Plasma Physics

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Module No. # 01
Lecture No. # 25
Tokamak

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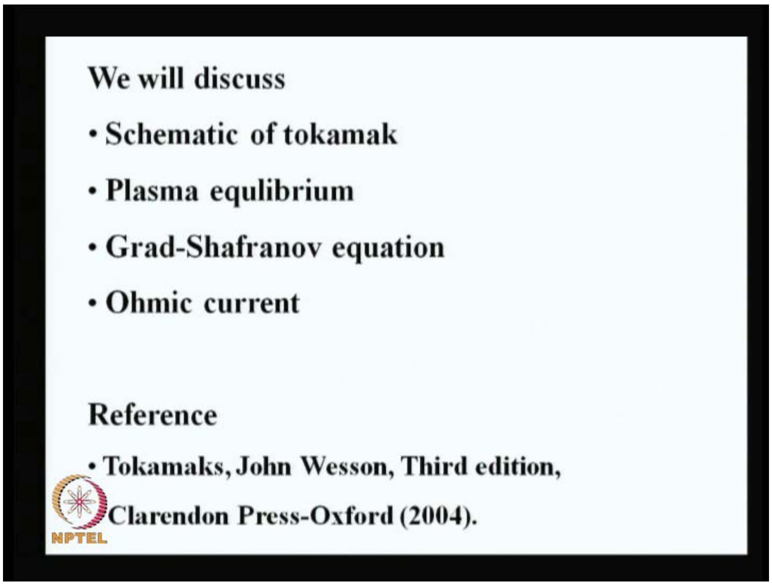

Today, we will discuss the toroidal plasma device called tokamak in detail. We shall discuss the schematic of a tokamak then, discuss the plasma equilibrium first on the basis of a single particle motion and the constant of motion that we deduce last time.

Then, we will go over to Grad-Shafranov equation that talks about a general equilibrium of a plasma and if time permits I will discuss how would you produce current by using induction effect in a plasma for both purposes for the purpose of plasma confinement as well as for the purpose of plasma heating.

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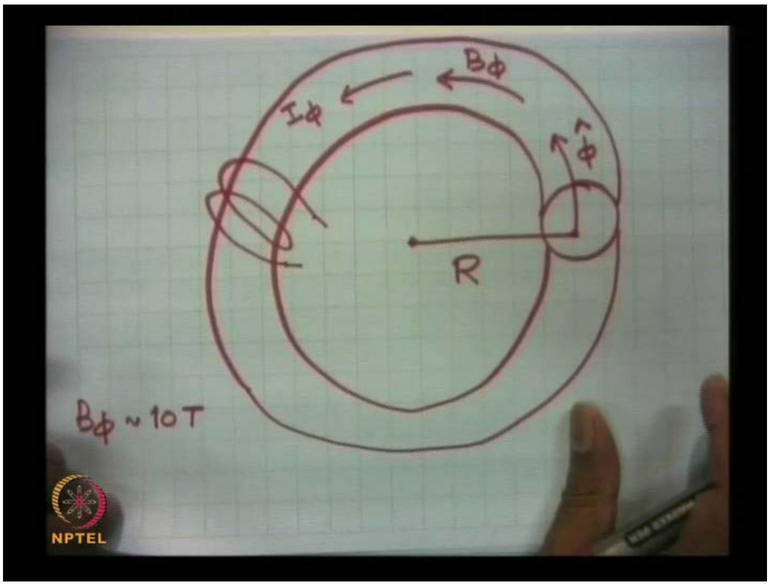

Let me begin with the schematic of a tokomak that we had we have discussing last time. So, tokamak is a torrel device. It has some cross section may be one can choose a tokamak with a circular cross section it could be different in circle also and then it can go like this and go like this.

The distance from the axis of the system to the center of the plasma is called measure radius and is R. But, I will keep R as a variable it can be from here to here anywhere in the plasma. I will be using a cylindrical polar coordinate system in which this direction here is my phi direction. So, this is my phi direction.

Plasma will be allowed to have a current in this direction and also have magnetic field in this direction B phi and also a current I phi. B phi is produced by a set of coils. Schematically I will denote them as these kind of turns, but, basically these are very thick single coils because the current that you want to pass through them is hundreds of amperes may be close to several hundreds of amperes or glow amperes. Current has to pass in them because we want to produce a magnetic field typically of the order of 10 teslas.

The magnetic fields that are envisaged on heater the biggest machine that will be typically of the order of 10 teslas which is huge. So, you require the large very special kind of coils and people are rather using super conducting field coils to carry huge current in them. A current is needed here that will produce a colloidal magnetic field to compensate for two drifts which are dangerous curvature drift and grad B drift of electrons. iI this is not there then the plasma cannot have a equilibrium. So, we would like to examine that issue.

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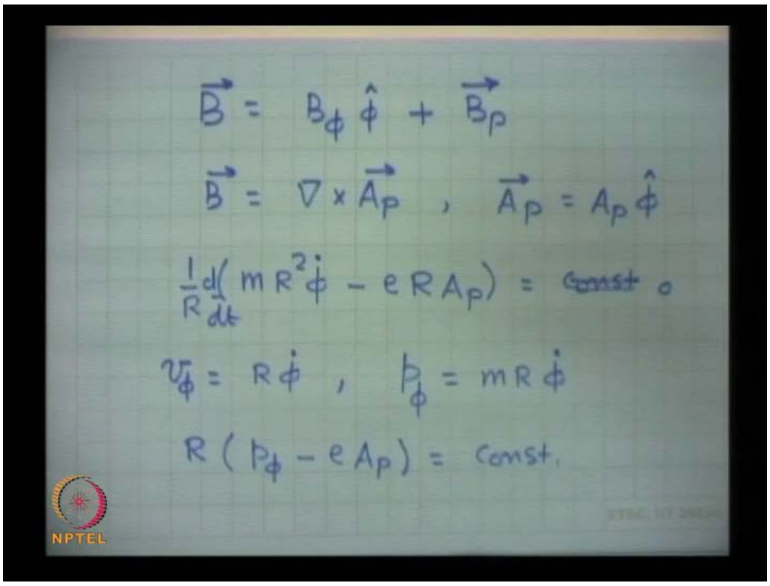

Well in this configuration I had written the magnetic field in the plasma as B is equal to the azimuthally magnetic field B phi, that is produced by external coils plus a colloidal magnetic field produced by the plasma current I phi. This B p was expressed as curl of a vector potential A p and A p was having only phi component. So, this I wrote down as A p into phi.

