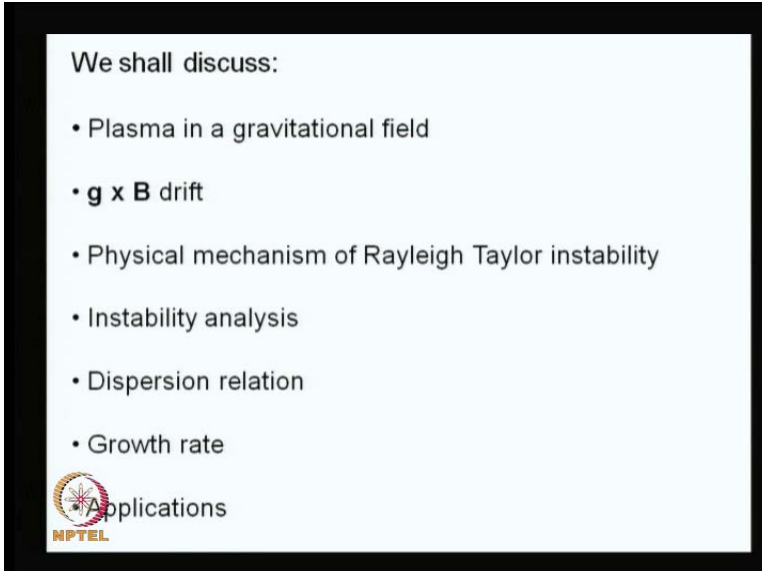


Plasma Physics
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Module No. # 01
Lecture No. # 19
Rayleigh Taylor Instability

Today I would like to talk about another instability which is very extensively studied in plasmas as well as in fluids is called Rayleigh Taylor instability.

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We shall discuss:

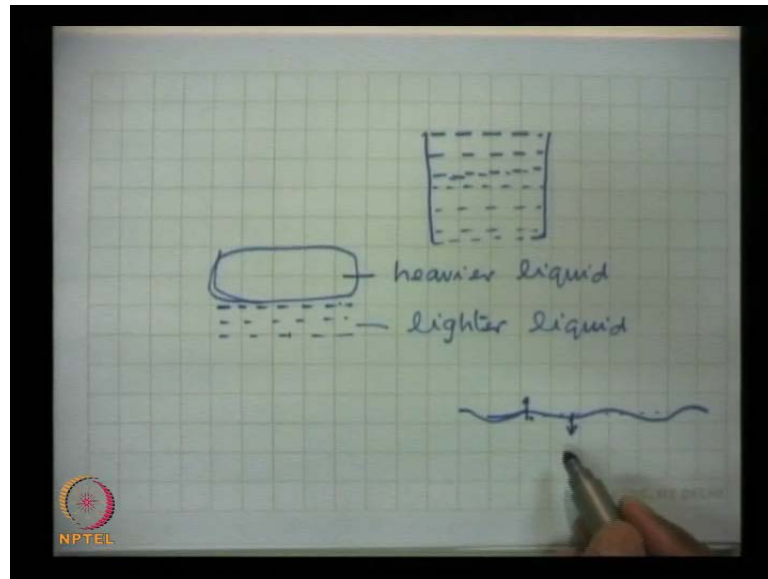
- Plasma in a gravitational field
- $\mathbf{g} \times \mathbf{B}$ drift
- Physical mechanism of Rayleigh Taylor instability
- Instability analysis
- Dispersion relation
- Growth rate

Applications
NPTEL

We shall discuss plasma in a gravitational magnetic field, how can you sustain a plasma in a gravitational magnetic field. The plasma electrons and ions will experience a drift called \mathbf{G} cross \mathbf{B} drift. Then we will discuss, the physical mechanism of Rayleigh Taylor instability, we will carry out a instability analysis using fluid theory obtain a dispersant relation and the growth rate.

Then we will discuss an applications, this instability has been found to be important in ionosphere f region of the ionosphere. And also in a different form, it has been found to be relevant important in mirror machine in tokomak.

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Well, let us consider, a liquid this is a lighter liquid and on top of this there is a heavier liquid. For instance, if I put a stone on water and if in a column there is no point, no region for the water to get up. And the stone exactly fits the channel then the stone will compress the water.

Stone density is larger than the water density and if the stone is very hard, it does not allow the penetration of the water through it, then you can have a stone swimming over water. Because, if I have a container, exactly tide container, in which water is filled there, but, a stone is just exactly fitting in here, then there is a possibility that stone can be there.

However, if there are no walls here, then water can go up from the sides and stone will drown in water. Now, consider a situation, suppose I have a lighter fluid somewhere and a heavier fluid. So, this is lighter fluid and this is a heavier fluid filling in this region.

In principle, you can have equilibrium when the upward force by the liquid balances, the gravitational pulled downward on the heavier liquid here this is a heavier liquid and the lighter liquid.

What will happen certainly there is equilibrium? Possible the issue, is if suppose there is a small perturbation, what will happen? So, on the interface between a heavier and lighter fluid if this is the interface and suppose there is a perturbation like this.

