Plasma Physics

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Module No. # 01
Lecture No. # 02
Plasma Response to Fields: Fluid Equations

Well, friends today I am going to talk about plasma response to fields. These could be external fields or self fields produced inside the plasma and the basic formalism would be involving the fluid equations.

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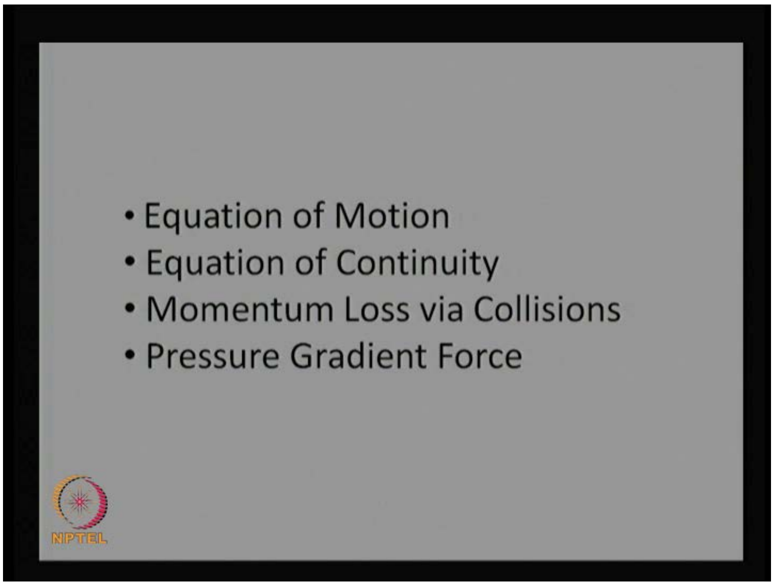

So, today I will give you a brief derivation of equation of motion, equation of continuity and these will involve the concept of momentum loss via collisions and force experienced by particles due to pressure gradient.

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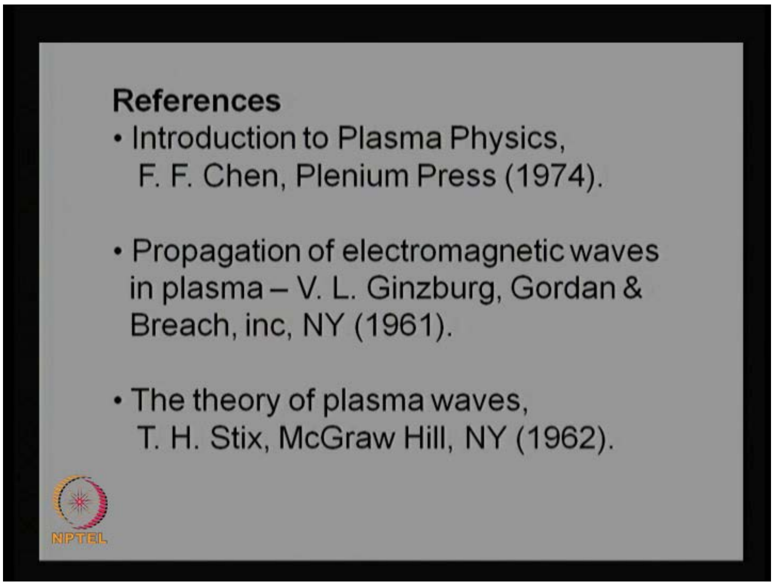

Well, these are the references ; three books one by F F chen the other one by V L ginzburg and the third one is by thomas H stix.

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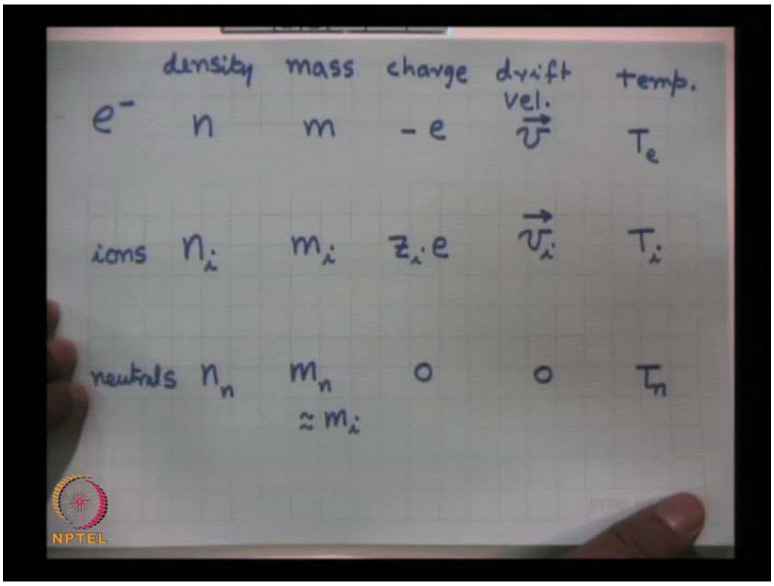

Well, the plasma response; as you know plasma comprises three kinds of species namely the electrons, ions and the neutrals . Neutrals are largely atoms some times in some cases they may be ions molecules also, but we will consider primarily the atoms.

Now, these species are characterized by some microscopic quantities like density then mass of its carrier or particle then charge of the particle, then we talk about the drift velocity and temperature. For electrons, we characterize write these quantities as density as n, mass as m, charge as minus e ,drift velocity as v vector which is the average velocity of particles and temperature is T sometimes we will write as T e.

For ions, density will be denoted as n i ,mass as m i ,charge as z i into e where, z i is called the charge number. For singly ionized ions z i is unity drift velocity as v i and temperature as T i.

For neutrals, the density will be designated as n subscript n , mass as m subscript n which will be nearly equal to m i ; mass of the ion , of the neutrals is 0 , drift velocity often will be 0 sometimes this we finites will refer that later and temperature of the neutrals will be T n.

