

**Plasma Physics**  
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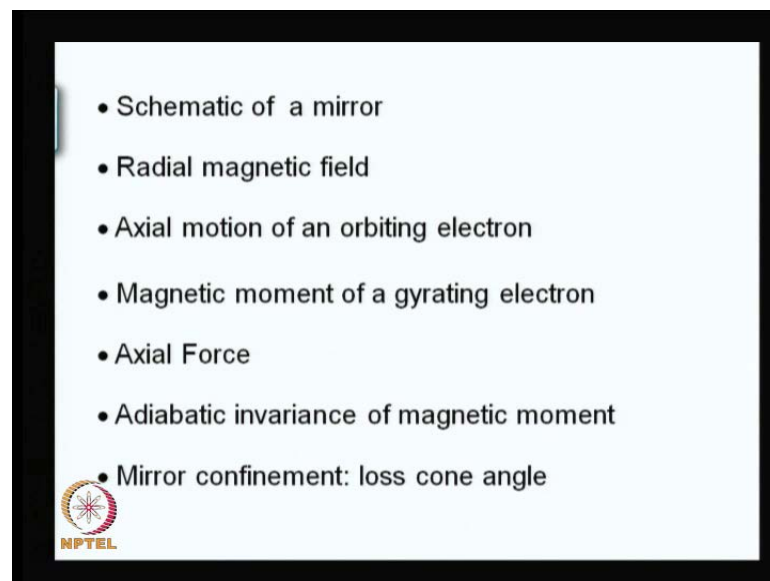
**Module No. # 01**

**Lecture No. # 22**

**Adiabatic Invariance of Magnetic Moment and Mirror Confinement**

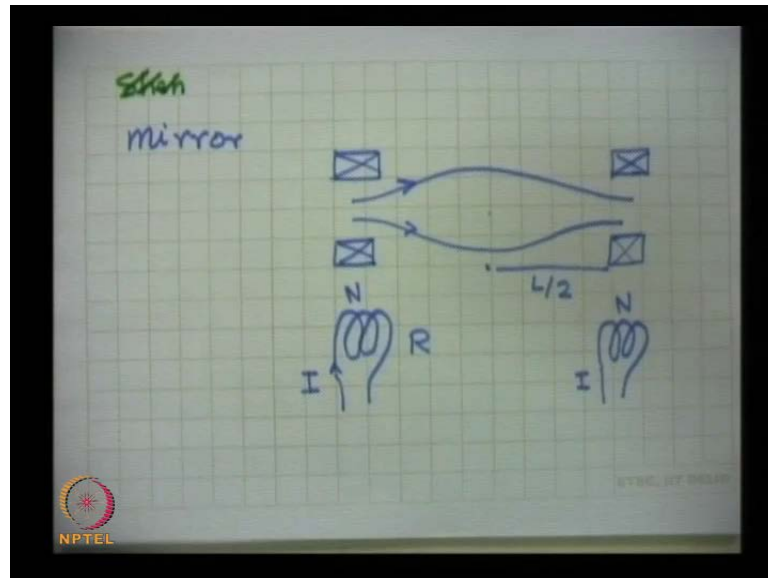
Today, we will discuss adiabatic invariance of a very important quantity called magnetic moment and mirror confinement in a plasma, which is a key to a very important machine called mirror machine.

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We shall discuss schematic of a mirror machine and then deduce an expression for radial magnetic field in the vicinity of axis of the machine. And we will consider the axial motion of an electron which is orbiting about the magnetic lines of force. And define magnetic moment of a gyrating electron, deduce axial force and hence, show the adiabatic invariance of magnetic moment and then we will discuss mirror confinement and obtain an expression for loss cone angle.

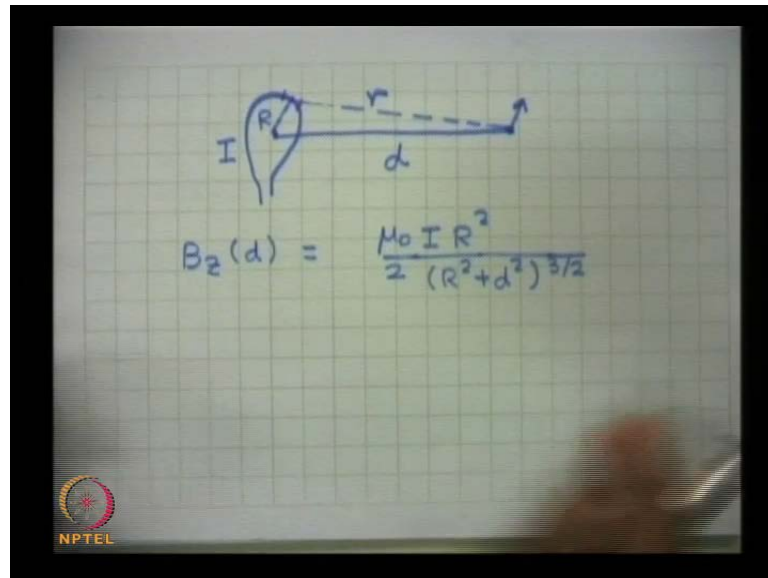
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Well, the schematic of a mirror is like this. Mirror is simply two coils placed at a distance much larger than the radius of these coils. This one coil here is schematic way of showing it. This is actually a circular coil placed here; another circular coil is placed here. And the lines of force that they produce are like these. If one consider, these are the lines of force, these coils are something like these carrying current. In the same sense, the number of the current in each coil is say  $I$  and number of terms is say  $N$ . They are identical coils and the radius of each coil is say  $R$ . And the distance from the center of their machine to the end or called throat of the machine is say  $L$  by 2.

And the magnetic field that this produces if you go from the center of the machine towards the end or throat, the magnetic field increases. So, there is a minimum magnetic field on the axis. However, if you go in the transverse direction the field falls off magnitude wise. So, in order to produce a minimum, absolute minimum magnetic field here; that is minimum in the  $z$  direction as well as minimum in the transverse direction. One normally places some current carrying rods here and those details we will discuss afterwards.