However, A p depends on R and z it has no phi dependence because we were talking about axis symmetric plasma confinement and in that configuration we found that a quantity called 1upon R into m R square phi dot minus e then it was R A p is equal to constant of motion. Actually this not one upon R.

D d t of this quantity was 0 hence, this quantity inside must be a constant of motion and if I define a quantity azimuthally velocity of the particle is equal to R. This is a phi dot here phi dot is the time derivative of phi and if I define p phi the momentum of the electron due to azimuthally motion then this quantity is m R phi dot in. So, when I write down this constant of motion I get I can rewrite this as R common multiplied by p phi minus e A p and A p is also in the phi direction is a constant of motion.

Now, the implication of this is important we simply says that if somewhere R is varies very little if particle if the particle goes from one region in tokomak to another region and A p changes then p phi must also change. However, in a magnetic field when a charge particle moves magnitude of momentum does not change, only direction of momentum changes.

Consequently the maximum change p phi can undertake as equal to particle magnitude of momentum and that is fixed. Consequently all regions of the tokomak are not accessible with the particle because A p changes too much from one region to another. So, that puts a restriction on particle motion and confines the particles.

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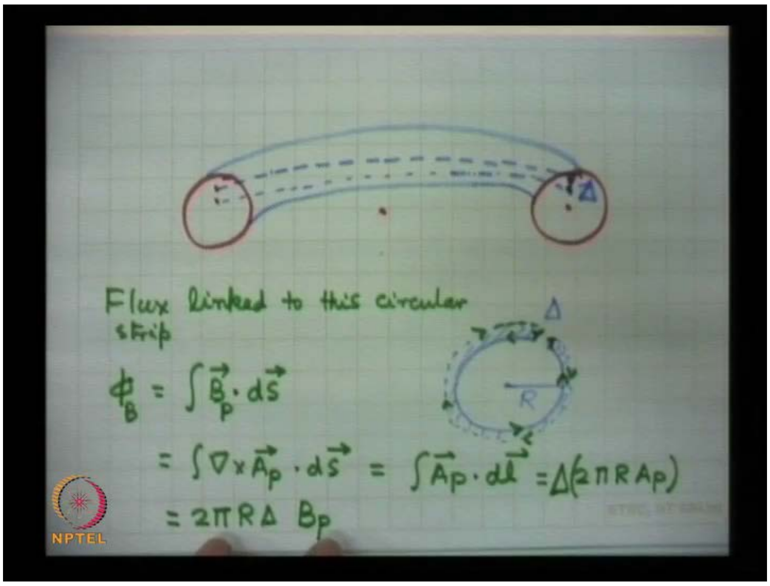

Let me make some estimate how much A p can change. For that I consider a special thing. Let me consider my tokamak cross section like this. Now please this is center of the plasma center of the or axis of the tokomak passes through here. This is capital R.

Now, please consider any point two points here, these two points here and visualize because as you move away from here, if there is a current in the plasma which is perpendicular to the cross section. Here axial because the phi direction is perpendicular to is phi direction is here which is perpendicular to this plane. So, what I am saying here is that if I consider a loop other cross section of the tokomak will be like here.

Let me just draw this here. So, if I consider some two points here means draw two circles round the tokamak one at this height and another one at this height. So, consider two pass two circular pass and those pass are just joint with each other. They are cut, but, I am suggesting is that please consider on the torus is like this one this just portion goes like this. This is the kind of torus we are having, This is the close curve.

So, please consider two circles one above the other. If this is my z axis I am suggesting that consider a circle like this of radius capital R another circle also of capital radius R, but, shifted in z by certain amount delta. So, this is the displacement in z direction and I will call as delta z or delta.

So, please consider two loops because I am expecting that when the particle from one position in the cross section moves to another position say in the z direction. We have learnt that the electron will undergo curvature drift or grad B drift in the minus z direction and ions in the positive z direction. So, I am considering when the particle goes in the z direction how much a will change in order to estimate the change in a I am considering a loop which is made of like this is a one loop in the x y plane another loop also in the x y plane, but, shifted in z.

So, this is may be z equal to 0 plane another loop shifted, but, slightly above this that would be an above this. So, two loops are shifted from each other by distance delta.Now, consider the flux magnetic flux linked with this surface. The total length of this path of radius R would be 2 phi R and height is delta. So, total area would be two pi R delta of this. I want to calculate the magnetic flux linked with this. Please understand I am considering this circle along the phi axis of the tokamak and this is at different. So, the radius of the both circles are same only difference is that one is at z equal to 0 plane and the other one is at a height z equal to delta

Now, let me calculate this integral flux linked to this loop this loop I will consider like any path I can consider like this and going backward on the other one. So, in the inner curve here and then path goes line integral is like this and surface integral is over the area.

So, flux linked to this is strip circular strip let me call this flux by some number may be phi B this quantity would be magnetic field dot the area of their strip now this B because this entire strip is tangential to phi and hence only colloidal magnetic field will contribute to this, but, B p is expressible as curl of A p potential vector potential dot d s using a stokes theorem I can converts into line integral which is equal to A p dot d l this is a line integral.

The path of the line integral I will demonstrated here this path goes here you may note this is a closed path it moves in the anti clockwise on one circle and in the clockwise session in the other circle one circle is in the z equal to 0 plane and other circle as at a height delta along z axis.