So, these are the microscopic parameters that characterize a species .The issue is that, when we apply an electric field for instance ; what is the equation that will governed the drift velocity of particles? what is the equation that will governed the evolution of density of particles? these two quantities n and v are characterized or governed by two equations they are known as the equation of continuity and equation of motion.

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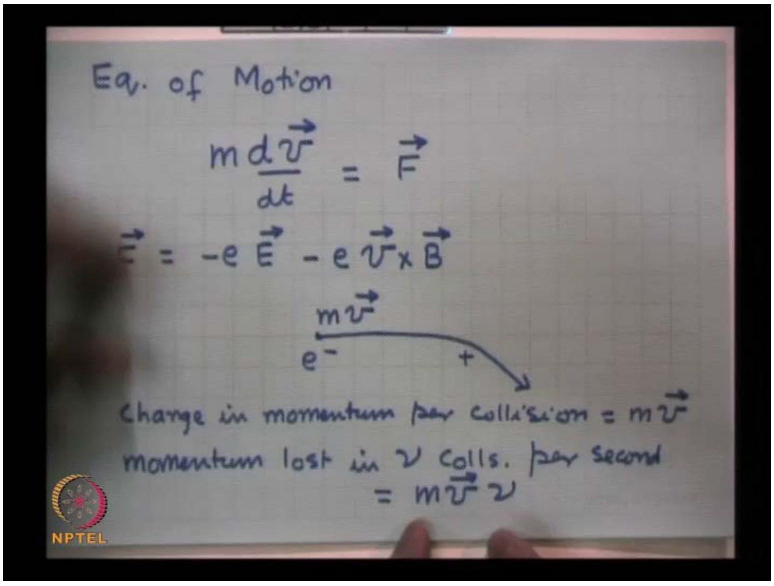

Let me, begin with the equation of motion .Equation of motion, for instance I will write the equation of motion for electrons; which says that rate of change of momentum m d v by d t is equal to the applied force or force experience by the particle.

Now, the force can be of many kinds, if it is ; there is an electric field in the system then an magnetic field in the system then the F would be equal to charge of the electron into the electric field e and there is a magnetic field the force will be minus e the charge into v cross v. So, this is known as the electromagnetic force on the particle on the electron.

Besides this, there is another force on the electron that could be due to collisional drag. The collisional drag, let us understand what is this; When the electrons move the acquire a drift velocity then this on their way they encounter a heavy particle means an ion or a neutral atoms.

For instance, this is a particle here ion, electron is coming from here as it comes close by it experiences the attractive force and sparks as deviated like this. So, the electron which was coming like this after passing through the vicinity of the ion as changes direction of momentum. So there is a loss of momentum or momentum change in each collision ,what we do we say that on an average in a collision momentum is randomized means, in some collision the change of momentum is negligible in some the change of momentum is two m v if electron is coming with a momentum m v then the change in momentum per collision is approximately equal to m v . Change in momentum per collision is equal to m v and if there are new collision per seconds then the momentum lost in collision per second will be nu times m v. So, momentum lost in nu collisions per second this would be equal to m v into nu.

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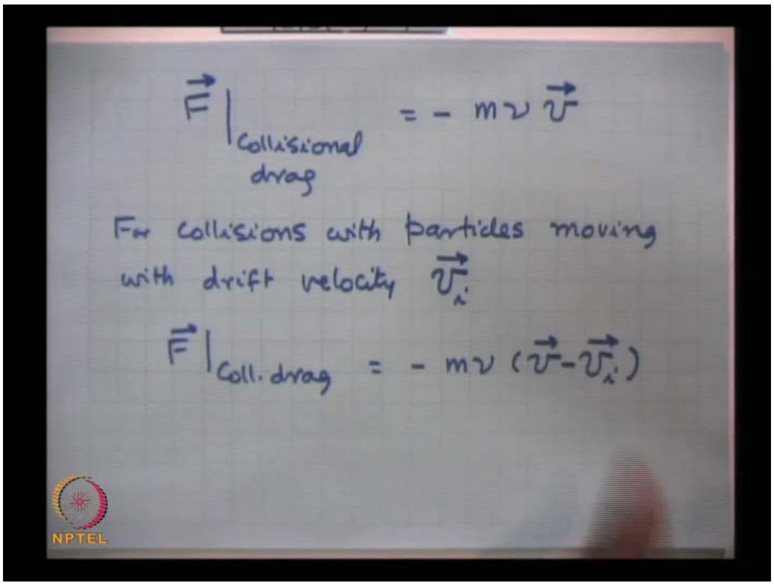

But, this is a loss of momentum not gain of momentum. So, equivalent collisional drag force of electron would be f due to collisional drag, would be equal to minus m nu v this I have written in the assuming that the scatter the ion or neutral is not moving.

In case, there is a drift of the scattering a scatter particle, then this is modified by for collisions with particles moving with drift velocity v i for instance, the F due to collisional drag would be minus m nu into v minus v i. So, this is a important loss mechanism for momentum.

So, electrons gain momentum from electric and magnetic fields, but they lose momentum via collisions to ions and neutral particles.

Usually, v i is a smallest compare to v and we can ignore this v i here, but in some cases this is required. So, unless we are considering a case where ion velocity is significant they will be essentially using this expression m nu v for momentum loss per collision per second.

