Plasma Physics

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Lecture No. # 24

Thermonuclear Fusion

Today, I would like to begin discussing a very major application of plasma physics that is thermonuclear fusion.

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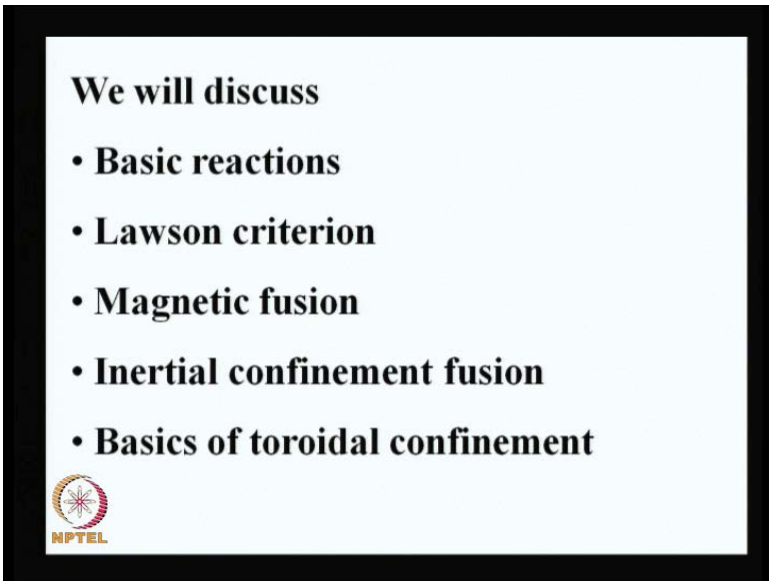

In this we will discuss basic fusion reactions, primarily the deuterium tritium reaction that is used to produce energy. And we will discuss Lawson criterion for energy breakeven and then we will discuss two approaches to fusion. One is called magnetic fusion and second one is called inertial confinement fusion also known as laser driven fusion. And then we will begin discussing some basics of toroidal plasma confinement, which is in at the forefront of fusion ratio is a device called tokomak.

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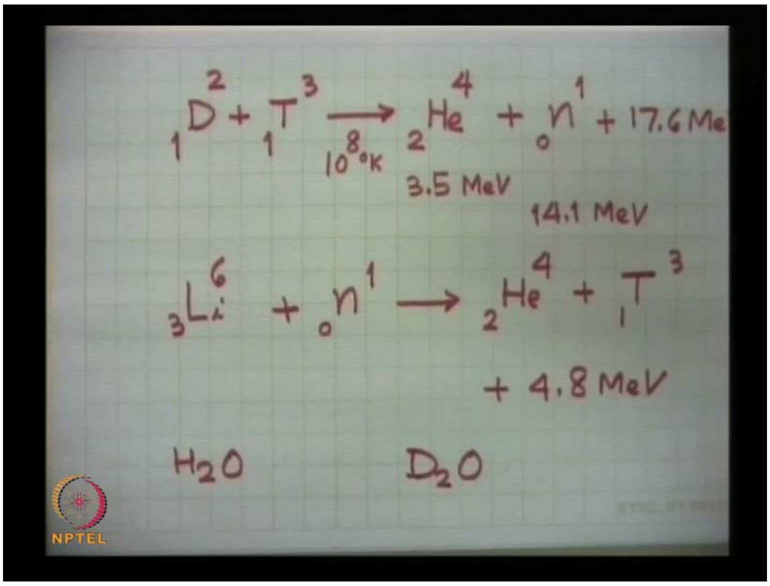

Let me begin with the basic equation, deuterium and tritium are considered to be the major source of a major thermonuclear fuel. If they can be fused the nuclei of deuterium and tritium they could be fused then, they will produce a helium nucleus called alpha particle with atomic number 2 and atomic weight 4. Deuterium has atomic number 1, atomic weight 2, tritium has 1 and mass number is 3. Deuterium and tritium are isotopes of hydrogen, if they can fuse, the nucleus can fuse with each other. Then, they can produce a helium nucleus plus a neutron which has 0 charge, but mass number is unity and in this process 17.6 M e V of energy is released. The energy that goes to alpha particle is 3.5 M e V and the energy that goes to the neutron is more 14.1 M e V.

Well, deuterium and tritium are isotopes of hydrogen. Deuterium could be recovered from sea water with at reasonable cost. And tritium can be recovered from lithium, which is quite in abundance in the soil and about 7 percent of lithium is 3 Li 6. If you bombard this with a neutron, then this will produce a alpha particle 2 He 4 and tritium nucleus which is 1 T 3, in the process 4.8 M e V energy is released. And very significant amount of sea water is deuterium, normally water is H 2 O, but typically around 0.1 percent or slightly less than that of sea water is due to D 2 O. So, deuterium is available in sea water and one can recover it at not too higher cost.

The problem is that, if you want to undertake this reaction, then the fusion has to, the nuclei have to overcome a very strong repulsive force between the two, because both are positively charged. This has a proton in the nucleus, this also has a proton, when they come very close to each other, they repel via column force. So, they have to be brought in within the range of nuclear force, so that they could fuse for there the temperature that are needed are typically of the order of 10 to the power 8 degrees Kelvin. So that, the nuclei could cross the column barrier and can fuse. A important quantity of consideration is called fusion cross action, which depend is strongly on the energy.

