

Plasma Physics
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Lecture No. # 17

Free Electron Laser: Wiggler Tapering and Compton Regime Operation

Well this will be probably my last lecture on free electron laser. In this lecture, we will discuss the energy gain by the laser radiation from electron beam in a conventional rather untapered free electron laser. And then the enhance energy enhancement by tapering the wiggler and also, we would like to view free electron laser from the prospective of a stream electron component scattering.

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We shall discuss:

- Energy gain in untapered FEL
- Tapered wiggler
- Potential energy buckets
- Phase space trajectories
- Energy gain by radiation
- Compton regime operation

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We shall essentially discuss: the energy gain untapered FEL that we were discussing last time, but we could not complete the discussions we will continue with the discussion. And we will go over to see apart a tapered wiggler is and obtain potential energy buckets due to the ponder motive force created on the or accelerated on the electrons by the laser field and the wiggler magnetic field. Whose, wave number is a function of position and then what kind of phase space trajectories electrons would have in that kind of ponder

motive force and what is the estimate of energy gain by radiation. And then we will talk about Compton regime operation.

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$v_0 \hat{z}$
 $\vec{B}_w = A_w(\hat{x} + i\hat{y}) e^{ik_w z}$
 $\vec{E}_L = A_L(\hat{x} - i\hat{y}) e^{-i(\omega t - k z)}$
 $\vec{B}_L = i \frac{k}{\omega} \vec{E}_L$
 $\vec{v}_p = \frac{\omega}{R + k_w}$
 $\vec{F}_p = -e \vec{v}_w \times \vec{B}_L \parallel \hat{z}$
 $\gamma_r = \frac{1}{\sqrt{1 - \omega^2 / k_p^2 c^2}}, k_p = R + k_w$
 $\gamma = \gamma_r + \Gamma$
 $\psi = k_p z - \omega t$
 $\frac{d\Gamma}{dz} = \frac{F_{pz}}{mc^2}, \frac{d\psi}{dz} = k_p - \frac{\omega}{v_2}$

Well; let me very briefly review that we are talking about the propagation of an electron beam in the interaction region of a free electron laser, which has a wiggler magnetic field and which also has a radiation field. And the electron energy which is resonant the electron velocity, which is resonant with the beam particles is defined as v with the ponder motive wave is called $B P$, which is equal to ω upon k plus $k W$ please remember. That we are talking about the wiggler magnetic field of this form $B W$, which has some amplitude $A W$ polarize in the right handed sense and has a phase variation as exponential $I K W Z$.

And the laser field is left circularly polarized, which is equal to $A L x$ minus $i y$ exponential minus $i \omega t$ minus $k z$. So, in presence of these two fields obviously, the laser also is a magnetic field, which is $B L$ which is equal to $i k$ upon ω into $E L$. So, in presence of these fields the electron which is injected from outside with a velocity v_0 in the z direction, this acquires velocities $v W$ and $v L$ and the Lorentz force due to cross field terms. Essentially $F P$ on the electron is caused by minus $e v$ due to the wiggler cross magnetic field of the laser this force is parallel to z axis, which is the direction of propagation this is my z direction.

So, the electrons experience a force and in presence of this force we define a quantity called gamma r. If the electron is moving with a velocity equal to the phase velocity of this force, which is the phase velocity of this force then, I call this energy equal to gamma r, which I defined as one upon under root of 1 minus omega square by k P square c square. Where k P I defined as the sum of the laser wave number vector plus the wiggler wave number. So, gamma r certainly is a quantity quite large. If omega upon k P is very close to c. Then we defined a quantity gamma rather we expressed particle energy gamma as gamma r plus some small departure from this value.

And we wrote down two equations one governing gamma, which was d gamma by d z is equal to F P z over m c square this is the energy equation. And one was the equation governing the evaluation of phase of the wave, phase of the wave, we defined as k P z minus omega t and we wrote down an equation force d psi by d z and, which was like k P minus omega by v z, then we wrote down v z in terms of capital gamma. So, finally, we got two coupled equations, one for particle energy gamma and then phase of the wave as seen by the electron beam and these equations in dimensionless forms were of this form.