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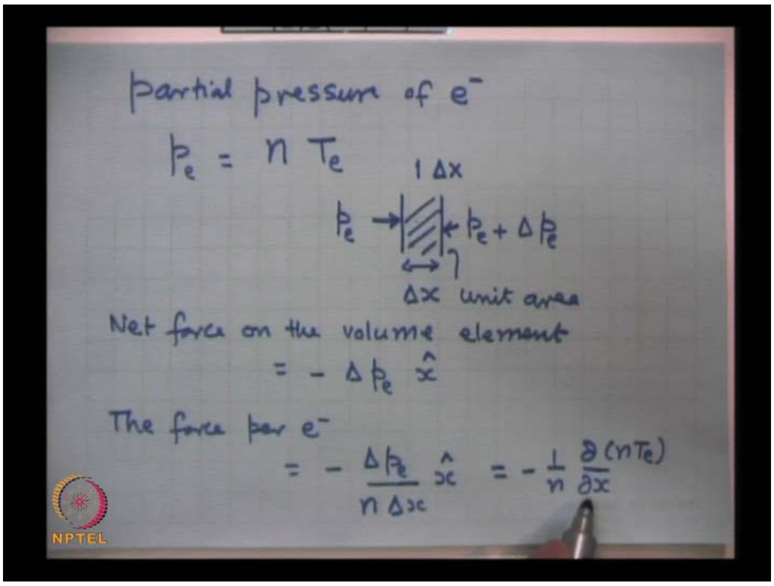

Then, there is another force on the electrons because they have finite temperature and hence they possessed finite partial pressure, partial pressure of electrons is written as product of electron density into temperature of the electrons. Here, boltzmann constant is written in T e its basically, you are familiar with pressure of a maxwellian gas as product of density into boltzmann constant and temperature.

For, electrons density is n and k t v we called k v t ,we call it simply t v this is T temperature and energy units. Now the problem is that, when we apply certain fields on the particles especially the fields of waves the particles acquired drift velocity and the velocity is not uniform everywhere, it may vary from point to point as a result density may change. So, pressure will change the issue is, if there is a non uniformity in pressure I will call this pressure as p e for instance is case filler quantity.

Now, consider a simple case ; suppose, I consider a region of space of unit area of cross section this is unit area and the distance between this is say delta x. So, consider a volume of plasma of unit area of cross section and length delta x and suppose the pressure varies with x. So, pressure on the left is p e and pressure on the right is p e plus delta p e because the pressure is changing.

So, it will be different here then there .Now, what will be the pressure does? pressure means the force exerted by the gas on the left on this. So, this force will be in this direction whereas, the gas or the electron gas on the right hand side will exert a force in the leftward direction. So, the rightward force is p e , leftward force is p e plus delta p e and hence the net force on the volume element is equal to minus delta p e in the x direction, but how many particles are filling this force.

The number of particles that are contained in this volume. If n is the density of electrons in unit volume. Then, the volume of this small space is this region the volume is one into d x. So, the total number of particles here would be density times delta x. So, the force per particle per electron would be then minus delta p e divided by the number of particles in this volume element, which is n times delta x and that is in the x direction.

This is, written as minus 1 up on n delta delta x of p e and p e is n t. So, its n times T e. So, this is an expression for the pressure gradient force expressed by each electron. So, now, we have found typically 4 forces that in electron will experience and I can summarize them

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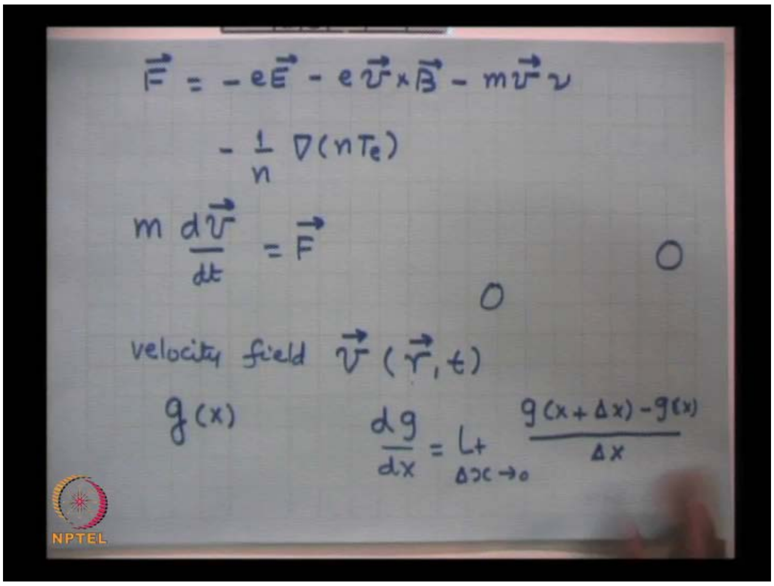

F s the electric force ,the magnetic force v cross v then there is a force due to collision drag which is m v nu and then there is a force due to pressure gradient which is minus 1 upon n gradient of n T e here , I have generalize the pressure gradient force in my derivation . I had presume that the pressure varies only with x, but if pressure varies in general with x y z then the force will be express as a gradient of pressure.

Now, the issue is on the left hand side; the equation of motion we had m d v divided by d t what is d v divided by d t ? let us understand this, please we are dealing here with millions and trillions of electrons in a system in a plasma .There are lot of particles and we are talking about average velocities, but the system may not have the average velocity if you take average velocity of particles in this region in a small volume around this point it could be different. Somewhere here it is different, then if you consider a volume element here. So, number of particles here if you count and measure their velocities and take their average the value of v will be different here then there.

So, rather than specifying the velocity of individual particle we define v as a quantity called velocity field, this means that I can divide my system by plasma into a large number of small volume elements and in each volume there may be large number of particles. So, find their average velocities, but this average will depend on time as well as position.

So, this velocity field is the average velocity of particles look at a position r at time t .If you have a function ,which depends on one variable then you can define that differential coefficient of that function. Simply, suppose there is a function say g ,some function of x then we define d g divided by d x as limit x going to 0 g at x plus delta x delta x going to 0 rather minus g attacks upon delta x this is the definition of differential coefficient of a function of one variable.

Here, we are dealing with velocity field which is a function of r n t r means x y and z. So, three variables and t. So, it is a four variable function. So, here we have to careful about finding the differential coefficient .