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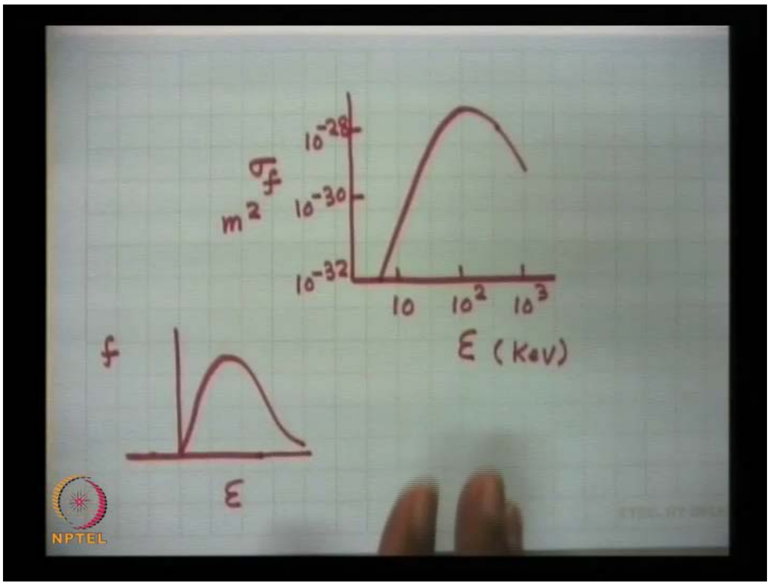

So, if I plot here a graph fusion energy as a function sorry; fusion cross section, this is I am plotting in per meter cube; in meter square. And I plot energy of deuterium when it is bombarding or coming close to the tritium and this I write in K e V, then suppose this 10 here, this is 100 here and this is 1000 here, this is logarithmic scale. And the fusion cross section, if I write here 10 to minus 32 meter square and let me write 10 to the power minus 30 here and let me write 10 to minus 28 here.

Fusion cross section below 10 K e V energy is small. So, it begins from here, it peaks around 100 typically like this, the curve was like this and then falls off something like this. So, what is required is, that you must have enough kinetic energy of tritium nucleus may be 10 K e V or higher so that, the fusion cross section is substantial. Even, if the plasma temperature is 10 K e V in the Maxwellian tail, if I plot the distribution function, energy distribution function f of electrons as a function of energy then this goes like this. So, there are substantial numbers of atoms or ions in the Maxwellian tail.

So, even if the average energy at which f peaks could be around 10 K e V, the number of ions with 20 or 30 K e V energy are also quite is very substantial and then they can cause thermonuclear fusion. But 10 K e V certainly is some sort of a guideline that one should aim to achieve. So, people have focused their attention in achieving plasma temperatures of about 10 K e V and please remember 1 K e V corresponds to about 10000 degrees Kelvin. And hence 10 K e V temperature is around 10 to the power 8 degree Kelvin, 10 crore degree Kelvin which is a huge temperature. No container can sustain such high temperatures and hence one should worry about the confinement of such hot plasma, all state of matter is in the plasma state at such high temperature.

So, primarily we are dealing with deuterium tritium plasma undergoing thermonuclear reactions of deuterium and tritium, and producing alpha particles neutrons by other reactions. Well, two things are important in here, number one that one should be able to heat this mixture to such a high temperature, this is first requirement and second thing is that once the mixture has reached that temperature of 10 K e V or so. And fusion reaction is started taking place, then the alpha particles that are produced in the reaction, they should be trapped within the plasma and they should be able to heat the plasma or sustain the heating, sustain the temperature that is called ignition.

So, two important considerations are that one should achieve breakeven, means the nuclear energy produced via D T reactions should exceed the amount of energy that you have put in to heat the plasma. And secondly, the alpha particles that you produce should be able to sustain the temperature.

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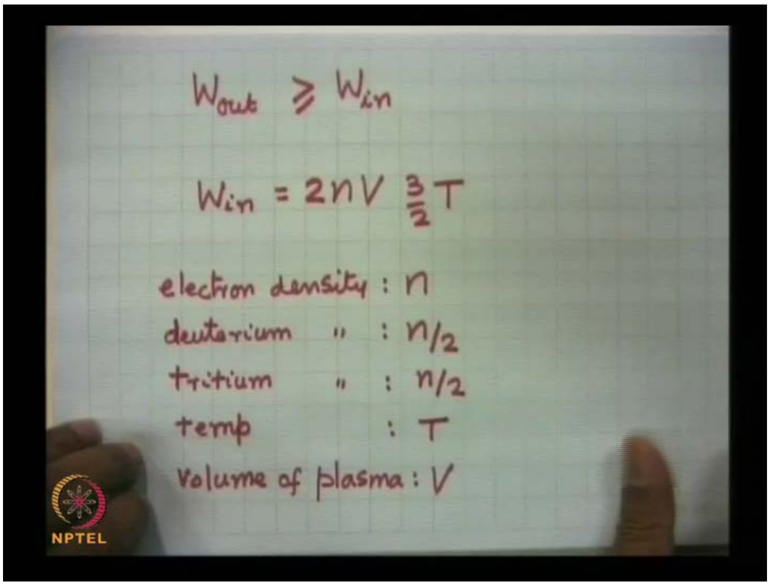

Now, let me make some estimate of energy equilibration or energy equality, that I want that the energy output to be equal or greater than energy in, if W in is the energy input to heat the plasma and this is the energy output. Energy input you can certainly think in terms of particle density etcetera in temperature. If your system has supposed electron density as n, obviously deuterium density would be half of it, because half the electrons are released from by deuterium and half are released by the tritium so, this will be n by 2. Tritium density I can take to be n by 2 also and temperature of suppose these 3 spaces they may differ, that suppose typical temperature I choose to be equal to T and the volume of the plasma is suppose V. (No audio from: 11:24 to 11:36)

So, the total number of particles in the system are n V and the kinetic energy of every particle, if I assume them to be a Maxwellian having; Maxwellian distribution function is 3 by 2 into T, where Boltzmann constant is hidden here, but this is a number of electrons. If you count the total number of particles; electrons plus deuterium ions plus tritium ions, then this becomes 2 n so, this is the right equation. So, total energy input that you have to provide to initiate the process is so much, where T has to exceed 10 K e V. Now, let us see how much energy one can produce, if the confinement time of the system is tau. Suppose, this mixture is tends there for a duration of the order of tau.

