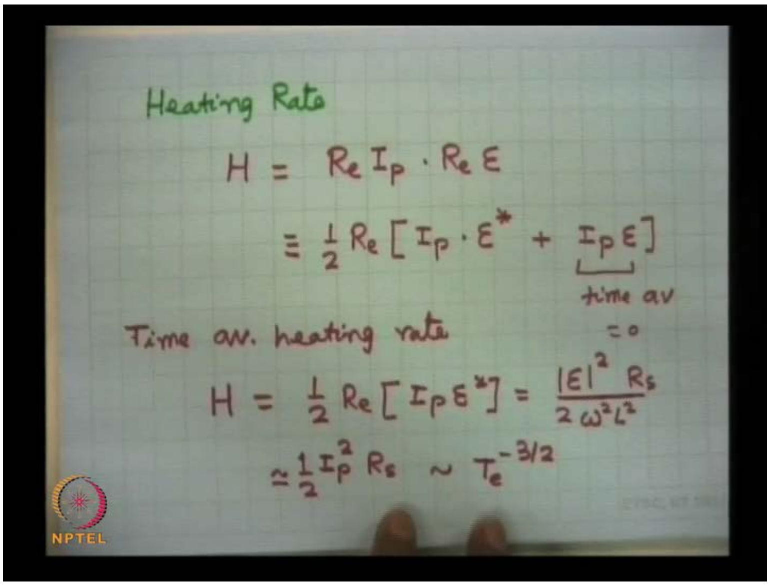

Heating Rate

$$H = R_e I_p \cdot R_e \mathcal{E}$$

$$\equiv \frac{1}{2} R_e \left[I_p \cdot \mathcal{E}^* + \underbrace{I_p \mathcal{E}}_{\substack{\text{time av} \\ = 0}} \right]$$

Time av. heating rate

$$H = \frac{1}{2} R_e \left[I_p \mathcal{E}^* \right] = \frac{|\mathcal{E}|^2 R_s}{2 \omega^2 L^2}$$

$$\simeq \frac{1}{2} I_p^2 R_s \sim T_e^{-3/2}$$

Let us see, what is the consequence of this term? An important quantity is the plasma heating rate. Heating rate is defined as the system if the system is resistance R s then the heating rate h is the product of real part of I p into real part of e m f I have used complex rotations otherwise this is actual current multiplied by actual e m f the product of the two is known as the heat dissipation per unit volume or rather the work done by the e m f per unit volume of heat dissipation.

However, the time average and its quantity is important which is called heat dissipation per unit volume, heat dissipation rate. So, time average and its quantity if I want then this can be written as half; real part of I p into e m f complex conjugate this is I p into e m f. This identity we had proved once that real part of a complex number into real part of another complex number is always equal to half real part of the product of the numbers complex numbers plus product of one number into complex conjugate of the other number like this.

But, both are varying in time exponential minus I omega t. So, time average of this turns out to be 0 while the exponentials will cancel here so time average of this is 0. So, if I take time average heating rate I can forget then, the last term and I get H is equal to half real part of I p into e star and if I put the numbers there then it turns out to be I p was this becomes E star or e m f square upon 2 omega square l square into R s.

In terms of current, if I wanted to write then this is nearly equal to I p square into R s divided by 2. So, heating rate is proportional to R s the resistance of the plasma which scales as temperature to the power minus 3 by 2 electron temperature to the minus 3 by 2 this is a very serious matter that this ohmic heating is important when the plasma temperature is low so when you are creating a plasma initially you produce a plasma in the Tokamak by some r m discharge but, that the temperature of the plasma is only few electron volt then you use this induction effect and you or transformer effect and erase the temperature. So, as the temperature builds up heating is very effective in the beginning but, as the temperature goes up it becomes less and less effective. Just by reaching temperature of the order of 1 or 2 kilo electron volt, this becomes quite ineffective. Now, this is a very serious matter. How, will you heat the plasma from 1 or 2 kilo electron volt to 10 kilo electron volt. That is a very major issue and we have to provide some supplementary or auxiliary scheme of heating.

