Plasma Physics

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Module No. # 01

Lecture No. # 21

Plasma Physics Grad B and Curvature Drifts

In this lecture we will continue discussing the electron and ion motion in a magnetic field. But, today we will permit the magnetic field to be inhomogeneous and curved.

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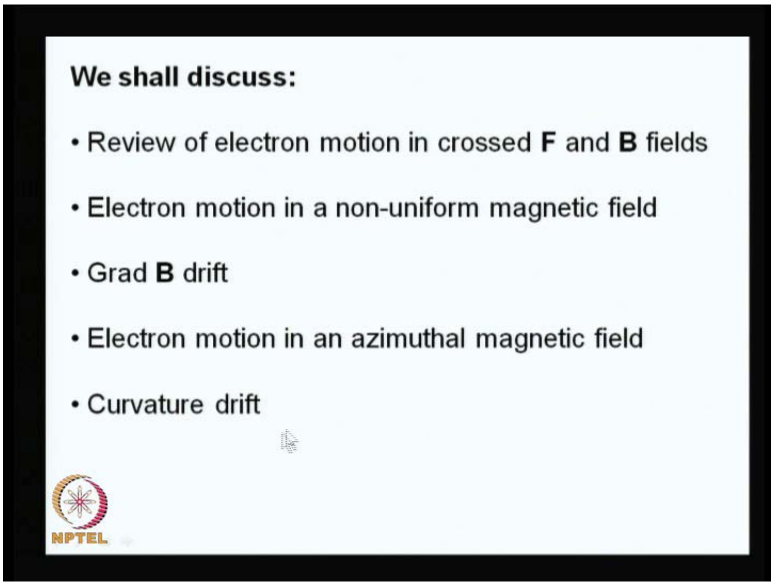

We shall discuss the electron motion in crossed F and B fields that we were talking about in the last lecture. Then we will talk about electron motion in a non uniform magnetic field with parallel lines of force and introduce the grad B drift. Then we will go over to discuss the electron motion in an azimuthally magnetic field and introduce a quantity called curvature drift and then if time permits I will discuss some relevance of these drifts.

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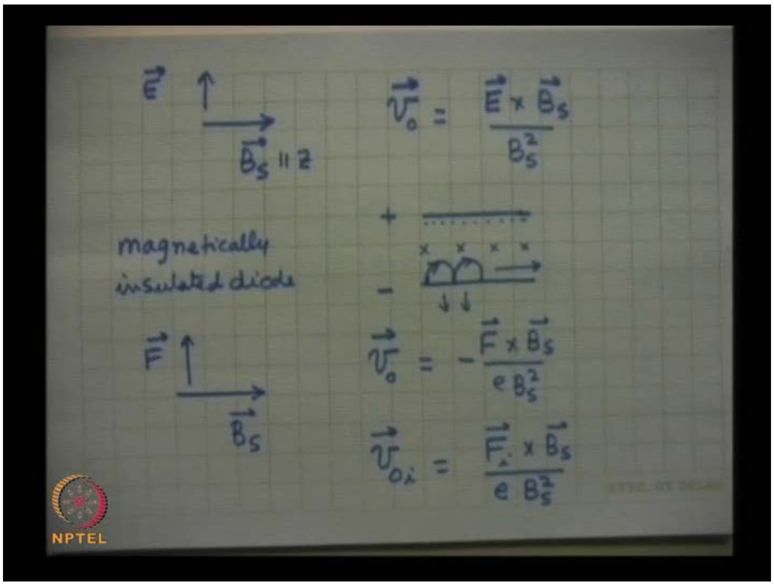

Well, we were talking about the crossed electric and magnetic fields in plasma. If we have a d c magnetic field here like in the z direction and electric field in this direction in the x direction E. In that case we found that the electrons and ions will move together in a plasma perpendicular to E and B s both and this drift was found to be v 0 the drift due to these d c fields having a d c component which was E cross B s upon B s square.

A typical example of relevance of this geometry is not only plasma but, is a non plasma system also called magnetically insulated diode. A magnetically insulated diode such a diode would have a cathode with a negative potential and an anode with a large positive potential.

These sorts of diodes are essentially used to produce ion beams. What do you have if you have created a small plasma here near the anode and if have apply a potential difference of very large value like potential difference of the order of 1 million volt the gap between these two may be of the order of few millimeters. Then what you want that if there are holes in the cathode this ions should be pulled by the negative potential and they should get out here. So, the ions from plasma should be pulled by the cathode they should get out.

But the electrons which are produced near the cathode surface by a field emission they should not be able to reach their otherwise the electron beam lighter and mass they can carry more current and major part of the power is then going not in the ions. But, in the electrons which is a bad thing. We do not want it. So, for production of energetic ions you want the ion acceleration electrons not to be accelerated. Then what you people do they apply a magnetic field perpendicular to the plane of this board.