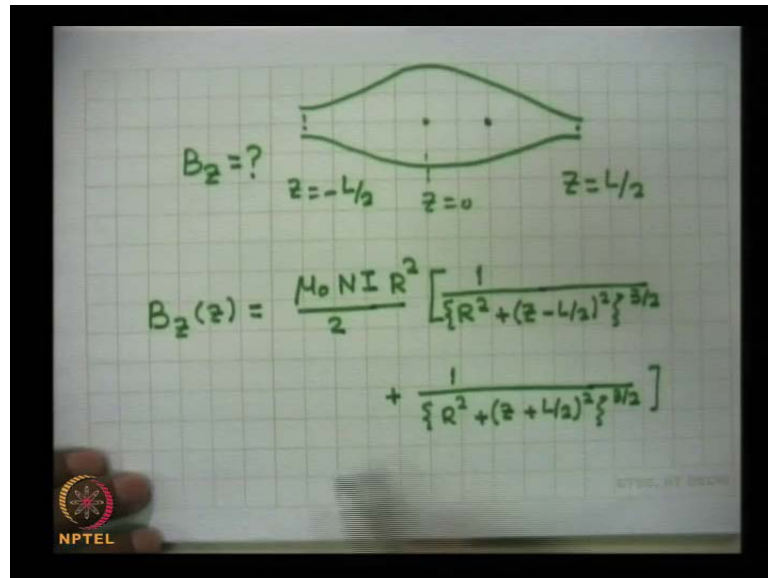
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If you consider, the magnetic field of a simple coil. Suppose, this is a single coil like this, circular coil and I want to find out the magnetic field at a point. Say  $d$  distance away from the coil of radius say  $R$ , this radius is  $R$ . Then the magnetic field at this point due to this element would be if I draw this, join this point this distance say  $R$ . Then this will be a perpendicular to in this direction. So, but when you sum over the contributors two magnetic field due to the all current elements of the wire which is carrying current say  $I$ . Then it so happens, that normal component cancel out only axial component survives. And  $B_z$  turns out to be at a distance  $d$  from the coil is equal to  $\mu_0$  upon  $2 I R$  square divided by  $R$  square plus  $d$  square to the power  $3$  by  $2$ .

This is the magnetic field which is axial on the axis of machine. To calculate magnetic field at any point in the vicinity is quite complicated. If I want to use the  $(( ))$  will be complicated. So, I want to do that, but let me write down the magnetic fields due to the mirror coils on the axis of machine and it is turns out to be.

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Let me write this  $B_z$  well. Let us keep this in view so, mirror is like this. So, this is my point  $z$  equal to 0. This is my  $z$  equal to 0 plane and here it is  $z$  is equal to  $L$  by 2. And this is the plane  $z$  is equal to minus  $L$  by 2 and I want to find out what is  $B_z$ ? At any point on the axis of machine. The distance of this point from this coil is  $z$  plus  $L$  by 2 and from this coil is  $z$  minus  $L$  by 2. Just keeping that in view, I can write down the magnetic field at any point,  $z$  away from the axis of the center of the mirror. This turns out to be is equal to  $\mu_0 N I$  where  $N$  is the number of turns in each coil divided by 2 into  $R$  square multiplied by  $1$  upon  $R$  square plus  $z$  minus  $L$  by 2 whole square to the power 3 half plus  $1$  upon  $R$  square plus  $z$  plus  $L$  by 2 whole square 3 half.

Now, this is the magnetic field at any point  $z$  on the axis. One may find that if  $L$  by 2, the length from the center to the throat from the mid plane to the throat is much bigger than  $R$  by  $R$ . Then this has a minimum on the; in the mid plane and maximum at the throat. So, these two fields one is called  $B_{\text{minimum}}$  and another one is called  $B_{\text{maximum}}$  can be easily obtained from here.

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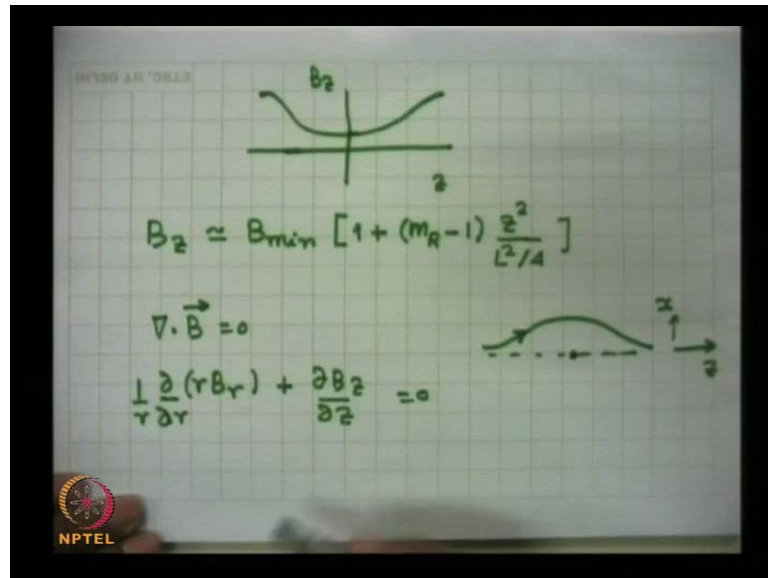
The image shows handwritten mathematical formulas on a grid background. The first formula is  $B_{\min} = \frac{\mu_0 N I R^2}{(R^2 + L^2/4)^{3/2}}$ . The second formula is  $B_{\max} = \frac{\mu_0 N I R^2}{2} \left[ \frac{1}{R^3} + \frac{1}{(R^2 + L^2)^{3/2}} \right]$ . The third formula, labeled 'Mirror ratio', is  $m_R = \frac{B_{\max}}{B_{\min}} \approx \frac{(R^2 + L^2/4)^{3/2}}{2R^3} \sim \frac{L^3}{16R^3}$ . An NPTEL logo is visible in the bottom left corner of the grid.

And one obtains  $B_{\min}$  turns out to be is equal to  $\mu_0 N I R^2$  divided by  $R^2 + L^2/4$  to the power  $3/2$ . And  $B_{\max}$  turns out to be at  $z$  is equal to  $L/2$ . It turns out to be  $\mu_0 N I R^2$  divided by 2 into  $1/R^3 + 1/(R^2 + L^2)^{3/2}$ . Usually, since  $L$  is bigger than quite big as compare to  $R$ , this term is quite small and hence, one can approximate this quantity as  $L/R^3$ . Anyway, the mirror ratio is defined as mirror ratio is very important parameter. That we shall recognize in a little while, but its definition is  $m_R$  the mirror ratio is the ratio of  $B_{\max}$  to  $B_{\min}$ . Well, if I really done this the value turns out to be is equal to  $(R^2 + L^2/4)^{3/2} / 2R^3$ .

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Of the order if I ignore this term then this of this order divided by  $2R^3$  which is of the order of. If I take this to be substantially bigger than  $R^2$  then this is really like  $L^3$ . Then through the machine divided by  $16R^3$ . One can achieve on long mirrors  $m_R$  maybe 2, 3, 4, 5, even 8. One can achieve and if I plot the magnetic field  $B_z$  as a function of  $z$ .