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Handwritten equations on a grid background:

$$\frac{dP}{dz} = -A \cos \psi$$

$$\frac{d\psi}{dz} = P$$

A vector diagram showing a right-angled triangle with hypotenuse P , vertical side P_0^2 , and horizontal side $2 \frac{A P}{P_0^3}$.

$$P^2 = P_0^2 + 2A (\sin \psi_{in} - \sin \psi)$$

An NPTEL logo is visible in the bottom left corner of the image.

Let me just write this (No audio from 06:59 to 07:07) d psi by d x i is equal to P and the other equation was d P by d x i equal to minus A Cos psi. These two equations, we combined together and we deduce a quantity an expression for P in terms of psi and that equation was of this form. P was equal to P 0 square minus twice A P rather a not a P is a

upon P_0 cube into no I am sorry this is not right let me delete this the equation that we deduced was P square is equal to P_0 square plus twice $A \sin \psi_i n$ minus $\sin \psi$ to the power half; where P_0 is the initial energy with which the electron initial value of P with, which the electron is launched into the machine.

A is the normalized amplitude of the ponder motive force, $\psi_i n$ is the initial phase of the wave as seen by the electron when it is injected to the interaction region and ψ is the phase of the wave at an instant of time as seen by the particle. And I mention that we can solve this differential equation for ψ and this equation for P , which is a major of particle energy iteratively. So, what I do that I take the square root of this it is not a square root here; this is just this quantity. So, we take the square root to the right hand side and express P in terms of or powers of A and that we did last time.

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$$P = P_0 - \frac{A}{P_0} (\sin \psi - \sin \psi_{in}) - \frac{1}{2} \frac{A^2}{P_0^3} (\sin \psi - \sin \psi_{in})^2$$

$$\frac{d\psi}{dz} = P$$

To the zeroth order (in powers of A)

$$P = P_0, \quad \psi_0 = \psi_{in} + P z$$

So, what we obtained was this. Let me write on the separate sheet. P is equal to P_0 minus A upon P_0 within bracket $\sin \psi$ minus $\sin \psi_i n$ minus half A square upon P_0 cube $\sin \psi$ minus $\sin \psi_i n$ square. What you have to do? The equation that we have to solve along with this one is $d \psi$ by $d x$ is equal to p . First we ignore A terms here so, to the zeroth order in powers of A is the amplitude of the ponder motive force. So, if ponder motive force is not there, if A is 0 then P equal to P_0 and if I use this P equal to P_0 then, you can differentiate this equation integrate this equation to obtain ψ is equal to

psi initial plus P times x i. This is what you will get? Here I presumed that at x i equal to 0 or z equal to 0 the phase of the wave as seen by the particles psi i n.

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To the next order
 use $\psi = \psi_0$ in P
 we get

$$P = P_0 - \frac{A}{P_0} [\sin \psi_0 - \sin \psi_{in}]$$

$$\psi = \psi_0 + \psi_1$$

$$\psi_1 = \frac{A}{P_0} \left[\frac{1}{P_0} \{ \cos \psi_0 - \cos \psi_{in} \} + x \sin \psi_{in} \right]$$

And then we go over to the next order; let me call this result as psi 0. So, this is the zeroth order P and zeroth order psi when effect of wave ponderomotive wave is not there on particle trajectory. To the next order (No audio from 12:23 to 12:31) we use the value of psi equal to psi 0 in the expression for P use psi equal to psi 0 in P expression for P and we get P is equal to P 0 minus a upon P 0 sin psi 0 minus sin psi i n. And if you use this value of P in the psi equation you will get psi 1 psi becomes then psi 0 plus psi 1; where psi 1 turns out to be equal to a P rather A upon P sorry, not a P A upon P 0 multiplied by 1 upon P 0 into Cos psi 0 minus Cos psi i n plus x i sin psi I n.