So, the electrons which are emitted from here they want to go from here towards the positive potential. But, because of the v cross B force they acquire a drift and they move like this. So, the electrons they just go like this in this direction and their velocity is given by this.

So, they cannot reach the positive electrode and hence there is no power loss to electrons. That is the beauty here. So, to stop the electrons flow you apply each perpendicular rather the magnetic field perpendicular to the electric field this is the very important configuration.

But, I will not go into the detail of this device. In a plasma you have electrons and ions both and sometimes you are encountered with the generation of electric field in the system by charge separation or something and sometimes you are encounters with ome other forces and we discuss that if the system as a magnetic field here say B s and a force perpendicular to magnetic field here F. Then the electrons acquire a drift v 0 which is equal to F cross B s upon e B s square with a negative sign this the electron drift.

And the ion drift was v 0 i the ion drift was d c drift component of the ion was force on the ions cross B s upon charge of the ion magnitude wise into B s square. So, these 2 quantities are very important in the case of gravitational field for instance.

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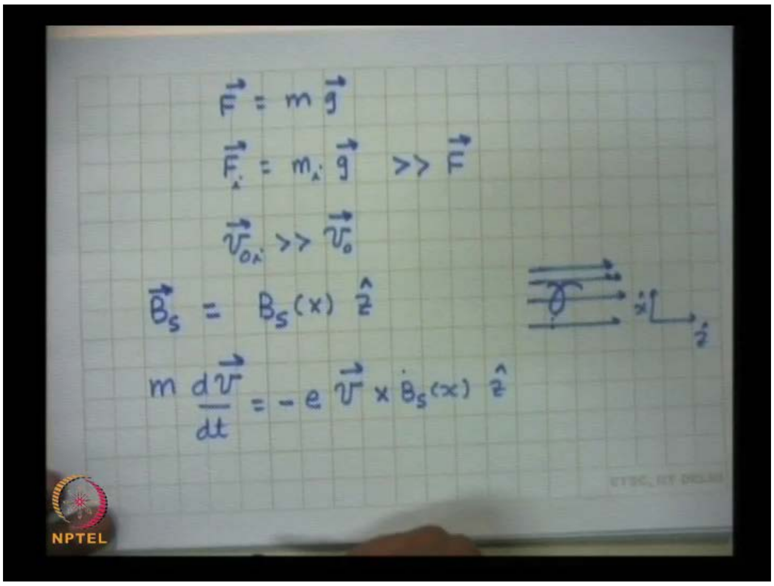

In the ionosphere the plasma electrons and ions experience a downward gravitational field g. But, the force due to the gravity on electrons is F is equal to mass of the electron into g where F force on the ions is mass of the ion into gravitational field g. g is the acceleration into gravity and you may note that this is much bigger than F and in that case the ion drift is much bigger than the electron drift. That is a important thing.

That if plasma is having only a gravitational field then this is the ion drift which is significant and that can cause instabilities like Rayleigh Taylor instability that we talked about earlier.

But today I would like to see suppose there is no gravitational field, no electric field, is it possible that the curvature in the magnetic field acts like some external force or in homogeneity in the lines of force magnetic field also as the same effect as a transverse force. Let us examine this issue.

So, I would like to consider the case when my esthetic magnetic field B s spell to z axis but, magnitude of B s depends on suppose x and so, the direction of my esthetic magnetic field is z. But, I am permitting the magnetic field magnitude to have a x dependence. Now let us examine what will happen.

I have to solve the equation of motion m d v by d t is equal to minus the electron charge electron velocity cross B s. Now this B s is a function of X. This is the serious issue. x depends on time because the force on the particle which is at some instance look at a position x will be given by this at a later instance of time when x exchanges for the particle position changes force will change because B s depends on x.

So, this becomes a serious issue such kind of magnetic field I can demonstrate like this suppose I have lines of force further from each other then become closer to each then closer to each other then this is the situation where the magnetic field magnitude is increasing with x this is my z direction this is my x direction.

So, this is the situation where lines of force. Normally a density of lines of force per unit area is plotted for proportional to the magnitude of magnetic field. So, if the lines of force are closer here it means B field is larger there.

In such a situation if the electron rotates about the any line of force, suppose electron is rotating like this then in some part of the orbit it is seeing a lower value of magnetic field and in Some part of the orbit it is seeing a larger magnetic field. So, the magnetic field value on the electron orbit is changing.

However, if the change in magnetic field is very gradual then the change in magnetic field in two different parts of the orbit will be different will be a small and in that limit I can solve this equation iteratively. Means I say that suppose my magnetic field were uniform I solve the equation of motion then I say that let me permit the magnetic field to have some slow x dependence and what is the consequence of that. So, what I am doing first I consider the electron motion about certain guardian center x g by g when treating B s to be uniform.