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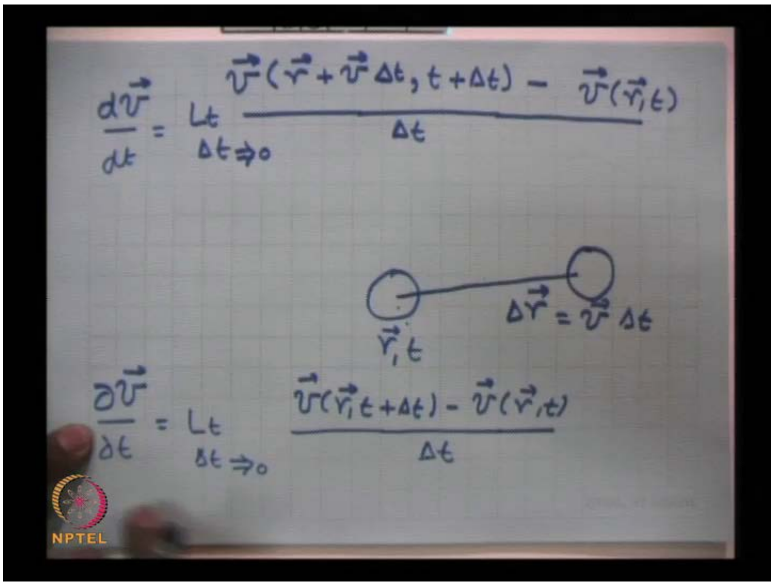

Now, total time derivative implies that if you are obtaining d v divided by d t then what should you do measure the velocity of an electron at r position at time t . Suppose this is my volume element r t look at r t. So, consider the number of particles here, find their average velocity at time t after a while this particles because they are moving with a finite drift velocity. So, they are moved out somewhere here. So, this position would be moved by distance delta r which will be equal to v velocity times delta t.

So, rather than obtaining the velocity at time t plus d t .Here, you measure the velocity at this time at this position. So, find the velocity at position r plus v d t. Because, this is the new position this particles have moved and a time t plus delta t and take the difference of this velocity here from the velocity that was here. So, your eye is fixed on the same particles which were here then they move to the here and divide this by delta t and then take the limit delta t going to 0 tending to 0.

So, this is the definition of total time derivative in contrast if, I had kept my eye fixed at this point; measure the velocity of particle at position r at time t then later at same point at time t plus delta t. I will get a quantity called delta v divided by delta t which is the limit delta t tending to 0.

I am, keeping my eye fixed on the volume element. So, v at r at time t plus delta t minus v at time at position r at time t upon delta t, please understand one thing that my eye is fixed on the volume element not on the particles. When, I calculate the partial time derivative of v. So, these two derivatives are different this is called total time derivative with respect to time this is called partial time derivative of velocity they are not equal they are same.

Now, we can understand that this v is a function of four variables and we can employ taylor expansion to simplify this expression , I will do that.

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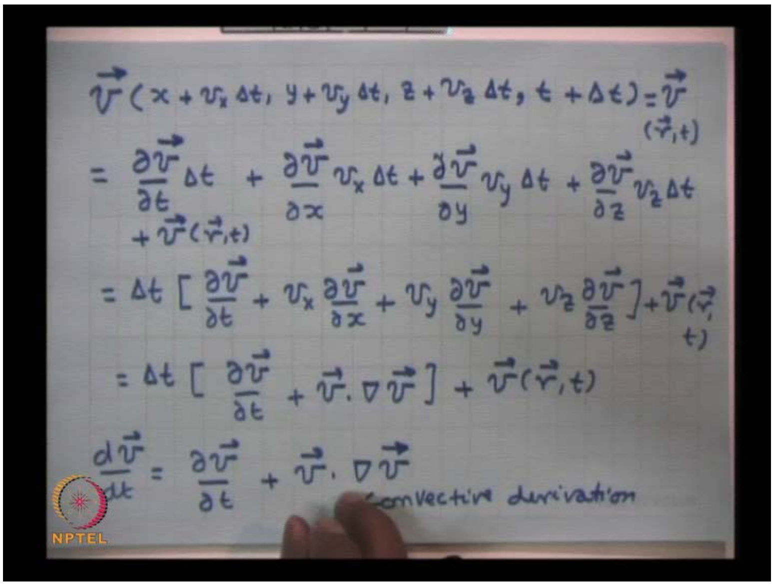

So, I can write simply v as a function of x plus v x delta t, y plus v divided by delta t , z plus v z delta t , t plus delta t this is my v at position r plus v delta t and t plus delta t . I am using the taylor expansion this gives me because this is a function of four variables. So, I will differentiate v with respect to each variable one by one delta v by delta t into change in variable t which by delta t this is the first thing.

Then, I differentiate v with respect to this variable. So, delta v upon delta x into increase in this variable by amount v x delta t. plus thus next variable is y. So, delta v upon delta y v divided by delta t, the last one is variation with respect to z. So, delta v upon delta z into v z delta t. Now delta t is common in all of these. So, I can take it common and I can write this as delta v by delta t plus v x delta v by delta x plus v y delta v upon delta y plus v z delta v upon delta z.

Now, delta delta x ,delta delta y and delta delta z are three components of del operator. So, I can write this in a simpler way as gradient delta t multiplied by delta v divided by delta t plus v multiply del and v and if I use this in my definition of d v divided by d t I obtain d v by d t is equal to delta v divided by delta t because, I have to divide this sorry I forgot one thing is very important.

This is the change in v, but I should write this a term here , I should have added a term here, which is v at r and t the initial value . I forgot to write that, I am sorry with that. So, just add this term here plus v at r and t. So, plus v at r and t plus v at r and t

So, when you substitute this in the definition of d v divided by d t is turns to be delta v by delta t plus v dot del v. So, there is a connection between total time derivative and partial time derivative, this is a additional term that comes over here it is called convective derivative this term is finite, only when velocity or its average velocity depends on position. If you apply a uniform field in to the all electrons, in the plasma and if v does not depend on position then; obviously, this can be zero.