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Then what you get is if I plot  $B_z$  here and  $z$  here. The field shelled has a minimum, goes like this and then here also it goes like this symmetric. So, in this region well, you want to localize the plasma. Magnetic field can be modeled as  $B_z$  typically of the order of  $B_{\text{min}}$  multiplied by 1 minus rather 1 plus  $m_R$  minus 1.  $m_R$  is the mirror ratio into  $z$  square by  $L$  square by 4. This is not exact expressions. Just a quite reasonable representation of magnetic field with  $z$  saying that this is minimum here and have some sort of a parabolic profile, but thing is that whenever magnetic field is a function of  $z$ . If you keep in view the second Maxwell equation divergence of  $B$  equal to 0.

And recognizing the symmetry of the problem that you are expecting the magnetic field to vary with  $R$  and  $z$  only not with the azimuthal coordinate  $\phi$ . Then this equation demands that  $\frac{1}{r} \frac{\partial}{\partial r} (rB_r) + \frac{\partial B_z}{\partial z} = 0$ . So, if  $\frac{\partial B_z}{\partial z}$  is finite because  $B_z$  is a function of  $z$ . This term must also be finite means  $B_r$  must exist that is important. So, any line of force which is like this or indeed has if it has a; this is my  $z$  direction and this is my transverse direction or  $r$  direction. Or you can call  $x$  direction and then there is a symmetry about  $z$  axis. So, what you are having is that your magnetic field indeed is travelling not parallel to  $z$  axis, but it is a magnetic to  $z$  axis. So, it suddenly will have a axial component and  $z$  component and a transverse component.

And we have to evaluate this quantity. I mention to you that evaluation of  $B_r$  from (C) is bit complicated. However, if I use this equation it is easy. If I recognize that  $\Delta B_z$  by  $\Delta z$  may be treated as a some sort of a constant near the axis of the machine. So, this is the center of the plasma. I want to find out magnetic field away from the axis. So, I take this term on the right hand side and integrate this equation.

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$$\frac{1}{r} \frac{\partial}{\partial r} (r B_r) = - \frac{\partial B_z}{\partial z}$$

$$r B_r = - \frac{\partial B_z}{\partial z} \cdot \frac{r^2}{2} + C_1$$

$$C_1 = 0$$

$$B_r = - \frac{\partial B_z}{\partial z} \frac{r}{2}$$

$-e v_\phi B_r$

$\theta$

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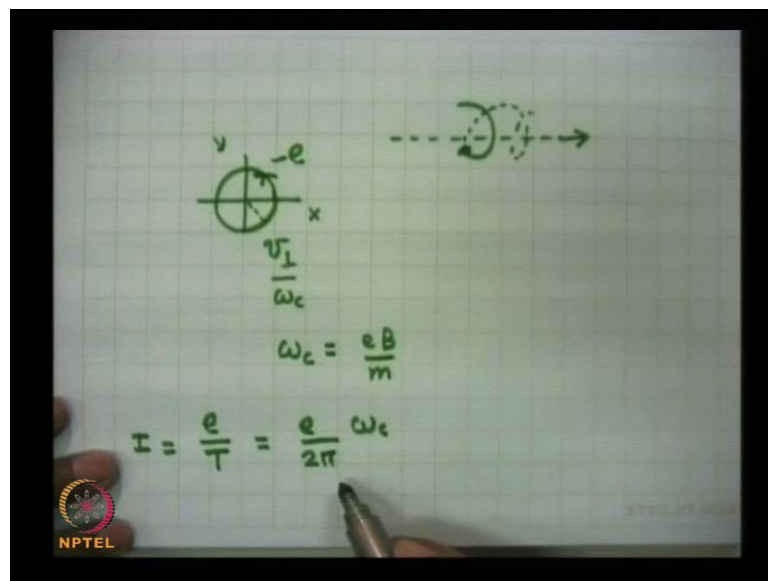
And I obtain,  $\frac{1}{r} \frac{\partial}{\partial r} (r B_r)$  is equal to minus  $\Delta B_z$  by  $\Delta z$ . Take this  $r$  here  $d r$  also there and integrate you will obtain  $r B_r$  is equal to minus  $\Delta B_z$  by  $\Delta z$  multiplied by  $r$  square by 2. For  $r B_r$  when goes there, it will be this plus a constant of integration  $C_1$ . Now, this equation should be satisfied at all values of  $r$  including  $r$  equal to 0, but at  $r$  equal to 0 of this term is 0 this is also 0. So,  $C_1$  must to be 0. So, once you take  $C_1$  equal to 0 then  $B_r$  can be written as radial magnetic field as minus  $\Delta B_z$  by  $\Delta z$  into  $r$  by 2. This  $B_r$  is very important in the determining the motion of the gyrating electrons and ions in a mirror machine. This is very important.

The reason is that if any line of force is going like this with a finite radial components. Suppose, this is a radial, this is my line of force or magnetic field then it the radial component is downward and if I consider the motion of an electron like this. So, when the electron moves around any line of force and that point if there is a radial component of magnetic field. So, if the electron has a  $\phi$  velocity if it is rotating about the line of force with some velocity say  $v_\phi$ . And there is a magnetic field  $B_r$  then the  $v$  cross  $B$

force on this electron would be in the z direction and if you take into account the charge also.

Then this force is in the minus z direction. And so, as the electron tries to or rather gyrates around the line of force with finite velocity in the z direction. As it goes from the center of the machine towards the throat, it explicit about a force and that I would like to calculate. Before, I calculate this reversible this axial force on the electron that we define a quantity called magnetic moment of the rotating electron.

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So, I am trying to consider the motion of an electron about the axis. So, this is the axis of the machine and electron is gyrating like this. It is moving forward as well, this go like this. Because we have seen that if the magnetic field is in this direction in the zeroth order approximation is the magnetic field is in the z direction. Then the electron will gyrate about the line of force in the right handed sense just like as if you take a right handed screw or rotate it. Then the direction of electron motion will be through the fingers and the direction of advancement of the screw will be the direction of magnetic field. So, this is the electron motion.

And if you examine the electron motion in the transverse plane in the x y plane what really has happening? If electron is rotating in this; is if I plot here x and y then the electron is going over the circle like this. Is the electron motion? The radius of the



electron is  $v_{\perp}$ , upon  $\omega_c$  called gyration radius, radius of gyration. And the frequency with which this rotates is cyclotron frequency  $\omega_c$  which I call as  $eB$  upon  $m$  and I am presuming that near the axis  $B_z$  is much bigger than  $B_r$ . So, primarily  $\omega_c$  is decided by  $B_z$ . Now, just like in an atom we considered the rotating electron like a current loop.