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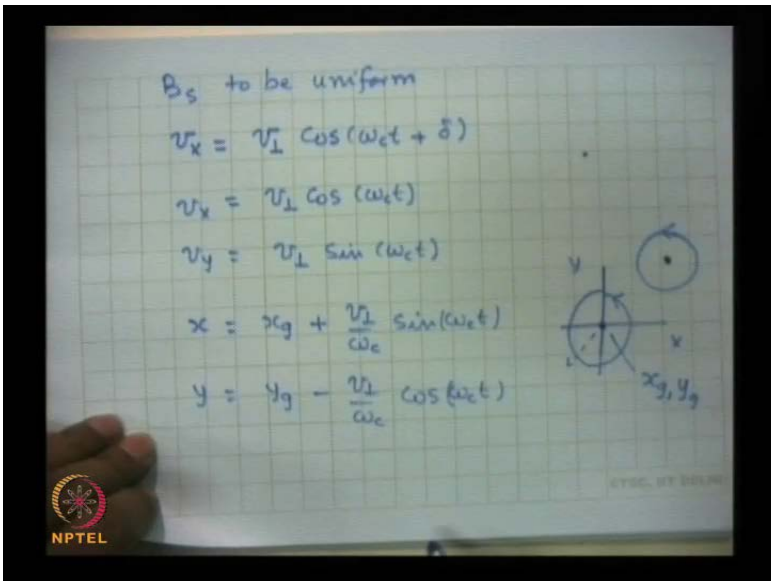

So, if I choose B s to be uniform and then that case we know that v X that we obtained earlier is equal to some perpendicular velocity of the electron into Cos omega c t plus delta for the sake of simplicity and without any loss of generality I will take delta to be 0 by choosing a proper origin of time.

So, I simply write down this is equal to v perp Cos omega c t. Just by choosing proper origin of time this is my v X and v y is equal to v perp sin omega c t and putting this is equal to d x by d t and integrating i get x is equal to x g plus v perp upon omega c sin omega c t and y is equal to y g minus v perp upon omega c Cos omega c t.

These are the solutions of equation of motion that we had discussed in our last lecture on taking delta equal to 0 delta is the initial phase of e x at time t equal to 0.

You may note here that this is a equation of a circle in x y plane with x g y g as the center x g y g are called coordinates of the guiding center. So, in x y plane the electron orbit was like this if x is plotted here y is here and z axis is perpendicular plane of the board like this. Then the electrons were rotating like this in this sense this electron motion.

If this is the center of guiding, center is actually it is not origin this is x g y g this is the otherwise well you have to would have been here in the electron motion will be like this. So, about the guiding center the electrons are rotating like this.

My point is that if I expand my magnetic field around the guiding center because this is region where the magnetic field is relevant. If, the electron is rotating here then the magnetic field in the vicinity of this electron orbit is important. If, electrons rotating here then this is magnetic field here is important. If, it is rotating here then the magnetic field value at this point is important in this region is important.

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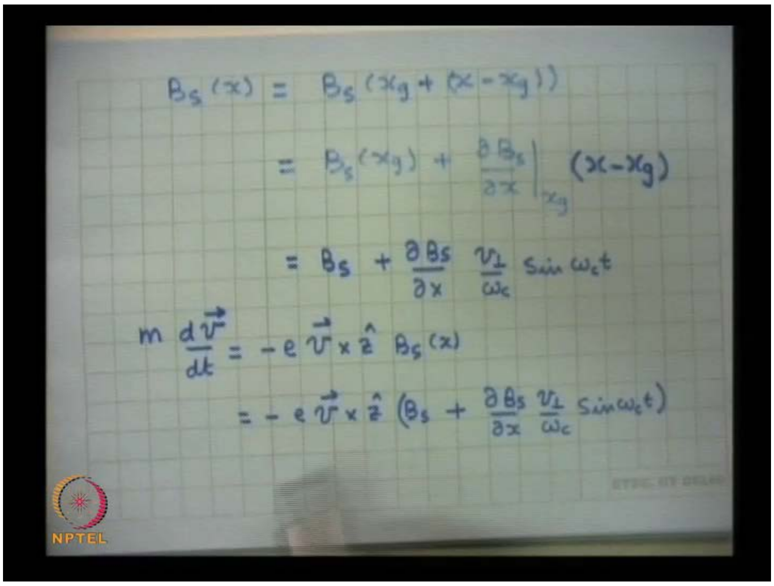

So, what we say that we will permit B s to have a x dependence and expand this about the guiding center. So, B s at x can be written as B s at guiding center x g plus x minus x g same thing x g x g will cancel out. So, this is but, x minus x g from the earlier expression that I showed to you for particle trajectory. This is B s at x g plus delta B s delta B s by delta x evaluated at x g into x minus x g x minus x g.