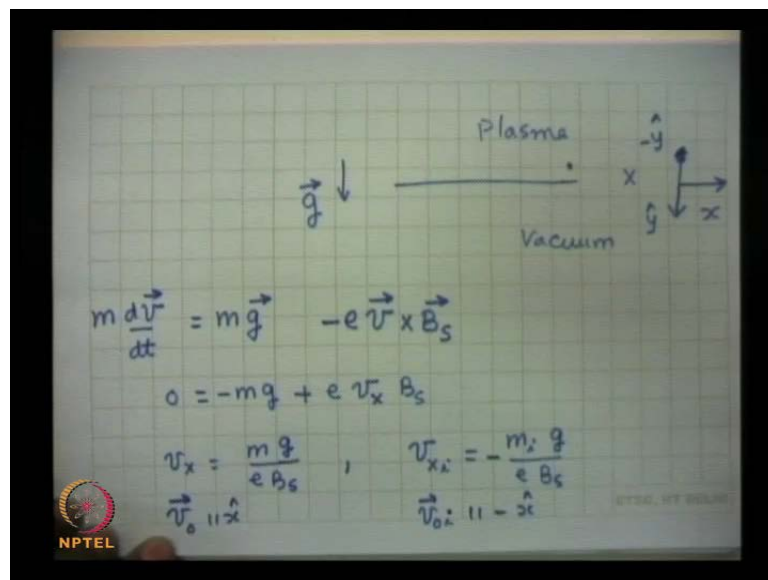
Then if the heavier fluid comes down somewhere, if there is a ripple in the surface. Then there is no equality of pressure in this region is less pressure, in this region higher and as a consequence.

The gravitational force on this portion will not be balanced by the pressure alone and this liquid will start coming down and it will going up here.

So, the ripple in the surface dividing the two liquids will grow. And the liquid in those smaller, deeper portions or turfs will keep on going down and here it will going up. So, liquid will move up lighter, liquid will move up and heavier, liquid will move down and this situation is called unstable situation. Laterally examine this issue, this problem.

And you cannot have a stable equilibrium with lighter liquid being below and heavier. Liquid being on top and this phenomenon gives this gives rise to the phenomenon of convection, this is a conventional thing in liquids.

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Now, we want to examine, what will happen in a plasma? Is it possible to contain a plasma? Suppose there is a plasma here and there is a weapon here, but, this is placed in a gravity like ionosphere is a region of space plasma, which is have of the surface of the earth. So, earth can pull it. So, if I well there is no vacuum here, the air, but, suppose I have a plasma which is support against gravity, gravity is in downward direction with acceleration due to gravity g.

Well, if there is no magnetic field, then certainly plasma will fall down. So, suppose I create a magnetic field which is perpendicular to the plane of this paper. Then what will happen? Any plasma electron or ion when it sees a gravitational force downward $m g$ and it also sees it is starts moving down.

Then it also sees a magnetic force, which is the product of charge velocity and magnetic field. If $-e$ is the electron charge, v is the electron velocity, B_s is the static magnetic field, then net force on the electron would be this plus this and this will be rate of change of momentum.

One of the possible equilibrium is that velocity becomes constant after a while and electron will acquire a velocity given by this equation, this equal to zero. So, if I put this right hand side is equal to 0, it you get a velocity that is called g cross B drift.

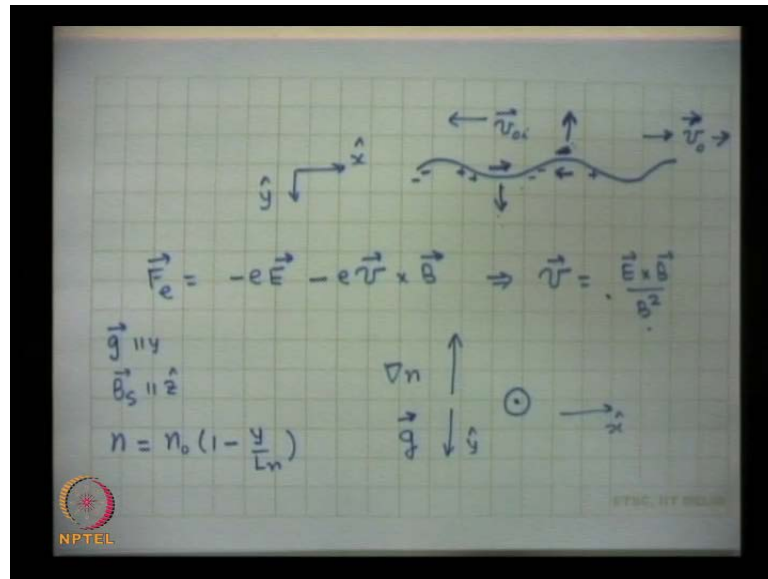
And for instance, if I take g in the downward direction, let me specify my coordinate system this horizontal direction. I will take to be x vertical direction, I will take to be y and well I think in some literature people take y downward direction as the direction of gravity is y . So, this is minus y upward and plus y downwards.

So, if I choose these situations. So, g if I take in the minus y direction, then this is equal to minus $m g$. If I write down the y component, this term, this will be $e v_z$, this is in the z direction. So, this will be y component I want. So, I will this get v_x and z , but, plus sign here equal to 0. This is when I equate the y component to this equation to 0, I get v_x is equal to m upon $e B_s$ and g .

Similarly, if you calculate for ions v_x , for the ions would be mass of the ion g upon $e B_s$, but, sign will be opposite, because charge of the ion is opposite to the charge of the electron, this is called g cross B drift. Because, it is perpendicular to gravity which is in y direction and the magnetic field which is in the z direction, g cross B_x cross z , oh sorry, y cross z . Did I make a sign error of sign, let me just check.

G is in the y direction. So, y component of this will be z x minus. So, this is plus this is correct, I think this is not wrong here this is fine. I will call this velocity of electrons as v_0 . This v_x is equal to v_0 and this velocity, I will call as v_0 I the ion velocity. I would like to examine, if I perturbed this equilibrium, what is going to happen? Please note electrons are going in one direction, positive axis direction, which is parallel to x axis and this is parallel to minus x direction.

