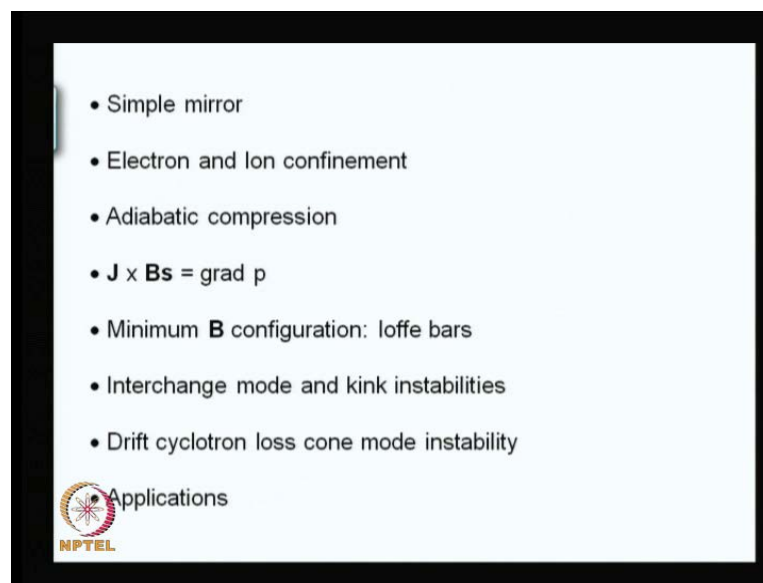


**Plasma Physics**  
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**Module No. # 01**  
**Lecture No. # 23**  
**Mirror Machine**

(Refer Slide Time: 00:31)



Well this will be the last lecture on mirror machine. In this, we will discuss a simple mirror configuration then we will review electron ion confinement and find a scheme of plasma heating and important scheme is adiabatic compression. Then we shall discuss or deduce rather an important equation that  $\mathbf{J} \times \mathbf{B}$  should be equal to gradient of pressure or plasma confinement and to achieve minimum  $\mathbf{B}$  configuration. What kind of arrangements we make, we use rather Ioffe bars and there are two important instabilities in cylindrical plasma systems. One is interchange mode and the other is kink instability.

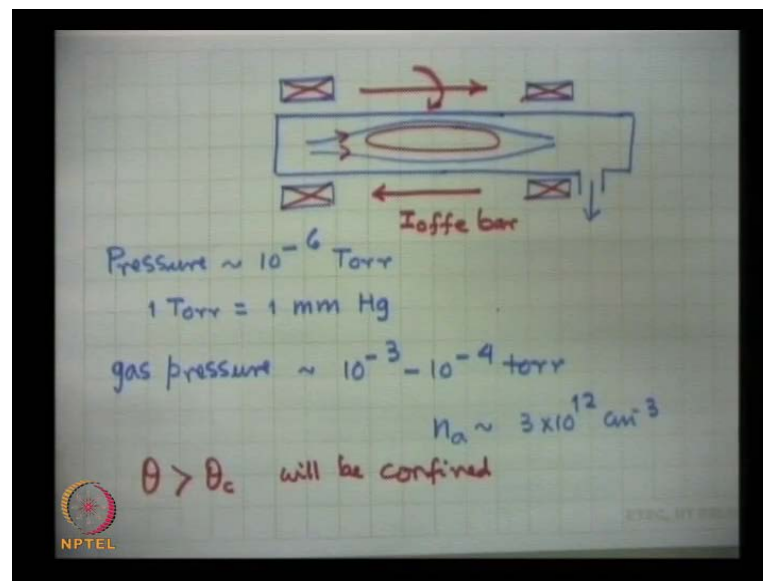
So, we shall discuss a little bit of these instabilities and as I was mentioning drift cyclotron loss cone mode instability is also serious concern in a mirror machine we shall discuss that and then we will deduce discuss some applications.

Two prime applications are always to be **captain** view one is the application in a fusion device mirror has a fusion machine actually, if it has a tremendous attraction at one time the interest as declined significantly over the years in comparison to a new device called tokomak, but a still there is lot of interesting interest in this machine.

Another important application of mirror machine is in producing plasmas of very high density for plasma processing of materials.

And third one in there is a natural mirror near the pole in the polar region of ionosphere and where the charge particles coming from the sun emitted from there as solar bus they precipitate near the polar regions near the poles, where the magnetic fields lines of force concentrate and in that case the precipitation gives rise to concentration of those particles and they give rise to a very special kind of radiations in the aurora region of ionosphere. So, those are some interesting phenomena we shall if time permits we shall discuss them.

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Now, let me begin with a simple schematic of a mirror well mirror essentially is a tube long tube several meter long and it as a connection to a vacuum system vacuum pump and these pumps create an evacuation a initially evacuate the system to a pressure how is typically 10 to the power minus 6 torr and one must remember one tort of pressure is equal to 1 millimeter of mercury.

$h d g$  is the pressure where  $d$  is the density  $h$  is the height of the mercury column and  $g$  is the acceleration due to gravity. So, 1 millimeter of mercury means  $h$  is 1 millimeter  $d$  is 13.6 gram per centimeter cube and  $g$  is around 980 centimeter per Second Square. So, from there you can deduce in dynes per centimeter square the pressure.

So, well at this pressure if you calculate the number of atoms it is quite a small then you fill in a desired gas here, for fusion you would like to fill in this with deuterium station plasma, but for other experiments you can use other gases like nitrogen helium etcetera and I just like to mention that typical pressures that which you fill in the gas in these machines. So, gas pressure is in the range  $10$  to minus  $3$  to  $10$  to minus  $4$  torr.

At  $10$  to minus  $4$  torts the atom density is turns out to be typically of the order of  $3$  into  $10$  to the power  $12$  per centimeter cube. So, the mirror machine plasma if it is fully ionized would have electron density around  $10$  to  $12$  into  $13$  electrons per centimeter cube. So, this is the vacuum system and then there are field coils here that will produce a mirror magnetic field and typically magnetic fields around a kilogauss a few kilogauss is created in the central region this region and plasma is contained in this region the lines of force as I have demonstrated earlier go like this and plasma is confined in this region. So, plasma is I can show something like here.