Well, this is a very serious matter and I think I going to talk about it separately. Nevertheless, I just or like to mention that there are two schemes of auxiliary heating that have been tried and have been tried very successfully. One of them is known as the radio frequency heating in which you use electromagnetic waves from outside into the system. You rather, you introduce electromagnetic power into the system and somehow those waves have to be chosen properly so that, they can heat the electrons or ions. If they can heat ions that is even preferable because, it is the thermonuclear fusion in which deuterium tritium ions are involved, not the electrons.

Secondly, people are employing neutral beams - electron beams and ion beams cannot be used. Because, they cannot penetrate into the plasma perpendicular to the magnetic field because there is already very strong toroidal magnetic field and one cannot launch charge particles perpendicular to magnetic field. However, one can use neutral particle; so people use neutral beams for this purpose. However, before I close today’s discussion I would like to mention something about the plasma current.

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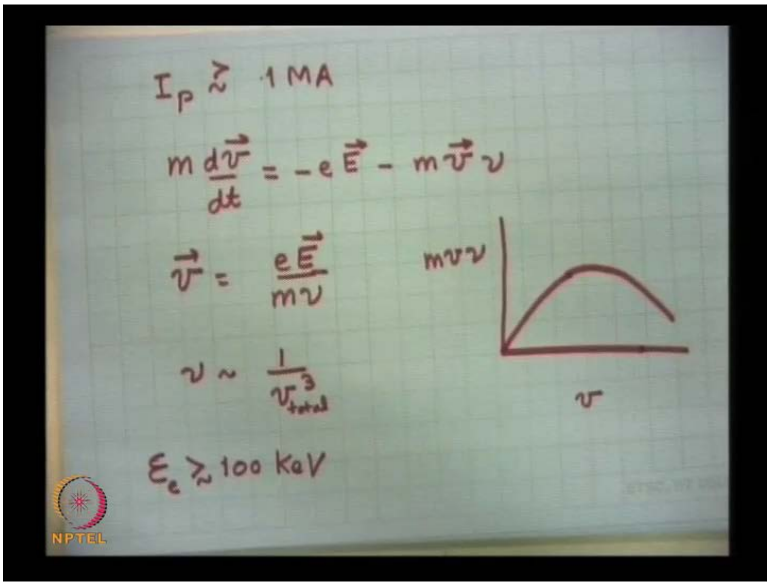

$I_p \gtrsim 1$ MA

$$m\frac{d\vec{v}}{dt} = -e\vec{E} - m\vec{v}\nu$$

$$\vec{v} = \frac{e\vec{E}}{m\nu}$$

$$\nu \sim \frac{1}{v_{total}^3}$$

$\mathcal{E}_e \gtrsim 100$ KeV

When you are talking about current I p of the order of or bigger than 1 mega ampere, several mega ampere current we are talking about voltage or e m f loop voltage may be only few tens of volts. But, the current is very huge the question arises is every electron contributing the current or only some electrons are contributing to the current.

Now, let us examine the character of current or rather the electron response to a d c field, a field of the low frequency field. I am talking about the rate of change of momentum of a particle m d v by d t. In the presence of a field electric field in the system is equal to the electric force on the particle minus m v. This is the average momentum drift momentum as the particle multiplied by the collision frequency this is the average momentum they lose per collision.

This collision frequency depends on the magnitude of velocity which you can consider to be a some of random velocity plus drift velocity. Now, whenever you apply a high electric field this is the momentum the electrons has is getting from the electric field per second and this is the momentum its loosing so in the quasi steady state what you have is that they balance each other and consequently you get velocity is equal to e E upon m nu. Please note the electrons which have a larger value of collision frequency which are very hot. They will acquire a larger drift velocity and this is important because, if the drift velocity becomes comparable or larger in thermal velocity this nu itself will depend on velocity. So, if I can plot a quantity here collisional momentum loss per second this is called m v nu plot m v nu as a function of particle velocity v please remember that collision frequency nu is proportional to 1 upon v to the power 3. Now, this v is sum of drift velocity plus random velocity v total. I mean, let me call v total; so, initially when drift velocity is low v is the drift velocity or average velocity of particles then this quantity will increase.