If i substitute the value of x minus x g I get this is equal to B s implied at x g. So, wont explicitly write this x g. So, imply it plus delta B s upon delta x into x minus x g. If, I put the value it is equal to v perp upon omega c sin omega c t. This is the value of the magnetic field as seen by a gyrating electron. At different times the value will be different. This is a constant but, this is the only time dependence.

So, in different times the field will be either higher than B s and or less than B s where B s is the value of magnetic field at the center of the gyration point. Now let us substitute this in the equation of motion.

Your equation of motion is m d v by d t is equal to minus e v cross z cap into B s B s at x i want to write. So, I will write Some of these 2 terms it becomes minus e v cross z cap into B s plus delta B s upon delta x v perp upon omega c sin omega c t this what we get.

I am going to write the components of this equation. Let me write down the x component recognizing that this is a scalar quantity. So, you have to worry about the x component of this then y component of this. Let us see.

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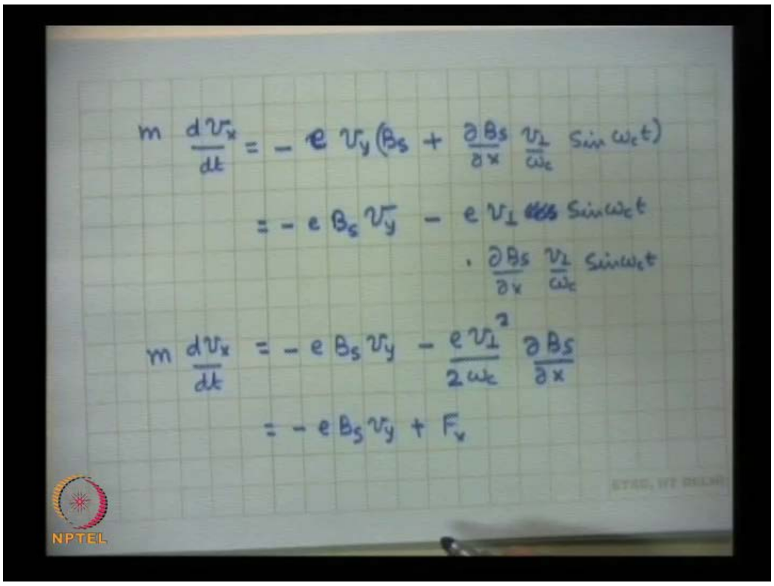

I get m d v X by d t is equal to minus charge of the electron into v y B s plus delta B s upon delta x v perp upon omega c sin omega c t. Now this v y into B s will be all write but, the second term is a small term because this is a perturbed term small term. I am considering change in magnetic field rate of change of magnetic field to be small. So, this is a small term. So, in here the value of v y that I will substitute is the 0th order value of v y I will substitute. So, I can write down this equation as minus e B s as such into v y let me remain keep this term as such.

Second term is important which is equal to minus e v y i had written as v perp Cos sorry v y was not Cos was Sin omega c t. This was the value of v y then multiplied by this term delta B s by delta x into v perp upon omega c into sin omega c t. If you multiply these 2 terms they give you sin square omega c t and sin square omega c t. You can write down in terms of 1 minus Cos 2 omega c t. So, they give you d c term plus a c term.

D c term is my concerned because of the d c force particles acquire a net drift and hence I will written only the d c part of this term which turns out to be equal to minus e B s v y minus e v perp square upon omega c sin square omega c t. As I mention to you is time average is half so, 2 will be there into delta B s upon delta x and this m d v X by d t.

You can treat this something like minus e e or this whole quality something like F X force in the x direction. So, this is equal to minus e v s v y minus some force in the x plus some force in the x directions. Additional force that is appearing there esthetic force though there is no electric field or gravitational field but, the in homogeneity in magnetic field manifest it is in such a way that it appears like as if the electrons are experiencing in net d c force which is perpendicular to the d c magnetic field whether is z direction and this force is in the x direction time average force is finite d c force is finite.

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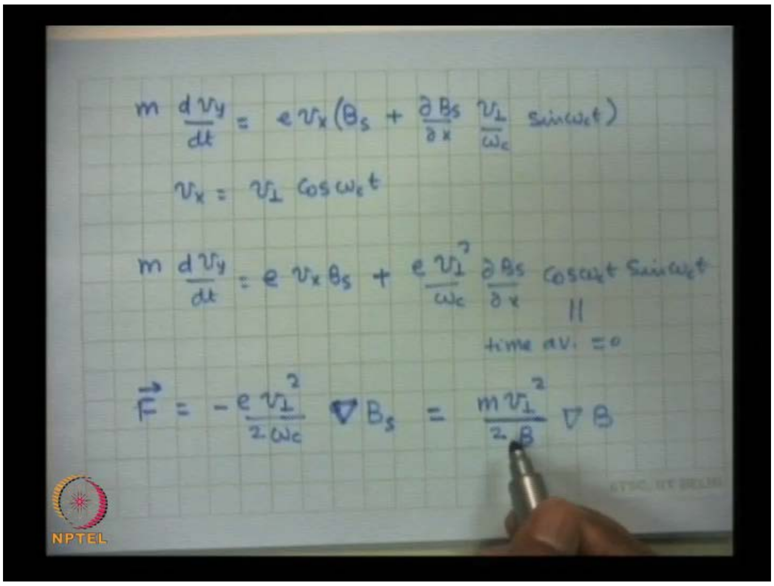