So, what do I get here A p dot d l, d l is the length primary length of this coil is in the phi direction and A p is independent of phi. So, it is a constant. So, when you move on one circle A p will have one value and when you written on the other circle at a certain height A p will have a different value and the contributions through a line integral from these two arms will cancel out because the direction in one arm is upward and the other is downward. So, they will cancel out.

So, the basic contribution to this integral comes from these long curved paths and this turns out to be equal to total length is 2 pi R into change in A p difference in this quantity. So, let me call delta or change in this quantity because it change in A p there will be some finiteness in this integral.

So, the flux linked with a strip circular strip of height delta z height delta and radius R turns out to be So much, but, actually is how much just in this strip suppose the value of colloidal magnetic field is B p and the area is two pi R delta. So, just put this is equal to 2 pi R into delta into colloidal magnetic field means when the electron goes a distance delta in the z direction then the vector potential changes by So much amount given by this and I get the value of delta A p change in A p is equal to two pi R will cancel out.

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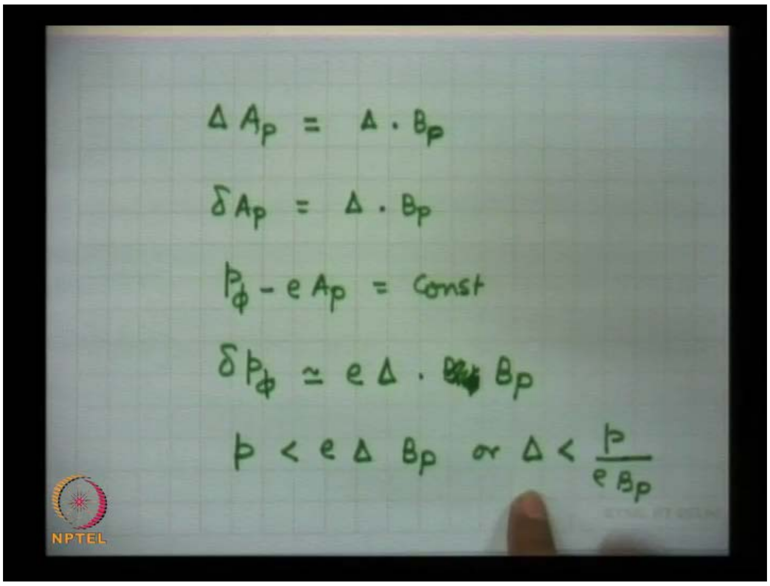

So, change in vector potential A p in going from one value of z to another value of displacement delta this is equal to delta times into B p this delta is not a product with a p. So, change in A p may be I will it will better if I call this as delta A p is equal to this delta is the displacement between the two loops into B p.

So, when the electron goes due to curvature drift or grad B drift along z direction by a distance delta it sees a change in vector potential by this amount and when vector potential changes then, obviously, p phi must change because we know at p phi minus e A p is a constant of motion. So, any change in A p must result in change in delta p. So, delta p phi will be of the order of e time is delta into B p sorry B p.

But the maximum value this quantity can undertake is actual momentum of the particle hence this delta has to be satisfying this condition delta into B p or delta must always be less than p upon e B p this is the maximum displacement a particle can go from the center of tokomak if they have is rotating around the center of tokomak in the beginning then it cannot move too much away from there in the z direction or away from the in any direction it will not be moving.

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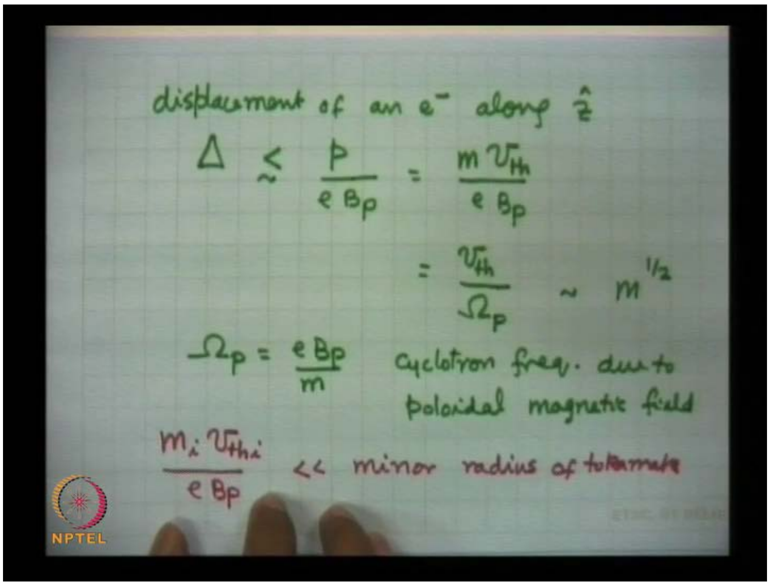

Now, let me calculate this quantity for a average particle moving with thermal velocity then what you find is delta the displacement of an electron along z direction would be typically delta or always be less than or of the order of p upon e B poloidal if I write down p as m into thermal velocity of electrons then this is e B p,e B p upon m I will call as the cyclotron frequency due to poloidal magnetic field and I can write down this is v thermal upon omega p with cyclotron frequency due to poloidal magnetic field.

So, if I am divining this is equal to e B p upon m cyclotron frequency of electrons due to poloidal magnetic field this is that magnetic field that is produced by azimuthal current and is perpendicular to the phi cap direction it is a important thing you may note that this excursion delta could be more for the ion because v thermal goes as one upon under root mass. So, this whole quantity is proportional to root mass.

So, this entire quantity is proportional to mass of the particle to the power half means electrons will be largely confined to within the center of the plasma, but, ions can go longer distance and hence this condition should be one must satisfy a condition that this delta should be much less than the minor radius of a tokomak. So, for confinement the essential condition is that mass of the ion into thermal velocity of ion upon e B p should be less than much less than the minor radius of tokomak it is a very interesting thing.