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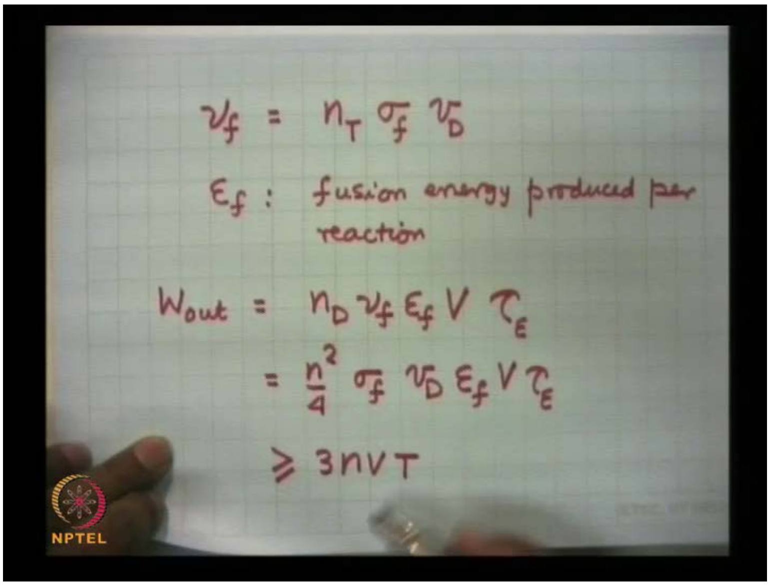

Let me estimate that, the number of fusion reactions a particle will go, it is a deuterium will undergo with tritium is defined as I will call this nu f, which is equal to the number of tritium ions per unit volume into fusion cross section into velocity of deuterium, thermal velocity of deuterium. Collision frequency normally is a product of density collision cross section into thermal velocity this is the kind of thing. And in each reaction you are producing energy say E f so, let me call E f as the fusion energy per reaction which is 17.6 M e V, fusion energy produced per reaction, nu f is the fusion collisions each deuterium ion is undergoing with the tritium ion.

Now, to calculate the energy output W out, this should be equal to total number of fusion reactions. Now, in a unit volume the number of tritium ions is n D, they go nu f each of them undergoes nu f collision per second fusion cross section collision per second and in the process produces so much energy. So, this is the amount of fusion energy produced perturbed volume multiply by the volume, this is the total energy output. If I put n D is equal to n by 2, where n is the electron density. And n T also as n by 2, then this can be written as n square by 4, n by 2 from here, n by 2 from there, multiplied by sigma f into v D into E f into V. And I want this to exceed the energy, that you have put in which is equal to n V into 3 times T.

Please remember, sigma f is a very strong function of energy. And v D also depends on temperature it goes as under root of temperature. So, this equality if I put and please I made a mistake here. This W out is the amount of energy produced per second, if the confinement time of the system is tau. Energy confinement time I may call it tau E, then I must multiply tau here also, this gives me one n will cancel out, a product of density of electrons into energy confinement time of the plasma, this should satisfy certain inequality. And if I take the equal sign here, then this is called the Lawson criteria.

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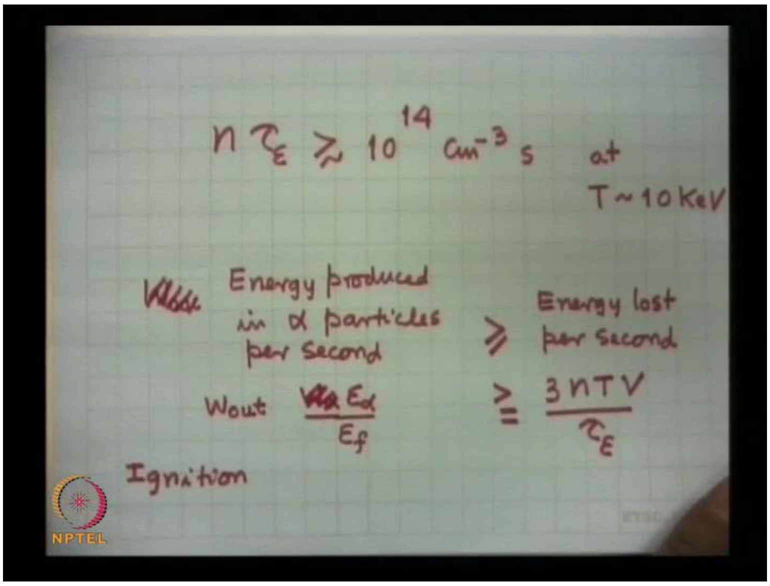

And when you put these numbers here, what you find is, that density and energy confinement in time product must exceed or should be of the order of 10 to the power 14 per centimeter cube into second at an electron temperature of or plasma temperature of 10 K e V, at other temperature this number will be different. But primary goal of fusion research in last 30, 40 years has been to achieve plasma density into confinement time product satisfying this condition.

Well, as far as ignition is concerned, if you want to have the burn continue by extracting energy from the alpha particles, what you require. That whatever energy is produced, W out that you call sorry, let me reframe this, what I want is that the energy produced in alpha particles per second, this must be equal or bigger than the energy lost via radiation or other process is (No audio from: 18:27 to 18: 42). If the plasma energy that we call as energy in, which was 3 n T into V, where V is the plasma volume, n is the density of the electrons in the plasma and T is the plasma temperature. This is the energy that is at any instant of time contained in the plasma particles.

If the plasma confinement time is tau E so that, is that much time so much energy is lost so, let me define a energy confinement time as tau E, then this is the energy lost per second, typically this is the estimate. And energy produced per second will be how much, I just estimated this quantity. So, that quantity was W out is the total energy produced in alpha particles plus neutrons 17.6 M e V W f. If I divide this by multiply by this by factor of energy of a alpha particle divided by rather let me put some symbol here, E alpha upon energy released in a few reaction.

This is 17.6 M e V and alpha particle is 3 point M e V so, this quantity is like one fifth so, this if this exceeds this number, then we say that we have achieved ignition. And you do not require any more heating mechanism, but this factor energy released per nuclear fusion reaction is E alpha for the alpha particle and total energy released is E f which is like 17.6 M e V. So, this ratio is one fifth so what you require is this. But first generation experiments on fusion focused on achieving Lawson criteria this requires higher value of n tau product.

