

**Plasma Physics**  
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**Module No. # 01**

**Lecture No. # 20**


**Single Particle Motion in Static Magnetic and Electric Fields**

Well today I would like to begin our discussion on plasma confinement, which is a very serious issue because plasma is a hot state of matter with very large pressure. And it has a tendency to expand so, if you want to confine their plasma you have to think of some non traditional ways. And in that context I would like to discuss single particle motion in a static magnetic field. Today we will consider uniform magnetic field and also, if there is a transverse electric field present in the system we would like to examine that also.

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**We shall discuss:**

- Need for plasma confinement
- Electron motion in uniform magnetic field
- Electron motion in crossed electric and magnetic fields
- $\mathbf{E} \times \mathbf{B}$  drift
- Magnetically insulated diode

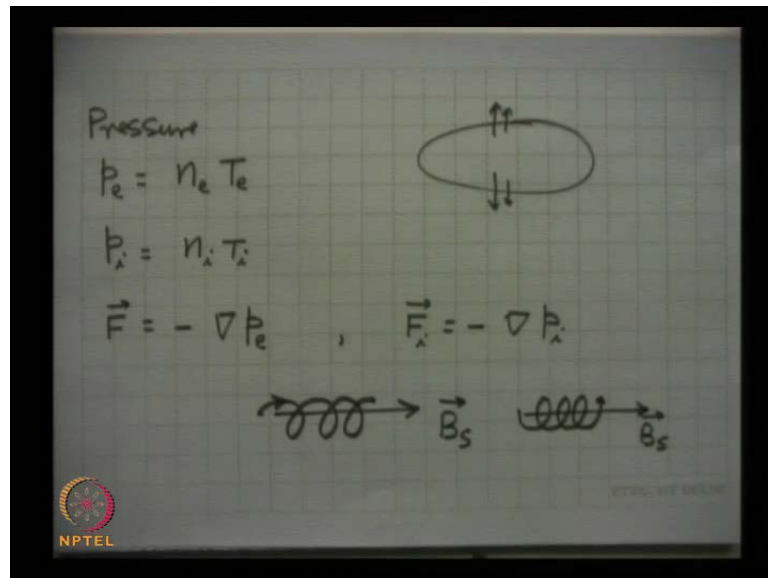


$\mathbf{E} \times \mathbf{B}$  drift

So, basically we will be discussing what the needs for plasma confinement are, and then we will discuss electron motion in uniform magnetic field, electron motion in crossed electric and magnetic fields that will give rise to  $\mathbf{E}$  cross  $\mathbf{B}$  drift. And then one of the major applications of this drift is in magnetically insulated diode. We shall learn what is

this diode. And then there is a  $\mathbf{g} \times \mathbf{B}$  drift due to gravity or gravitation field being perpendicular to magnetic field that also, causes  $\mathbf{g} \times \mathbf{B}$  drift.

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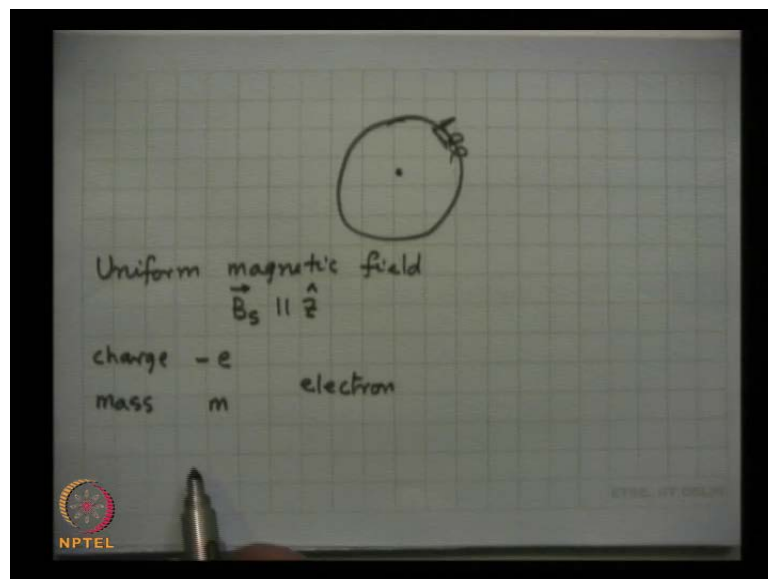
Well, I would like to begin with the need for plasma confinement. Suppose, I have a plasma confined produce somewhere here. Then this plasma would have a pressure partial pressure for electrons would be  $p_e$ ; which I can write down as a product of density of electrons and temperature of electrons, where Boltzmann constant is hidden in temperature. Similarly, for ions I can have a partial pressure  $p_i$  which is the product of ion density and ion temperature. Usually,  $n_e$  is equal to  $n_i$  but,  $T_e$  and  $T_i$  are different. What will happen that if I produce that plasma somewhere then the plasma electrons will experience a force, which will be equal to minus gradient of pressure.

And similarly, the ions will also, experience a force, which will be equal to minus gradient of ion pressure as a consequence of which the plasma will try to move electrons will go in these directions ions will also follow them. So way, because pressure is higher here less outside because plasma particles are not there. So, plasma will try to expand. If you want to stop this process of expansion you should compensate for these forces and that is a serious issue. Before we delve into the issue of how to compensate for it we would like to physically explore the possibilities that, is it possible that by the application of a magnetic field, can you stop the motion of plasma from high pressure region to low pressure region. That clue that we get from here from the particle motion the magnetic

field is that suppose, I have a magnetic field somewhere a static magnetic field as  $B_s$ . And if I have a fully ionized plasma, then if I leave an electron somewhere with some finite velocity, then it will try to move along the field lines like this rotates so, the plasma is the electron is confined across the line of force. It cannot move perpendicular to line of force too much it will just gyrate about the line of force and but, it can freely move along the line of force.