If you write down the y component of this equation of motion you obtain m d v y by d t is equal to e v X into B s which is equal to B s at the guiding center x g plus delta B s upon delta x into v perp upon omega c into sin omega c t. Since v X is equal to v perp Cos omega c t from your ever 0th order solution when magnetic field was uniform.

In that case, if I substitute this in the second term of this bracket then, my equation will become m d v y by d t is equal to e into v X into B s. This term if you get as if there is no even if there is no inhomogenity in the magnetic field plus additional term if I put this here it will become plus e v perp square by omega c delta B s by delta x into Cos omega c t into sin omega c t. But, you know if you average it out over time it will become 0 So, time average is 0. So, there is no d c part here because this is sin 2 omega c t divide by half by divide by 2 and sin omega c t 2 omega c t. When you average over time period a cyclotron period this will become 0. So, this does not contribute means inhomogenity in the magnetic field. If it is in the x direction then exerts a force in the x direction if there was a inhomogenity in the y direction then it will exert a force in the y direction.

So, the net force the magnetic field as really exerts is in matter of form I can write down this is equal to minus e v perp square by 2 omega c into del grad B. If magnetic field is in the x direction then this is simply delta x of B. In general so, this we have generalize this and put value of v omega c as e v upon mass. So, it becomes m v perp square upon twice B s into grad B.

In many books subscript s is suppressed. If you want to suppress you can do this then this is the force on the electron due to inhomogenity in magnetic field. This is a time average force. It is proportional to mass of the particle. Velocity square is independent of charge. Some force ions will experience and what is going to happen because of this there will acquire a drift as I just mentioned.

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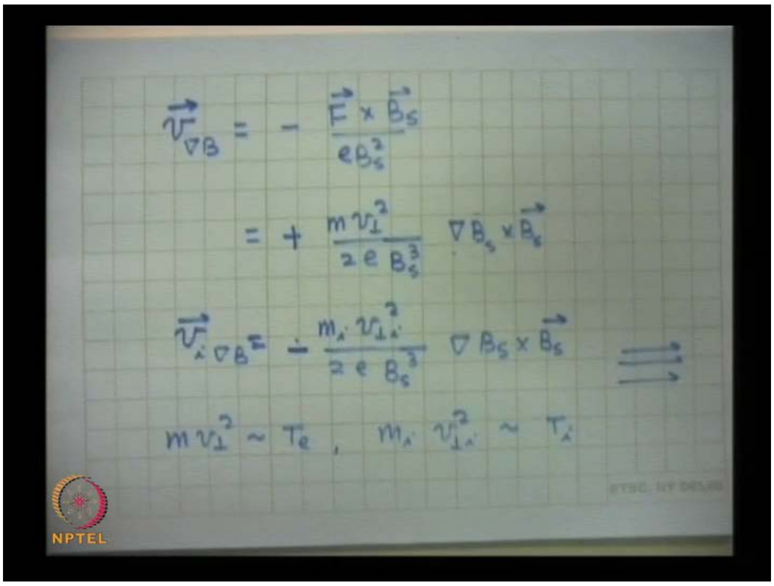

The electron drift is and this is called grad B drift. So, we put a subscript here which is equal to minus F cross B s. Upon B s square, I will suppress this subscript B s and what you get here is into charge of the electron was also there. So, this equal to minus m v perp square upon twice e and then one this is B cube actually or B s cube may be you can put B s cube grad B cross B. This is the kind of force that the just a velocity due to gradient in magnetic field electrons will acquire this is perpendicular to gradient in magnetic field and d c magnetic field.

If the magnetic field as a variation in x direction then this force is in the y direction and how about the ions. So, ion grad B drift would be replace charge of the electron by ion charge and mass by ion mass. Then you will get minus becomes plus. So, m i v perp i square upon twice e B s cube gradient of B s cross B s vector. These 2 drifts are in opposite directions and if it is a maxwellian plasma then the if you take the average drifts over all particles then m v perp square is like temperature.

So m v perp square like temperature of the electrons and m i v perp i square is like ion temperature in with bold man constant observed in T e or T i. Then you may note if the plasmas have equal or comparable T e and T i then these 2 are equal. So, electrons go in a one direction and ions go in a different direction. As I showed to you that if my lines of force were like these then the electrons will move may be perpendicular to the plane of this paper and ions will go downward and that is very dangerous because if electrons and ions move in the different directions then they will create a space charges separation and a space charge field is created. That is space charge filed which is created in this case in the vertical direction will at create a e cross B drift of electrons and ions in the same directions and push the particles away. So, if there is a electric field produced perpendicular to this when the electrons e cross B drift and ion e cross B drift will be in the x direction and plasma will be pushed out.