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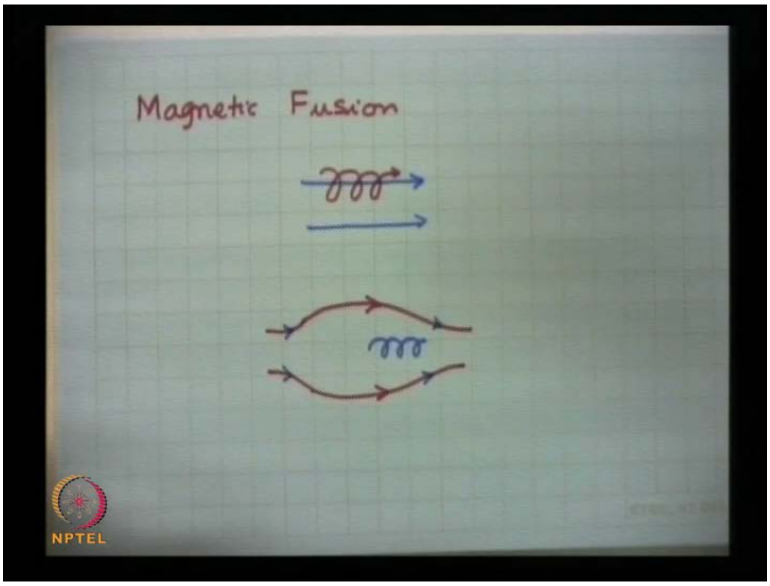

Now, there are two schemes to achieve this product, one is called magnetic fusion, where you employ magnetic field to confine particles. We have already seen that, if you have straight lines of force like these magnetic lines of force, then the charged particles may be electron, if they move they are introduced in the system. They will be confined across the line of force though they can they are free to move along the lines of force.

So, there is a confinement in two dimension two transverse dimensions, but there is no confinement if the lines of force are straight forward in this direction along the length of the lines of force. Then, we came across the mirror confinement that even if the lines of force are straight forward, but if you can create a lines of force like this that they converge like an a mirror machine, then the electrons and ions which are gyrating over the field lines here. When they come close to the throat and if they have large pitch angle, they can come back and they will be confined in the system.

So, magnetic field can provide confinement in three dimension, it is a different matter that this is a not a stable confinement. Because any small perturbation called interchanging perturbation gives rise to disruption of the plasma confinement or disturbance or destabilization of plasma confinement. But certainly one possibilities like this, that you can confine the plasma by using magnetic field such that the lines of force converge towards the end of the machine and plasma is filled in here. Then, one can think of a confinement by using close lines of force and that is called toroidal confinement.

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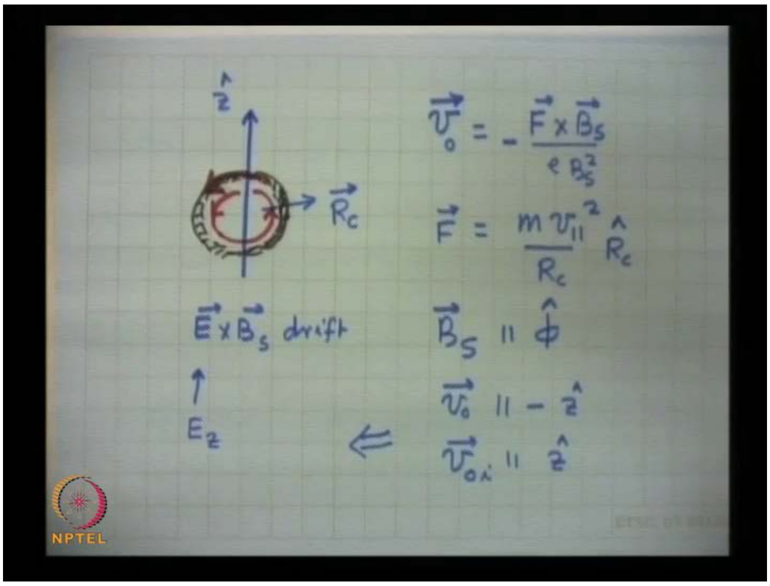

The toroidal confinement is like this. Suppose, I take a current carrying wire of very large length, then the lines of force along the wire are circular, they close on themselves. And if you can create a plasma somewhere here in this region, suppose I create a plasma in this region. So, the plasma if I can create in this region, one would expect that the particles in this region, because they are moving along the field lines and they will be gyrating about the field lines and may be confined. But what we had seen that in such a situation, because the lines of force have a curvature R c was in the outward direction and they also have a in homogeneity in the magnetic field and that gives rise to drifts.

The electrons move in one direction and ions in the other direction perpendicular to the plane of this board by the v cross B force and that produces a vertical electric field perpendicular to the plane of the board. So, what really happens is that, the electrons undergo a force a drift which is given as F cross B s, upon e B s square and this drift is 0th order drift. And this force is primarily the centrifugal force, which is m v parallel square upon radius of curvature R c of the line of force where the particle is located and that is in the radial direction R c. So, what really happens here is that, under this force there is a negative sign here, that if magnetic field is in this case is in the z direction.

Suppose, I choose a magnetic field in the z direction like this; sorry not magnetic field, magnetic field is in the toroidal direction, B s in this case is parallel to phi direction. The current that produces this magnetic field is in the z direction, in that case, this is in the phi direction; this force is in the radial direction so, R c radial cross phi is in the z direction this is minus z direction. So, electron drift is parallel to minus z cap and ion drift is parallel to z cap. So, a very simple configuration that you have a current carrying conductor in the z direction perpendicular to the plane of this board, it will produce magnetic field a circular line or close lines of force. But this will produce a drift of electrons in the minus z direction and ion drift in the opposite direction.

So, there will be a charge separation created between these two if you do not compensate for it. And that charge separation will produce an electric field E z and this electric field will give rise to E cross v drift of electrons ions both in the same direction and that turns out to be in the radial outward direction and plasma confinement is not possible. So though, there is a good possibility that if you can compensate for these drifts, you can have particle confinement in the close lines of force. However, this is a very serious challenge to compensate for these drifts of electrons and ions caused by curvature drift and grad B drift. Grad B drift is in the same direction as curvature drift is just adds to it so, this is the serious matter.