So, at least a uniform magnetic field appears to provide a confinement for electrons in the transverse direction or in two dimension transverse dimensions. But it provides no confinement along the magnetic field if it is uniform. Similarly, for the ions if you launch an ion the ion motion will be in the anti clockwise sense we shall learn about it in little while, and this goes like this. This is ion motion in the magnetic field which is also, localized about the line of force. So, at least in two dimensions two transverse dimensions you expect that the electrons are ions will be localized. Obviously a central theme central issue would be is it to confine the particles along the field lines. One possible scenario is that you can have a close line of force like this.

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This kind of line of force you can produce by having a current carrying wire here. Suppose, there is a current carrying wire carrying current perpendicular to the plane of this in this direction away from the screen towards you. Then the line of force will be like this so, it is curve line of force, then newly one can visualize that if the electron is

gyrating of the line of force like this. If it is going like this, then maybe it will keep on moving along the lines of force, well they will have the gyration about the line of force. And it will be coming back from here to here, but the serious problem is that the line of force is curved.

There is a curvature on the line of force it is not a straight line of force so, we have must examine whether this curvature on the line of force causes some departure from this circular motion. Another thing is that this magnetic field due to a wire current carrying stage wire is non uniform it is higher closer to wire. And you as you move away from the wire the field amplitude decreases. So, the issue arises whether this non uniformity in the magnetic field magnitude will cause some departure in particle orbit. And may be is it that this non uniformity and curvature in the magnetic field causes the electrons to leave the line of force and move away.

If it happens then there is a very serious matter so, a basic problem in magnetic confinement is to understand electron and ion motion in magnetic fields. But, before we take up the problem of electron motion or ion motion in non uniform and curved magnetic fields we must examine the particle motion in a uniform magnetic field in some detail. So, I will consider a uniform magnetic field for the sake of specificity. I will choose the z axis to be parallel to magnetic field. So, I choose my static magnetic field along z direction.

And let me characterize my electrons by charge minus  $e$  and mass small  $m$  here also, the electrons. So, I am trying to understand a single particle motion in a uniform magnetic field which is applied along z direction charge of the electron is minus  $e$  and mass is  $m$ .

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$$m \frac{d\vec{v}}{dt} = -e\vec{v} \times \vec{B}_s, \quad \omega_c = \frac{eB_s}{m}$$
$$\frac{d\vec{v}}{dt} = -\omega_c \vec{v} \times \hat{z}$$

x comp:  $\frac{dv_x}{dt} = -\omega_c v_y$

y comp:  $\frac{dv_y}{dt} = \omega_c v_x$

$$\frac{d^2v_x}{dt^2} = -\omega_c^2 v_x$$

Let us write down the equation of motion further electron rate of change of momentum and  $m \frac{d\vec{v}}{dt}$  is equal to charge into  $\vec{v} \times \vec{B}$  the magnetic force on the electron. I divide this equation by mass and define a quantity  $\omega_c$ , which is equal to  $eB_s$  magnitude upon  $m$ . I will call this quantity as  $\omega_c$  and we shall recognize that this is the frequency of rotation of electrons around the line of force. We shall learn this in a little while. So, let me cause this quantity  $eB_s$  magnitude by  $m$  as  $\omega_c$  then, this equation becomes on dividing by  $v$   $m \frac{d\vec{v}}{dt}$  is equal to minus  $\omega_c$  into  $\vec{v} \times \hat{z}$  because  $\vec{B}_s$  is in the  $z$  direction.

Now let me write down the components of this equation,  $x$  component will give me  $\frac{dv_x}{dt}$  is equal to minus  $\omega_c$  and  $x$  component of this which turns out to be equal to  $v_y$ . The  $y$  component of this equation will give me  $\frac{dv_y}{dt}$  is equal to  $\omega_c v_x$ . These two equations are coupled  $v_x$  equation has  $v_y$   $v_y$  equation has  $v_x$ . However, you can decouple them by differentiating either of the two equations once before to time. I will differentiate this equations  $(( ))$  to time then I will obtain  $\frac{d^2v_x}{dt^2}$  is equal to minus  $\omega_c^2 v_x$ . If I use this equation becomes  $v \omega_c^2 v_x$  this is a equation  $v_x$  itself and it resembles simple harmonic oscillator equation and it have the simple solution.

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$$v_x = A \cos(\omega_c t + \delta)$$

$$v_y = A \sin(\omega_c t + \delta)$$

$$v_x^2 + v_y^2 = A^2 \equiv v_{\perp}^2$$

$$v_x = v_{\perp} \cos(\omega_c t + \delta)$$

$$v_y = v_{\perp} \sin(\omega_c t + \delta)$$

$$m \frac{dv^2}{dt} = -e(\vec{v} \times \vec{B}_s)_z = 0$$

RCP

The simple solution of this equation is  $v_x$  is equal to some constant into  $\cos \omega_c t$  plus constant  $\delta$ . So, there are two constants of integration it was a second order differential equation. And hence will, have in general two constant of motion and a  $\delta$  they can take any values depending on the initial conditions.  $\delta$  can be take into be 0 by proper choice of time the origin of time where do you choose then you can take  $\delta$  to be 0 and if I use this equation this value of  $v_x$  in the  $y$  component of equation of motion. You obtain  $v_y$  is equal to a  $\sin \omega_c t$  plus  $\delta$  one can do one thing in here. If you square and add these two components you will, find that  $v_x^2 + v_y^2$  is equal to a square and this is a constant of motion.