So, particles are primarily confined by the azimuthal magnetic field if you can choose typical radius of major radius of a tokomak as around a meter or more and aspect ratio means the ratio of minor radius to major radius is around one third.So, if you are talking of a tokomak whose minor radius is about 30 40 centimeters 40 centimeter typically then this quantity should be having a value may be less than point three sorry or may be one third one half or may be one tenth of minor radius which is putting a lot of restriction on B p that a certain minimum value of B p is required and B p is produced by the azimuthally current and typical currents that can satisfy this condition are in the order of mega ampere.

So, this condition provides an estimate of minimum magnetic field required for plasma confinement in a tokomak you require a toroidal current of the order of may be half mega ampere or mega ampere to have ion and electron confinement in the plasma.

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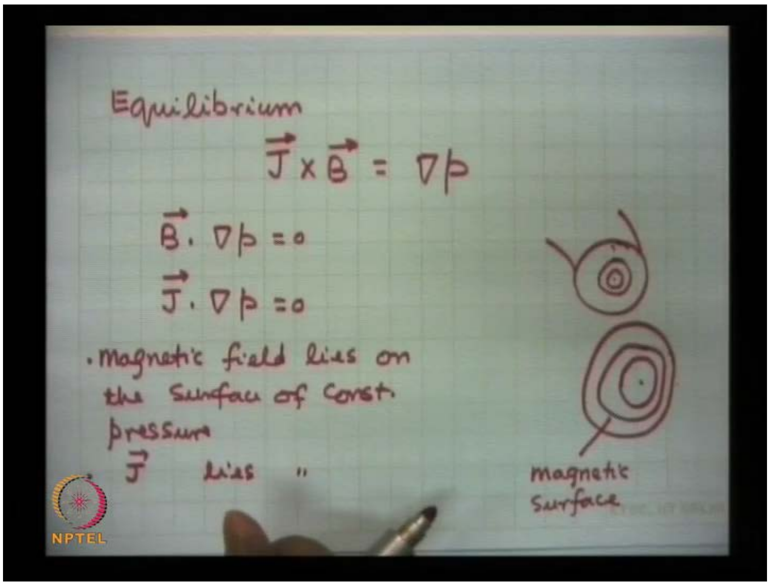

I think now I would like to talk of plasma equilibrium in some general term in one of the lectures I have talked about equilibrium that if I want plasma to be in equilibrium then or time independent equilibrium then J cross B should be equal to gradient of pressure where J is the current density in the plasma B is the magnetic field in the plasma total magnetic field by external currents and internal currents and p is the pressure in the plasma we have deduce this equation.

I would like to draw some important criteria for plasma confinement from this equation first thing that when we notice from here is that B dot grad p is 0 and similarly, if I take J dot of this equation then J dot grad p is 0 what are we expecting inside a tokomak always keep the view this is the cross section of a tokomak and we are expecting that whenever you move in the phi direction this phi direction, wherever you go the pressure will not change current magnitude will not change you are excepting B magnitude also not to change. So, we are talking of excess symmetric equilibrium in which p can vary only with z the vertical coordinate actually in this will be z coordinate along from here center to the rim and one of them would be r.

So, what I am saying here is that B field must lie on a surface of constant pressure grad p means this is a vector quantity perpendicular to the surface of constant pressure. So, B field must lie on the surface of constant pressure and similarly, this says that J must lie on the surface of constant pressure. So, a surface of constant pressure that may whatever shape it may have it may have a circular shape like this they are the surfaces of constant pressure or it may have a shape like this it may have a shape like this.

The surfaces may be coming close to each other here they may be farther from here, but, they cannot cut each other. So, if I have an any surface of constant pressure on the cross sectional plane a plane perpendicular to phi cap then J and B must lay on this surface the surface of constant pressure will also be a surface of constant because B field will be lying parallel to that. So, this is called magnetic surface.

So, first of all we are saying that B will lay the lines of force will always be tangential to the surface of constant pressure a surface of constant pressure. So, write these in words magnetic field lies on the surface of constant pressure the same thing happens with J also lays on the surface of constant pressure. So, any surface of constant pressure is also known as magnetic surface magnetic field is tangential to the surface this one very important thing that we have seen that is why simple consideration we note this.

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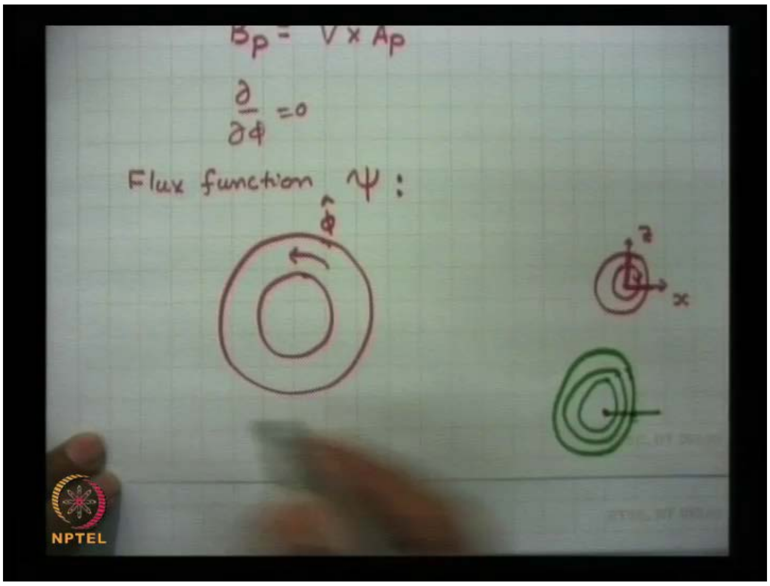

And since we have been writing B poloidal is equal to curl of A p we have seen that there is a constant of motion which was in terms of a quantity R a p. So, I would like to first of all express this p though in general they are functions of R and z J is also a function of R and z and B is also a function of R and z there is no phi dependence for ax symmetric system.