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Let me consider a perturbation in the surface, suppose this is the boundary between the lower boundary of the plasma this is vacuum here and plasma there.

Please note, electrons are travelling in this direction, this is direction x-axis. So, this v_0 of the electrons and ions are travelling in the opposite direction v_0 i. sorry arrow is here. So, ions are traveling in this direction and magnetic field is in the z direction, z direction is where let me just understand this.

X direction is here, y direction I am taking downwards. So, x cross y is evolve the paper in the vertically upward direction like this perpendicular to the plane of the board. And this will be upwards x cross y. No, into the plane of the television x cross y, will be right handed screw going to this board.

Now, when the electrons and ions move in opposite directions the look and this region. Here the ions will come somewhere here and electrons will be appearing there. So, when they will produce an electric field in this direction.

Now, this is the electric field in x direction magnetic field is in the z direction. So, x cross z, if you calculate the force, the electric force will be minus e E on the electron and magnetic force would be minus e v cross B. This is the net force on the electrons due to the electric field plus the values that is generated by the v cross B motion.

So, from here, calculate the value of v , this we I call as the electron velocity, that it will acquire because of the electric field. That velocity you will obtain from here and if I take balance the two, then this will give me v which is equal to e cross B . you can just verify this upon B square.

So, what is the consequence? The consequence is that, the force on this will be e cross B upon B square electric field is in the x direction, B is in the z direction. So, this turns out to be minus y , I am bit surprised.

The electric charge here is negative, this charge is positive here, sorry in this region, the electric field will be in this direction, sorry, this will be where the field will be in the minus x direction. So, the force will be probably I made a mistake in the drift, if the charges positive this is this kind of thing and this is this way.

Well it. So, happens that, the this velocity should be downward here and upward here. I might have made some error in calculating the g cross B drift direction. But, essentially is what happens is that the space charge motion the electron and ion motion in opposite directions, when you perturbed a boundary between plasma and free space.

Then you create a space charges on the boundary and that produces an net electric field. And that e cross B drift pulls the electrons and ions together, this is independent of a charge. And that pulls the plasma down in the trough direction and pulls the plasma up in this direction enhancing the perturbation, this is called the relatively instability.

So, a plasma supported against gravity by a d c magnetic field is unstable. Just like a heavier fluid supported by a lighter fluid is unstable here a plasma support a magnetic field is unstable. Now, in order to carry out the mathematical analysis of this process, we will consider a simpler module rather than considering a sharp boundary. We will consider a diffuse boundary a slow boundary.

So, I will consider a situation, that I have a density gradient in the vertically upward direction and gravity in the downward direction and static magnetic field perpendicular plane of the paper.

So, I am considering a geometry in which g is parallel to y . I am taking y as the downward direction and this is my x direction. And my static magnetic field is in the z direction and I will consider a specific variation of density with distance. And I will consider to be increasing function of height.

So, I will consider something like $n = n_0 (1 - y/L)$, because, I am choosing y direction downwards. So, as you go up, you are going to higher and higher values of negative y . So,

density will increase because y is negative and consequences increases. So, this is just choose this and if I do this then let me begin with my equilibrium first.

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Equilibrium
 $T_e = 0, T_i = 0, \text{ no collisions}$
 $+m_i g \hat{y} + e \vec{v}_{0i} \times \vec{B}_s = 0$
 y comp.
 $m_i g - e v_{0i} B_s = 0$
 $\vec{v}_{0i} = \frac{m_i g}{e B_s} \hat{y} \times \hat{z}$
 $\vec{v}_{0e} = -\frac{m_e g}{e B_s} \hat{y} \times \hat{z} = 0$

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For the sake of simplicity, I will consider the plasma to be having no temperature. So, electron temperature is not there, ion temperature is not there, no collisions. I will not considering any collision effects to be. I am considering a plasma in which the gravitational force which is minus m or the plus $m g$ in the y direction is balanced for ions like that let me write down for ions plus $e v_0$ cross B_s .

Now, let me equate this to 0 in equilibrium. So, the value of v_0 that I get you can obtain by taking y component of this equation. $M g$ this is x this is z . So, minus $e v_0 \times B_s$ equal to 0, I think that is the mistake I had made earlier.

So, you get v_0 is equal to $m g$, this is ion motion, ion considering. So, put a ion mass here, ion mass upon $e B_s$ this is the equilibrium velocity of ions. And put a subscript i here because, I am correct motion ions. So, put a subscript i to characterize this ion motion.

So, ion drift velocity is in the y direction, no x direction. And similarly, if you calculate the electron drift velocity v_{0e} it will be mass of the electron into g upon $e B_s$, but, the sign of charge is opposite.

Please note that the electron drift velocity is very very small as compare to ion drift velocity. Because, electron mass is very small as compare to ion mass and hence, this I can ignore. So, ignore this. So, this is my equilibrium and I would like to perturb this equilibrium by a

perturbation. I want the perturbation to have a k vector on the x direction because, this like a ripple in the x direction.

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$$\phi = A e^{-i(\omega t - kx)}$$
 Eq. of motion
 Eq. of cont.] for e^- and ions
 Poisson's Eq.
Electrons

$$m \left(\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) = -e \vec{E} - e \vec{v} \times \vec{B}_0$$

$$\vec{E} = -\nabla \phi, \quad \vec{B} = 0$$

$$\vec{v} = 0 + \vec{v}_1$$

So, consider a perturbation of electro potential phi is equal to A exponential minus i omega t minus k x. I am perturbing my equilibrium by this electrostatic perturbation and like to find out whether this perturbation will grow. Means omega will be complex with a positive imaginary part or not.