And what is  $v_x^2 + v_y^2$   $v_x$  is the perpendicular component of velocity in the  $x$  direction and  $v_y$  is the component of velocity in the  $y$  direction. These two together when square and added are denoted by a symbol  $v_{\perp}^2$  and this is a constant of motion. So, if I call  $a$  is equal to  $v_{\perp}$  then these equations can be written as  $v_x$  is equal to  $v_{\perp} \cos \omega_c t + \delta$  and  $v_y$  is equal to  $v_{\perp} \sin \omega_c t + \delta$ . One can note one thing in here that, if I plot a graph with  $v_x$  on the  $x$  axis and  $v_y$  on the  $y$  axis what do we get at certain instant of time when this argument is 0.  $v_x$  is equal to  $v_{\perp}$ .

Suppose this is a value  $v_{\perp}$  from here to here and at that instant of time  $v_y$  will be 0 because if the argument is 0 this is 0 then  $\sin 0$  is 0 so this is point is located here. At a

later time when this argument time increases this quantity will, acquire some finite value and  $\cos$  will, decrease from its value unity to some smaller value and this will increase so, the point will be somewhere here. Later in time when this quantity becomes equal to  $\pi/2$  this is also  $\pi/2$ .  $v_x$  becomes 0  $v_y$  becomes  $v_{\text{perp}}$ . So, we are coming to a point here means if you put points at different institutes of time you will, draw equation of a circle obviously.

This is the equation of a circle and you are moving in this direction as time goes on. This is a motion of the particle velocity in the anti clockwise sense. But, the beauty here is that if I take a right handed screw my electron is rotating in the right hand sense like this and if I rotate my right handed screw along the direction of velocity change. This is the arrow in which the velocity curve is evolving. In that case the direction of advancement of the screw will be the z axis and that is the direction of magnetic field. Means if I see in the direction of magnetic field then the electron orbit will look like moving in this way.

This is the electron orbit just like, if I advance rotate a right handed screw like this then the direction of advancement of this screw will be the direction of magnetic field. So, I will call this motion as R C P right circularly polarized motion. So, the electrons in a magnetic field gyrate about the line of force in the right handed sense. Just like a right handed screw moves and how about the z component of equation of motion because, the z component of equation of motion, if you examine  $m \frac{d v_z}{d t}$ , which is equal to minus  $e \mathbf{v} \times \mathbf{B}$  this is the force. I want z components since  $\mathbf{B}$  is in the z direction this quantity is 0 this tells you that  $v_z$  is a constant of motion.

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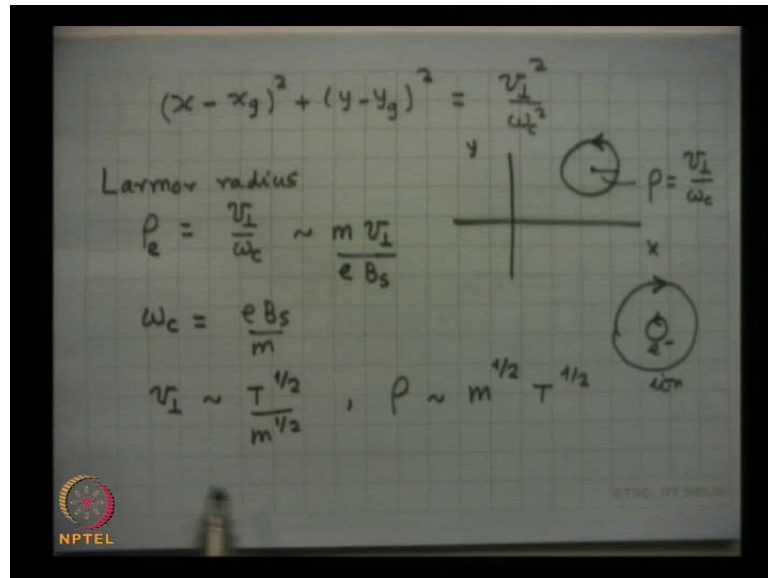
The image shows a whiteboard with handwritten mathematical equations. At the top, it states  $v_z = v_{0z}$  with the note "Constant of motion". Below this, the velocity components are given as  $\frac{dx}{dt} = v_x = v_{\perp} \cos(\omega_c t + \delta)$  and  $\frac{dy}{dt} = v_y = v_{\perp} \sin(\omega_c t + \delta)$ . The corresponding position equations are  $x = x_g + \frac{v_{\perp}}{\omega_c} \sin(\omega_c t + \delta)$  and  $y = y_g - \frac{v_{\perp}}{\omega_c} \cos(\omega_c t + \delta)$ . An NPTEL logo is visible in the bottom left corner of the whiteboard image.

So, let me write this,  $v_z$  is a constant of motion let me call this as  $v_{0z}$  it is a constant of motion. So, what we have learnt that the electron velocity of the line of force is unaffected by the magnetic field it remains as such forever, and the transverse velocity magnitude wise remains constant. But, its direction changes it moves in a circular path in a circular manner. Now I want to find out the actual trajectory. The  $x$  and  $y$  coordinates of the particle to do that I use this definition that  $\frac{dx}{dt}$  is equal to  $v_x$  particle velocity in the  $x$  direction. And which was equal to  $v_{\perp} \cos \omega_c t + \delta$ .

And similarly,  $\frac{dy}{dt}$  which was the  $y$  component of electron velocity equal to  $v_{\perp} \sin \omega_c t + \delta$ . You can easily integrate these equations if I integrate this first equation I will get  $x$  is equal to some constant of integration. If I call  $x_g$  plus  $v_{\perp}$  upon  $\omega_c \sin \omega_c t + \delta$  and the second equation will, give me  $y$  is equal to some constant of integration.  $y_g$  plus this will be minus sorry let me remove this minus  $v_{\perp}$  upon  $\omega_c \cos \omega_c t + \delta$ . There is a beauty in these equations. If I take  $x$  minus  $x_g$  and  $y$  minus  $y_g$  and square them and add.