So, magnitude wise what is the current in this electron? If electron is moving in a circular orbit with charge minus  $e$  magnitude is  $e$ . Then  $e$  upon time periods for one revolution say  $t$ . This is the equivalent current in this loop. Magnitude of current is  $e$  upon  $t$  and  $e$  upon  $t$  turns out to be, if I put  $t$  is equal to  $2\pi$  by  $\omega_c$   $e$  upon  $2\pi$  upon  $\omega_c$ , this is the current. And for any current loop we define a magnetic moment as the product of area into current. So, let me define that (No audio from 19:48 to 19:55) and it so happens.

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Magnetic moment

$$\begin{aligned} \mu &= I \times \text{area} \\ &= \frac{e \omega_c}{2\pi} \pi \frac{v_{\perp}^2}{\omega_c^2} \\ &= \frac{m v_{\perp}^2}{2B} \end{aligned}$$

Flux linked with the gyrating  $e^-$

$$\phi = B_z \pi \frac{v_{\perp}^2}{\omega_c^2} = \frac{2\pi m v_{\perp}^2}{2B e^2} m$$

$i$   $v_{\perp}/\omega_c$

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So, magnetic moment (No audio from 20:02 to 20:10)  $\mu$  is equal to current into area and that turns out to be current. In this case is  $e \omega_c$  upon  $2\pi$  and area is  $\pi$  into larmor radius square which is  $v_{\perp}$  square upon  $\omega_c$  square. And if you cancel out these quantities one  $\omega_c$  will cancel out,  $\pi$  will cancel out and remaining  $\omega_c$  is  $e v_{\perp}$  upon  $m$ . It turns out to be equal to  $m v_{\perp}$  square upon twice  $B$ . This is a important quantity and we would like to learn about the nature of  $\mu$ .

As the particle moves from center of the machine towards the throat; does it change or does it remain constant? One may also recognize an important quantity, if the electron is rotating about the line of force. Suppose, the electron is rotating about the line of force with the radius which is equal to  $v \perp$  upon  $\omega c$ . What is the magnetic flux linked with this? The magnetic flux would be the magnetic flux linked with the gyrating electron. It is a product of magnetic field into area. So, which is equal to  $\phi$  is equal to magnetic field is  $B_z$  into area is  $\pi$  into  $v \perp$  square upon  $\omega c$  square. One may recognize that this quantity if I put  $\omega c$  as  $e B$  upon  $m$  can be also written as  $\pi m v \perp$  square upon  $v$ . One  $v$  will cancel out, one  $v$  will survive.

Then you will be having some constants like  $e$  square,  $e$  square and  $1/m$  belongs to be there I guess. My point is that  $\phi$  is related to  $\mu$  beside some constants.  $M v \perp$  square by  $2 B$  is called  $\mu$ . So, I can write down this quantity by multiplying  $2$  here and multiplying  $2$  here. As this central quantity is magnetic moment this entire quantity is magnetic moment. So, the remaining quantity, these quantities  $2 \pi m$  upon  $e$  square is simply constants for a particle. And hence, if we can show that  $\mu$  is a constant of motion then the flux linked with the gyrating electron will also be a constant. And that would be a very valuable thing. We shall deduce some important results from here. From this if we can show that  $\mu$  is a constant of motion.

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$$\begin{aligned}
 m \frac{dv_z}{dt} &= -e(\vec{v} \times \vec{B})_z \\
 &= -e(v_r B_\phi - v_\phi B_r) \\
 &= e v_\phi B_r, \quad B_r = -\frac{r}{2} \frac{\partial B_z}{\partial r} \\
 r = \rho &= \frac{v_\perp}{\omega_c} \\
 &= -e \frac{v_\perp^2}{2\omega_c} \frac{\partial B_z}{\partial r} = -\frac{m v_\perp^2}{2B} \frac{\partial B}{\partial r}
 \end{aligned}$$

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So, I would like to go over to discuss the particle motion in the axial direction and deduce the constancy of  $\mu$ . My equation of motion is  $m \frac{dv_z}{dt}$ , a rate of change of momentum or mass into acceleration.  $z$  component of acceleration is equal to charge of the electron into  $v$  cross  $B$  force on the electron. And I want only  $z$  component of this force, this is a  $z$  component of equation of motion. Now, this can be written as minus  $e v$ ,  $z$  component is  $v_r B_\phi$  minus  $v_\phi B_r$ . By symmetry, there is no magnetic field in the  $\phi$  direction. I can ignore this term; this is the only term that survives. I can write down this is equal to  $e v_\phi$  into  $B_r$ , but  $B_r$  we already obtained  $B_r$  was is equal to minus  $R$  by  $2 \Delta B_z$  upon  $\Delta z$  what is  $r$ ?

Your electron we are considering to be gyrating with radius, larmor radius and that is  $r$ .  $r$  is the location of the electron away from the axis which is equal to larmor radius. So, if I put  $r$  is equal to  $\rho$  which is equal to  $v_{\perp} \text{ upon } \omega c$ . Then the right hand side of this equation turns out to be this equation turns out to be equal to  $e v_\phi$ . I can simply write as  $v_{\perp}$  into  $B_r$  which is minus. So, minus sign is here and  $R$  is  $v_{\perp}$  by  $\omega c$ . So, it becomes  $c_{\perp}^2 \text{ upon } 2$ . This  $2$  and this  $\omega c$  into  $\Delta B_z \text{ upon } \Delta z$  and this  $\omega c$ . If I put in terms of  $e B \text{ upon } m$ , then this turns out to be equal to minus  $e$ .  $e$  will cancel out,  $m v_{\perp}^2 \text{ upon } 2$  by  $2 B$  is called  $\mu$ . So, what we have found that,  $m \frac{dv_z}{dt}$  for a single particle is equal to  $\mu \Delta B_z \text{ by } \Delta z$ , this is a important result.

Here, I am approximating  $B_z$  by  $B$  because  $B_r$  is very tiny. So,  $B_z^2$  is certainly much bigger than  $B_r^2$  which is approximately, this is alright. Now, this  $m v_{\perp}^2 \text{ by } 2$  by  $2 B$  is called  $\mu$ . So, what we have found that,  $m \frac{dv_z}{dt}$  for a single particle is equal to  $\mu \Delta B_z \text{ by } \Delta z$ , this is a important result.