To examine that I need to find out, I need to solve the equation of motion and equation of continuity for electrons and ions both, and then I need to solve the Poisson equation, this has to do.

So, let me write down, the equation of motion for electrons this is mass into delta v by delta t plus v dot Del v is equal to minus e E, the electric force minus e v cross B the magnetic force, this I have to consider.

In the system, there is only one magnetic field, there is a static magnetic field which is in the z direction, the perturbation is purely electrostatic. So, electric field is minus grad phi. And there are no magnetic perturbations. So, B field of this wave is 0 because curl of e is 0.

Another thing for electrons this velocity I can write down as a equilibrium velocity which is nearly 0 personally perturbation quantity p v 1. And when I substitute this here, I ignore the product of perturbed quantity. So, this term does not contribute at all and this term survives, this will survive, this will also survive this v 1 cross B s.

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The image shows a hand-drawn derivation on a grid background. The equations are as follows:

$$\frac{\partial \vec{v}_1}{\partial t} = + \frac{e \nabla \phi}{m} - \omega_c \vec{v}_1 \times \hat{z}, \quad \omega_c = \frac{e B_s}{m}$$

$$-i\omega \vec{v}_1 + \omega_c \vec{v}_1 \times \hat{z} = i \frac{e k \phi}{m}$$

x comp.

$$-i\omega v_{1x} + \omega_c v_{1y} = i \frac{e k \phi}{m}$$

y comp.

$$-i\omega v_{1y} - \omega_c v_{1x} = 0 \Rightarrow v_{1y} = i \frac{\omega_c}{\omega} v_{1x}$$

$$v_{1x} = - \frac{e k \omega \phi}{m(\omega^2 - \omega_c^2)} \approx + \frac{e k \omega \phi}{m \omega_c^2}$$

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So, let me write this equation for the perturbed velocity as Δv_1 by Δt is equal to minus $e \text{ grad } \phi$ sign reverses upon m minus $\omega_c v_1$ cross \hat{z} cap. Where I have defined ω_c as $e B_s$ upon m . E is the magnitude of electron charge, m is the electron mass and B_s is the magnitude of a static magnetic field. So, I have written that equation in this form, this is the perturbed velocity.

Replace $\Delta \Delta t$ by minus ω because, ϕ varies in that fashion which that divert for this velocity. So, you get minus $i \omega v_1$ bring this term also in the left hand side. So, it becomes plus $\omega_c v_1$ cross \hat{z} cap is equal to right hand side will be $i e k \phi$ upon m . Write down it is components x components and z component or y component.

X component would be minus $i \omega v_{1x}$, this will give you plus $\omega_c v_{1y}$ is equal to $i e k \phi$ upon m , because k is in the x direction. For the y component, you will get y, this equation will be minus $i \omega v_{1y}$ minus $\omega_c v_{1x}$ is equal to 0. Because, there is no y component of the right hand side.

This equation gives you v_{1y} is equal to $i \omega_c$ upon ω v_{1x} take this one to the right hand side divide by minus $i \omega$. And you will get this result and if I use this in this equation I will get v_{1x} also.

So, v_{1x} is equal to minus, if you just be careful, you will obtain this $e k$ upon $m \omega^2$ minus ω_c^2 into ω up here, because one ω was there. So, you just obtain this, you can just verify this minus $e k \phi$ upon m this expression you will get.

For this wave, if ω is much less than ω_c which indeed it is, then this is approximately equal to ignore this. You will get minus $e k \omega \phi$ upon $m \omega_c^2$ and sign becomes positive. So, electron density perturbation, velocity perturbation is simply this expression and let me calculate the density perturbation also.

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$$\frac{\partial n}{\partial t} + \nabla \cdot (n \vec{v}) = 0$$

$$n = n_0 + n_1 \quad n_0 = n_0^0 \left(1 - \frac{y}{L_n}\right)$$

$$\left[\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot \vec{v}_1 \right] = 0 \quad n_1, v_1 \sim e^{-i(\omega t - kx)}$$

$$\frac{\partial n_1}{\partial t} + \nabla \cdot (n_0 \vec{v}_1) = 0$$

$$-i\omega n_1 + \frac{\partial n_0}{\partial y} v_{1y} + n_0 \frac{\partial v_{1x}}{\partial x} = 0$$

$$n_1 = \frac{1}{i\omega} \left[-\frac{1}{L_n} n_0^0 v_{1y} + i k n_0 v_{1x} \right]$$

The density perturbation turns out to be, how can be obtained from the equation of continuity Δn by Δt plus divergence of $n v$ is equal to 0. N express as n_0 plus n_1 , use this here and linearism, you will get Δn_1 by Δt plus $n_0 \text{ del dot } v_1$ equal to 0, replace del by $i k$. Sorry, I made a mistake here, n_0 is not a constant quantity this is wrong. So, let me delete this, this not write.

Remembering that n_0 depends on y , it is increasing in the vertically upward direction n_0 . Please, remember is equal to $n_0^0 (1 - y/L_n)$. Sorry, I made a mistake n_0 should be in the interior, I will get Δn_1 by Δt plus divergence of $n_0 v_1$ equal to 0.

This equation please remembers n_0 depends on y and v_1 depends only on x . So, this will have 2 terms and this is I can replace by minus $i \omega$ because perturbed density will vary perturbed quantities are all vary as exponential minus $i \omega t$ minus $k x$. But, n_0 is not a perturbed quantity it does not vary in the same fashion.