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I will get  $(x - x_g)^2 + (y - y_g)^2 = \frac{v_{\perp}^2}{\omega_c^2}$ . This  $v_{\perp}$ , please remember is the magnitude of perpendicular velocity where perpendicular refers to is based to magnetic field. This is an equation of a circle whose center is  $x_g, y_g$ . Electron does not reach the center, but the electron's coordinates are  $x$  and  $y$ . So, if I plot a graph here suppose  $x_g, y_g$  being constant of motion could be anywhere suppose this is here I am plotting  $x$  here  $y$  there. This is an equation of a circle like this.

And the radius of the circle is I called as, a  $\rho$  which is equal to  $v_{\perp} / \omega_c$ . So, the electron will move in the clockwise in this, anti clockwise sense on this plane or this is called right circular in R C P way right circular polarized way like this. It is a circle of radius  $\rho$ . Which is the ratio of perpendicular velocity of the electron to  $\omega_c$  and it is called Larmor radius for electrons this  $\rho$  is given use the symbol subscript e.  $\rho_e$  is equal to  $v_{\perp} / \omega_c$  you may avoid this subscript also does not matter. Well this is a quantity that is inversely proportional to cyclotron frequency.

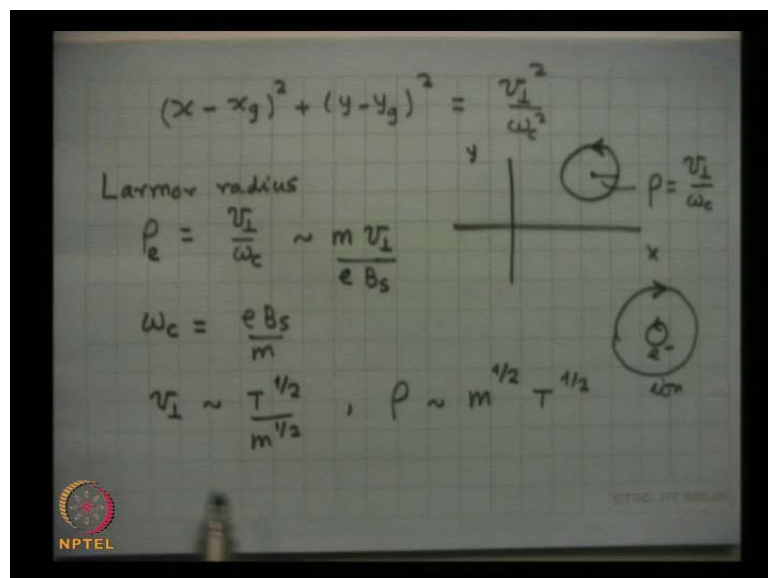
And if you look at the expression for  $\omega_c$  which is equal to  $e B_s / m$  this is how I defined  $\omega_c$  this quantity is inversely proportional to mass. And hence  $\rho_e$  is proportional to mass into perpendicular velocity. So, this goes as mass of the particle into velocity of the particle  $v_{\perp}$  obviously. It will go as inversely as charge and magnetic field basic thing is in here that in a plasma at finite temperature. Electron speed scales as

temperature to the power half. And consequently the larmor radius rho scales as and this is a promotional to m minus half also velocity scales as under root of temperature upon mass.

So, if I put this in here i get at the Lamar radius will be scale as m to the power half and temperature to the power half. This is the scaling this will have charge dependence on end magnetic. Will dependence as well but, as far as the dependence on mass is concerned this goes like this. What I wanted to tell you is that if I had considered the ion motion then the ion will also undergo a gyration about the lines of force. But, the larmor radius will be much bigger because mass is bigger for ions. If the temperature and of ions and electrons are comparable then ions will have a much larger larmor radius.

They will rotate over a very large circle and the electrons. If they are rotating on this small circle ions will rotate over a very large circle and the rotation will be opposite sense we shall learn about this in a little while. This is the ion motion this is the electron motion. So, in magnetic field plasma electrons rotate in the right handed sense about the lines of force or about the magnetic field, ions rotate in the left handed sense with a much larger larmor radius and a word about omega c why it is called as ideal frequency.

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If you look at the expression for x that we had written like, x g plus rho sin omega c t plus delta and y is equal to y g minus rho Cos omega c t plus delta. These are the

coordinates of an electron that are evolve may with time, but you see here that whatever is the value of  $x$  at certain time  $t$  the same value of  $x$  will be repeated at a time  $t$  plus  $2\pi$  by  $\omega_c$  because, if I add  $2\pi$  in this phase then  $\sin \theta$  and  $\sin \theta + 2\pi$  are same  $\cos \theta$  and  $\cos \theta + 2\pi$  are same they are repeated. So, in place of  $t$  if I replace  $t$  by this quantity then  $x$  and  $y$  do not change.

Means that the particle is reaching same starting from some point goes round and comes back to the same point. After a time  $2\pi$  by  $\omega_c$ . So, this is called the time period of rotation of electron  $t$  is equal to  $2\pi$  upon  $\omega_c$  inverse of this period is called frequency in hertz. So, frequency of rotation in hertz is equal to  $1$  upon  $t$  and this can be written as  $\omega_c$  upon  $2\pi$  it is in hertz cycles per second, and if I want to put a radian per second then the frequency of rotation in radian per second is  $\omega_c$ , which is equal to  $2\pi$  by into  $f$  frequency of rotation in of gyration rather in radian per second.