So, electrons and as most will be pushed out. So, that is a very dangerous thing and this inhomogenity in magnetic field you cannot avoid it certainly there. Whenever after all you have to create a plasma and you do not want after for production of magnetic field, you have to have a current and if you want to produce magnetic field over a very large region then you require very large currents and that is very expensive.

So, if you want to confine a plasma it is preferable to produce magnetic fields only in that region where plasma is required, in the vicinity of that. Not everywhere. So, whenever magnetic field is finite in Somewhere and small elsewhere then there is a gradient in magnetic field and one should find the way out to avoid plasma motion under this grad B drift. So, this is a important thing that I wanted to mention.

I think I made a little mistake here. From the last page when I borrowed this grad B drift this expressions all right. When I put the value of F i made a mistake in the previous last step. This sign is positive and this sign is negative. Please make this correction. I am Sorry the grad B drift on electrons is given by this expression with a positive sign and on the ions with the negative sign. This difference is important. Now I would like to go over to discuss a another scenario when the lines of force are curved what happens?

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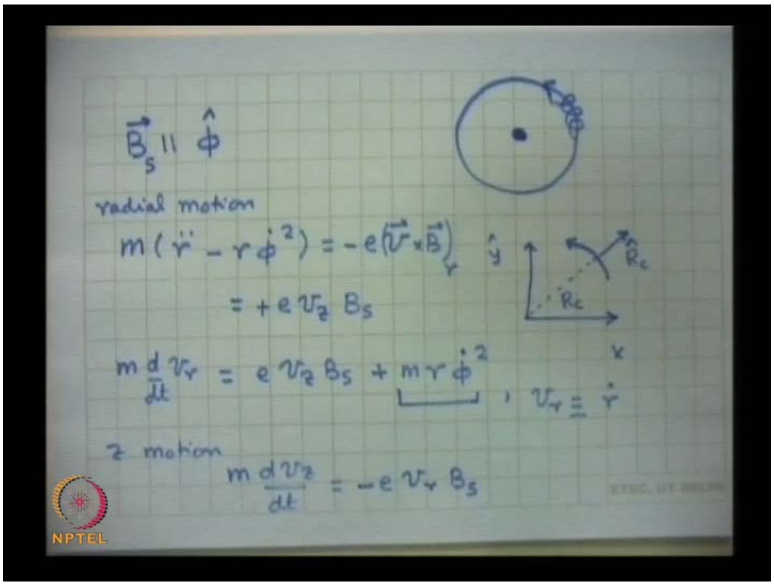

You know I mention to you that, if you want to confine a plasma, one possibility is that you have lines of force which are closed because then there is a possibility. That means a simplest possible notion will tell you that if you have a lines of force then the particles which are gyrating over the lines of force like this when the gyrate about the lines of force and move probably the distribute the line of force and come back.

But grad B drift we have already seen because, if you have a current carrying wire perpendicular to the plane of the board then, there will be a drift of electrons perpendicular to the plane in one direction and of ions in the opposite direction. But, what about the curvature is it going to do Something? Let me examine the role of a curvature.

To understand this issue, I will consider without a special case of magnetic field suppose magnetic field is parallel to azimuthally direction phi. Means I am considering a cylindrical polar coordinate system. Suppose, this is my x axis this is my y axis and z axis is perpendicular to the plane of the screen.

Let me consider a line of force which is going like this in the phi direction and the radius of curvature is this. The value of R c and the direction normal to the line of force is unit vector R c cap. So, I want to find out if my line of force is a curvature with a radius of curvature R c. What is the equation of motion for the electron there is no other field just only a magnetic field. So, this is a esthetic magnetic field I will put a subscript s.

My equation of motion would be mass I will write down components of equation of motion. So, it is radial equation of motion radial motion m r double dot. If r is the cylindrical polar coordinate of an electron then the radial acceleration is r double dot minus r phi dot square. This is equal to radial force which is minus e v cross B s and i must write down the radial component of this force.

Let me write down this B is in the phi direction, v has all x Sorry radial azimuthal and a z components. So, I want radial component then I should use here v phi or v z and when you multiply this turns out to be equal to minus e and this will give you v z and v phi with the sign positive. So, this is what you get for the radial equation of motion and what I can do I can take this term on the right hand side.

So, if I take this term on the right hand side this equation I can write down this as m and r double dot as d d t of v r. If I call r dot is equal to v r then this becomes is equal to e v z B s plus m r phi dot square. This is the additional term that you get there where I defined v r is equal to r dot this is my definition.