So, n_1, v_1 they go as this, but, not n_0 that was the error I wanted to I was going to make. So, for perturbed quantities $\Delta \Delta t$ can be replaced by minus $i \omega n_1$. And that will give me

2 terms here plus δn_0 into v_{1y} plus δn_0 into δv_{1x} and this is equal to 0.

δn_0 by δy if I evaluate is like $\frac{1}{L} \int n_0 dy$. So, I will just write this as $\frac{1}{L} \int n_0 dy$ and that gives me how much.

n_1 is equal to $\frac{1}{i\omega}$ and here you are going to get minus $\frac{1}{L} \int n_0 dy$ this term. And this will give me $ik \int n_0 v_{1x}$, when I have taken these terms on right hand side this is the kind of equation I get.

So, density perturbation for electrons I have obtained, now I will put the values of v_{1y} and v_{1x} and we obtain if I put the value of v_{1y} .

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$$n_1 = \frac{1}{i\omega} \left[-\frac{1}{L} n_0 \frac{i\omega e}{\omega} v_{1x} + ik n_0 v_{1y} \right]$$

$$= \frac{n_0}{i\omega} \left[-\frac{i\omega e}{\omega L n} + ik \right] \frac{e k \omega \phi}{m \omega_c^2}$$

Ions

$$m_i \left(\frac{\partial \vec{v}_i}{\partial t} + \vec{v}_i \cdot \nabla \vec{v}_i \right) = -e \nabla \phi + e \vec{v}_i \times \vec{B}_0 + m_i \vec{g}$$

$$\vec{v}_i = v_{0i} \hat{x} + \vec{v}_{1i}$$

$$\frac{\partial \vec{v}_{1i}}{\partial t} + v_{0i} \cdot \frac{\partial \vec{v}_{1i}}{\partial x} = -e \nabla \phi + e \vec{v}_{1i} \times \vec{B}_0$$

Let me just write this n_1 is equal to $\frac{1}{i\omega}$ within this will be minus $\frac{1}{L} \int n_0 dy$ and this is $i\omega c$ by ωc by ω rather into v_{1x} plus $ik \int n_0 v_{1y}$. I can write actually as $\frac{1}{i\omega}$ of the order of $\int n_0 v_{1x}$. And I can take, if I please note here k is bigger than $\frac{1}{L}$. But, ωc by ω is additional factor that comes over which is really very huge.

ωc by ω is may be six out of ten orders of or eight orders of magnitude for some reasonable parameters of ionosphere for instance, there is a huge number. So, usually this term dominates over this term, but, let us retains both the streams and put the value. So, what you get is $\frac{1}{i\omega}$ n_0 , I will take common. So, put n_0 out and you will get minus $i\omega c$ upon ωL plus ik multiplied by v_{1x} whose value turns out to be $e k \omega \phi$ upon $m \omega c^2$.

This is the dominant term actually in this expression, this is my perturbed electron density, then I must calculate the perturbed ion density. So, for the ion response I have to retain the effect of B_0 because the d c drift is there. So, for ions my equation of motion will be difference.

This is $m \Delta v$ by Δt plus $v \cdot \text{Del} v$ is equal to this ion motion. So, put I subscript everywhere i here i here is equal to e minus $e \text{ grad } \phi$ this is the ion charge plus e and minus $\text{grad } \phi$ is the value of the electric field minus plus $e v \text{ cross } B$ the magnetic force. So, when you solve this equation put ion velocity is equal to v_0 ion the equilibrium velocity which was in the x direction plus a perturbed quantity v_1 i and use this in this equation and linearism this equation.

Well I forget to write one more term here, that is the plus $m g$ term, sorry, let me write down this here mass of the ion and gravity. This term should be written there when we substitute this here, the $v_0 \text{ cross } B_s$ term exactly cancels with this term. So, gravity does not really play any role in the perturbed velocity equation. Because, $v_0 \text{ cross } B$ exactly cancels with this by definition.

And this equation of linearization gives Δv_1 i upon Δt plus v_0 i dot Δx of v_1 i is equal to minus $e \text{ gradient of } \phi$. And this term will give you e plus v_1 of ion cross B_s . This equation we have to solve it is simple replace Δt by minus $i \omega \Delta t$ by $i k$ and then this equation take the following form.

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The image shows a whiteboard with the following handwritten equations:

$$-i \omega' \vec{v}_{1x} - \vec{v}_{1x} \times \vec{\omega}_c = -\frac{e}{m_i} i k \phi$$

$$\omega' = \omega - k v_{0x}, \quad \omega_c = \frac{e B_s}{m_i}$$

$$v_{1y} = -i \frac{\omega_c}{\omega'} v_{1x}$$

$$v_{1x} = -\frac{e k \omega' \phi}{m_i \omega_c^2}$$

The whiteboard also features an NPTEL logo in the bottom left corner and a hand holding a pen at the bottom center.

Minus $i \omega$, I have made a mistake, I must put a ion mass in the denominator. Because, this mass I am dividing, this equation becomes minus $i \omega v_1$ minus v_1 cross ω_c ion cyclotron frequency. I think, I made a mistake over mass here as well there is mass there also.

So, what you get here is right hand side of the equation of motion becomes minus e upon $m_i k \phi$, this is prime here actually and ω' is equal to $\omega - k v_0$. So, this the only modification, this is the same equation as electron equation of motion with charge of the electron replaced by ion charge.