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$x = x_g + \rho \sin(\omega_c t + \delta)$   
 $y = y_g - \rho \cos(\omega_c t + \delta)$   
 $t \rightarrow t + \frac{2\pi}{\omega_c}$

Time period of rotation  $T = \frac{2\pi}{\omega_c}$

$f = \frac{1}{T} = \frac{\omega_c}{2\pi} \text{ Hz}$

$\omega_c = 2\pi f$  freq. of gyration in rad./s

And it is useful to have some estimate of this magnetic field this cyclotron frequency  $\omega_c$  which is equal to  $e B$  upon  $m$  if I choose  $B$  is equal to 1 tesla this is a unit of magnetic field and n k system of units and 1 tesla is  $10^4$  gauss. Gauss is a unit of magnetic field in c g s systems. So, if I choose  $B$  is 1 tesla  $\omega_c$  turns out to be electron charge is  $1.6 \times 10^{-19}$  column  $B$  s e choose one mass of the electron is  $9.1 \times 10^{-31}$  kilogram. Then this is a radian per second. Which is of the order of if I choose this something like  $10$  then this becomes  $30$ .

So, ten if I take here 10 this become 30. 30 when goes up becomes 11, 11. So, this becomes typically 1 point may be 7, 1.7 into 10 to the power 11 radian per second. If you divided this by 2 pi you will, get in hertz 2 pi is like 6. This turns out to be like three typically of the order of 3 3 30 gigahertz. So, when I divide this by 2 pi which is I take like 6, then 1.7 divided by 6 is like, 0.3 into 10 to 11. If I put in terms of gigahertz giga means 10 to the power 9 hertz then this is like 30 gigahertz. So, very interesting estimate for omega c is, at 1 tesla the cyclotron frequency of electrons is 30 gigahertz.

Now, when you are trying to heat plasma by using electromagnetic waves via cyclotron resonance heating then we would like to match the frequency of the wave to electron cyclotron frequency. And then you should be choosing waves of frequencies around 30 gigahertz if the plasma is a magnetic field of about 1 tesla. If the plasma wave has a magnetic field of about 10 tesla like a large tokomak has then you should be aiming at wave frequency about 300 gigahertz. So, that is a thing that you must keep in view that what kind of waves are required, if you want to employ electron cyclotron resonance heating of plasma so omega c is a important parameter.

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The image shows a handwritten calculation on a grid background. The equations are as follows:

$$\omega_c = \frac{e B_s}{m}$$

$$B_s = 1 \text{ Tesla} \quad (10^4 \text{ Gauss})$$

$$\omega_c = \frac{1.6 \times 10^{-19} \times 1}{9.1 \times 10^{-31}} \text{ rad/s}$$

$$\approx 1.7 \times 10^{11} \text{ rad/s}$$

$$\approx 30 \text{ GHz}$$

In the bottom left corner of the grid, there is a small circular logo with the text 'NPTEL' below it.

Well a word about the ion motion the ion motion is governed by this equation mass of the ion into d v by d t for the ion charge of the ion is plus e not minus e. And v cross B s this v is the ion velocity you can put a subscript, I may not subscript does not matter the important difference is the charge sign. So, here I define omega c i as the magnitude of

this charge of the ion if it is simply ionized into B s upon m i, then what happens this equation takes the form d v by d t is equal to omega c i into v cross z cap which is the direction of magnetic field.

Compare this equation with the electron equation of motion which was for the ions in the electrons we had d v by d t is equal to minus omega c into v perp v cross z cap means the solutions of this equation that we had obtained in those solutions, if I replace omega c by minus omega c i should recover the same result the results for ions.

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Ions

$$v_x = v_{\perp} \cos(\omega_{ci} t + \delta)$$

$$v_y = -v_{\perp} \sin(\omega_{ci} t + \delta)$$

$$x = x_g + \frac{v_{\perp}}{\omega_{ci}} \sin(\omega_{ci} t + \delta)$$

$$y = y_g + \frac{v_{\perp}}{\omega_{ci}} \cos(\omega_{ci} t + \delta)$$

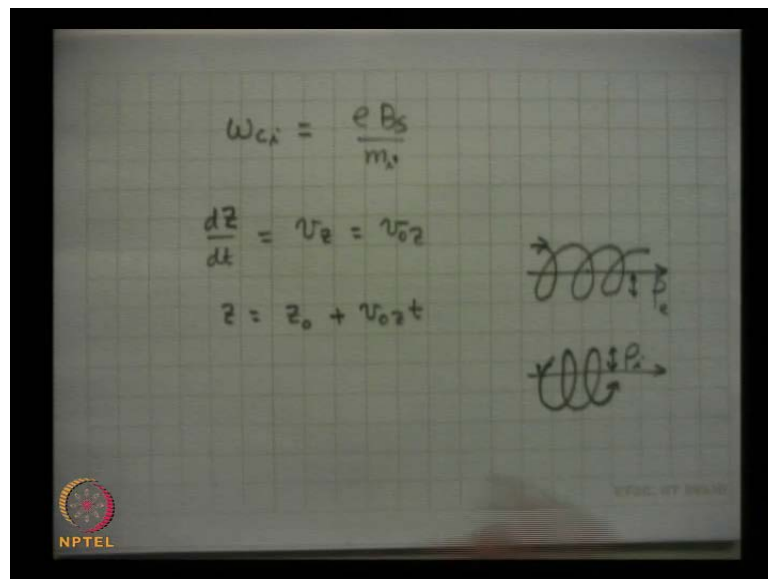
$$\rho_i = \frac{v_{\perp}}{\omega_{ci}} = \frac{m_i v_{\perp}}{e B_s} \sim m_i \frac{v_{\perp}^2}{T_e}$$