So, delta delta phi is 0 in view of this I would like to introduce a quantity called flux function psi let me define this. This is something very important in plasma jargon in plasma literature this is very often used and I would like to drawn something deduce some write lets understand the meaning of this quantity.

But I am saying is that in a tokomak when you move this is the cross section please keep this in view and also keep this picture of a tokomak like this. So, if I draw any plane any where perpendicular to the phi direction this is my phi direction. So, draw a plane somewhere here or anywhere suppose this is one of the planes that I have drawn which is certainly perpendicular to phi axis.

So, this may be my x direction and this my z direction because phi axis I have chosen perpendicular to this now what I am saying that in this plane if I can think of you know the lines of force that you are expecting are all like this magnetic lines of force will be like this circles and they will not cut each other they may be close to each other here, but, may be farther from each other here, but, if you take any cross section any draw any line from here to here or from here to here the number of lines of force that will cross this will be same.

So, let me draw this bigger picture here if I take any point here on x axis then the lines of force could be like this and now please visualize a circular strip that goes rounds this is the cross sectional view here and go round the torus when you go round the torus in phi direction and if you go from center of the plasma to any surface here this surface or this surface anywhere on the surface. So, choose any surface then the total magnetic flux linked with this strip would be how much this is the quantity I call psi prime.

The total magnetic flux linked their strip the total length of this strip will be two pi R kind of thing because I am going to two pi R distance here multiplied by the width, but, the length also varies. So, let me call a quantity flux linked with this strip of some inner radius R and outer radius R dash etcetera.

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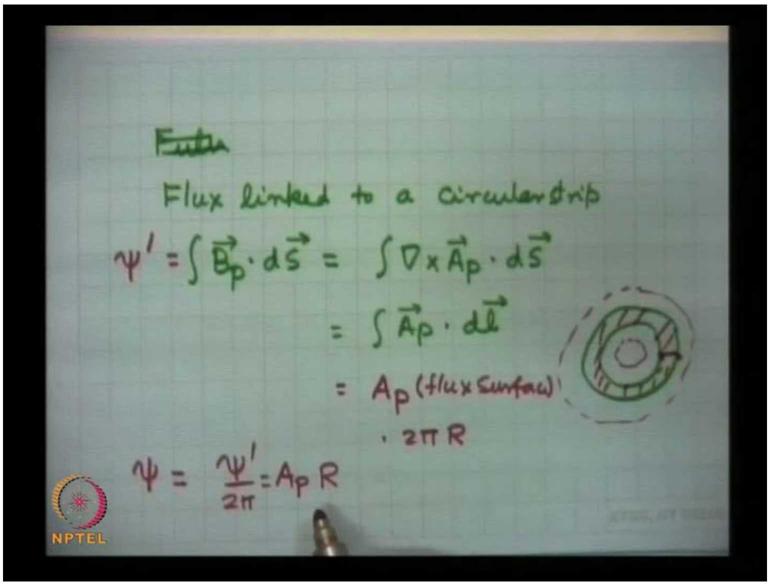

So, that let me write this on a separate sheet the flux sorry flux linked to a circular strip just is the kind of a strip I have drawn in my earlier discussion of constant of motion or you could have considered a strip which is horizontal that was vertical strip that was horizontal strip does not matter.

Then, I am talking of poloidal magnetic field dot d s and this is equal to curl of A p dot d s by using the stokes theorem I can write down this is A p dot d l. So, I am having a strip circular strip like this inside a tokomak, tokomak may be your actual tokomak is something like this. This is not a proportionate plot this is the inner side of the tokomak this is outer side of the tokamak and I am considering center of the plasma is here. So, I am considering a step of this. This area this is the strip on which I am calculating my integral.

But I am saying here is that this area that I am considering here I can just have a small cut here as showed this will be a strip over which I am evaluating this quantity and this quantity is the flux linked called as psi dash and this quantity turns out to be because A p does not change with phi with the phi. So, there will be A p in the inner in the central region and outer region, but, in the center because you can take A p to be 0 may there is no magnetic field there and you can take A p there is always a constant available there and you can take to be 0.

So, the A p will be basically A p value at the outer circle or in the flux surface multiplied with total length which is equal to two pi r. So, if I consider any flux surface flux. Surface please understand is a surface of constant pressure on which lines of force are tangential then the vector potential multiplied by R is the total flux linked with the strip is starting from the center of plasma to that surface and hence if I divide this by two pi I call the quantity flux parameter which is psi dash upon two pi into A p which is equal to A p into r.

So, rather than talking about A p magnitude of A p which is A p phi I should be talking about A p R which is a more reasonable thing. So, what really we are saying is that though p B and J they depend on z and r, but, it should be possible for us to have their dependence only a one quantity called psi. So, flux is I want to introduce as a parameter rather than x and z as or R and z as parameters I want to introduce psi as a parameter.

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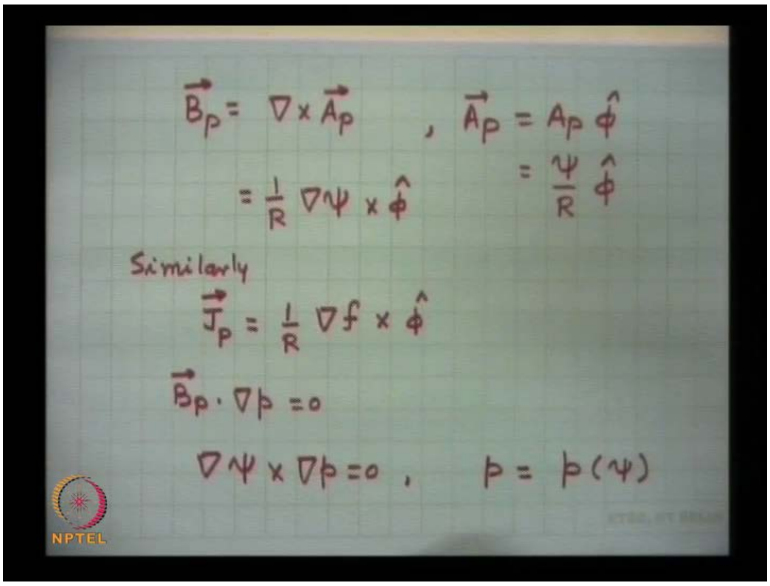

Well once this is there let me express poloidal magnetic field which is curl of A p and if you put the value of A p take A p in the phi direction and A p I am writing as psi by R because R A p I called as psi. So, A p as psi by R phi cap and substituted here and evaluate this quantity it turns out to be equal to one upon R gradient of psi which is a scalar quantity cross phi cap just by substituting this in here I can write down my poloidal magnetic field as gradient of psi cross phi cap.