However it has been and we shall learn today that mirror reflection alone is not sufficient for plasma confinement mirror reflection simply tells that the electrons which have an angle which angle greater than critical of the also an angle  $\theta_c$  these ions and electrons will be confined will be confined.

But there can be a instabilities that will not help that will not let this equilibrium persist for long, but people found that a suitable configuration for plasma confinement is the one for which there is a minimum in a magnetic field. So, magnetic field should be minimum when you move in the  $z$  direction or along the excess of machine and it should be minimum when you moved in the transverse direction.

To achieve that people employ four bars called offer bars one is here carries current in one direction another is here, that carries called other direction called offer bars and there are two coils one on top and below.

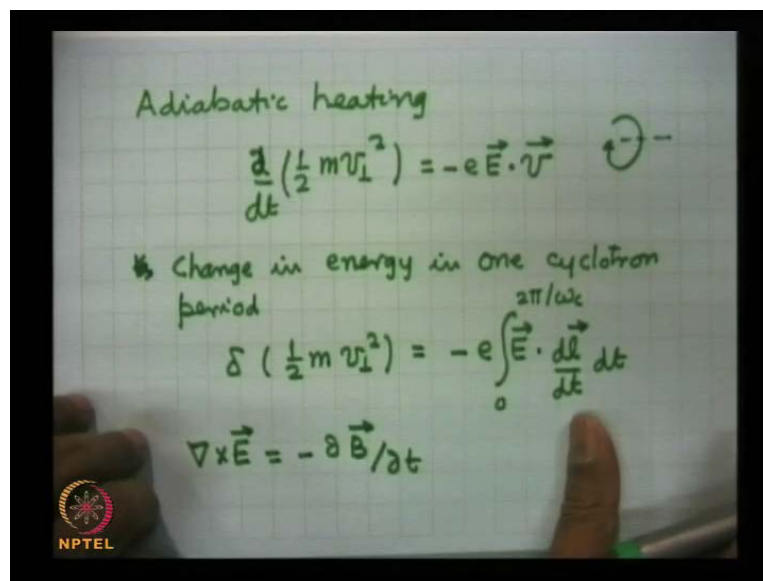
So, there are four current carrying wires that will produce a magnetic field that will, super impose with the magnetic field produce by these coils these are the actual field coils and they will produce man field like this these will produce magnetic field like this and one can achieve a minimum B configuration for plasma confinement.

Well as for as the heating is concerned what really happens that in order to increase the temperature of the system you need to supply energy from outside initial heating is achieved by what we call as adiabatic compression initially, you apply well first if you all you create a plasma here by R if it is charge when the these fields are very small.

Then suddenly you increase the magnetic field when magnetic field increases we shall learn a little while then mu still is a constant of motion means the magnetic flux linked by linked with every orbiting electron is a constant of motion. So, when B field increases v prep must also increase and in this case energy is not a constant of motion mu is a constant of motion.

So, when B field increases in time then any time where a magnetic field will produce in electric field and that will give rise give energy to the particle and its kinetic energy increases and from the electric field that is produced by the time varying magnetic field. So, plasma is heated. So, we shall teach about adiabatic heating let me look into this problem of adiabatic heating.

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Let me look into the energy gain you know rate of change of energy  $\frac{d}{dt}$  for instance of  $\frac{1}{2} m v^2$  this was a kinetic energy of the electron. So, I want to find out how this is I am considering an electron which is gyrating in a magnetic field say  $B_z$  around the axis and when the  $B$  field increases in time its magnitude increases in time. I would like to find out how much the energy of the electron increases then the  $B$  field will time varying  $B$  field will produce an electric field suppose the value of the electric field is  $e$  into this is the force on the electron which is gyrating.

So, if the electric field if the electron is gyrating in the  $\phi$  direction if there is an electric field is in the  $\phi$  direction then there will be work done by this per second would be product of these two the electric force multiplied by the displacement per second is called the work done by this electric force on the electron and hence it should be equal the energy increase in rate of increase in energy of the electron.

So, if I want to find out the work done in one cyclotron revolution. So, work done or change in energy rather change in energy in one cyclotron period I am considering that the magnetic field is changing very gradually.

So, let me calculate the change in energy of the particle in one cyclotron period how much that would be I must multiply by this  $dt$  and integrate minus  $e \mathbf{E} \cdot \mathbf{v}$  I can I do not  $d l dt$  if the electron and this  $d l$  by  $dt$  is for  $v$  into  $dt$  and let me integrate this from time 0 to  $2\pi$  upon  $\omega c$ . Let me calculate this quantity and you know the electric field satisfy this equation curl of  $e$  is equal to minus  $\frac{\delta B}{\delta t}$  from the Faraday's law of electromagnetic induction.

So, what I am going to do  $d t d t$  will cancel out this is a line integral which by using a Stokes theorem I can write down in terms of curl of  $e$  and let us see what do I get.

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$$\delta\left(\frac{1}{2} m v_{\perp}^2\right) = -e \int \nabla \times \vec{E} \cdot d\vec{S}$$

$$= e \int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

$$\delta(\mu B) \approx e \frac{\partial B}{\partial t} \pi \frac{v_{\perp}^2}{\omega_c^2} \quad \left| \begin{array}{l} \text{in one} \\ \text{cyclotron} \\ \text{period} \end{array} \right.$$

$$\mu \delta B + B \delta \mu = \mu \delta B$$

$$\mu = \text{const. of motion}$$

So, change in energy in 1 cyclotron period is half  $m v_{\perp}^2$  is equal to minus  $e$  this becomes curl of  $e$  dot  $dS$  the area and curl of  $e$  will be  $\delta B$  by  $\delta t$ . So, this becomes is equal to  $e \delta B$  by  $\delta t$  into  $dS$ .