And when you do this you obtain for the velocity of ions so, for ions i obtain velocity is equal to v perp of the ion, into if I take Cos omega c t plus some constant to delta this delta may not be the same as the original delta. But it is a constant of motion this is the sorry this is I am writing v x remove this arrow from here v x is so much. Then v y turns out to be minus v perp this is i let me put a subscript i here omega c i t plus delta so, ions have a cyclotron frequency much less than the electron cyclotron frequency, and the sense of rotation is also different. If I plot a graph here with v x on the x axis for the ions and v y on the y axis, then the ion suppose initially this argument is 0 v x is equal to v perp v i is 0, but later on at subsequent times this point will move. The locus will move on a circle in this way so, it will go in this sense in this clockwise sense. Electron was in the anti clockwise sense. So, the rotation of ions is different; similarly, if you write down the x coordinate it will be x g and then it was plus v perp upon omega c i, it was if I write

this Cos just a second let me see if I put this  $\frac{dx}{dt}$  then this turns out to be so, much into Cos oh sorry this becomes  $\sin \omega_{ci} t + \delta$  and  $y$  is equal to  $y_g + v_{\perp}$  upon  $\omega_{ci}$  Cos  $\omega_{ci} t + \delta$ .

So, if I plot  $x$  here  $x$  coordinates ion here and  $y$  coordinate here this is my point  $x_g y_g$  called the guiding center. And then the ion rotation will be like this the radius of rotation would be  $\rho_i$  and  $\rho_i$ . I will define as the ion perpendicular velocity upon  $\omega_{ci}$ . This quantity will scale as because mass of the ion into perpendicular velocity of the ion obviously upon charge of the ion magnitude into  $B_s$  this is the exactly equal. And if, I consider my plasma to be maxwellian then  $v_{\perp}$  is related to ion temperature and ion mass. So, this scales as mass to the power half and ion temperature to the power half. So, ion larmor it is certainly is very large. So, there are two important differences in electron motion ion motion ion cyclotron frequency is very low because  $\omega_{ci}$ .

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Depends on ion mass inversely so, if you want to heat ions by using ion cyclotron resonance by electromagnetic waves. Then you need waves of much lower frequencies that is a very important thing because, ion mass is even for a proton plasma if you have is about 2000 times bigger  $\omega_{ci}$  is 2000 times bigger than the electron mass. So, you require a wave frequency 2000 times a smaller, if you are having deuterium tritium plasma. Then you are having two kinds of ions where their cyclotron frequencies

obviously different.  $\Omega_c$  for deuterium is different than  $\omega_c$  for tritium and they are 4000 or 6000 times smaller than the electron cyclotron frequency.

So, whenever you aim at heating the ions you should aim at having electromagnetic waves or electrostatic waves of frequency is close to  $\omega_c$ . Second thing is that the ion Larmor radius is much bigger than the electron Larmor radius. Well this is primarily the motion in a uniform magnetic field which is unbounded in the  $z$  direction so,  $z = z_0 + v_{z0} t$  if I write down it is equal to  $v_z$  which is a constant equal to  $v_{z0}$  that we have seen. And if we integrate this gives you  $z$  is equal to some constant initial value of  $z$  coordinate of the particle plus  $v_{z0} t$ . So, the particle will keep on moving in the  $z$  direction whereas, we will start keep on rotating about the guiding center. So, in the  $x-y$  plane the particle motion is a circle whereas, it has to move in the  $z$  direction.

So, if I have a line of force like this then the electron which is if it has no velocity  $v_{z0}$  velocity then will also rotate in a circle but, because of this  $z$  this will keep on moving like this. So, this is how it will move this is the electron motion ion motion will be of this form this will be like this. Well the ion will be moving slower than the electron, but the ion Larmor rate is much bigger I have by mistake. I have drawn this to be of same comparable size but the electron Larmor radius is much smaller than the ion Larmor radius may be a 50 times or 100 times smaller than this radius from here to here this is called Larmor radius  $\rho_i$ . Larmor radius is here is  $\rho_e$  electron Larmor radius ions move with the larger Larmor radius.

Well when we encounter magnetic fields which are curved or inhomogeneous then the motion of equation of motion becomes difficult to solve. However, if we understand the particle motion in crossed field means suppose, there is a magnetic field along  $z$  direction. And if you apply an electric field or a gravitational field or any other force perpendicular to magnetic field and if you can understand the dynamics of electron and ion in these two fields. Then based on our understanding of this problem we can understand the particle motion and curved and inhomogeneous magnetic fields very easily.


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$\vec{E} \parallel \hat{x}$   
 $\vec{B}_s \parallel \hat{z}$       Crossed electric and magnetic fields

Electron Motion  
 $m \frac{d\vec{v}}{dt} = -e\vec{E} - e\vec{v} \times \vec{B}_s$

x Comp:  $m \frac{dv_x}{dt} = -e(E_x + v_y B_s)$

y Comp:  $m \frac{dv_y}{dt} = +e v_x B_s$



So, before I delve into the particle motion and curved fields curved magnetic fields. I would like to consider the case when there is a electric field in the system suppose, this is parallel to x axis and there is a magnetic field in the system which is in the z direction these are called crossed electric in magnetic fields and magnetic fields. I would like to delve into this problem both I will consider to be electrons to be a static independent of time and uniform equation of motion for electrons. I will write this is mass into acceleration is equal to charge of the electron minus E into the electric field e minus E B cross B s this is the magnetic force this is the electric force in the electron.