And by similarity I can introduce another function f such that J the current density which is poloidal current density in the plasma can be written as one upon R gradient of another function f cross phi cap I do not know what I f I should choose, but, just as for B p I have chosen a vector potential in terms of a scalar function psi I think I believe that I can write down J p also in terms of scalar function f, but, I should find a proper scalar function f that is the issue.

Now, what you can see here that B p dot gradient of p 0 why B p because J cross B was gradient of pressure and gradient of pressure has no phi component. So, this should be true and if I substitute for B p this expression then one can deduce from here that gradient of psi this gradient of psi cross gradient of p equal to 0 just by substitution you can get this.

So, the surface of constant psi and the surface of constant pressure this essentially says that must be same. So, p should be expressible because if p is a function of psi only then this will hold. So, I am saying that this equation implies that I can write down this p as a function of psi rather than calling this to be a function of R and z I will say that my pressure is a function of psi.

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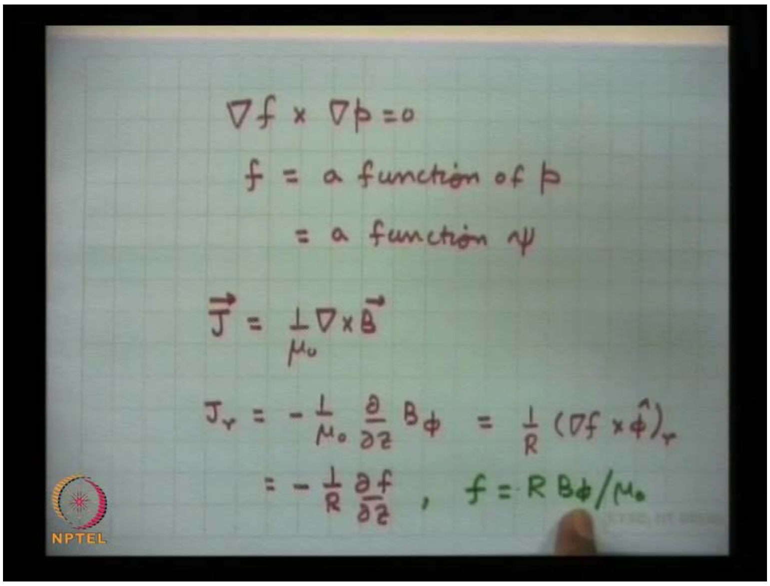

And similarly, because J dot grad p is also 0 I get gradient of f cross gradient of p also equal to 0 and this says that f is a function of p and since p is a function of psi I must say that is a function of psi now these are two very vital things.

So, rather than saying that my current magnetic field etcetera are functions of R and z I say that they are functions of psi the magnetic flux. So, I will treat magnetic flux as a variable a space variable, but, still I would like to find out in what way it depends on R and z once the dependence of R and z psi on R and z is known to me I can write down my magnetic poloidal magnetic field and so on.

So, let me go over to deduce a general equation governing psi because if psi is known to me everything else is known to me now in order to arrive at the equation governing psi I again this start with the equilibrium condition I think before I do that let me also realize a connection between this function f and magnetic field B phi is it related to that let us examine this issue.

You know J the current density is related to curl of B as one upon mu zero. So, from here if I write down say radial component of current density this turns out to be equal to minus mu 0 one upon mu 0 delta delta z of B phi, but, B phi we have already written sorry J R we have already written as one upon R gradient of f cross phi cap. So, I should take radial component of this which turns out to be equal to minus one upon R delta f by delta z because f is a still something unclear to me, but, by equating these two I get a connection between f and B because one upon delta delta z is there and R and z are independent variables.

So, I get a connection between these three these two phi and f and it turns out to be that f is equal to from here R B phi upon mu 0 where mu 0 is the magnetic movement sorry magnetic permeability of free space. So, f which in terms of which I am define my current J is equal to one upon R grad f cross phi this same f is related to azimuthally or toroidal magnetic field B phi this is the relationship.

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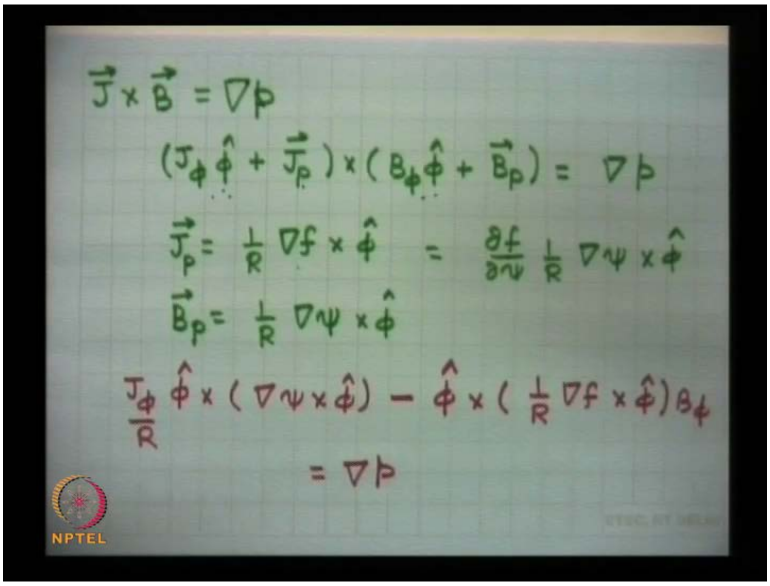

Now, let me written back to my pressure balance equation J cross B equal to grad p equation and what I say is this grad sorry grad p is equal to J cross B this is my initial equation for equilibrium of a plasma this is a essential condition time independent equilibrium J I write down as J phi cap plus J poloidal there are two kinds of currents in general possible cross B also as two terms a toroidal magnetic field plus a poloidal magnetic field and this is equal to gradient of p.