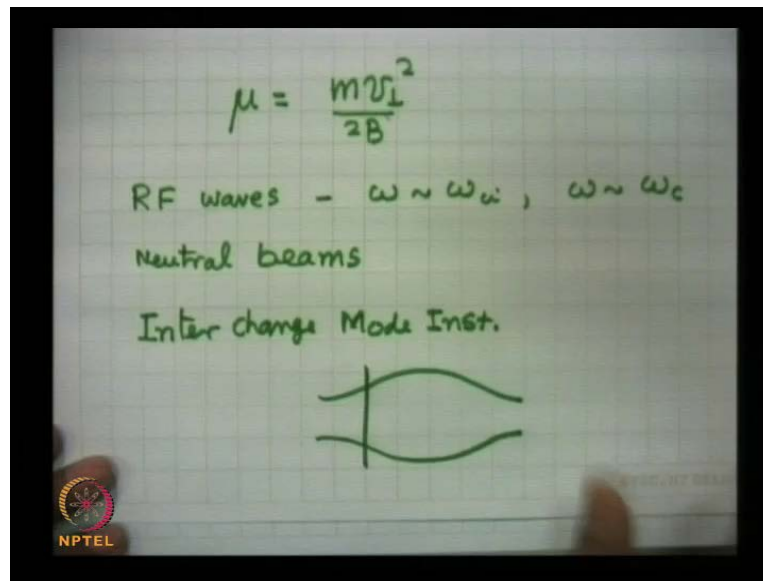
Now, let this quantity  $\delta B$  by  $\delta t$  is fairly constant over the orbit of the electron over the cross section over which the electron is rotating. So, I can approximately write this as  $e \delta B$  by  $\delta t$  and  $dS$  is  $\pi$  normally  $d^2 c^2 v_{\perp}^2$  upon  $\omega_c^2$  and this quantity on the left hand side is change in if I write down this in terms of  $\mu$  then this is  $\mu$  into  $\delta B$ .

So,  $\mu B$  is equal to this quantity and this is in one this is quantity I must evaluate change in this quantity in one cyclotron period in one period. So, what really happens is this is the quantity which is equal to  $\mu$  into  $\delta B$  in one cyclotron period  $\delta B$  by  $\delta t$  in change in this quantity in one cyclotron period and multiplied by one cyclotron period. So, this becomes the change in  $v$  in cyclotron period and this gives you  $\mu$  is a constant of motion.

This is a very important observation that earlier we have drive  $\mu$  as a constant of motion for a time independent magnetic field which was inhomogeneous  $B_z$  was a function of  $z$  and we found that as the electron goes from the middle of the mirror towards the throat then  $\mu$  remains constant.

But here we have found that an electron gyrating above the line of force even in the center of the machine it may not have any z direction or z velocity, but it will gain energy from the electric field because, magnetic field is changing with time and magnetic field when changes with time then, if the magnetic field changes by delta B energy will increases by this amount. So, mu really remains a constant because this you can write down as mu delta B plus B delta mu.

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And what does it tell that if mu is a Constance of motion adiabatic invariant this is equal to m v prep square upon 2 B. So, whenever B increases with time v prep must also increase. So, energy increases. So, if you can increase the magnetic field by a factor of say ten your are increasing the particle temperature by factor of ten because this is like temperature of the electron or ion depending on what particle you are considering

So, that is a important result. So, adiabatic compression means whenever you increase suddenly the or the electric magnetic field in the field coils then the plasma which is already there will acquire higher and higher temperature and this way you can achieve temperatures to be tune of may be hundred electron volt.

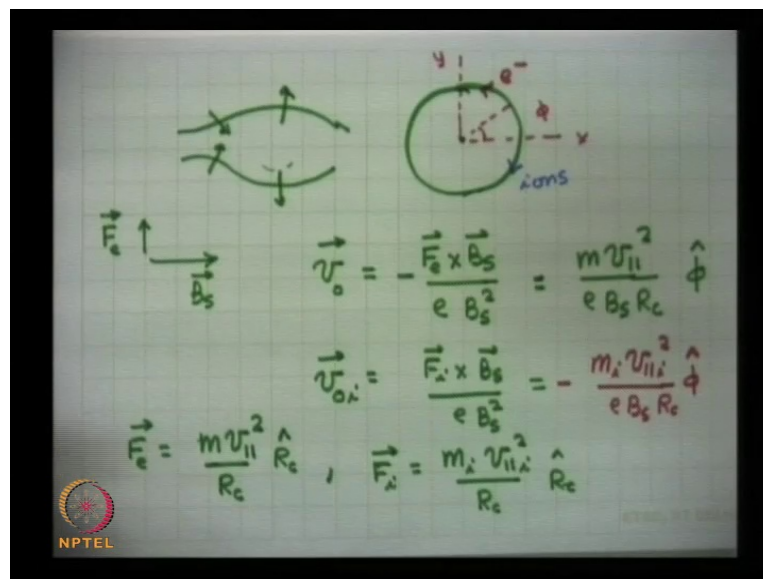
The issue is how you will increase the temperature beyond that point you require radio frequency waves or you require neutral particles neutral beams and both have been employed in number of machines.

Let have been lot of experiments in heating by using waves electromagnetic waves of frequencies around ion cyclotron frequency or electron cyclotron frequency. So, of the wave frequency  $\omega$  around  $\omega_c$ .

$\omega$  and  $\omega_c$  of the order of  $\omega_c$  this is the ion cyclotron frequency and this is the electron cyclotron frequency these waves can preferentially heat ions or electrons depending on whether the frequencies close to ion cyclotron frequency or electron cyclotron frequency and neutral beams can be launched into the machine and by charge exchange collisions they get ionized over there in the system and they deposit there energy on the plasma ions and heat the ions.