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$$\begin{aligned} \text{Energy } E &= \frac{1}{2} m v_x^2 + \frac{1}{2} m v_z^2 \\ &= \mu B \\ m \frac{d\vec{v}}{dt} &= -e \vec{v} \times \vec{B} \\ m \vec{v} \cdot \frac{d\vec{v}}{dt} &= 0 \\ \frac{1}{2} m v^2 &= \text{Const. of motion} \\ \frac{d\mu B}{dt} &= - \frac{d}{dt} \frac{m v_z^2}{2} = m v_z \frac{d v_z}{dt} \end{aligned}$$

Now, we can proceed further recognizing that for an electron moving in a magnetic field energy, total energy of the particle is a constant of motion. And energy is defined as half  $m v_{\text{perp}}^2$  which is half  $m v_x^2$  plus  $v_y^2$  and plus half  $m v_z^2$ . Certainly, this is a total energy, it is a constant of motion and this you can easily see, from the equation of motion. Which says that  $m \frac{d\vec{v}}{dt}$  is equal to minus  $e \vec{v} \times \vec{B}$ . Identify take dot of this equation with respect to  $\vec{v}$ . Then I get  $m \vec{v} \cdot \frac{d\vec{v}}{dt}$  is equal to  $\vec{v} \cdot$  of this quantity which is perpendicular to  $\vec{v}$ . So, this is 0 exactly and hence this result, that  $e$  is a constant of motion. So, we get  $m v^2$  half  $m v^2$  is a constant of motion in a magnetic field if there is no electric field in the system.

Now, if I take  $e$  as a constant of motion then what I can do? I can differentiate this equation with respect to time and let us see what do I get? First of all half  $m v_{\text{perp}}^2$ ; I can write down in terms of  $\mu$  because, I have defined  $\mu$  as the ratio of this quantity to magnetic field. So, this quantity I can write down is equal to  $\mu$  into  $B$  and if I use this as  $\mu B$ . And the differentiate this equation I get  $\frac{d}{dt}$  of  $\mu B$  is equal to minus  $\frac{d}{dt}$  of  $m v_z^2$ . Just put this  $\frac{d}{dt}$  equal to 0 which means that  $\frac{d}{dt}$  of this should be minus of this quantity, derivative of this quantity into half. Now, you can  $m$  by 2 is a constant. So, this becomes is equal to  $m v_z \frac{d v_z}{dt}$  and  $\frac{d v_z}{dt}$ , I have already obtained in terms of  $\mu \Delta B_z$  by  $\Delta t \Delta z$ . So, I let me substituted for this.

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The whiteboard shows the following equations:

$$\frac{d}{dt}(\mu B) = -m v_{\perp}^2 \left(-\frac{\mu}{m} \frac{\partial B_z}{\partial z}\right)$$

$$\mu \frac{dB}{dt} + B \frac{d\mu}{dt} = + \mu \frac{dB}{dt}$$

$$\frac{d\mu}{dt} = 0$$

$$\mu = \frac{m v_{\perp}^2}{2B} = \text{Constant of motion}$$

$$E = \frac{1}{2} m v_{\perp}^2 + \frac{1}{2} m v_{\parallel}^2 - \text{---}$$

An NPTEL logo is visible in the bottom left corner of the whiteboard image.

And what do I get? I get  $\frac{d}{dt}(\mu B)$  is equal to  $m v_{\perp}^2 \frac{d v_{\perp}^2}{d t}$  which I found to be equal to  $-\mu \frac{dB_z}{dt}$  because  $B_z$  is like  $B$ . This is what we got. What I can do?  $v_{\perp}^2$  I can write down as  $B_z \frac{d v_{\perp}^2}{d t}$  and then this derivative of  $\frac{d v_{\perp}^2}{d t}$  into  $\frac{d B_z}{d t}$  can be combined together. And I can write down this; why this  $m$  is there  $m$  should not be there. Actually, this divided by  $m$  this was the value of  $\frac{dB_z}{dt}$ . So,  $\frac{d v_{\perp}^2}{d t}$  was this only this  $m$  was also there. So, this  $m$  will cancel out it becomes  $-\mu \frac{d v_{\perp}^2}{d t}$ .

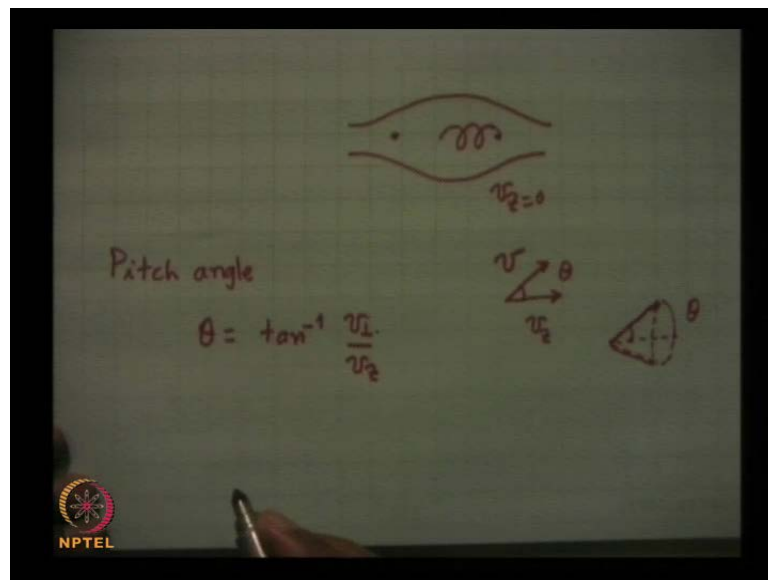
If I break the left hand side by using the product rule for differentiation, I get two terms here; one is, sorry, there was a negative sign also there outside there was a negative sign. So, this becomes plus and you will get  $\mu \frac{dB}{dt} + B \frac{d\mu}{dt}$  is equal to this. The first term here will cancel the term on the right hand side and this gives you  $\frac{d\mu}{dt}$  is equal to 0. So, as the electron travels inside the machine,  $\mu$  must be a constant of motion. And  $\mu$  as defined as,  $\frac{m v_{\perp}^2}{2B}$  is a constant of motion (No audio from 33:05 to 33:13). It is a very important deduction, that when an electron or ion. It moves in the machine, from the center of the machine towards the throat or comes by whatever  $\mu$  remains a constant.

Now, please see it is consequence that as the electron which is gyrating about the field lines and is going towards the throat, it is going to higher and higher value of  $B$ . So, what is really physically happens? That when  $B$  increases  $v_{\perp}$  must also increase to keep

mu constant. But, we also know that energy is a constant of motion, which energy is a sum of half m v perp square plus half m v z square. So, if v perp increases, v z must decrease to keep e constant. So, these two important constants mu and e. They tell us that whenever an electron travels from center of the mirror machine towards the throat, it is B will increase, hence v perp will increase. And when v perp increases, v z must be decrease.