So, whenever your force that is producing the average velocity is space dependence then this term may be important and one as to be careful one cannot ignore it. So, well this is the total time derivative of velocity and now I will use this in the equation of motion and write the equation of motion like this.

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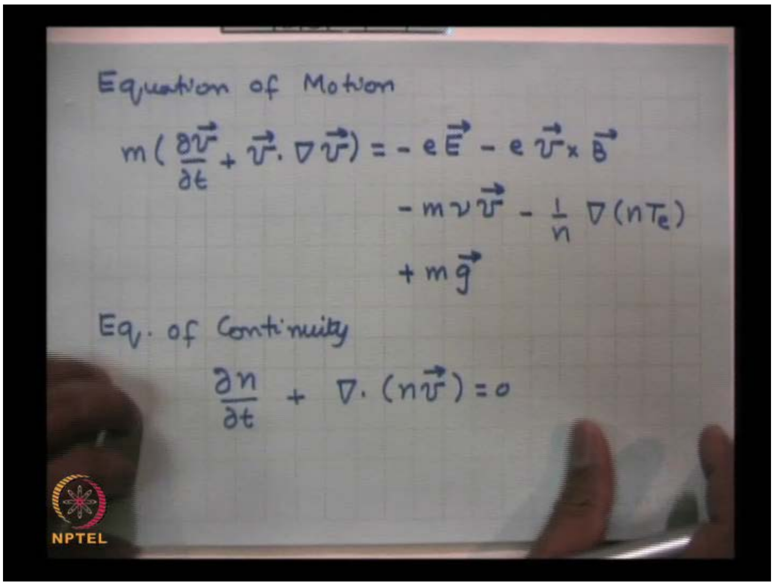

So, equation of motion can be written as m delta v divided by delta t plus v multiply del v is equal to minus e E minus e v multiply B the magnetic force minus m nu v collisional drag force and minus 1 upon n gradient of n T e ,the pressure gradient force this is our complete equation of motion.

Well, in some cases like if you are talking of plasmas in the stars and like sun then there is a gravitational force also. So, to the right hand side when must add a term if gravity is there , say gravity is g then plus m g a force due to gravitation also be added there.

But, in most laboratory plasmas of interest m g s not significant you can ignore this and hence we will be dealing largely with the these terms remaining terms in this equation, but in some cases of interest we will be adding this m g term also.

Now, this is one equation which has been very widely used , you may note one thing it will be simple .Suppose, the plasma has only electric field forget this magnetic force term forget the gravity term, forget the pressure gradient term and suppose the collisions are not there then forget this term also.

Then, the response of electrons to an electric field is given by this equation v is called the response and e is called the source that acts on the particles the thing is that the response has a single term v here, but it is a product of response terms this products of v is called non-linear term and this is a typical characteristic of plasmas that is many applications this non-linear terms become important and this is largely the source of non-linear phenomena like harmonic generation, parametric instabilities and many other phenomena.

Similarly, if there is a magnetic field of the wave of instance then this v multiply v term we cannot, ignore and this also is called non-linear term because the force involves the response itself. So, whenever this is a product of response to the source then the term is called non-linear term and this also is responsible for a large number of phenomena non-linear phenomena.

Sometimes , the product of density and temperature is may cause in nonlinearity or collision frequency with velocity product because collision frequency also depends on velocity of the particles not only average velocity, but on thermal velocity also.

So, they are also sources of nonlinearity in plasmas. So, basic thing is that plasma response in general to any field is non-linear; however, if the fields are weak you can ignore the non-linear terms and the process of neglecting these non-linear terms is called linearization. So, we will solve these equations in little while; however, let me go over to another important equation and that is called the equation of continuity.

Equation of continuity can be written as rate of change or partial rate of change of density with time delta n divided by delta t plus divergence of n v is equal to 0. I think, I need to explain few things in here ,what is n v? and why this divergence of n v is related to density ?

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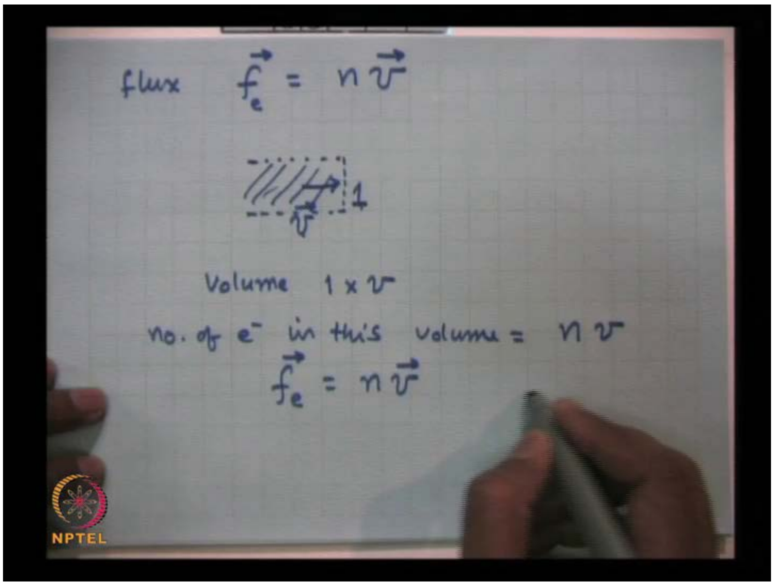

let me explain, this product of n v is usually called the flux of particles and let me denote this by f e a vector which is equal to n v let us understand, what is this quantity?

Suppose, electrons have a average velocity in certain direction; suppose, this is direction of average velocity of electrons consider a unit area perpendicular to v a question would arise how many particles will cross this area in one second ?

These, the average velocity with which the electrons move means the distance they travel in one second. So, all those electrons which are at a distance equal or less than v will be able to cross this because in one second they will travel a distance v.