So, temperatures of the order of clon electron volts can be achieved by these techniques. Well the two issues are important in here, that if the system microscopically is stable and I would like to bring out two important instabilities whether qualitatively today 1 is very similar to Rayleigh Taylor instability and this is called interchange mode instability please keep in view that we are always talking about a axially symmetric mirror for instance and if I have a plasma here am expecting the plasma to have some of azimuthally symmetry. So, the plasma is the symmetry, if I cut any cross section here suppose I look at this cross section here the plasma here will look circular.

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So, let me make a circle here this is the circular cross section. So, I can consider my plasma number of uniform density I am saying that the plasma is filled inside this and there is no plasma outside and your magnetic field is perpendicular to the plane of the board going upwards. So, now, this is the direction of magnetic field.

Now, please note here the mirror machine has curvature the lines of force have curvature here the curvature is in the radial direction outward whereas, near the throat the curvature is in this direction inward. So, what really expecting here this is the situation where that after all plasma is of high density placed here density increases when once goes towards this center of the plasma and here you have curvature opposite to the density gradient where as in this in this region where the curvature is towards the plasma and density gradient is also towards the plasma.

So, they are in the same direction and in a little while we shall learn that this region is called a good curvature region and this is called a bad curvature region because there is a possibility of growth of a perturbation let us see how it happens. We have learnt earlier that if there is a system in which there is a magnetic field  $B_s$  and if there is a force exerted on a particle electron for instance  $F_e$  then this electron will move with a drift velocity  $v_0$  which is equal to  $F_e \text{ cross } B_s$  upon charge of the electron minus  $e$  into  $B_s^2$  similarly, if ion experiences a force  $f_I$  then  $v_0$  of the ions would be  $f_I$  of the ion cross  $B_s$  magnetic field esthetic magnetic field upon  $e B_s^2$  with a positive sign.

The interesting part here is that the curvature differ instance of there will curvature in this magnetic field. So, curvature drift has  $F_e$  for electrons this force is equal to mass of the electron into parallel velocity of the electron along the line of force square upon this divided by the radius of curvature and this in the direction of radius of curvature.

A similar expression I can write down for  $f_I$  and this is equal to mass of the ion  $v$  parallel of the ion square upon radius of curvature and  $R_c$  cap you may note here, that the electrons if  $R_c$  is in the direction like this in the radial direction and this magnetic field is in the  $z$  direction then  $R \text{ cross } z$  would be in the  $\phi$  direction minus  $\phi$ . So, this becomes parallel to  $\phi$  the value would be  $m v$  parallel square upon  $e B_s$  into  $R_c$  and that will be in the  $\phi$  direction.

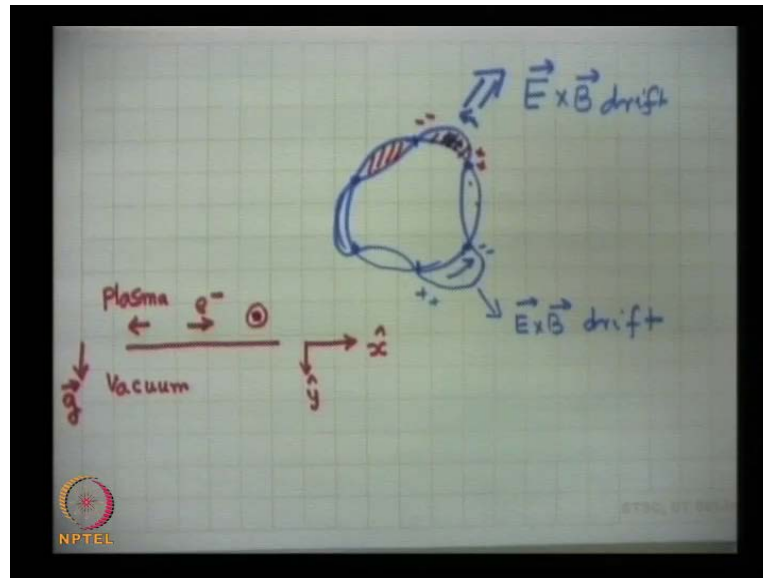
And for the ions this would be if I put this expression this is radial direction this is the z direction. So,  $R \times z$  is minus. So, minus  $m I v_{\parallel} / I^2$  upon  $e B s R c$  into  $\phi$  cap means the electrons and ions in the plasma here, they are besides going  $(\odot)$  about the lines of force will try to move this is please note here the electrons and  $(\odot)$  are gyrating over the lines of force like this, but then they will be having a  $\phi$  motion. So, they will be going electrons will be going in 1 direction the ions will be going in the opposite direction like this.

So, if I plot this cross sectional in the cross sectional plane this is my x-axis this is my y-axis I measure  $\phi$  suppose electrons are travelling here this angle is  $\phi$  direction is here. So, electrons are going this way and ions are going this way this is electrons go this way and ions travel in the opposite direction.

I think I can draw ions like this. So, in the plasma because of curvature the electrons and ions will go round in these directions in opposite directions and this is just because of the curvature you can add grad B drift also that reinforces with this the directions are same.

So, for the electrons the curvature drift and grad B drift are in the same direction  $\phi$  and for ions also curvature and grad B drifts are in minus  $\phi$  directions now what is going to happen if I perturbed this isobaric surface the surface over which the plasma pressure is uniform if I perturb this suppose the plasma is shall I display interchanges position let us see what I mean.

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I am considering that suppose this is the cross section of the plasma and let me consider well let me divide this let me just see what kind of picture I want to draw yeah let me draw a point here a point here a point here a point here. So, let me draw a picture like this goes and then comes back like this and let me take this like this.

Suppose the should be symmetric I have not been careful in this means if I displays suppose there is some displacement some perturbation in the plasma moves from here to here and the plasma from here moves there.

What I am saying is that the electrons are rotating in this direction. So, they will create a space charge a negative space charge here and a ion positive charge will appear here.