So, that e remains constant and it is possible, if v perp is quite large in the beginning and v z is small, that v z decreases to 0 somewhere. And beyond that point, the electron will not travel or ion will not travel.

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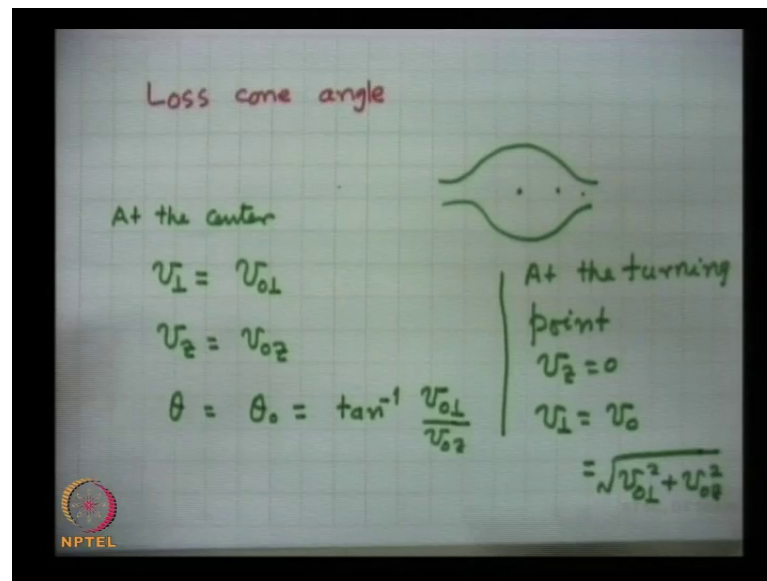
So, what is happening is that this is kind of your mirror machine. When the electron goes from here towards here, mu remains constant and somewhere the electron may encounter v z equal to 0. From that point on time, the electron will reverse its path and come back. And when it comes back, it goes in the other direction and it may go up to a similar point on other side. And will bounce back and forth between two turning points on the two sides of the mid plane. Such electrons are called trapped electrons, they are trapped inside the mirror machine or confined. However, if initially the electron has large axial velocity, large v z and a small v perp, then v z may decrease.

Substantially, when the electron goes from center towards the throat, but may not become 0, come to 0 and such electrons will be lost from the machine. They are called the electrons that escape from the machine. Well, there is not a special name for them, but those electrons certainly will have a large  $v_z$  and a small  $v_{\perp}$  and we define an angle called pitch angle (No audio from 36:22 to 36:27). Is now we will defined theta, in such a fashion, that suppose at any instant an electron has a z component of velocity or suppose, the electron is moving in some velocity  $v$ , with at angle  $\theta$  to z axis. Then this is called z component of velocity and there will be transverse component of velocity.

So, this is  $v_{\perp}$  upon  $v_z$  is called  $\tan \theta$  or  $\theta$  is called pitch angle which is  $\tan^{-1}$  of  $v_{\perp}$  upon  $v_z$ . On a graph, on which you denote z axis as  $v_z$ , when because the electron is gyrating so, what will happen? The electron velocity will change direction in such a fashion. There is suppose my z axis and the electron is initially suppose which is here, then this will rotate over the z axis like this. So, this will move on a cone, this is the kind of cone it will move. So, the velocity vector will move on a cone, but the angle this will make with the z axis is, they remain in the same. This is called pitch angle  $\theta$  at some location. However, as this electron goes from the central region towards the throat, this  $\theta$  will increase because  $v_{\perp}$  increases,  $v_z$  decreases.

So, now I would like to deduce an expression, for what is the value of pitch angle  $\theta$  at the center of the mirror machine. That when the electron goes towards the throat, it suffers a reflection. What is the minimum or the larger sorry minimum value of  $\theta$  that is permissible? If  $\theta$  were large, then certainly there will be reflection. But, if  $\theta$  is a small, there is no reflection. So, what is that critical angle or called loss cone angle, critical value of  $\theta$  for which, the reflection occurs from where the field is maximum.

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So, let me calculate this quantity and this is called loss cone angle (No audio from 38:41 to 38:52). Loss cone angle I want to deduce, what we are expecting? Always keep this picture in view (no audio from 39:01 to 39:07), that your electron is going from here towards this somewhere here. In the center or at the center (No audio from 39:17 to 39:21), we are saying that the electron has say,  $v_{\perp}$  is equal to  $v_{0\perp}$  and it has  $v_{\parallel}$  is equal to  $v_{0\parallel}$ . And theta let me call as,  $\theta_0$  which is equal to  $\tan^{-1} v_{0\perp}$  upon  $v_{0\parallel}$ , this is my pitch angle.

As the electron goes reaches the turning point, what I am expecting the turning point? At the turning point (No audio from 40:11 to 40:18), I am expecting  $v_{\parallel}$  equal to 0. And  $v_{\perp}$  should become equal to  $v_0$ , because total magnitude of velocity is a constant. So,  $v_{\perp}$  should be equal to  $v_0$  which is the sum of square of  $v_{0\perp}$  and  $v_{0\parallel}$  and under root. So, which is equal to under root of  $v_{0\perp}^2$  plus  $v_{0\parallel}^2$ . But, in the 2 cases magnetic moment here and magnetic moment here this should be same, because it is a constant of motion.



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$$\begin{aligned} \text{At } z=0 \quad \mu &= \frac{m v_{0\perp}^2}{2 B_{\min}} & \text{At } z=L/2 \quad \mu &= \frac{m v_0^2}{2 B_{\max}} \\ \frac{v_{0\perp}^2}{v_0^2} &= \frac{B_{\min}}{B_{\max}} = \frac{1}{m_R} \\ \sin \theta_0 &= \left( \frac{1}{m_R} \right)^{1/2} \\ \theta_0 &\equiv \theta_c \end{aligned}$$

So, let me write down the magnetic moment. At  $z$  equal to 0 (No audio from 41:06 to 41:11), magnetic moment  $\mu$  is equal to  $m v_{0\perp}^2$  upon 2 into  $B_{\min}$  because magnetic field there is be minimum. And at  $z$  is equal to  $L/2$ , what has happening?  $\mu$  is equal to the total perpendicular energy is  $m v_0^2$  by 2 and  $v$  field is  $B_{\max}$ . If we equate these 2, I get a value of  $v_{0\perp}^2$  upon  $v_0^2$  and it turns out to be equal to  $B_{\min}$  upon  $B_{\max}$ . And this ratio we call as mirror ratio, 1 upon mirror ratio.