Now, please note that we have been saying that J is equal to one upon R gradient of f cross phi cap and B p we have already said is equal to one upon R gradient of psi cross phi cap. So, J and B p are like this first of all you may note that if I take cross product of the first term be the first term here will be 0 because phi cap cross phi cap is zero.

Secondly you may note here that if I take phi J p cross B p will be how much this is J p actually this J p cross B p if I look here please note this quantity grad f because this depends on psi is the same thing as delta f by delta psi or d f by d psi whatever you write it into one upon R gradient of psi cross phi because f depends on psi alone and through psi it depends on another quantities.

So, this I can write down like this and then same direction it means that the cross product of the first term here with this is 0 and the cross product of the last term here with last term here is also zero. So, only cross terms survive and this equation becomes rather interesting simple equation and that gives me let me write this.

As this cross this. So, J phi and phi cross B p B p is this term let me write down B p as one upon r. So, this is grad psi cross phi cap and then this cross with this now this is J p cross phi cap which I cannot do minus phi cap cross J p. So, J p I will write down this expression J p is one upon R grad f cross phi cap and then there was B phi left out. So, B phi and this is equal to right hand side grad p this is the equation when I substitute for J phi and B phi in terms of psi this is known as the grad shafranov equation which is a very important equation, but, let me move gradually.

This grad phi I will write down as grad psi. So, now, what I going to do everywhere I am going to get some sort of a grad term now please note this is like a cross B cross c you can always write down a cross B cross c as B into a dot c let me just mention this identity.

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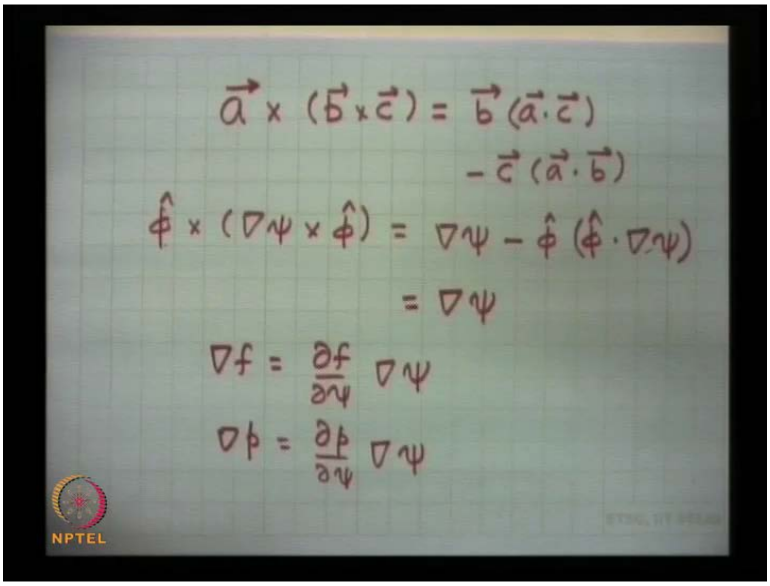

If there is a cross B cross c this is always equal to B vector a dot c minus c vector a dot B here we are having in terms like phi cap cross grad psi cross phi cap this is equal to grad psi phi cap dot phi cap which is unity then minus phi cap into dot of these two.

So, minus phi cap into phi dot this quantity is means let me write down phi dot grad psi, but, as I mention to you there is no variation in phi. So, this grad Del has no phi components. So, this is 0. So, this is equal to simply gradient of psi. So, what I can do if I write this grad f is equal to delta f by delta psi into gradient of psi because f does not depend on anything else and grad p is equal to delta p upon delta psi into gradient of psi then in that equation all terms have grad psi common and you can cancel that then this equation takes the following form.

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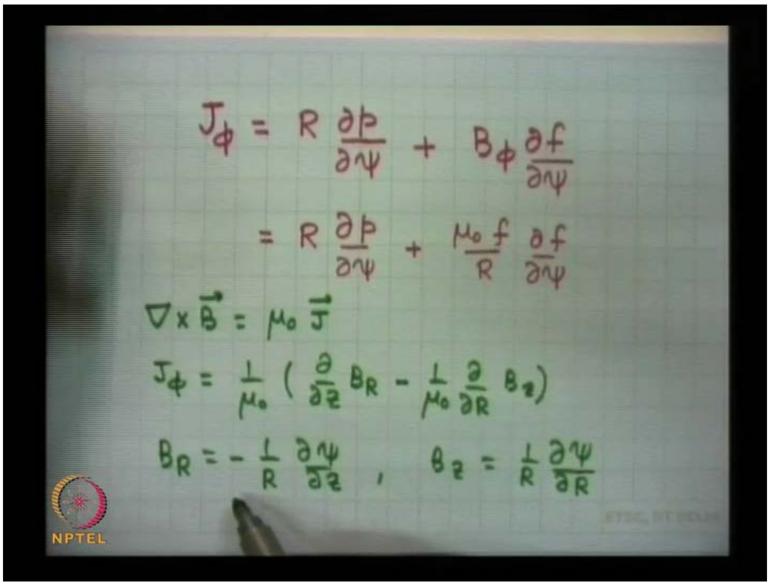

So, let me write down that equation J phi turns out to be equal to R delta pressure by delta psi actually I could have written d d T d d psi also plus B phi into delta f by delta psi, but, B phi I can write down in terms of mu 0 f by R as proven before. So, R delta p by delta psi plus mu 0 f upon R into delta f by delta psi and how about J phi J phi we will use this equation the last Maxwell equation or ampere's law will says that curl of B is equal to mu 0 j.