What was really found in 1950s late 50s was a theorist. (( )) That if you can have a current in the plasma in the azimuthally direction in the phi direction. The same direction in which a DC magnetic field, azimuthally magnetic field is produced by this current carrying wire. Then there is a possibility to compensate for to negate these drifts and that is a very important contribution, we shall look into this in detail either today or in our next lecture. But that is a very important contribution to plasma physics that just by passing a current in the system in that azimuthally direction just in the plasma have a current in the same direction in which the magnetic field exist in this direction, have a current in the plasma then, you can have this.

This device is a; will produce at toroidal plasma and a more effective or more economic way of producing such a magnetic field is not to have a single wire along z direction.

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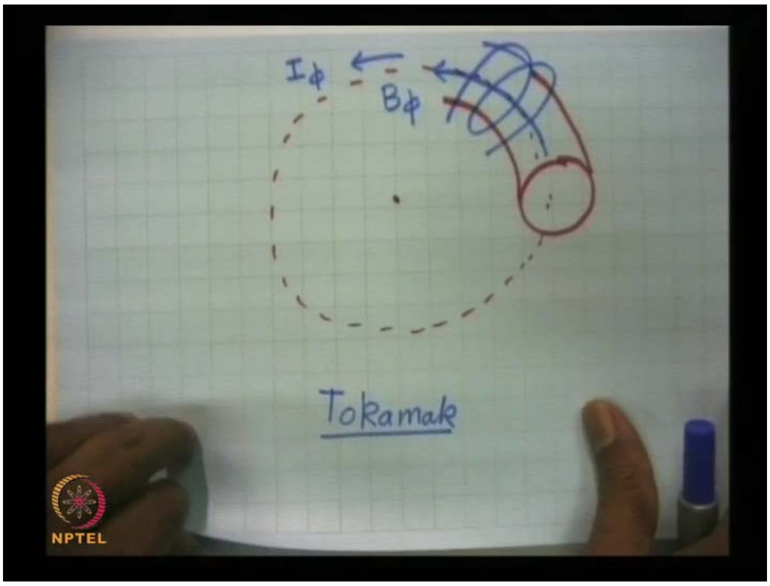

Rather a configuration that has come up is like this that you have a torus, torus is like this. So, close this on itself, close this curve on itself like this and this one. This is a torus like a bicycle tube which if you straighten will be cylinder and if you bend the cylinder, it will bend on itself to form a torus. So, plasma is filled in this here and by using magnetic field coils which go like this schematically. You will produce a magnetic field in this direction, this is the direction of static magnetic field called B phi. And you do not require a conductor passing through the center of the torus, rather these field coils are very close to the plasma to this region, where you want to create a plasma they can produce a magnetic more effectively.

So, and if you can treat this entire torus as a secondary of a transformer, it should be possible to induce a current also. So, current is also you can create here in the same direction and let me call as I phi, plasma current. That can produce a suitable magnetic field that can neutralize the drifts caused by grad B drift and centrifugal force or curvature drift. So, this is the kind of more economic configuration for toroidal confinement and this device is known as Tokomak and this is at the forefront of fusion research.

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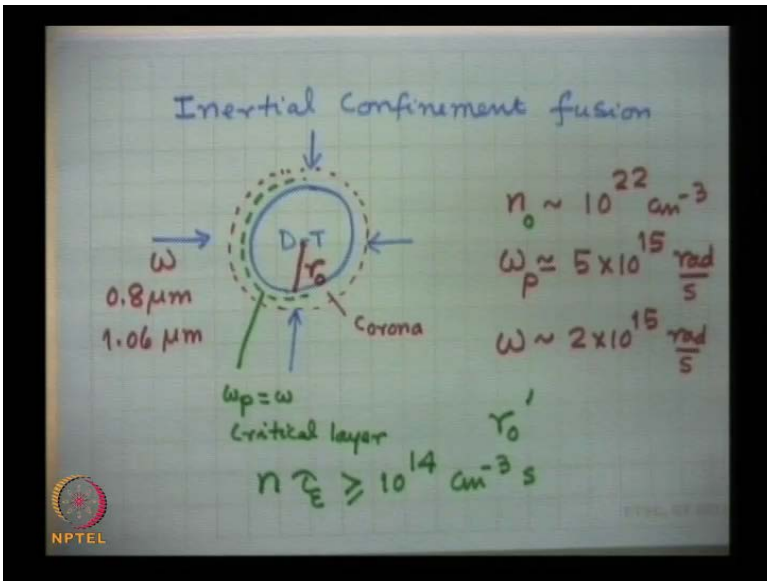

However, let me before I delve into Tokomak. Let me also, talk to you about the other scheme of fusion which is called inertial confinement fusion. (No audio from: 32:46 to 36:58) In this scheme, one employer one considers a palate, a spherical palate of deuterium, tritium. And you shine laser light from all directions, usually 16 beams, laser beams are employed they are launched from 16 different directions on to a deuterium tritium palates, whose radius could be of the order of a centimeter. And it is a deuterium tritium mixture, where if you calculate the atomic density of deuterium tritium, then that will be typically of the order of 10 to the power 22 atoms per centimeter cube.

So, if you convert this deuterium tritium when the laser arrives here, it converts this deuterium tritium mixture. And this radius of this from here to here is probably, I call this as r 0 is around a centimeter, then a hot plasma is created here. And as soon as the plasma is created, this density is more than the critical density for this laser. Suppose, the laser frequency is omega, then the plasma frequency that you produce is around 5 into 10 to the power 15 radian per second and if it is a few times 10 to 22, then this will be slightly more sorry this is omega p. The frequency of the laser that you employ here, either have 0.8 micron wave length, they are called T I suffice laser or 1.06 micron wavelength called neodymium glass laser.

So, typically the wave length is around 1 micron. So, if I take 1 micron here, then omega turns out to be typically 2 into 10 to the power 15 radian per second. Obviously, this omega is less than omega p so, the laser will not penetrate once the plasma is created and once the plasma is created, it will deposits energy only on the surface and surface will expand. So, when the surface expands, you have a lower density outer region called corona and this is this happens within a fraction of a peak of second. So, once the laser arrives here, it quickly converts the plasma this into a plasma and a hot outer region is; are called corona is produced.