And  $v_{0\perp}$  by definition is  $v_0 \sin \theta_0$ . So, what we get here is,  $\sin \theta_0$  is equal to 1 upon  $m_R$  under root. And this angle is a very special angle, because here the reflection we are taking to occur at the throat  $L/2$ . And this angle is called loss cone angle, this  $\theta_0$ , we give a special name as  $\theta_c$  critical.

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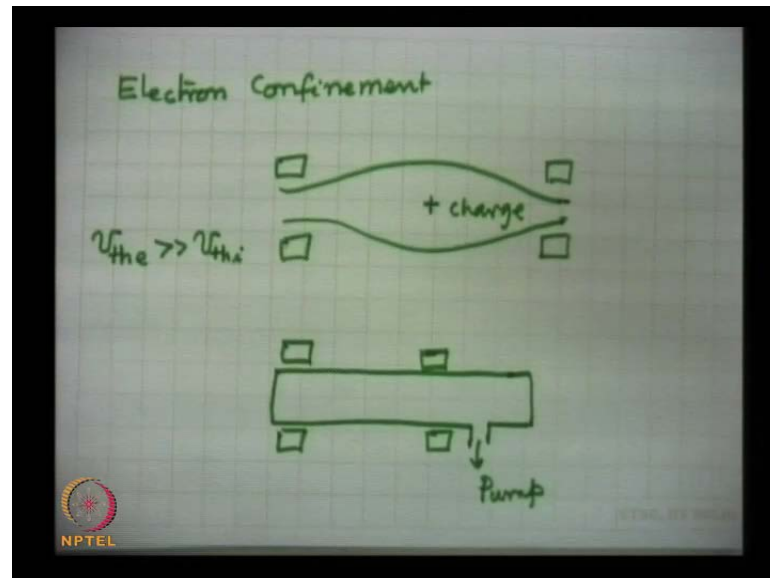
$$\theta_c = \sin^{-1} \left( \frac{1}{m_R} \right)^{1/2}$$
$$m_R = 4$$
$$\theta_c = 30^\circ$$
$$v_z = v_0 \cos \theta_c = \frac{\sqrt{3}}{2} v_0$$
$$v_{\perp} = \frac{v_0}{2}$$
$$\theta_0 \geq \theta_c \text{ are confined}$$

The loss cone angle and that turns out to be (No audio from 43:08 to 43:14), theta c is equal Sin inverse 1 upon m R under root. For a typical mirror, with mirror ratio something like 4 suppose, I chose mirror ratio is equal to 4. In that case theta c is equal to sin inverse of half, sin inverse of half is 30 degrees. Which means, that those electrons for which v z is equal to v 0 Cos theta c is equal to Cos 30 is root 3 by 2 v 0 and v perp is v 0 by 2. These electrons are travelling at a pitch angle with a velocity, at an angle of 30 degrees to magnetic field on the axis is the mirror axis. So, the electrons which are launched here, at 30 degrees to mirror axis. These electrons when they go here and come back they will suffer a reflection at the throat, they will go like this and then come back from the throat, it is the path.

And they will go to the other side whereas; all electrons with theta bigger than theta c will be trapped. So, the electrons with theta 0 greater than or equal to theta c are confined, and theta 0 less than theta c will be lost. One must note, that this mirror ratio does not depend on the mass or charge of the particle, is the same for electrons and ions. If we ignore any space charge field created in the system. So, both electrons and ions can be trapped the lines of force are not closed. They are open lines of force, but a still the system has a interesting property, that the particles which are moving at large pitch angle, they are trapped inside the machine. That is a very interesting property (No audio from 45:56 to 46:01).

Now, I would like to make some deductions from here well, first of all I would like to examine the influence of mirror effect on plasma confinement in a mirror machine.

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Let me talk about electron confinement (No audio from 46:31 to 46:41). What happens? You have a mirror machine here, then the lines of force goes like this and your coils are here, then you create a plasma over here somehow. Normally in a mirror machine actually the whole thing is really placed in a cylinder schematic like this (No audio from 47:12 to 47:18). You have a field coils here, these are your field coils, this goes to the pump and you introduce a plasma in this system, you can create a plasma. First you create a vacuum and then you create a plasma here, I think those operations I will discuss later, but let me tell how the particles are confined here. Whenever you create a plasma in the machine, the plasma electrons and ions may have comparable temperatures.

But, the electrons for a same temperature will have a larger thermal velocity than the ion thermal velocity. So,  $v$  thermal of the electrons is much bigger than thermal velocity of ions. Consequent and user is the distribution function of electrons is like Maxwellian. So, there will be electrons with the small  $v_z$ , large  $v_z$  means small pitch angle, large pitch angle. The electrons with small pitch angle, will be lost from the machine very quickly and thus they will produce a net positive charge. So, there is a residual positive space charge, that space charge will stop further flow of electrons or further escape of

electrons. And the electrons which are left here, initially they have very few electrons with small  $v_z$ , I mean large  $v_z$ .

But, after electron-electron collisions, they will demise themselves and acquire some sort of a Maxwell distribution function. But, that is not these electrons are not lost because of the positive space charge. That is created here by the escape of some electrons initially. How about the ions? The ions which are filled in the system, those ions quickly initially will move out, if their angle is pitch angle is less than the loss cone angle  $\theta_c$  and other ions are localized. So, the two space is the electrons primarily in a mirror machine have a Maxwellian distribution function whereas, the ions have a non Maxwellian distribution function. And this is called loss cone distribution function.