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To the second order in A^2

$$P = P_0 + P_1 + P_2$$

$$P_2 = -\frac{A^2}{2P_0^3} (\sin \psi_0 - \sin \psi_{in})^2 - \frac{A}{P_0} \cos \psi_0 \cdot \psi_1$$

$$\psi_0 = \psi_{in} + P_0 z$$

Normalized energy gain by radiation

$$= \int_0^{2\pi} -P(z=1) d\psi_{in} / 2\pi$$

$$= -\int_0^{2\pi} P_2(z=1) d\psi_{in} / 2\pi$$

Then we go to the second order, and when you go to the second order in a square. When you retain these terms P can be written as P_0 plus P_1 plus P_2 . P_1 and P_0 I had already given. So, P_2 turns out to be from this expression for P square you get P is equal to P_2 is equal to minus a square upon twice P_0 cube into $\sin \psi_0$ minus $\sin \psi_{in}$ square minus A upon P_0 $\cos \psi_0$ into ψ_1 . This sort of expression you get for P_2 . This is an important thing where ψ_0 is related to the initial phase ψ_{in} plus initial normalized energy into distance of propagation, normalized distance of propagation.

So, we have really got the particle sort of energy as a function of x , x_i is coming in a little complicated fashion through x_i^0 there is no explicit dependence on ψ . It is coming slightly in a complicated fashion through ψ . So, x_i dependence or z dependence of P_2 is through ψ_0 or x_i . Now, the issue is I want to find out what is the energy gain of the electron or energy loss of the electron net energy loss by electron. So, net rather I would like to find out normalized energy gain by radiation. If I treat P as some sort of a normalized energy with respect to γ_r then the normalized energy gain by radiation is the same thing as the degree initial energy of the electron minus P at a value x_i is equal to 1 because x_i equal to 1 means z equal to 1.

So, this is the particle energy in the beginning this is the particle energy this is end so, this is energy going to radiation. And I would like to obtain this quantity as an average over all the particles electrons that are entering the interacting chamber, between phases

0 to 2 pi. So, psi i n is varying from 0 to 2 pi. So, what I should do? I get this quantity integrated over d psi i n and integrate from 0 to 2 pi and divided by 2 pi. This is the normalized average energy gain by radiation per particle when it is averaged over all the phases of the particles.

So, this should be within the bracket this is P 0 minus P evaluated at x i equal to 1. I can rewrite this expression please note here if I put the value of P here P 0 minus P 0 will cancel out P 1 if you evaluate the expression for P 1 d psi i n is time average is 0 if you just look at the expression for P 1. And if you this is only P 2 that survives here and that gives, rise to finite energy gain by the radiation that is why I have to go to evaluate P 2.

So, it is this term, which is important in energy transfer from particles to wave and this turns out to be simply equal to put this here, this is P 2 with a negative sign of course, and of I am evaluating this x i is equal to 1 multiplied by d psi i n upon 2 pi and 0 to 2 pi. There is a negative sign already in minus outside here and these terms and you can take care so these become positive. And let us evaluate this quantity and let us see what do we get?

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Energy gain (Normalized) by radiation

$$\langle E \rangle = \frac{A_0^2}{P_0^3} \left[2 \sin^2 \frac{P_0}{2} - P_0 \sin \frac{P_0}{2} \cdot \cos \frac{P_0}{2} \right]$$

$$\frac{P_0}{2} = x = \frac{L k_p (\gamma_0 - \gamma_r)}{2 (\gamma_r^2 - 1) \gamma_r}$$

$$\langle E \rangle = \frac{A^2}{8} G(x)$$

$$G(x) = -\frac{d}{dx} \left(\frac{\sin^2 x}{x^2} \right)$$

It peaks at $x = 1.5$, $G = .6$

This can be done easily and the result turns out to be the energy gain by radiation energy gain normalized by radiation. This quantity is let me call energy gain may be delta e per particle. It turns out to be A P square upon P 0 cube and 2 sin square P 0 by 2 minus P 0 sin P 0 by 2 into Cos P 0 by 2. And the interesting thing is that this quantity can be

expressed in a compact form if I define P_0 by 2 as X suppose, I define a quantity X like P_0 by 2. Then the energy gain turns out to be equal to actually, this is normalized energy gain so let me put a prime somewhere because it is normalized quantity so put a prime there.