So, what you are expecting is that in one second all those particles that are filled in a volume of length unity of area of cross section unity and length v they will be able to cross this and how many particles are there in this volume? the volume of this the space is this region is length in length v into cross section one.

So, volume is one into v and the number of particles in this would be number of electrons in this volume would be electron density n into volume of this region which is v. So, this is the flux. So, flux essentially implies the number of particles crossing unit area per unit time if they are moving with drift velocity v because this is vector quantity. So, we call this f e as n v as I written, I have. So, this is called the particle flux crossing unit area per unit time.

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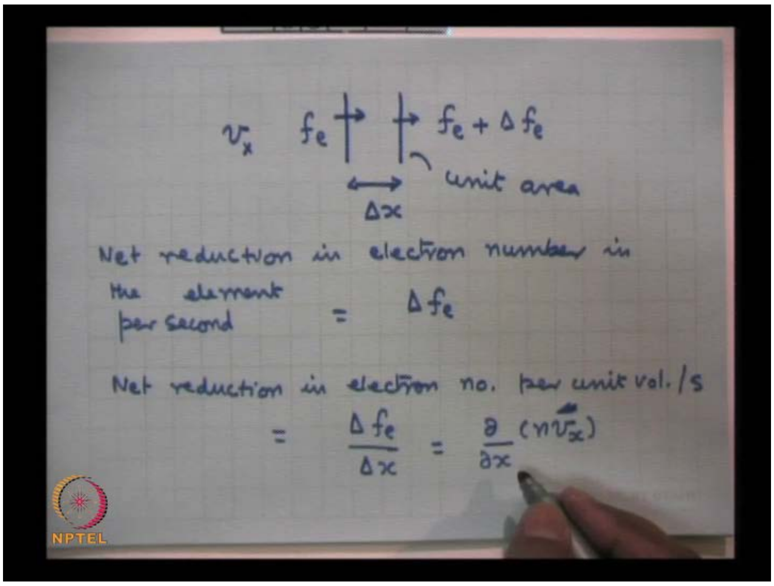

The issue is in a system if this quantity is not uniform it varies to be simple. Consider a region, let me go to the next page consider a small volume of unit area of cross section and separation of width like delta x and suppose the velocity is in x direction. So, this is my velocity v x if particles are flowing here with a flux f e the flux with which they are moving out could be f e plus change in f e . Suppose in change in f e is delta f e. So, the number of electrons entering per second this volume element is f e the number of electrons leaving this unit area per second is f e plus delta f e.

So, if this quantity is positive more particles are leaving less are coming in and hence there is a net reduction and density particle number here. So, net reduction in electron number in the element is equal to delta f e per second is delta f e and the total volume is how much unit area unit area I am considering. So, area of this element is delta x. So, this is the number of electrons reduced in volume delta x.

So, net reduction in electron density or electron number in the electron number per unit volume per second would be delta f e upon volume which is delta x. I can write this quantity as delta delta x of f e, but f e is a product of n v and v as I am taking in external direction n v x .

You can, generalize this to three dimensions rather than considering a volume element perpendicular to x access .It is wherein general direction then particles are entering there the volume elements could be a box like this.

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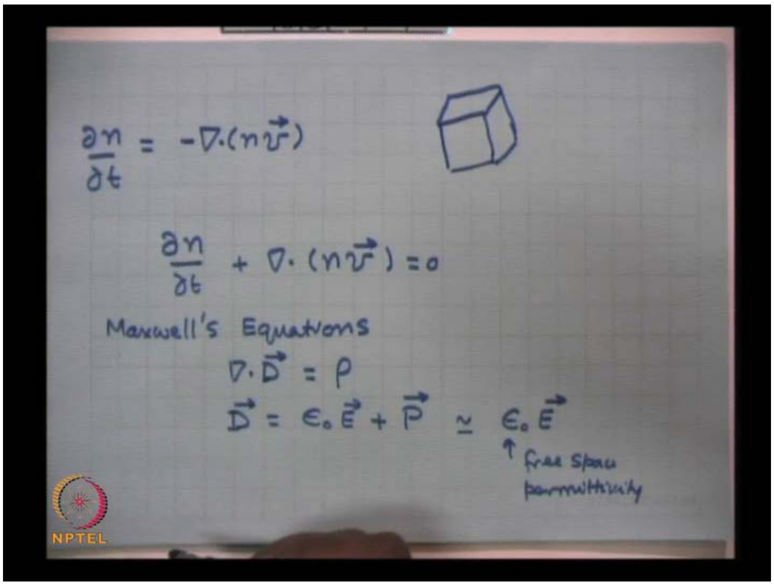

So, particle can enter and leave. So, in three dimensions , when particles are moving then the net number of particles reduced per unit volume is gradient of n v.

Please understand , this quantity is the reduction. So, if I have to write the increase then I should write a minus quantity. Whenever this quantity is negative , then there is increase then more particles enter less leave then there is a increase. So, in that case a negative.

So, then just put this equal to rate of change of density and this is precisely the equation of continuity.

So, the equation of continuity, let me rewrite is delta n by delta t plus divergence of sorry there is a dot product required here n v is equal to 0. So, I think I have given you a simple derivation of two important equations in plasma dynamics that controls the dynamics. The dynamics of electrons and a similarly you can write the equations for ions also.

So, we treat ions as a mixture of two fluids ; electron fluid and ion fluid and you tried separate equations for electron density and electron velocity and similarly ion density and ion velocity. But , whenever the electrons move they constitute a current. So, when there is a current then the current will produce from maxwell's equations magnetic field and electric field or wherever chargers move they also produce electric and electric field even if the density is there. So, when there is accumulation of charges somewhere or refraction of charges that will produce electric field. So, one should combine these equations the equation of motion continuity with the maxwell's equation and then they form a complete set to describe a system.