Now, let me write down the phi components of this equation it turns out to be J phi is equal to one upon mu 0 is goes here and take the phi component which as delta z of B radial component minus one upon mu 0 delta R of B z and since we know we are by definition is equal to minus one upon R delta psi by delta z and B z is equal to one upon R delta psi by delta R.

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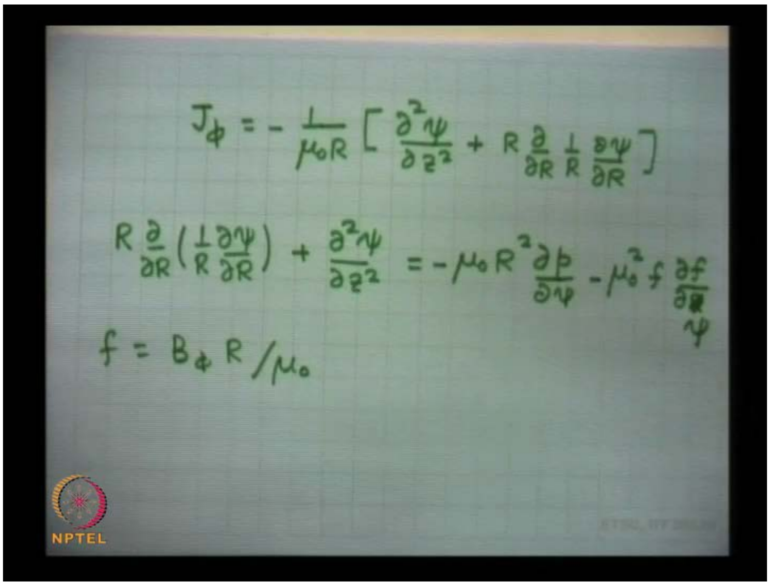

From the definition of psi we get this substitute this here you get J phi as minus one upon mu 0 into R multiplied by d two psi by d z square plus R delta delta R of one upon R delta psi by delta R and using this in the equation for J phi that I had written I get the grad shafranov equation which is R delta delta R of one upon R delta psi by delta R plus d two psi by d is d z square is equal to minus mu 0 R square d p by d psi minus mu square f delta f by delta R.

This is a general equation that will govern in psi. So, if I presume some profile of pressure as a function of psi I can write some expression here and also assume f which is the B phi how does it vary with R because this f is related to B phi I just mention that B phi is equal to or f is equal to B phi into R upon mu 0.

So, if I presume any functional dependence for f on psi or if we can take f to be constant as well in this case if f is a constant independent of R then this is 0 this or this is delta delta psi not R this is psi here. So, if I take delta f by delta psi to be 0 and choose some value some functional dependence of p on psi this equation can be solved numerically to find out psi as a function of R and z and that will give you. So, for different profiles of p verses psi you will get different singed of equilibrium.

Now, if psi is known to you as a function of R and z p is known to you as a function of R and z because I have already assumed a functional dependence for p and once this is done then J also I have known because if I have obtain defined some specific value of or dependence of f on psi then was J was related to gradient of f cross phi cap.

So, everything is determined self consistently and I will just give you a typical numerical solution of this equation that people have carried out many other solutions have been carried out let me just mention this for one profile of or dependence of p versus psi and f versus psi people generate this kind of plot.

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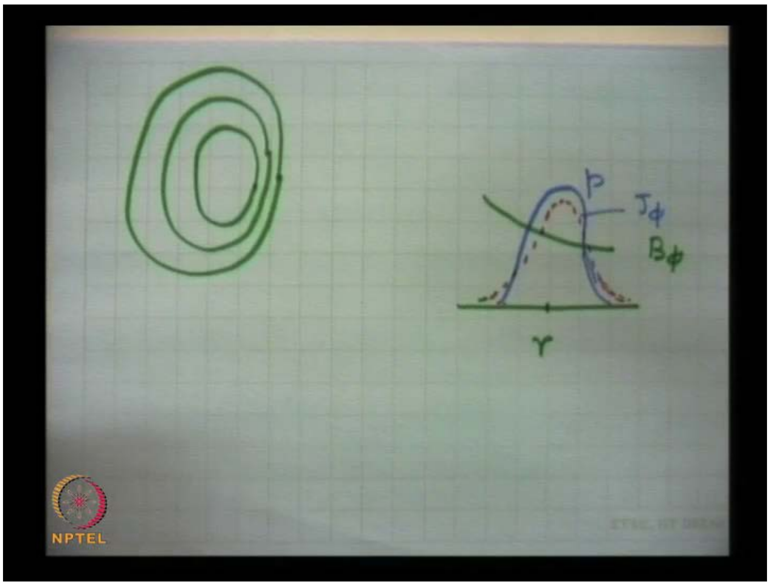

So, like cube pressure surfaces are like this they are also called magnetic surfaces this sort of magnetic surface people get and if I plot as a function of this is called minor radius treat the center of the plasma as the origin and move along outsides. So, r.

If I plot as a function of R then or x whatever then the B phi profile is like this. B phi is a function of r and the profiles turns out to be current density profile is like this is the current density slightly tilted outwards and the pressure profile is also similar let me drawn by a different color this comes like this something like that fairly I would say that this is the profile for p, this is the profile for toroidal current J phi which is not uniform and you can in the central region you can take it to be something like parabolic current density profile pressure profile is also like parabolic and B field profile (( )) as you move away towards the outer side of the tokamak I think a h I will stop here and we shall discuss other aspects of tokomak in our next lecture. Thank you.