Similarly, if I write down the z component of motion then m d v z by d t this quantity is equal to z components of this force z component will be r here and phi there. So, minus e v r and B phi which is B s. Please look at these 2 equations. We consider the case of crossed F and B earlier this is like a addition a force transfers to magnetic field which is the phi direction. This is radial directions you can keep this like a radial force because radial equation of motion.

So, what will you get here is that the presence that the curvature in the line of force and I am considering the particle motion at a distance r from the centre and at this point the line of force I am considering to have radius curvature r.

So, this is primarily r is the radius of curvature the line of force then your electron is experiencing a net force due to curvature in the magnetic field and besides that if magnetic field as a dependence on coordinates then that will appear through B s also. That effect is different. Inhomogenity in magnetic field appear through this B s but, besides that because of curvature of the line of force you are getting a additional force here.

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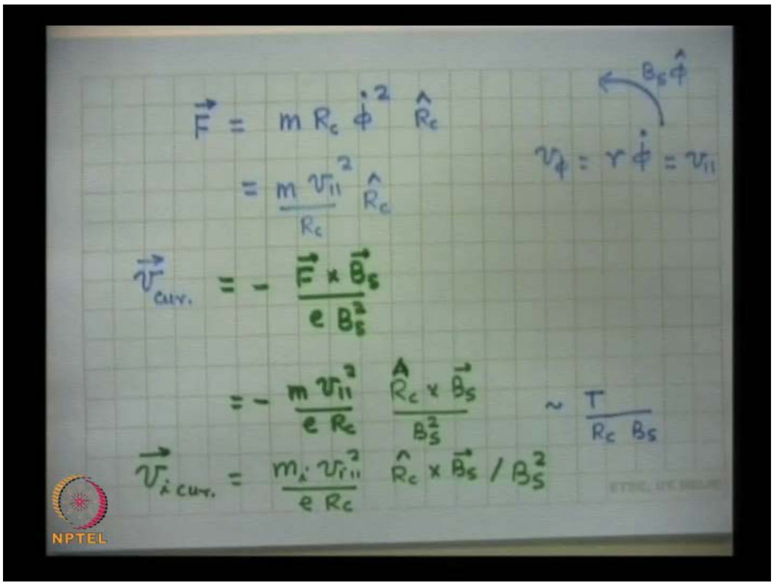

So, let me write down this force vector form. Electrons experience a force F which is in the direction of radial direction which is direction R c. m R c phi dot square multiplied by R c cap. If the electrons are rotating in that direction now let me write down please understand we are having a system in which lines of force are rotating. So, this is the phi direction. B s is the direction in the direction phi. So, this is primarily the and the electron velocity v phi is r phi dot. This is the azimuthal velocity which I call actually as v parallel because this is parallel to magnetic field. So, v phi is usually denoted by symbol v parallel because there is a direction of magnetic field, parallel refers to the direction parallel to magnetic field. So, if I do this then I can write down this quantity as m v parallel square divided by R c.

So if I phi dot if I put equal to v parallel by R and R is capital R c. Then this is the expression I get and the direction is along the radius of curvature and if force is known to me I can write down the electron drift due to curvature. Let me write down this as minus F cross magnetic field upon e B s square.

So, if I put the value of F here this is minus m v parallel square upon e R c. Then R c cap cross B s upon B s square this is the curvature drift of electrons. For ions it will be of opposite sign and let me write down for v i curvature which will be equal to m i v i parallel square upon e R c R c cap. Actually this is cap here cross B s upon B s square.

Since, m v parallel square is related to temperature of the electron and m i v i parallel square is related to ion temperature, these two drifts are also comparable when the 2 electrons temperatures are comparable.

If you have a toroidal magnetic field produced by either a current carrying wire or toroidal winding on torus. In that case these two drifts the curvature drift and grad v drifts are comparable and they cause particle motion particle velocity in the same direction. If you really look into a realistic situation then curvature drift and grad B drift are the same direction for electrons and same direction for ions. But, ion drift is in one direction electron drift is in the opposite direction and they both give rise to charge separation and charge separation produces electric field and creation of electric field is always a dangerous thing because that gives rise to e cross B drift of plasma and plasma particles move away from the vessel and electron ions move away together that is a dangerous thing.

So, we have noted one thing in here that these drifts are proportional to temperature. Obviously, in some plasmas there is temperature analyze trope the average kinetic energy of particles along the field lines is different or temperature in along field lines is different. Then perpendicular to filed lines in that case these drifts could be different but, if you take isotropic plasma then these drifts are comparable. So, this is proportional to temperature, inversely proportional to radius of curvature of the lines of force and inversely proportional to magnetic field. This is also the same way.

So, these drifts are stronger in hotter plasmas and obviously they can be suppressed with their radius of curvature is huge or magnetic field is quite a strong very strong but, as I mentioned that production of magnetic field is quite expensive and hence one should achromise the production of magnetic fields. So, one should find some ways to avoid these drifts.