Well let me write down this component x and y components x component would give me  $m \frac{dv_x}{dt}$  is equal to minus e E x. And this will give me if I can write minus e if I take common take the x components of this equation this turns out to be plus  $v_y B_s$ . And the y component of equation of motion is mass  $m \frac{dv_y}{dt}$  is equal to there is no y component of electric fields this terms is not there this gives me plus  $E v_x B_s$  you may note here, when there was no E x these two equations were similar we x equations contained  $v_y$ ,  $v_y$  contained  $v_x$  obviously signs were different in these two places.

But, now some additional term is coming in here which is d c term it does not depend on time or position. So, if I can redefine my  $v_y$  say  $v_y$  prime such that these  $(\ )$  can be clubbed together then what I can do.



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$$v_y' = v_y + \frac{E_x}{B_s}$$
$$v_y = v_y' - \frac{E_x}{B_s}$$
$$m \frac{dv_x}{dt} = -e v_y' B_s$$
$$m \frac{dv_y'}{dt} = e v_x B_s$$
$$v_x = A \cos(\omega_c t + \delta)$$
$$v_y' = A \sin(\omega_c t + \delta)$$
$$\omega_c = \frac{e B_s}{m}$$

So, let me define  $v_y'$  such that this is equal to  $v_y$  plus  $E_x$  upon  $B_s$  then so, means or  $v_y$  I am saying put  $v_y$  is equal to some quantity like  $v_y'$  minus  $E_x$  upon  $B_s$  if I define like this. So, if I substitute for  $v_y$  like this in both those equations then the  $x$  component equation becomes  $m \frac{dv_x}{dt}$  is equal to minus  $e v_y' B_s$  and the other equation becomes  $m \frac{dv_y'}{dt}$  is equal to  $e v_x B_s$ . These two equations are exactly same as if there were no electric field the only difference is that  $v_y$  is to be replaced by  $v_y'$ .

And we have already solved these equations earlier. So, I will simply write down the solution the solution turns out to be  $v_x$  is equal to some constant  $A \cos(\omega_c t + \delta)$  and  $v_y'$  is equal to  $A \sin(\omega_c t + \delta)$ . Where  $A$  and  $\delta$  are constants of integration to be evaluated by using the initial conditions and  $\omega_c$  is the same as before, which is charge of the electron magnitude wise into  $B_s$  upon mass of the electron. Now  $v_y'$  is really not the  $y$  component of velocity actual velocity is  $v_y$ . So, I should use this  $v_y'$  in this equation then my  $v_y$  becomes.

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$$v_y = A \sin(\omega_c t + \delta) - \frac{E_x}{B_s}$$
$$v_0 = -\frac{E_x}{B_s} = \left( \frac{\vec{E} \times \vec{B}_s}{B_s^2} \right)_y$$
$$\vec{v}_0 = \frac{\vec{E} \times \vec{B}_s}{B_s^2}$$

So, your  $v_y$  is equal to  $A \sin \omega_c t + \delta - \frac{E_x}{B_s}$  now please note this is a d c term so electron velocity besides having a oscillatory part has a d c drift, it is continuously drifting without changing sign it is continuously drifting in y direction with this velocity minus  $\frac{E_x}{B_s}$ . And if your electric field is in the x direction magnetic field is in the z direction this is your  $B_s$  and this is your  $E_x$  fields, then this velocities in the y direction y direction is z cross x or so, I can really write down this velocity as let me call this is  $v_0$  velocity. So, this  $v_0$  which is equal to minus  $\frac{E_x}{B_s}$  can be really called as  $\frac{\vec{E} \times \vec{B}_s}{B_s^2}$  because x cross z is minus y.

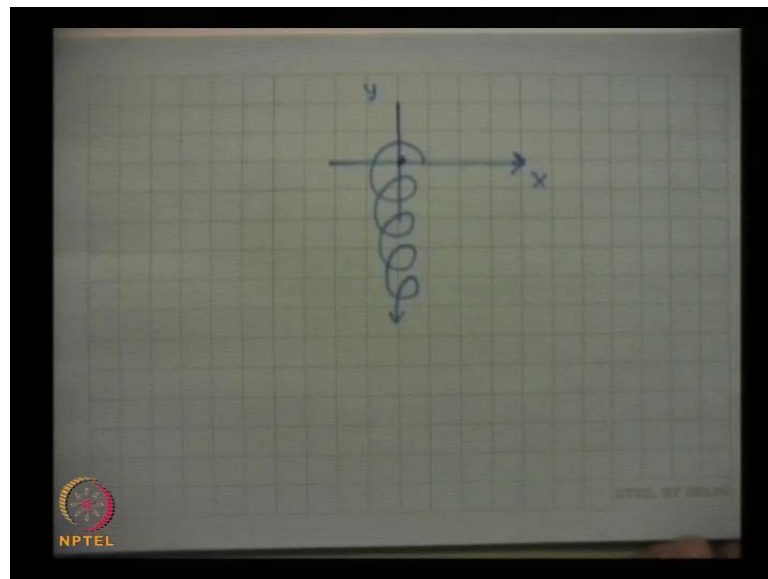
So, this is the same thing and I am writing the y component of this quantity actually there is only y component finite. So, what you are really getting that the presence of a transverse electric field in the system gives rise to a drift d c drift to particles and vector y is, if I had to write simply then the electrons will acquire a drift  $v_0$ , which is equal to  $\frac{\vec{E} \times \vec{B}_s}{B_s^2}$  please note this drift is independent of mass and independent of charge whether they are the electrons or ions they will move in the same direction in the same d c drift. So, if you have plasma somewhere placed somewhere here magnetic field is here and if there is an electric field that you have to applying the transverse direction.