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Electrons

$$f_e = n_0 \left( \frac{m}{2\pi T_e} \right)^{3/2} e^{-m v^2 / 2 T_e}$$

Ions

$$f_i = n_0 \left( \frac{m_i}{2\pi T_i} \right)^{3/2} \frac{m_i v_i^2}{2 T_i} e^{-m_i v_i^2 / 2 T_i}$$

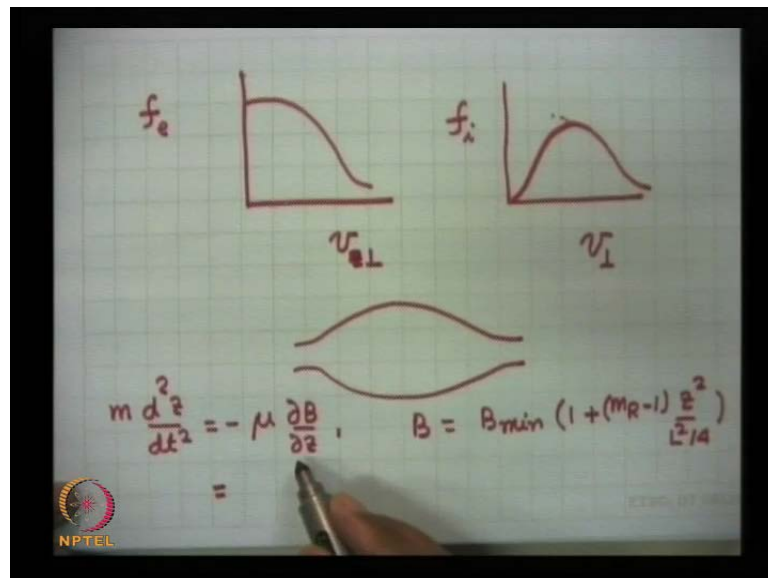
$e\phi / T_e \geq 1$

NPTEL

So, the distribution function for electrons is like this (No audio from 49:55 to 50:01), they have  $f$  for the electrons like, density of electrons  $n_0$ ,  $m$  mass of the electron  $2\pi$  into temperature of the electron, in units of energy means Boltzmann constant is hidden in there. This to the power  $3/2$  exponential minus  $m v^2$  by  $2 T_e$ . This is the electron distribution function in the machine. Whereas, for the ions the distribution function is  $f_i$  which is equal to  $n_0$ , mass of the ion upon  $2\pi T_i$  ion temperature, to the power  $3/2$  into  $m v^2$  by  $2 T_i$  exponential minus  $m_i v_i^2$  upon  $2 T_i$ . This is the additional factor here. This additional factor, change is the character of ion distribution function very significantly.

If I plot the two distribution functions, then their nature will be different. Before I do that, let me mention that this is the electron density, this is the ion density and I mentioned to you that the electron density slightly less than the ion density. Because, some electrons have escaped early. However, a very small difference in the 2 produces a very strong space charge field and that traps the electrons. So, this, the space charge potential in the plasma  $e \phi$  upon  $T_e$  is around 3 or 4 is certainly bigger than one. So, that the electrons in the machine are trapped by the electrostatic attraction by the space charge of ions. Whereas, the ions are trapped by mirror reflections and this is a factor here so, if I plot the distribution functions.

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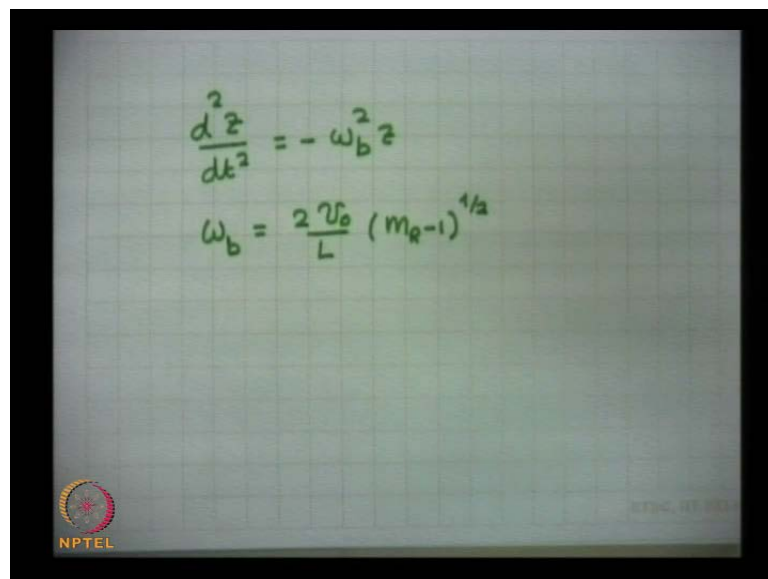


I have different kind of nature for  $f_e$  and  $f_i$ . The distribution function for electrons if I plot as a function of  $z$  velocity of, sorry perpendicular velocity or any velocity., I think I should plot  $v_{\perp}$ .  $v_{\perp}$  here then this will turn out to be like this for electrons. If I plot a similar thing for ions as a function of  $v_{\perp}$ , at  $v_{\perp}$  equal to 0 is not maximum 0 so, this goes like this. So, there is a region here in velocity space, where particles are lost and a system which is not maxwellian, that is not in the most probably state. So, every system has a tendency, to go to a most probably state and hence we can alternatively say that, this system is a free energy to drive instabilities. And this gives rise to some serious instabilities which is very detrimental for the plasma confinement in a mirror machine.

And this wave we shall learn in our next lecture, a very special kind of modes arises, because of this special nature of distribution function. That is called drift cyclotron loss cone mode, that becomes unstable and that produces Plasmon's or some sort of phonons. Or the plasma particle quasiparticles, that cause scattering and more and more ions suffer scattering and pitch from angle is scattering and initially that you might have large theta. But, then collision with these Plasmon's, there theta can become less than critic in this loss cone angle and they can be lost so, the plasma confinement is very lost.

This is a very serious matter I think in our next lecture, we shall learn about this important property. Before I close I would like to mention, that in a mirror machine, the particle which are confined whether electrons or ions. These particles suffer bounce motion here and bounce motion, if you examine the equation of motion for z component, then it turns out to be  $m \frac{d^2 z}{dt^2}$  is equal to minus  $\mu \frac{\Delta B}{\Delta z}$  and  $\frac{\Delta B}{\Delta z}$  if I model v as  $B_{\text{min}} \left( 1 + \frac{z^2}{L^2} \right)$  multiplied by  $\frac{z}{L^2}$  the model, then it turns out to have the character of simple harmonic motion.

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$$\frac{d^2 z}{dt^2} = -\omega_b^2 z$$

$$\omega_b = \frac{2 v_0}{L} (m_R - 1)^{1/2}$$

So, let me just write down this expression, this equation. It so happens that, this equation takes the form  $m \frac{d^2 z}{dt^2}$  is equal to minus  $\omega_b^2 z$  and bounce frequency of  $\omega_b$  of electron motion turns out to be, twice  $v_0$  upon  $L$  into mirror ratio minus 1 to the power half.

This is quite interesting; the bounce frequency for ions will be small. Because,  $v_0$  is a small for ions it will be large. And this gives rise to oscillations particles move around go back in force and if you impress any external force on the particles in the frequency of the force matches. With this frequency, you can expect some sort of a resonance, but I think those details will not discuss in these in the course of in this course. But there are certain interesting features of this motion also; I think I will stop at this point. Thank you.