But, as drift velocity becomes comparable to thermal velocity or larger then this dependence of this function on v will appear through nu also. And, this quantity will decrease; so this will have start decreasing that is a serious matter because this is the maximum momentum loss an electron can have in a plasma you cannot have higher than this momentum loss but, if you apply electro field larger this is the momentum gain by the electron from the electric field and if cannot lose that much momentum by a collisions. Then, the electron will be continuously accelerated such electrons I mentioned earlier are called runaway electrons.

The electrons for which this momentum loss is not balanced by the momentum gain; they are called runaway electrons and in Tokamak when you apply the kind of fields by transformer action. Those fields render electrons quite sum electrons which are initially large velocities they become runaway electrons because, for them the drift velocity the collisionality is very weak and they are the ones which carry large part of the current. So, most of the current in Tokamaks is carried by electrons with energy - electron energy greater than or of the order of 100 kilo electron volt.

Now, that is a very big advantage because if your current is carried by low energy electrons for them, collision frequency is large and this term is very huge; this momentum loss is large. However, if the current is carried this momentum is lost means that, the electric field is doing more work. The work done by the electric force in making an electron move is given by H is equal to the electrical force into drift velocity particle velocity.

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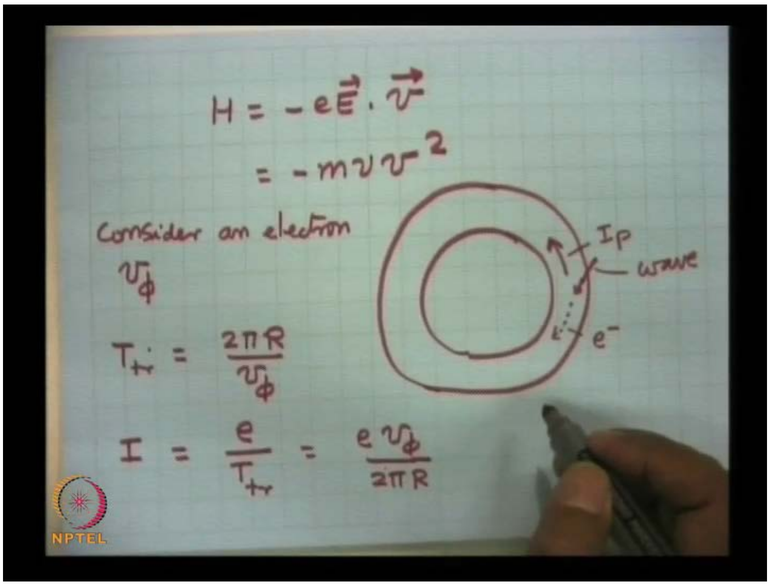

$$H = -e\vec{E}\cdot\vec{v}$$

$$= -m\nu v^2$$

Consider an electron

v_ϕ

$$T_{tr} = \frac{2\pi R}{v_\phi}$$

$$I = \frac{e}{T_{tr}} = \frac{e v_\phi}{2\pi R}$$

This is the work done by the electric force in making the current driven to drive the current. So, each particle if it is to acquire a drift velocity move then, this is the work done by the electric field on this and you would like to have a for a given current this quantity to be minimum. So, what really is happening here that the particles with larger velocity have less damping in because this quantity is equal to minus m nu v and v dot v so v square and this nu goes as 1 upon v cube; so this quantity decreases. Means, more the energetic particles, less the energy wastage in driving a particular current and that is an important issue. So, one clue that we get from our discussion is that it will be preferable in a Tokamak though the average temperature of the plasma is like 10 kilo electron volt. But, you would like to have current driven by energetic electrons which have energies like 100 kilo electron volt. So, 10 times the average energy; the energy of these particles should be of that order that will be much preferential, much preferred.

For this purpose, people realize that waves can ((do)) this job better because, in transformer action actually, you are really driving a substantial part of current by the low energy electrons and some part of current by the high energy electrons. However, even if you want to do this process by using radio frequency waves then, the waves can preferentially heat the or give their momentum at energy to energetic particles and can drive a current.

I would like to give you an assessment of what kind of R F powers will be required for a current drive. Let me give you some estimate; more details we will discuss in our next lecture.