Well we will discuss in the search for an equilibrium to counter these drifts came the proposal for tokomak. I think we will have exclusively separate lecture for tokomak confinement in a mirror machine which is also very fantastic thing a very interesting device though not much in fashion in these days but, I would like to mention.

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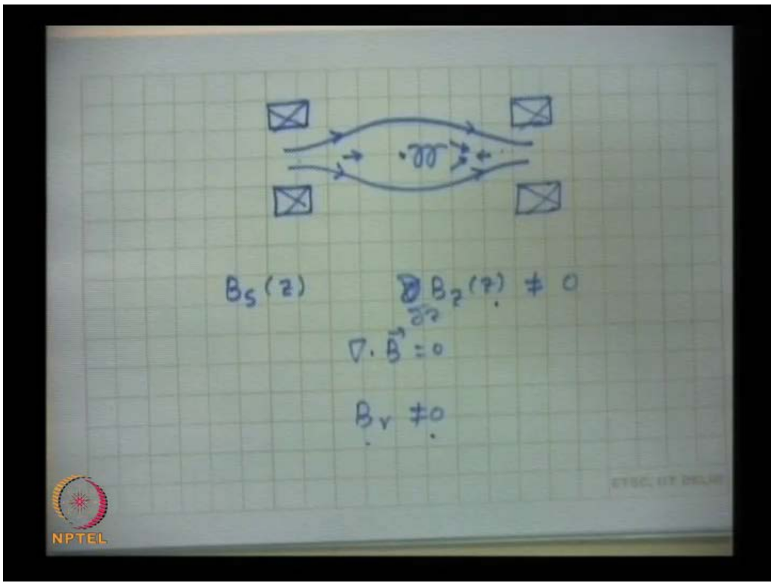

In a mirror machine what do you have in a mirror machine you have a some sort of a tangential coil a field coil. So, if you have a field coil with winding like this circular winding then this will produce a magnetic field which is large here and the filed falls off in this direction. Then we have another field coil here, same field coil similar field coil circular coil. Then what you do you encounter you produce a magnetic field whose lines of force go like this.

So, this is simplest configuration of magnetic field that you have magnetic field is large here, small here, large there. This is a situation where magnetic field in the actual region a has B z finite and this depends on z.

So, this is a inhomogeneous magnetic field along the field lines but, what happens whenever the magnetic field has a B z a function of z since divergence of B has to be 0.

So, whenever this is grad B z or delta B z is non 0, this field must have a radial component of B field.

So, B r turns out to be non 0 in the plasma may be 0 in the axis. But, elsewhere is finite then there is a interesting thing that if I have a charge particle here which is rotating over the line of force like this. But, the lines suppose are curved inclined because of the radial component like suppose they are inclined like these lines of force then if the electron is moving in the phi direction radial electric field is at the B r component then, the v cross B force will have a finite z component and it so happens that the force is in the backward direction that is a very interesting situation.

So, whenever you have a magnetic field in mirror machine a function of z B r turns out to be finite and that exerts a force on particles for them to return back towards the middle of the machine and that situation is very helpful for plasma confinement.

So, you can find a plasma in a magnetic field through the inclinational lines of force. If the lines of force are parallel then the plasma will move away along the line of force but, when the lines of force are coming close through finite curvature then they have a tendency to pull the plasma back into the center of the machine and that is an interesting situation.

We will introduce this machine we will talk about this machine exclusively in one lecture and we shall learn that the grad B can cause reflection of particles or mirror reflection of particles. We shall discuss this but, before that I would like to introduce a constant of motion called magnetic moment as a constant of motion and then we shall discuss this device. So, in our next lecture we shall do that.

So, today we have studied two important things number one that whenever there is a inhomogenity in magnetic field in a direction perpendicular to field itself in that case electrons and ions acquire a drift. The drifts are comparable in magnitude but, opposite in direction for electrons and ions and secondly when the lines of force have a finite curvature then also the electrons and ions acquire drifts of comparable of equal magnitude but in opposite directions and on any particle the curvature drift and grad B drift usually or may be often act the same direction.

Hence for a confinement of a plasma its mandatory that we should find ways to avoid these drifts or counter balance these drifts so that a space charge separation does not occur in the plasma.

One thing finally I would like to mention that people often sight similarity in the curvature drift and what we call as the g cross B drift whenever there is a curved curvature in the line of force the force after all F that I have mention to you appears and that forces time independent and is very similar to gravitational force.

The only difference is that the gravitational drift on electrons was much smaller than that on the ions but, the curvature drift is comparable on electrons and ions but, the same kind of instabilities like the instability Rayleigh Taylor instability which was driven by g cross B drift. That instability can be driven by curvature drift and that is a very important instability in mirror machine and other devices. I think I would like to stop at this point and next time we come over to discuss mirror machine. Thank you.