So, minus sign is there, electron mass is to be ion mass ω_{ci} is the ion cyclotron frequency. Which is defined as $e v_0$ upon m_i and ω' is some sort of Doppler shifted frequency of the wave of perturbation, as seen by the drifting ions.

And you can simplify this to obtain v_{1y} which turns out to be equal to minus $i \omega_{ci}$ upon $\omega' - \omega_{ci}$ of the ion and v_{1x} rather the same thing is equal to x component of v_{1i} turns out to be equal to minus $e k \phi$ upon $m_i (\omega' - \omega_{ci})^2$, this is what you get.

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$$\frac{\partial n_{i1}}{\partial t} + \nabla \cdot (n_0 \vec{v}_{i1} + n_{i1} \vec{v}_{0i}) = 0$$

$$n_i = n_{0i} + n_{i1} \quad n_{i1} \equiv n_{i1}$$

$$\vec{v}_{i1} = \vec{v}_{0i} + \vec{v}_{i1}$$

$$-i\omega' n_{i1} = -\nabla \cdot (n_0 \vec{v}_{i1})$$

$$= -\frac{\partial n_0}{\partial y} v_{i1y} + i k n_0 v_{i1x}$$

$$n_{i1} = -\frac{n_0}{i\omega'} \left[+ \frac{i\omega_{ci}}{\omega - \omega_{ci}} + i k \right] \frac{e k \omega' \phi}{m_i \omega_{ci}^2}$$

So, the ion velocity perturbation has primarily ω replaced by ω' which is the Doppler shifted frequency of the perturbation as seen by the ions. Then I go to solve the equation of continuity. The equation of continuity also gets modified for the ions.

The equation of continuity is n_i by Δt plus divergence of $n_0 v_1$ of the ions plus there should be a term here plus n_1 perturbation of ion density into v_0 . This is n_1 please, what I have done here is actually n_i have written as n_0 plus n_1 or n_{i1} whatever you call.

So, when I substitute this in the equation of continuity, $\frac{\delta n_0}{\delta t}$ is 0. So, this is only term that survives. And because, I have already written velocity as of the ions is equal to v_0 of ions plus v_1 of ions. So, you get two terms here.

This is the additional term that was missing in the electron equation of motion. And the consequence of this is that Δ this is a perturbed quantity. So, when you operate this with Δ operator it will be replaced by ik . So, this n_1 and this n_1 they will combine together actually n_1 is the same thing as n_1 by mistake. I have written this like this.

So, this term and these two terms combine together to give you minus $i\omega'$ into n_1 . This is the means replace this by minus $i\omega'$ this Δ operator operating over this term by ik . And you will get the simply ω' and this is equal to minus, this term taken on the right hand side divergence of $n_0 v_1$. Which again you will have 2 terms minus Δ operating over n_0 first, then this quantity.

So, which is equal to $\frac{\delta n_0}{\delta y}$ of ions the same thing as electrons also into v_1 . Because, if I take y component of this, then y as to be there and then you will get a term plus ik this is the x directions. So, n_0 of ions multiplied by v_1 .

So, I will substitute the values of v_1 and $v_1 x$ etcetera. And the result that I will write is this is very similar to the electron result is ω replaced by ω' . And it turns out to be n_1 , let me write down.

n_1 is equal to n_0 , ω upon $i\omega'$ within the bracket is. So, ω is ω' and this is plus $i\omega c$ upon ωl plus ik then $e k \omega$ dash ϕ upon $m i \omega c^2$ and sign is opposite.

This expression I have written from the electron expression replacing ω by ω' and electron parameters by ion parameters charge and mass in similar expression. After I have obtained the perturbations or in density of electrons and ions, it is simple to obtain the dispersion relation by using the Poisson equation. Which turns out to be divergence of d is equal to ρ , this is the Poisson equation.

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$$\begin{aligned}\nabla \cdot \vec{D} &= \rho, & \vec{D} &= \epsilon_0 \vec{E} \\ & & &= -\epsilon_0 \nabla \phi \\ \rho &= (n_{0e} + n_{1e})e \\ & & &- (n_{0i} + n_{1i})e \\ & & &= (n_{1e} - n_{1i})e \\ \nabla^2 \phi &= \frac{1}{\epsilon_0} (n_1 - n_{1i}) \\ \phi &= -\frac{1}{k^2 \epsilon_0} (n_1 - n_{1i}) \\ \Rightarrow n_1 &\approx n_{1i}\end{aligned}$$

When I substitute D is equal to $\epsilon_0 E$ and E as $-\nabla \phi$ with a negative sign and ρ which is the charge density of electrons and ions together. So, this is equal to $n_{0e} + n_{1e}$ this is into e is the ion charge density minus n_{0i} of the electrons plus n_{1i} of the electrons into e since n_{0e} and n_{0i} are the equal this is equal to $n_{1e} - n_{1i}$ into e sign is opposite minus of this.

So, use this in here, this equation takes the following form, $\nabla^2 \phi$ is equal to $1/\epsilon_0 (n_1 - n_{1i})$ again replace this ∇^2 by $-k^2$ and you will get $\phi = -1/k^2 \epsilon_0 (n_1 - n_{1i})$.

Just substitute the values of n_1 and n_{1i} . You will recognize that they also contain ϕ and every term that you get in $n_1 - n_{1i}$ is much bigger than ϕ . This is coefficient of ϕ is 1 there will be ϕ on the other side. Also, but, the coefficient of those terms is much bigger than one.