Well, the physics of this process we shall discuss in detail later, but it would suffices to say here, that the hot plasma which is that the plasma which is create outside. It will be part of it will be transparent to the laser, because there will be some layer here at which the laser frequency equals. This is the layer some sort of a layer here, at which omega p is equal to omega, inside this omega p is bigger than omega the laser frequency and hence that is a over dense plasma as far as laser is concerned. So, laser will penetrate up to the critical layer and it will deposit a synergy is if get heated. What you get a situation in which the outer region is very hot and hence pressure is very large, whereas, the inner region is relatively cool; cold.

And there is a sharp pressure gradient created near the critical layer and that will launch a shock wave inward and compress the core. So, suppose the core has been compressed to a radius r 0 prime, this is the core with in the process of compression will get heated and where you expect thermonuclear fusion reactions to take place. Suppose, you can confine this plasma where the density as also increase, when you compress the core the density which was initially is so much will increase, let me initial density let me denote by n 0, the new density I will call as n 0; as n.

So, the modified density and the confinement time of the compressed core, this should exceed 10 to the power 14 per centimeter cube into second to achieve for the Lawson criteria. This is what you require so, this is the density of the compressed core and this is the confinement time of the compressed core. Because compressed core after this compressed, it will start expanding because it has been heated. And typical confinement time, you can estimate by considering the process to be ambipolar diffusion process or ambipolar expansion.

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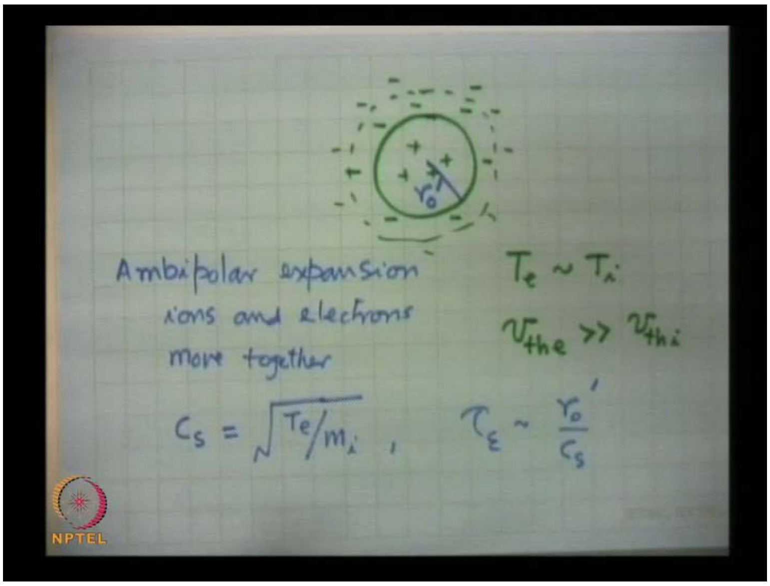

Let me explain what is ambipolar expansion, suppose I have a core of deuterium tritium heated, if I assume that the electrons and ions are at comparable temperatures. So, if T e is comparable to T i is still thermal velocity of electrons is much bigger than thermal velocity of ions due to their different masses. So, what will happen, the plasma which is filled in here have electrons moving with large thermal velocity so, these electrons will quickly get out they will come out here.

However, when they come out, there is a net positive charge left behind and that push these electrons back. So, the net positive charge is left out in the inside and that pushes these electrons back and does not allow the electrons to leave, a distance more than divide length. But this positive charge will repel the ions themselves and consequently ions will move out. So, once ion can go out to a some larger distance suppose, ions can come out to this region this distance, then the electrons can go further out they can move little more. And in this way the electrons and ions as a assembly moves this charge separation of the order of divide length which is quite small. So, this is called ambipolar diffusion.

And ambipolar for ambipolar expansion, in which the ions and electrons move together (No audio from: 40:34 to 40:42) they move together. And the velocity with which this happens, it turns out to be of the order of C s called sound velocity and which is equal to electron temperature upon ion mass. If the plasma is a mixture of deuterium tritium, then this ion mass is some sort of a geometric mean or some mean of tritium and deuterium masses. So, this is the kind of a speed one gets. And if, the initial radius of the plasma is suppose r 0 dash, then the time of confinement would be of the order of, if the radius of compressed core divided by C s.

Please note, that the original radius of the palate that we consider was around a centimeter, this compressed radius will be a only a few mille meter. Sound velocity, if you calculate at a temperature of about few K e V, then this will be of the order of few times 10 to 7 centimeter or may be close to 7, 10 to 8 centimeter per second so, this is typically of the order of a nanosecond. So, if it is like a nanosecond or few nanoseconds, then the density that you would require should be certainly more than 10 to 23 or so to achieve Lawson criteria, it means this compression is mandatory.

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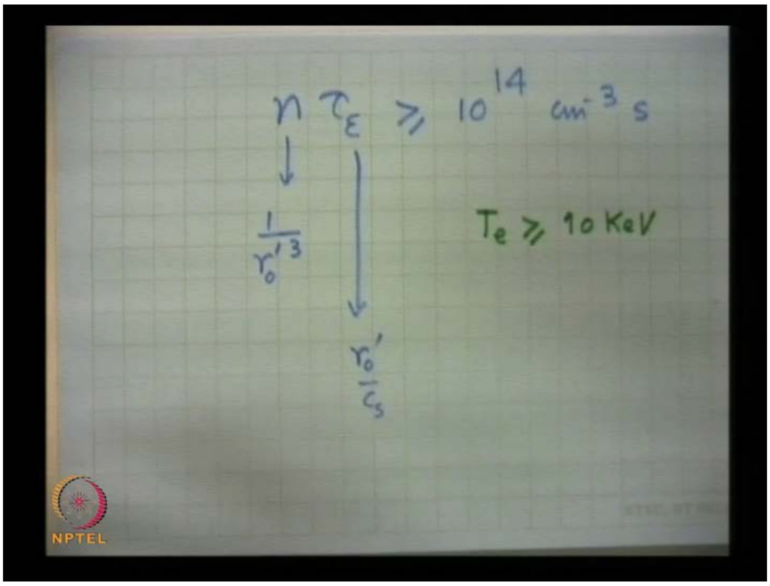

But there is a beauty in there that, in the Lawson criteria n tau product there is of our interest, which I want to be more than 10 to the power 14 per centimeter cube into second. This if the size decreases of the palate, then this will increase as one upon radius of the palate to the power 3 inversely to it is varies inversely with the radius. So, if the radius becomes half, the density will become 8 times if it becomes one third it becomes 20 times larger, whereas, the confinement time scales as r 0 dash by C s. So, r 0 dash means, when you take the product this scales as 1 upon r dash square means, compression is suddenly helpful to achieve this.