So, the issue that we are addressing is that we have a Tokamak and I want to drive a current by using a radio wave. Suppose, I want the electrons to go in this direction, sorry, current to flow in this direction, I want the electrons to travel in the opposite direction and I want my wave to launch my wave so that they come in this direction and the deposit momentum on the particles and particle then electrons continue to move in this direction; this is the motion of electrons.

This is the I p the plasma current that I want to sustain or I want to drive. so this is my wave that has to come from here this is my wave the momentum of the wave has to be having a component in the toroidal direction so that, it deposits a momentum on the particles well in order to have some feel for it. How much current is required? How much momentum transfer is required? Let us see. Suppose, I consider electron moving in a circular path and the velocity of the electron is v phi; suppose, consider an electron with velocity v phi moving then this electron will go around in a time T. well, T I am calling temperature; so, let me call this time of travel as T t, travel T t r; this will be equal to 2 pi R the distance travelled in one circle divided by v phi.

So, an electron will take so much time to go round the torus and the current it will induce I will be equal to charge of the electron which is e magnitude wise divided by T t r travelled time which is typically equal to e v phi upon 2 pi R. so, this is the current each electron will produce.

Now, the question arises; how much power, how much momentum it loses via collisions because, that much momentum has to be replenished by the wave. So, it loses a momentum per second momentum loss per second. If it suffers a collision then in a collision it loses momentum m v phi and with the collisions are nu per second; this is the momentum it will lose per second.

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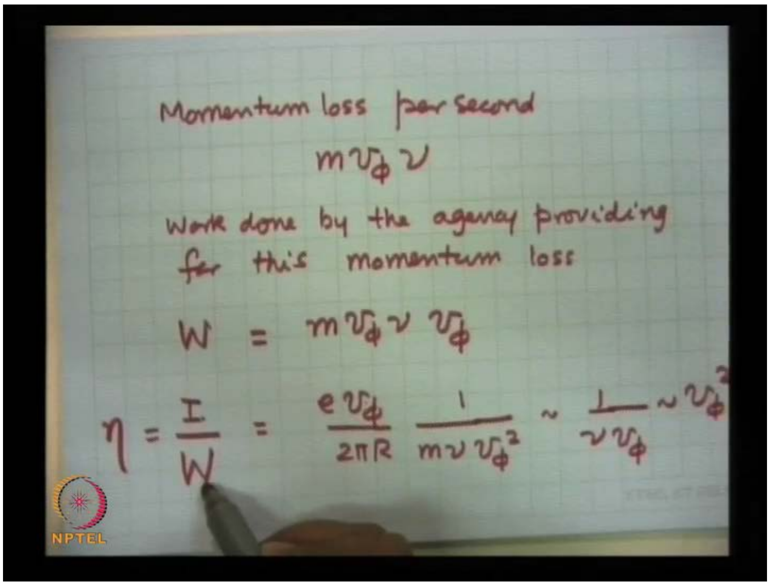

Momentum loss per second

$m v_\phi \nu$

Work done by the agency providing for this momentum loss

$W = m v_\phi \nu v_\phi$

$\eta = \frac{I}{W} = \frac{e v_\phi}{2\pi R} \frac{1}{m \nu v_\phi^2} \sim \frac{1}{\nu v_\phi} \sim v_\phi^2$

So much momentum should be provided to it to be so much work will be force momentum per loss per second is called the so much force must be exerted in the particle per second or so much momentum is to be replenished. So, work done by the by this force which is providing so much momentum by the agency providing for this momentum loss, will be force which is m v phi nu multiplied by the distance travelled per second in the direction of force which is B phi.

So, what you require is, this is the heat loss rate or work done W and the current I gave to you was I. So, for driving a I current the work that you do turns out to be equal to e v phi upon 2 pi r was the value of I that I wrote per particle and work that you do is m nu v phi square and this is called the figure of merit for current drive by a radio frequency wave. You may note here 1 v phi will cancel out this quantity scales as 1 upon nu into v phi square B phi and nu goes as 1 upon B phi to the power 3.

So, this goes as v phi square means, more energetic particles are preferred for current drive. If you want to have a higher efficiency of driven current to power loss then, you must have the current carried by particles with large velocity toroidal velocity. I think we will continue our discussion on current drive and heating in the last in the next lecture which will be probably the concluding lecture on Tokamak; thank you very much.