Hence I can ignore the term on the left hand side. So, this equation gives me that n_1 is nearly equal to n_{1i} . Because, the individual terms on the right hand side are too big as compared to unity or ϕ on the left hand side. So, I can ignore the left hand side term and in order to satisfy this equation n_1 is nearly n_{1i} and these perturbations, I have already obtained. So, let me write down the expressions n_1 and n_{1i} and what you get is this.

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$$\omega_p = \left(\frac{n_0 e^2}{m \epsilon_0} \right)^{1/2}$$

$$\omega_{pi} = \left(\frac{n_0 e^2}{m_i \epsilon_0} \right)^{1/2}$$

$$n_1 \approx \frac{n_0 e k^2 \phi}{m \omega_c^2} - \frac{e k \phi n_0}{m \omega_c^2 l_n \omega}$$

$$n_{1i} \approx -\frac{n_0 e k^2 \phi}{m_i \omega_{ci}^2} - \frac{e k \phi n_0}{m_i \omega_{ci} l_n (\omega - k v_{0i})}$$

$$n_1 \approx n_{1i} \quad (\omega - k v_{0i}) \omega = -\alpha^2$$

I just write down the expression. I think it will be useful. Please note, I am to divide those expressions by epsilon 0. And I will introduce a quantity called plasma frequency as $n_0 e^2$ upon $m \epsilon_0$ to the half, Where m is the electron mass similar quantity for ions. I can define as $n_0 e^2$ upon $m_i \epsilon_0$ to the power half. Then what you get when I put n_1 is of the order of n_{1i} , you can write down these expressions as follows.

n_1 is approximate is nearly equal to $n_0 e k^2 \phi$ upon $m \omega_c^2$ minus $e k \phi n_0$ upon $m \omega_c^2 l_n$ density scale length. And n_{1i} is equal to minus $n_0 e k^2 \phi$ upon $m_i \omega_{ci}^2$ minus $e k \phi n_0$. I think there is ω also here in this equation ω is there and. So, ω prime here $m_i \omega_{ci}^2$ into $\omega - k v_{0i}$ of ions what you get.

The expressions are very similar and you have to equate them. Once we equate them, you get a simple dispersion relation because, ϕ will cancel out from every term. So, n_1 approximately equal to n_{1i} lead to every simple dispersion relation. Which says that $\omega - k v_{0i}$ of ions multiplied by ω is equal to a quantity minus Alpha Square. This is the simple dispersion relation by equating these 2 you get.

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$$\alpha^2 = \frac{v_{0i}^2}{L_n} \frac{\omega_{pi}^2}{\omega_{ci}^2 (1 + \omega_{pi}^2/\omega_{ci}^2)}$$
$$\approx g/L_n$$
$$\omega^2 - \omega k v_{0i} + g/L_n = 0$$
$$\omega = \frac{1}{2} \left[k v_{0i} \pm \sqrt{k^2 v_{0i}^2 - 4g/L_n} \right]$$

For $k v_{0i} < \sqrt{4g/L_n}$, ω is complex
one root has +ve imaginary part

And alpha square turns out to be equal to v_{0i} upon density scale length into ω_{pi} square that I have defined earlier upon ω_{ci} into 1 plus ω_{pi} square upon ω_{ci} square.

And there is under root, I do not think this in under root and this turns out to be, if I put the value of v_{0i} as g upon ω_{ci} and if I in most plasmas ω_{pi} is much bigger than ω_{ci} . So, 1 can be ignored and this turns out to be simply g upon L_n g is the acceleration due to gravity and L_n is the density scale length.

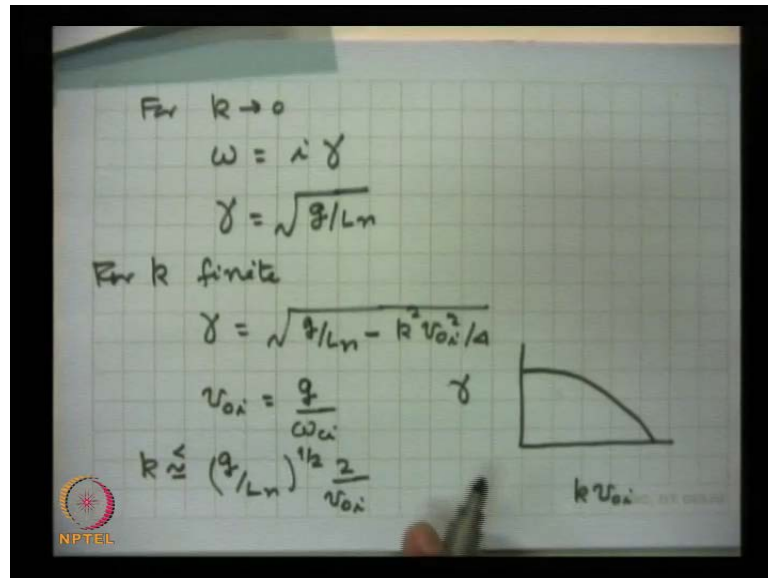
So, please note your expression the wave with the dispersion relation becomes ω square minus $\omega k v_{0i}$ plus g upon L_n equal to 0 a very simple dispersion relation you recover. And I found, if I can find the roots, this will give you ω is equal to half $k v_{0i}$ plus minus under root k square v_{0i} square minus four g upon L_n .

Well, this quantity could be bigger than one positive or negative. When this is positive ω is purely real and there is no instability. So, for large values of k this quantity is positive. And there is no instability means, if you are repel on the, of the perturbation in the plasma has a small k , large k , then there is no instability.

However, for $k v_{0i}$ less than $\sqrt{4g/L_n}$ under root, whenever this quantity is less than this ω is complex. And one of the roots, will have a positive imaginary part, one root has positive imaginary part and that gives the instability.