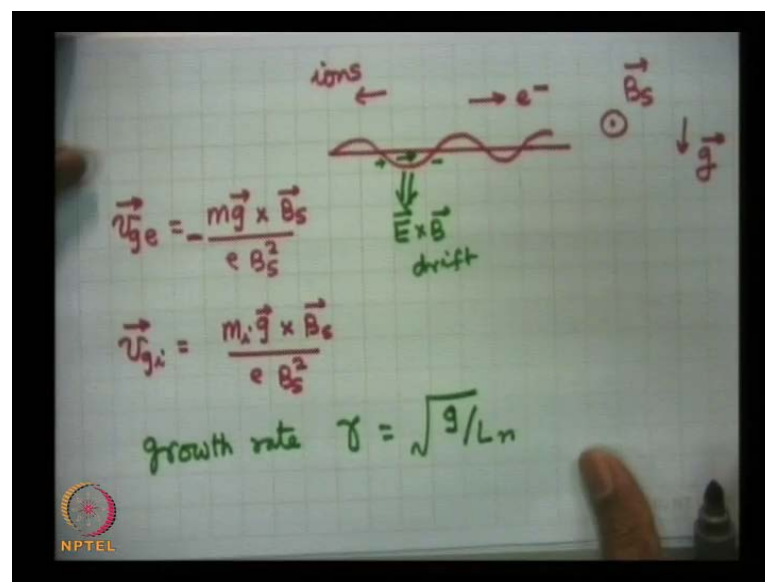
As a consequence there will be an electric field in this direction in this region there will be electric field in this direction electric field in this direction in the plasma and  $e$  cross  $B$  drift of electrons and ions will be in the outward direction similarly, here the electrons will charge will appear here ion charge will appear here there will be electric field in this direction and  $e$  cross  $B$  drift will be in this direction.

Thus these portions will bulge more and more. So, if there is a small bulging somewhere then this bulging will grow this ripple will grow and the system will be unstable. So, plasma will not be confined for a long time this instability is very similar to Rayleigh Taylor instability that we discussed one day.

In Rayleigh Taylor instability, but, what happens is this let me recall little bit in a Rayleigh Taylor mode we considered a plasma supported against vacuum or consider to gravity rather in vacuum gravity was downwards and this had a magnetic field perpendicular to the plane of the paper. So, if I consider this to be my x direction this was my y direction and B field was perpendicular to the plane of this board in z direction no x was y x cross y will be into the board. So, this will be dot here is into the board.

Now, what happens that because of this gravity being perpendicular to magnetic field the plasma which is filled in here it acquires drifts and the electron drift is in this direction and ion drift is in this direction. So, this is the electrons go in this direction and ions go here and let us examine what is the consequence here.

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In the case of Rayleigh Taylor instability if I have a small a original surface and if I have a ripple here then the electrons are going in this direction and ions going in this direction the drifts are simply  $v_g$  on the electrons is equal to  $m_e g$  the force cross  $B_s$  upon  $e B_s^2$  and  $v_g$  I is equal to  $m_i g$  cross  $B_s$  upon  $e B_s^2$  the first one will be the negative sign because ion charge in electron charge in negative.

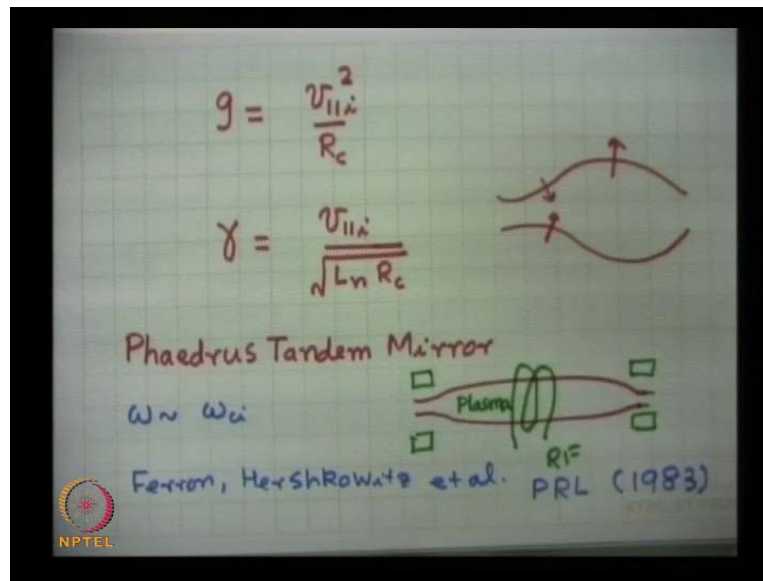
Now, please note here that mass of the ion is much bigger than the mass of the electron and consequently  $v_{g,e}$  is very tiny this is the direction of  $v_{g,e}$  never the less it is the relative velocity of the two that is important. So, what will happen ions will go in this

direction electrons will go in this direction and there is an esthetic magnetic field perpendicular to plane of the board and gravity is here this is esthetic magnetic field. What will happen that this portion will acquire a positive charge because ions will move. so, this becomes a positive charge here then negative there and there will be electric field here and this electric field will pull the plasma downward by  $e$  cross  $B$  drift.

So, what was happening in the Rayleigh Taylor instability situation same thing happens here the only problem here, is that the drifts due to curvature in magnetic field and inhomogeneous magnetic field are roughly equal for electrons and ions, but, they are opposite direction.

In this case of Rayleigh Taylor instability the growth rate was  $\gamma$  was under root  $g$  upon  $L \ln$  how about here what kind of  $g$   $v$  has here in the case of interchange instability you just write down the equivalent  $g$  which is equal to centrifugal acceleration for ions.