And one can achieve and people have achieved this product, but obviously you require this to be satisfied in conjunction with electron temperature also to exceed 10 K e V. So, this condition has to be satisfied in conjunction with electron temperature of this order. Well, this scheme we shall discuss separately in one of the lectures of inertial confinement fusion, it has made tremendous progress over the years and I think I will return to this in a separate lecture. Today, I would like to take you back to toroidal confinement and let me discuss some basic features of toroidal confinement and then we shall return to more details in our next lecture.

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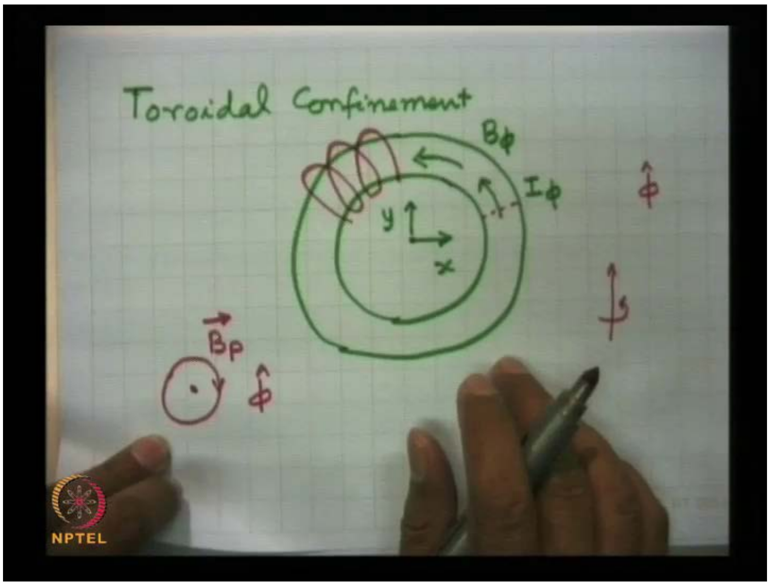

So, a primary thing in toroidal confinement is to counter the grad B and curvature drifts and for that people thought ((.)) less a plasma so, the tokomak if I have a schematic of a tokomak like this. Where plasma is this is the axis of the system, axis of symmetry perpendicular to the plane of the board, which I call as z axis. And let me these call this as my x direction, this is my y direction and we use a right handed coordinate system. So, what you will have, that I am having a plasma which has a toroidal magnetic field B phi and it also has a toroidal current I phi. I phi is not uniform, I do not know what kind of variation I phi I should have to have a optimum confinement, but what will I expect is that, this toroidal this azimuthal current I phi will produce magnetic field.

And the lines of force will be perpendicular to this direction, if I draw a plane perpendicular to phi direction, suppose I draw a plane like this, then the lines of force will be circles about this phi direction phi cap. So, I am expecting just like in case of a wire which was carrying current like this the lines of force were circles. Here also I am expecting the lines of force to be circles about these bent lines of force, this should be circles in a plane perpendicular to this, if I draw a plane perpendicular to phi direction then the lines of force will be like this. So, this is my phi direction normal to the plane of this board, then these will be the lines of force, I will call this magnetic field as B p poloidal magnetic field.

So, there are two kinds of magnetic fields in the system, one is a magnetic field produced by the external coils like these, they are the coils that will produce a magnetic field B phi and a current in the phi direction will produce a magnetic field that I call B p. I would like to discuss a constant of motion and that will give you as a clue of plasma confinement, just like in a mirror machine I introduced a quantity called magnetic moment. Here also I would like to introduce a similar quantity.

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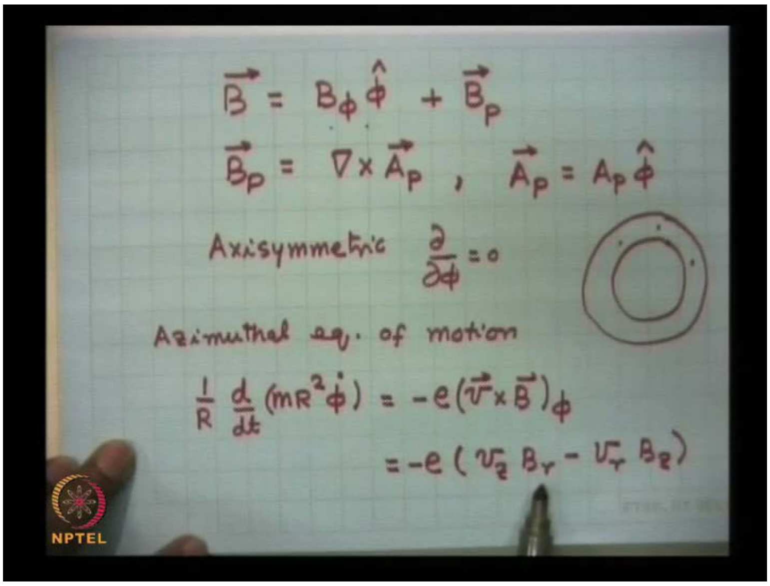

Let me write down the magnetic field in the plasma as p is equal to B phi, phi cap this is the toroidal magnetic field produced by external currents coil plus the magnetic field produced by the plasma current, which I call as B p also called poloidal magnetic field. Now, as magnetic field can always be expressed as curl of a vector potential, I can write down B p also as curl of a vector potential I will call as A p, the direction of A p is in the direction of plasma current. If I am taking the plasma current in the phi direction then I expect this A p to be also in the phi direction.