Then besides having the cyclotron motion about the field lines the plasma electrons will drift in the direction perpendicular to E and B both, if my electric field is here then the

plasma electrons though they will be gyrating of the line of force like this, but then they will be drifting themselves up and this is a constant velocity. So, what kind of trajectories I expect. The electron were suppose I think it will be better, if I plot z axis here and y axis is here y axis there then in the x y plane it will be x z y z plane, it will be better to plot my electric field is in the x direction and y cross z in the x. So, this is into the plane of this board into the board that is my direction of electric field.

So, normally I would expect that the electrons they will be having a minus z velocity and if they have a  $v_0 z$  also finite. Then they will be moving z direction also, they will be moving away if electric field were not there in the y z plane, what will happen the electron this is my x direction. Electron will be gyrating in the x y plane like this actually in this way electron will be gyrating like this. So, y will be moving like this but, now besides that they will be moving in this direction, because of  $v_0 z$  velocity and  $v_0 y$  velocity so, the electrons are gyrating away I could have plotted in the x y plane also.

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What will happen in the x y plane suppose this is my x direction y direction the electrons are gyrating about the line of force in this way in this fashion? But, as they go down they will go down like this is what happens. So, the electron is travelling actually with large this radius remains the same this is a round picture these are not a smaller radia. So, the electrons are moving away from the line of force, this line of line of force actually is this

one magnetic field is in z direction perpendicular to the plane of this board so, they are moving like this perpendicular direction.

So, e cross B drift takes electrons away from the line of force which is a very serious matter. One should never try to have a d c electric field created in the plasma by charge particle motion. Otherwise, that will take away electrons and ions both in the same direction, away from the lines of force or magnetic field region to outer region and causes loss of plasma confinement. In general if we had any other sort of force transverse to magnetic field what would happen let me just write down and give you some general expression.

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$$\vec{F} \perp \vec{B}_s$$

$$m \frac{d\vec{v}}{dt} = \vec{F} - e \vec{v} \times \vec{B}_s$$

$$\frac{\vec{B}_s \times (\vec{F} \times \vec{B}_s)}{B_s^2} = \vec{F} - \frac{\vec{B}_s \cdot \vec{B}_s}{B_s^2} \vec{F} - \frac{\vec{B}_s \cdot \vec{F} \cdot \vec{B}_s}{B_s^2}$$

$$-\frac{(\vec{F} \times \vec{B}_s) \times \vec{B}_s}{B_s^2} = \vec{F}$$

Suppose, I had a force F in general direction perpendicular to a static magnetic field then my equation of motion for electron would be m d v by d t is equal to F minus e v cross B s however, F I can certainly write without any lose of generality any vector F if I consider a quantity B s cross F cross B s upon B s square. Suppose, I examine this quantity how much is this we will note that this quantity is F vector into B s dot B s this is identity actually B s dot B s upon B s square minus this B s vector and F dot B s upon B s square.

However I have chosen F and B s to be perpendicular so, this term goes away and B s dot B s is simply B s square. So, this is identically equal to F means I can certainly write

this  $F$  as a cross product of this and this quantity, if I take  $B_s$  on the other side is simply equal to  $F \times B_s \times B_s$  upon  $B_s^2$  with a negative sign. This term I can certainly write like this  $a \times B$  is minus  $B \times a$  this is what I have done now what you can see here, if I put this  $F$  equal to this expression in here minus sign is there minus sign is there.

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$$m \frac{d\vec{v}}{dt} = -e \left[ \vec{v} + \frac{\vec{F} \times \vec{B}_s}{e B_s^2} \right] \times \vec{B}_s$$

$$\vec{v}' = \vec{v} + \frac{\vec{F} \times \vec{B}_s}{e B_s^2}$$

$$m \frac{d\vec{v}'}{dt} = -e \vec{v}' \times \vec{B}_s$$

$$\vec{v}_{drift} = - \frac{\vec{F} \times \vec{B}_s}{e B_s^2}, \quad \vec{v}_{drift} = \frac{\vec{F} \times \vec{B}_s}{e B_s^2}$$

Then my equation becomes  $m \frac{d\vec{v}}{dt}$  is equal to minus  $e$  if I take common then you will get in the interior one of the term will be  $\vec{v}$  plus  $\frac{\vec{F} \times \vec{B}_s}{e B_s^2}$  cross  $\vec{B}_s$  means I can certainly, define a  $\vec{v}'$  quantity is equal to  $\vec{v}$  plus a constant this constant  $\frac{\vec{F} \times \vec{B}_s}{e B_s^2}$  then the structure of this equation becomes  $m \frac{d\vec{v}'}{dt}$  just put  $\vec{v}$  equal to just minus this and differentiate this is a constant independent of time. Then this quantity becomes equal to minus  $e \vec{v}' \times \vec{B}_s$  as, if there is no  $F$  there is no other force except to d c magnetic field and the particle motion. That we studied in uniform magnetic field holds and consequently the solution turns out to be as if this quantity with the negative sign is called drift.

So, d c drift due to the force which is equal to minus  $\frac{\vec{F} \times \vec{B}_s}{e B_s^2}$  and for ions  $v_i$  drift would be  $\frac{\vec{F} \times \vec{B}_s}{q_i B_s^2}$  upon charge of the ion upon  $B_s^2$  this is a important issue this is the force on the electron this is the force on the ion these forces may be different. So, this is a important thing. I think we will be using this expression for drift due to a transverse force in the presence of d c magnetic field and when we discuss

the confinement of plasma or plasma motion in non uniform and curved magnetic fields these expressions will be important well I wanted to discuss with you the relevance of  $\mathbf{e} \times \mathbf{v}$  drift in magnetic lens of a diode probably I will defer this discussion to my next lecture. Thank you very much.