So, especially when k tends to 0 means when this quantity is negligible as compared to this the growth rate is simple. Because, you can ignore this term, you can ignore this term and the growth rate turns out to be.

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So, for k tending to 0 ω is simply equal to i times γ and growth rate is equal to under root of g by Ln or time period of growth is root Ln by g just like a simple pendulum.

This is the interesting part of it, that situation is very complex. It is a plasma with electrons and ions moving. And magnetic field stopping the plasma to fall down against gravity is still the expressions very simple, very similar to what we get in a simple pendulum.

And if k is not 0 for finite k , what you get the growth rate is as I mentioned there. The growth rate is equal to under root of 4 rather g upon Ln minus k square v_{0i} square by 4 . And please remember v_{0i} is the ion drift velocity, which is equal to g upon ωc_i .

So, this also depends on g , but, k is primary think here it will be useful to plot growth rate as a function of $k v_{0i}$. Normally people plot this upon this growth rate resolve some number, you will get this kind of behavior.

Now, this instability is important in f region of the ionosphere. And it gives rise to fluctuations in density perturbations of the order of with wave length k of the order of from here g upon Ln into 4 upon rather under root of this quantity into 2 upon v_{0i} .

And that turns out to be typically of the order of kilometer. So, in the ionosphere people have observed density fluctuations with a scale length of the order of eight kilometer may be half kilometer to three kilometers or. So, in that range and they are attributed to this Rayleigh Taylor instability. People also give this name as spread of instability.

Well here I have ignored the effect of collisions, if you include the collisions of electrons and ions then this growth rate is slightly modified. But, primarily the physics remains the same, mathematical treatment remains the same. And this growth rate, this instability has been observed always observed in ionosphere.

At times, people have observed very great depression and densities during especially during the sunset. people have observed order of magnitude reduction electron density because of this density so. But, that is, that requires little more complicated physics is not simply, this I have discussed the linear stage of this instability. That this instability will grow and it will cause a perturbation.

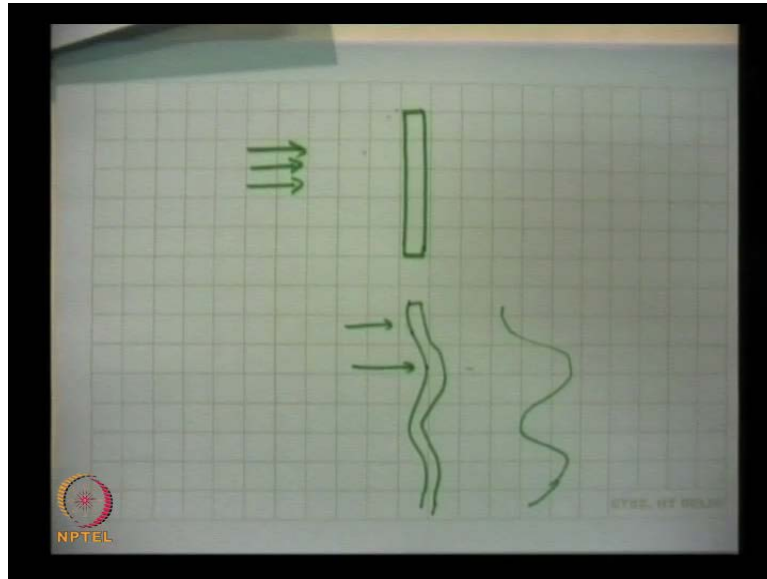
And however, what happens that you may ask a question that if the ionosphere is unstable to this instability, the plasma should fall down. Obviously, plasma will fall down, but, then the air neutral air will go up and again this will get ionize.

So, this is a dynamic equilibrium in the presence of the sun, that is why there is not a really fall down. In that sense plasma is coming down, the gas is going up and some sort of a equilibrium electron density is maintained in the ionosphere. But, these fluctuations are certainly there and they are responsible for causing scattering of radio waves.

And lot of work on this has been done in last several decades, and it is continuous to be important. I may also mention that in the equatorial ionosphere, there is a current called electro jet current. There is a electric field just like $\mathbf{g} \times \mathbf{B}$ drift can cause instability $\mathbf{e} \times \mathbf{B}$ drift also causes an instability. Which is similar to Rayleigh Taylor instability and that is also been observed that gives rise to shorter wave length fluctuations.

And I think with these two instabilities, we can understand lot of phenomenon in the ionosphere, I would like to also to mention that Rayleigh Taylor instability of late has been found to be very relevant in laser driven proton acceleration.

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People employ lasers to accelerate ions. For that, they take a very thin foil, very thin foil of sub micron width of a metal and shine laser. On this, the lasers pushes the electrons via the ponderomotive force or radiation pressure force. And when the electrons are pushed, they pull the ions also along with them.

But, it has been found that, if there is a ripple in the surface of this foil, then if the foil takes this form. Then the radiation pressure which is falling in here is different than here. The reason is that, this is curved when the laser foil, this aluminum foil or material foil is curved. The radiation pressure force exert in different, different positions.

The positions like these, they have not acquire larger radiation pressure force and these ranges less and consequently, this foil acquire the shape. So, the perturbation grows and this is also known as Rayleigh Taylor instability. And I think, it is one of the major concerns in laser driven proton acceleration at the moment. And I think, if time permits during our course of lectures, we will have some discussions on laser driven proton acceleration and the relevance of Rayleigh Taylor instability in there. I think I close at this point.

Thank you very much.