So, A p I will choose as A p and phi cap for an axisymmetric system and this is the approximation, we always make for tokomak. So, axisymmetric system is the one axisymmetric for which all the quantities like current magnetic field as they do not depend on phi. Means, all points inside the tokomak have the same likely or having same value of magnetic field here, here and here so, magnitude of this quantities do not change with phi so, choose delta phi equal to 0. Now, in this case I would like to write down the azimuthal equation of motion, azimuthal equation of motion. Which is 1 upon R d dt of m R square phi dot is equal to the force on the electron due to the magnetic field which is v cross B force and I must write down the phi component of this.

Now, this is the minus e is the charge of the electron, if I write down the components this will be v z and B r minus v r B z. B r and B z are both the components of magnetic field B p, because the toroidal magnetic field is the phi direction and we are not interested in that component. We are interested in the radial and z components of b field so, which are deducible from curl of A p.

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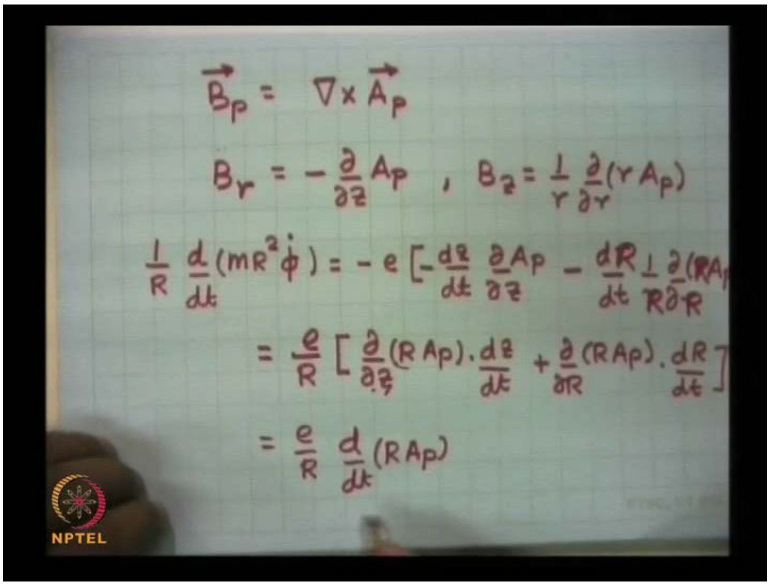

So, let me write down the radial and z component of magnetic field. So, B p we have; has curl of A p and B p; radial would be how much, this is simply called as B r radial component of B p is B r and this will be equal to r if I write down. I must write down a phi component here, z here, minus z here and phi there. So, this will turns out to minus delta delta z of A p this in the phi direction. And B z that I need will be equal to radial component of this operating over the phi component turns out to be equal to 1 upon r, delta delta r of r A p, A p is a phi so, these are the two components.

If, I substitute these two in the equation of azimuthal motion, I get 1 upon r sometimes people denote this a small r, sometime we call as capital R, we are writing as capital R is the simply the variable coordinate or measure radius coordinate. So, this is d dt of m R square phi dot is equal to minus e v z B r. Now, v z I write down as say dz by dt, this is my v z into B r, which is this quantity so minus delta delta z of A p. Then, a term minus v r, which I write down as dr by dt, this is for v r multiplied by d z, which is this term. So, it becomes 1 upon r delta delta r of r A p this is what I get. What I can do here? I can multiply this equation by R and divide by R, I can write down please this r and capital R no different.

So, let me use the same symbol here put a capital R here rather than a small r. So, if I do this and take small r common, I can take this is equal to a minus time also common, it becomes e upon capital R. And then this can be rewritten as delta delta z of R A p into dz by dt plus this term, which is delta delta R of R A p into dR by dt. Here R, I have taken inside the delta delta z operator, because R is independent variable independent of z so, I have taken this inside. You may treat this as a full derivative of R A p with time, because A p does not depend on time, explicitly it depends only on R and phi r and z and there is no phi dependence.

So, this entire quantity can be written as e upon R into d dt of R A p and if you look at the left hand side and right hand side, 1 upon R is common in both of them and consequently you get a constant of motion and which I would like to write down.

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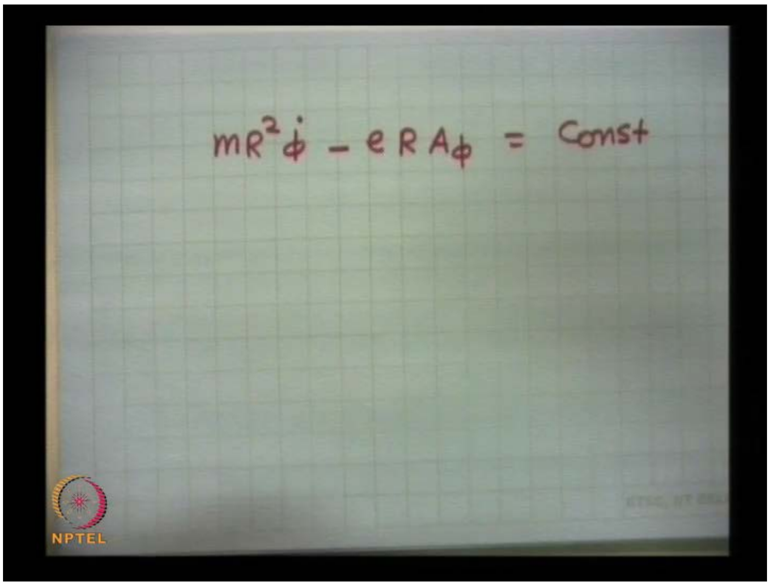

The constant of motion turns out to be m R square phi dot minus e R A phi is a constant of motion. This is very important constant of motion and it has tremendous implication for plasma confinement in a tokomak, just like magnetic moment has implication for plasma confinement in a mirror machine. And I think implications of this we shall discuss in our next lecture. We are coming close to end for todays talk and in the next talk we will discuss the implication of this on plasma confinement. Thank you very much.