So, let met write down the maxwell's equations also to complete our description. So, I am going to just briefly mention maxwell's equations . The first equation is known as divergence of D is equal to rho. Now D is relative to the electric field through this relation by definition D is equal to epsilon 0 E plus polarization P.

In gaseous plasmas, the polarization of atoms is very small and you can ignore this to capital P. So, for gaseous plasma this is typically of the order of epsilon 0 into E where epsilon 0 is called free space permittivity this is free space permittivity in case units. So, this is this displacement vector is related to electric field by this quantity epsilon 0 E.

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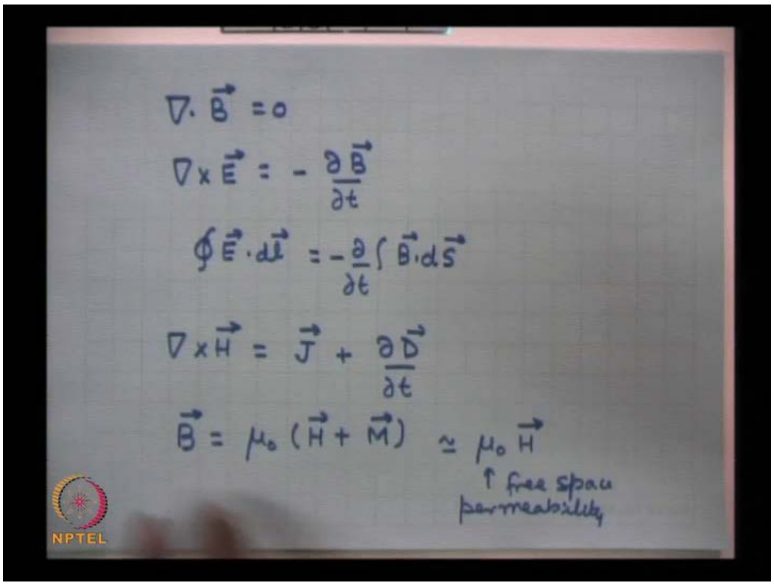

Another equation is, the divergence equation for magnetic field v we says divergence of v is equal to 0 v is the magnetic field third is the equation equivalent of faraday's law of electromagnetic conduction which is curl of E is equal to minus delta B by delta t.

In integral form, this equation sometimes is written and this is written as E multiply del this quantity over a closed path is called e m f. Just line integral of the electric field is equal to rate of change of flux link with the circuit. So, this is B dot d s is the flux linked with the circuit and rate of change of flux with time .

And the fourth maxwell equation is, generalization of empires law this says that curl of H is equal to J plus delta D by delta t J is known as the current density and H is related to the physical quantity B the magnetic field by this relation B is equal to mu 0 H plus M where H well mu 0 is the magnetic permeability of free space and M is the magnetization or dipole movement per unit volume.

For plasmas, this m is negligible and this is equal to mu 0 H. So, this is called free space permeability. So, these four maxwell's equations coupled with the equation of motion continuity form a complete set of equations governing the dynamics of plasmas.

Well, I would like to caution that these equations though they describe the plasma dynamics in most situations, but it still they are missing some important effects called kinetic effects because when we are dealing with the propagation of waves in particular some electrons maybe moving with the velocity of the wave phase velocity of the wave and they can interact very resonantly with the waves and can exchange momentum and energy with the waves very effectively.

So, we have to deal with different electrons moving with different velocities differently and for that one requires a kinetic description or the description based on velocity equation when we come to that stage we will discuss that equation also.

Well. So, I have given you four maxwell's equations that in congestion with the equation of motion and continuity complete the description of plasma many phenomenon plasmas; however, before I close I would like to say a word about collisions.

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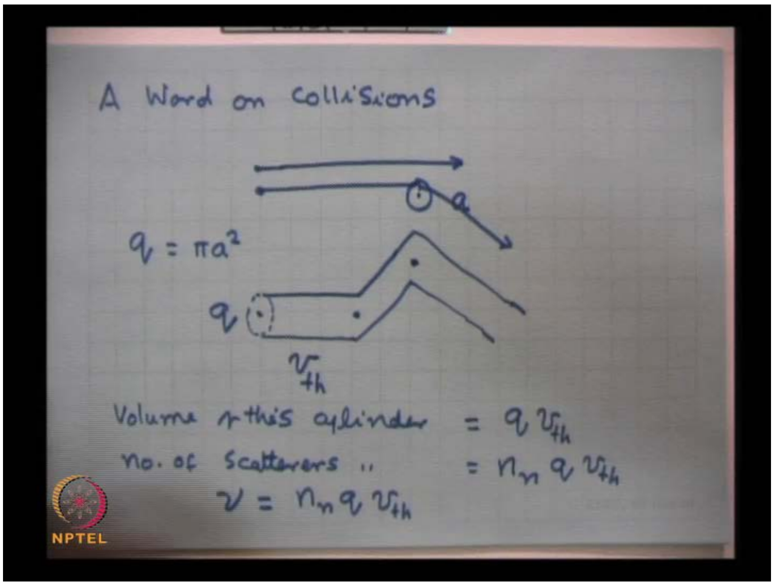

So, a word on collisions will be proper at this stage ;well ,what happens that when an electron moves in the vicinity of a scatter. If the electron moves quite far away from the scatter it does not suffer a collision it goes like this, but when it falls within certain a distance the distance of closest approach is called impact parameter.

For instance, if I have a typical radius of an atom like a then you can characterize the effective area of cross section of the atom to be q is equal to pi a square. So, whenever the electron falls on this area it will suffer a collision and after collision it will go like this if it moves away from this area then it suffers no collision.

So, for electron neutral collisions you can always assign a effective area with the atom. Now, I would like to see the problem in a different way rather than assigning a area to the a scatter I am assigning an effective area of cross section to the electron suppose the electron has a hallow around it of cross section q and when the electron travels a distance in one second is like thermal velocity v thermal.