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So, if I replace  $g$  by  $v$  parallel square upon  $R$  radius of curvature  $R_c$  just do this and you will get the growth rate for the interchange mode is equal to  $v$  parallel this is I am talking of ions ion divide by  $L \ln$  into  $R_c$  under root  $L \ln$  is the density scale length which is typically the radius of the machine which is like ten, fifteen centimeter or So, and  $R_c$  is the radius of curvature which is quite huge of the order of length of the machine or even bigger. So, this is a large quantity  $L \ln$  is a small quantity and  $v$  parallel  $I$  is the thermal

typically thermal velocity of ions. So, growth rate scales with under root of temperature and inversely proportional to the square root of minor radius of the machine which is of the order of  $L_n$  of that order and radius of curvature of the lines of force it is a serious matter.

If we please note here the instability grows only when the density gradient and gravity are opposite to each other or, but, if you reverse the direction then there is no instability that is why I was mentioning that in a mirror the lines of force are like these. So, there is a good curvature here this is a bad curvature here because here there will radius of curvature is outside in density gradient is inward.

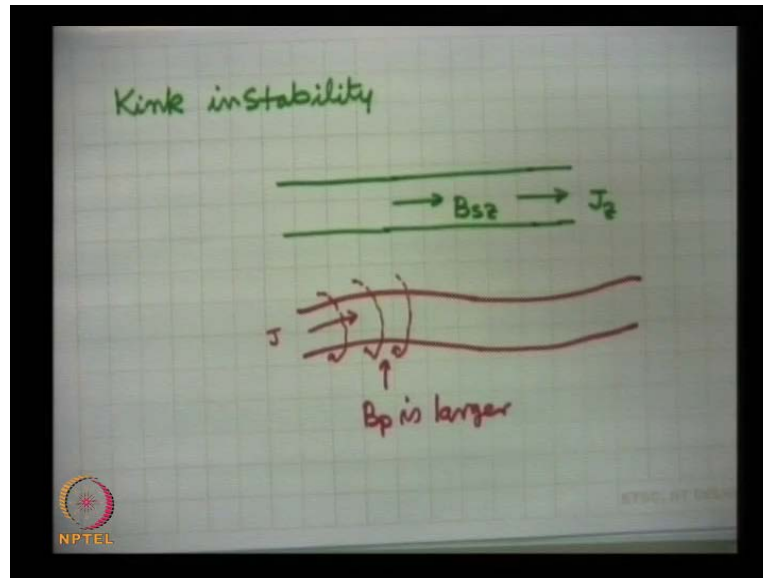
So, that is a bad curvature this is a good curvature and the region of good curvature is little the region of bad curvature is bigger and consequently system is unstable, but, if you put those offer bars then which is change this structure here and you can suppress this instability.

Well people tried different ways to suppress this instability one possible way is by using radio frequency waves there was an experiment conducted in one mirror called Phaedrus tandem mirror it was conducted many years ago I think in 1980s and the plasma of the Phaedrus mirror was tandem mirror they considered a plasma a very long mirror was there and they put a R f coil here in the middle the magnetic mirror coils were here and. So, on plasma was in here.

Well what they found was the fields that they implode were a frequency close to ion cyclotron frequency within 10 percent and those field were found to be effective in suppressing the interchange mode instability this is also known as flute instability and the main experiment list was this group was Ferro Hershkowitz and etal at the university of this concern and this was published in (( )) lattice 1983.

Well we developed an explanation a theory for understand these results and we found that if you can couple the unstable mode to a high frequency mode then the ponder motive force of the high frequency mode can stabilize by non-linear coupling can stabilize this instability. So, there are some efforts in stabilizing the macro instabilities in a mirror machine.

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However there is another instability that is we call as kink instability let me just mention what it is this instability is important in a machine may be not really. So, important in a mirror machine, but, in any device that contains an axial current.

So, suppose I have a plasma cylinder and there exist current in there suppose  $J_z$  current density is there in the system there is a magnetic field also here  $B_s$  in the  $z$  direction suppose the system the plasma very long plasma column and suppose this plasma somehow acquires a kink rather than going like this it becomes like this.

Suppose this is  $(( ))$  of repelling character along the length then what happens that the current is now shifting its directions  $J$  goes like this then this will produce a magnetic field obviously, any current which is excel will produce a colloidal magnetic field like this. So, these lines of force will be like this.

But, the lines of force will be away from here and they will come closer to her each other. So,  $B$  field is bigger here, colloidal magnetic field is larger here than here and magnetic pressure becomes more here than outside here and that tries to increase this ripple this bunching this is kink instability. This is of some concern in some machines which carry actual currents tokamak has a current and  $I$  should vary a instability similar to this instability there, but, I think mirror machine should not worry too much about this

instability a more dangerous instability in mirror machine is related to loss cone distribution function of ions.

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The image shows handwritten notes on a grid background. At the top, it says "Electrons" and "Maxwellian". Below that is the equation for the electron distribution function:  $f_e = n_0 \left( \frac{m}{2\pi T_e} \right)^{3/2} e^{-m v^2 / 2 T_e}$ . Below this, it says "Ions" and "Loss cone distribution". Below that is the equation for the ion distribution function:  $f_i = n_0 \left( \frac{m_i}{2\pi T_i} \right)^{3/2} \frac{m_i v_{\perp}^2}{2 T_i} e^{-m_i v^2 / 2 T_i}$ . In the bottom left corner, there is a logo for NPTEL.