So, in one second it will move like this, but before it covers one second it may collide with an atom and it may changes path it may go like this. So, the electron comes here and suffers a collision and goes in this direction and suppose another collision and goes in this direction.

Now, the mass of the electron is very tiny as compare to the mass of the scatter. So, primarily the energy transfer is very little the moment transfer is the significant quantity. So, these are called momentum randomizing collisions. So, what I can say that the electron is moving a distance of the order of v thermal, thermal velocity or thermally speed in one second. So, it will travels a zigzag kind of cylinder of cross section q and length v thermal and the number of collisions it will suffers in one second is the same as the number of its scatters, it will encounter on its way because the scatters which are outside will not collide with this.

So, only though this scatters which are lying inside this cylinder they will be colliding with this electron. So, what is the total volume of this electron cylinder volume of the cylinder which is a fictitious cylinder of cross section q the time assigning to an electron length is v thermal cross sectional area is q. So, q into v thermal is the volume and in this volume how many scattering particles will be there scatter will be there.

If , that neutral particles have a density n. Then, the number of scatters in this cylinder would be equal to their density into volume and this quantity is then called the collision frequency because with each particle electrons is going to suffer a collision. So, the number of collision the collision frequency nu is equal to n q into v thermal this is the collision frequency is a product of density of a scatters into collision cross section of the scatter into thermal velocity of the electron.

For neutral particles, q is simply equal to pi a square where, a is the radius of the atom which is of the order of an angstrom of few angstroms. So, q is typically ten to the power minus nineteen meter square, but the case of collisions of electrons with ions is very different let us understand that also .

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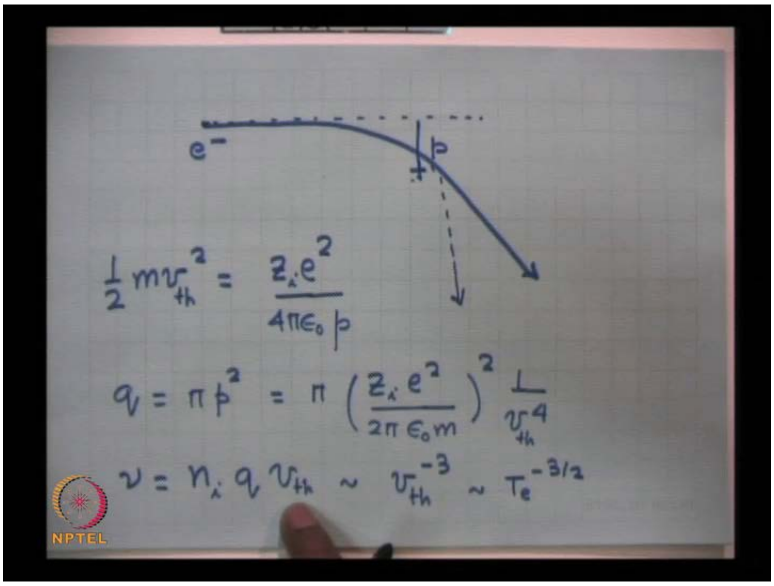

In the case of ions, ions exert a force on the electrons even if it is quite far away from here; obviously, if the distance of closest approach suppose the electron is moving like this then the distance of collision approach is called impact parameter p. If this electron is coming closer here, this will not go in this path. It will go like something like this will feel attracted and go like this. This is the actual path the electron will travels .

And you expect that larger the value of p weaker the effect of this attractive force or Coulomb force of the ion on the electron. So, the change in momentum or change in direction of the electron motion will be less effective when p is large, but p could be quite large not the angstrom it could be several angstroms tens of angstroms or hundreds of angstroms because, this is a slowly verifying the potential or the electrical force due to the ion on the electron is long range force so, but how do you define a effective collision frequency.

We define an effect collision frequency by saying that find out the impact parameter p for which the electron suffers a 90 degree collision it means, it goes like and comes like this. So, that the angle of scatter is 90 degrees and what has been found is a simple calculation that impact parameter p for which the kinetic energy of the electron which is half m v square equals the potential energy due to the ion if the ion charges z i into e electron charges is e then this quantity divide by p 4 pi epsilon 0 into p is the potential energy.

So, if I equate the kinetic energy of the electron to potential energy of the electron at the impact parameter p then the p that you get from here turns out to be the one that if an electron arrives with the impact parameter p given by this equation it suffers 90 degree collision.

So, we define an effective area of cross section q is equal to pi p square and if you put the value of p. From this equation, it turns out to be p is equal to. So, pi is there and two will cancel out. So, you will get z i e square upon 2 pi epsilon 0 m whole square into 1 upon v 4 this is a very v thermal rather this is thermal velocity.

I am presuming here, that my electron is moving with a velocity actual velocity is this v really is the actual velocity here I am presuming that the actual velocity is like thermal velocity typically some electrons may move faster some may move slowly. So, we are typically taking this like thermal velocity.

So, collision cross section q is inversely proportional to fourth power of velocity how would the collision frequency collision frequency is a product of ion density collision cross section and thermal velocity. So, when you put these number in here this is scales as thermal velocity of electron to the power minus three or temperature of the electron minus three by two this is a very important dependence means the electron temperature larger the electron temperature weaker the collision frequency .

And later on, we will learn that all major quantities like electrical conductivity, thermal conductivity , diffusion coefficient , etcetera this microscopic quantities depend on collision frequency and hence they depend on electron temperature very sensitively this is a very important dependence.

So, where is for the case of electron neutral collisions collision cross section q as a constant if q is a constant then the collision frequency will increase with thermal velocity like temperature to the power half.

Whereas, in a electro nine collision because q decreases with velocity of the electron. So, collision frequency scale says minus 3 divided by 2 this is something very significant and we will see its effect on response of plasma to electric field later. thank you very much .