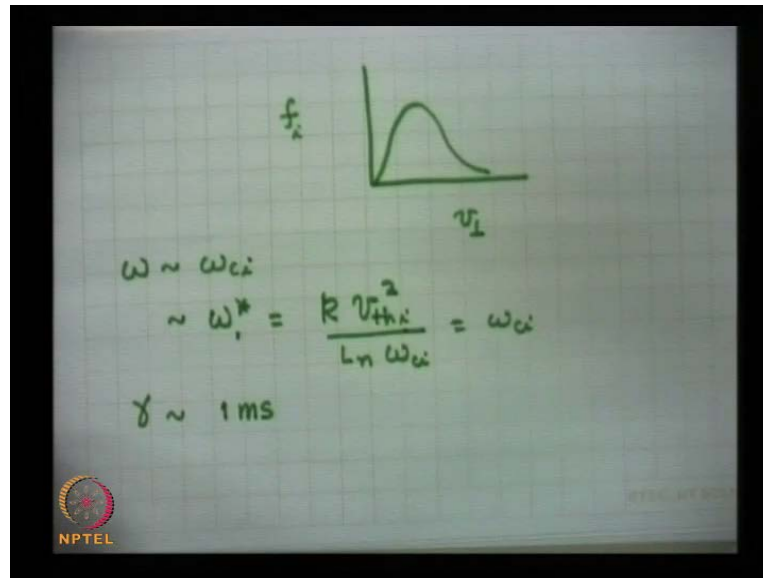
In my last lecture I mentioned that in mirror machine has electron distribution function, which is Maxwellian which is as I mentioned earlier  $f_e$  is equal to  $n_0 \left( \frac{m}{2\pi T_e} \right)^{3/2} e^{-m v^2 / 2 T_e}$  where  $v^2$  is  $v_{\perp}^2 + v_{\parallel}^2$ .

Ions on the other hand are non Maxwellian owing what we call as the loss cone distribution function loss cone distribution and if I plot this well let me write this.

$f_i$  is equal to  $n_0 \left( \frac{m_i}{2\pi T_i} \right)^{3/2} \frac{m_i v_{\perp}^2}{2 T_i} e^{-m_i v^2 / 2 T_i}$  this  $v^2$  is sum of  $v_{\perp}^2 + v_{\parallel}^2$  this is only  $v_{\perp}^2$  this extra term here tells you that particles with this small  $v_{\perp}$  are not there in the system they have lost well actually you can put some power here that depends on the mirror ratio.



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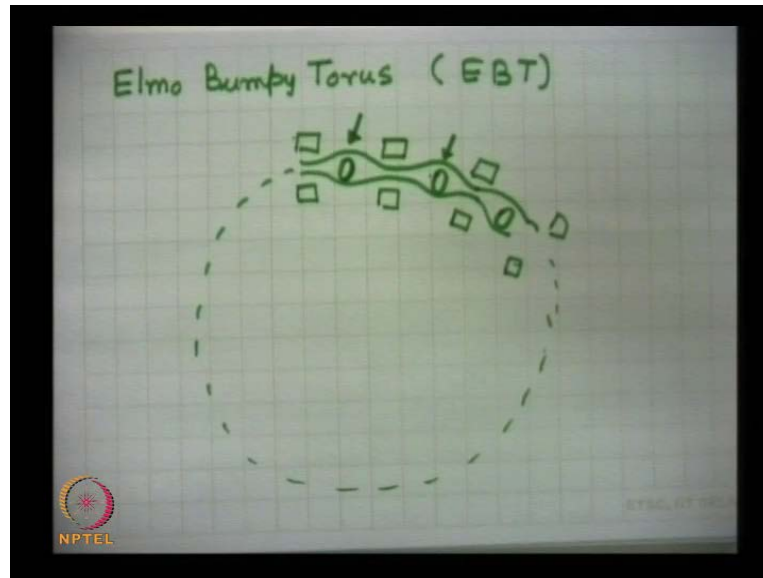
So, this is a typical loss cone distribution function that I have shown and graphically this goes like this plot  $v_{\perp}$  here  $f_i$  here and this is like this. So, this is a loss cone here these particles have lost and this gives rise to a instability whose frequency is very close to ion cyclotron frequency and also in the vicinity of diamagnetic drift frequency which is equal to  $k$  of the wave number  $k$  is the wave number of the unstable number wave into  $v_{thi}$  thermal of the ions thermal velocity of ion square upon density scale length in the machine into ion cyclotron frequency.

So, from this condition one can get the value of  $k$  frequency of the mode is very close to  $\omega_{ci}$  and wave number is such that when we put this is of the order of equal to  $\omega_{ci}$  you will get the value of  $k$  also.

So, this is a low wave length mode this is the wave length perpendicular to  $k$  and to  $d$  magnetic field and this is a very serious instability this can grow on the time scale of the order of a mille second unless you take remedial majors to suppress this instability one remedial major is that you try to fill the lost cone you filling particles to cover this loss suddenly there is one possibility, but, then you have to pumping particles additional particles again continuously in the system.

Second possibility was and which was very fascinating at one time there was very significant effort at oak ranch.

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And other places and the new device came into existence and they called it as Elmo bumpy torus in brief E B T what they was wanted to do that in a mirror machine we are losing particles from the throats.

So, if you put these mirrors on a torrid like this suppose I have a mirror here these one set of coils here then I have a torrid another torrid here then I have another torrid here and then just around this. So, what will happen you will have line of force like this and close it when is torrid like this.

So, if you have a large number of such mirror segments placed and joining over each other in a steroidal sense then the ions which are lost here will be filling the lost cone of the other mirror the lost which are here, and So, on and consequently they will be confined there and what was found that in a normally in a tokomak you require a steroidal current to confine the plasma in this we have found that, if we launch microwaves at second harmonic frequency cyclotron frequency when they can form here very hot rings near the mid planes of the mirrors and these rings provide the instability against many instabilities and plasma is confined.

So, this is a very interesting way to filling the loss cone that you are allowing the electron or ions lost from one machine to go into the other and from the other two comes to this one. So, there is a filling of the loss cone and hence v c l c mode can be stabilized

here that was a very interesting device I think still some work may be going on this machine Elmo bumpy torus and electrons cyclotron resonance heating had fundamental and second harmonic was found to be very useful especially second harmonic are found to be a very useful in heating the plasma.

Obviously electron cyclotron heating will heat the electrons and those electrons will have then had to transfer their energy via collisions to ions.

So, this is a very interesting way of avoiding the d c l c mode the drift cyclotron loss cone mode I think I skipped one important consideration in plasma confinement and that is the consideration for some macroscopic consideration for plasma confinement.

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$$\vec{J} \times \vec{B} = \nabla p$$

In the steady state  $\frac{d\vec{V}}{dt} = 0$

Electrons

$$0 = m \frac{d\vec{V}}{dt} = -e\vec{E} - e\vec{V} \times \vec{B} - m\nu(\vec{V} - \vec{V}_i) - \frac{1}{n_e} \nabla(n_e T_e)$$

Ions

$$0 = e\vec{E} + e\vec{V}_i \times \vec{B} + m\nu(\vec{V} - \vec{V}_i) - \frac{1}{n_i} \nabla(n_i T_i)$$

And that is essentially I want to deduce a general condition that J cross B should be equal to gradient of pressure this B is the esthetic B field and this is the plasma pressure.

For any confinement this condition must be satisfied the plasma is uniform there is no problem you do not to require a magnetic field, but, if the magnetic field if the pressure is not uniform you require a current in the system and B field in the system, let us see how this can happen what is the first of all let me deduce this and then discuss some implications of this.

First of all you know that if plasma has any pressure gradient or electric field it will give rise to drift velocities. So, in the quasi steady state or a steady state I will call this quasi steady state the drift velocity of particles should not depend on time. So,  $\frac{d}{dt}$  should be 0 for electrons as well as ions.

So, what I require  $m \frac{d\mathbf{v}}{dt}$  which is 0 for electrons if there is an electric field in the system then the electrons will experience this force electric force, if they are drifting with velocity  $\mathbf{v}$  and there is a magnetic field then they will experience a magnetic force and if there is a there are collisions between electrons and ions then they will lose momentum  $m n_e \mathbf{v} - \mathbf{v} I$  this is the relative velocity of the electrons and ions this is the momentum transfer from electrons to ions and if there is a gradient in pressure for electrons then this is  $n_e \nabla p_e$  this is the equation of motion for electrons in their steady state I put this equal to zero.

Similarly, for ions this 0 will be charge will be plus e. So,  $e E$  and this will be  $e \mathbf{v} \times \mathbf{B}$  cross magnetic field then the momentum that the electrons loses gained by the ions. So, put plus sign here  $m n_i \mathbf{v} - \mathbf{v} I$  is the collision momentum the ions gain from the electrons and similarly, a term  $n_i \nabla p_i$  gradient of  $n_i p_i$  this is the equation of motion for ions for electrons is this.

What you can do I want to avoid this  $e$  at the two equations and what you can do you can multiply the first equation by electron density and this by the ion density and recognizing that in plasmas quasi neutrality often holds. So,  $n_e$  equal to  $n_i$  if I take and add these 2 equations then this momentum terms cancel out electric field terms cancel out and these two terms after removing  $n_i$  and  $n_e$  from here they add up to give you gradient pressure and remaining terms give you current density  $e \mathbf{v} \times \mathbf{B}$ .

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$$\vec{J} = -n_e e \vec{v} + n_i e \vec{v}_i$$
$$n_e = n_i$$
$$\vec{J} \times \vec{B} = \nabla p$$
$$\nabla \times \vec{B} = \mu_0 \vec{J}$$
$$\frac{1}{\mu_0} \vec{B} \times (\nabla \times \vec{B}) + \nabla p = 0$$

$p(r), B(r)$

$\vec{r}$   
 $B_z$

So, if I define current density  $\vec{J}$  is equal to minus  $n_e e \vec{v}$  this is the electron current density and ion current density is plus  $n_i e \vec{v}_i$ . So, just by adding these 2 equations and recognizing that  $n_e e$  is equal to  $n_i e$  quasi neutrality condition then you get this condition  $\vec{J} \times \vec{B}$  is equal to  $\nabla p$ .

Now, I wanted to tell you that if I put  $\vec{J}$  from the equation of rather amperes law or Maxwell's equation which says that curl of  $\vec{h}$  or curl of  $\vec{B}$  rather are equal to  $\mu_0 \vec{J}$  in their study state when there is no displacement current this is the Maxwell's equation. So, if I put from here then I get interesting equation this says that  $\vec{B} \times \text{curl of } \vec{B}$  upon  $\mu_0$  plus gradient of  $p$  is equal to 0 where  $p$  is the sum of electron pressure plus ion pressure.

Well I think a detail discussion on this we will have later, but, here it will suffice for today that this equation if I consider a one dimensional situation suppose I have a plasma cylinder and my magnetic field is in the  $z$  direction, but, I want to confine the plasma. So, plasma pressure has a variation with  $R$  this is my radial direction. So, suppose I want to allow  $p$  as a function of  $R$  and  $B$  as a function of  $r$ , but, the direction of  $B$  is in the  $z$  direction just substitute this here this equation quickly gives.

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$$\frac{\partial}{\partial r} \left( \frac{B^2}{2\mu_0} + p \right) = 0$$
$$\frac{B^2}{2\mu_0} + p = \text{const}$$
$$\beta = \frac{p}{B^2/2\mu_0} < 1$$

You  $\frac{\partial}{\partial r}$  of  $B^2$  upon  $2\mu_0$  plus  $p$  is equal to 0 means  $B^2$  by  $2\mu_0$  plus  $p$  is a constant of constant in this particular one dimensional case when I have taken the pressure and magnetic field to vary with  $R$  and what we find that the region where plasma pressure is more  $B$  field has to be smaller and vice versa. So, what you require if I want to confine plasma in this region then the  $B$  field outside should be more than in the inside.

The minimum magnetic field in the plasma you can have a 0. So, the outside field has to be more than this value given by this equation. So, a certain minimum magnetic field is required and what you occasionally a very important quantity in plasma jargon is that called beta of the plasma which is kinetic pressure  $p$  upon magnetic pressure  $B^2$  by  $2\mu_0$  and this turns out to be less than one only then you can satisfy this condition. This is important in a mirror machine or in any device that is why a minimum  $B$  configuration is mandatory.

The plasma should have a minimum magnetic field where the pressure is large and I think with these few considerations I think we have had lot of interesting discussion and we shall return to this later. Thank you.