And this turns out to be equal to A^2 by mistake I have put a P , this is simply A , the amplitude square by 8 into G of function X where, G of x is $\sin^2 X$ upon X^2 . This is a very interesting function and if you plot this as a function of X please remembers X is the difference of initial X I think it will be useful if I write this X explicitly in terms of energy. This turns out to be equal to $L k p$ into γ of the electrons in the beginning minus the resonant γ divided by 2 γr^2 minus 1 into γr . So, this is the departure of electron initial energy from the resonant energy. This is the essentially the energy available sort of energy available with the particle if you multiply by $m c^2$.

So, this is the important parameter on, which this gain factor G depends. How much is the value of G ? If I plot this, it turns out to be like this, Plot G here and X there. The values may be like 2, may be here 4, may be there minus 2, here minus 4, there this function is 0, when this is 0 and the peak value is around 0.6 so, it is 0.6 and this is minus 0.6 here. So, this function has a peak of 0.6 around 1.5 so this is and this becomes 0 around three this goes like this the sort of behavior it gets. So, this function gain is positive only when X is positive and its peaks at a value of X around 1.5. It peaks at X equal to 1.5 and the maximum value of G at that point is around 0.6.

The net energy gain by radiation single particle energy gain will be proportional to the square of laser amplitude square of the wiggler amplitude and it will be this factor will be around 0.6. If you have chosen your initial energy of the electron beam in such a fashion that this quantity is around 1.5. That is a very important deduction from this analysis that one should always have a beam moving faster than the ponderomotive wave. So, that γ_0 is bigger than γ_r and these are the normalized parameters. You should choose that value now, in oscillator what really happens? Or even in amplifier what happens? As the radiation grows at the expense of beam energy because radiation is getting energy from millions of particles. So, it is amplitude really grows up.

Then A evolves this amplitude A evolves with distance and in this formalism we have not consider that so, people carryout this calculation numerically and then they can get the energy gain. And by this technique you can have energy gain typically of a few percent in a free electron laser may be percent or few percent of that of that order. In order to increase the efficiency of the free electron laser, you need to device ways to retard the electrons more efficiently, then by simply the ponder motive wave. Now, what can happen? We can get a clue for this in one way that the ponder motive force traps the electrons.

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Pond. wave $v_p = \frac{\omega}{k + k_w(z)}$

A tapered wiggler $\vec{B}_w = A_w (\hat{x} + i \hat{y}) e^{+i \int k_w dz}$

$\gamma_r = \frac{1}{\sqrt{1 - v_p^2/c^2}}, \quad \frac{d\gamma_r}{dz} > 0$

And if you can slow down the ponder motive wave, now, ponder motive wave can slow down means, what ponder motive wave moves with a phase velocity v_p which is ω upon $k + k_w$. If you can have a wiggler whose wave number can be modulated can be reduced with z wave number can increase this z suppose, I make this a function of z increasing function of z then v_p will go down. So, what will happen? This is the ponder motive wave which traps particles somewhere here and if you can slow down the ponder motive wave by making this ripples closer. So, the wave number increases then, the electrons they which are trapped by this wave they will be forced if you can trap the electrons in potential at the minima.

And those electrons are then retarded by force. So, by retarding the ponder motive wave by making k_w as a function z increasing function of z , you can get much higher

energies. And this scheme of retarding the ponderomotive wave is known as tapering and this kind of FEL is called tapered free electron laser. So, we would like to find out. What is the effect of making k_W as a function of z ? I do not think I am going to details of this, but the physics I can reveal in a simpler way. Consider a tapered FEL; tapered wiggler. I am choosing k_W as a function of z , I can write down the wiggler field as B_W is equal to some amplitude $A_W \cos(k_W z + \omega t)$ and this I will write down as $B_W = \int k_W dz$.

Because this is a function of z so this some sort of $A_W k_W B_W$ representation for the wiggler field. Some people make wiggler tapered by making A_W also is a function of z , but I will consider for the sake of simplicity in physics, let me consider this. Then what is happening? The resonant energy that we define γ_r this be function of z because γ_r we defined as $\sqrt{1 - v_p^2/c^2}$. So, whenever, the phase velocity of the ponderomotive wave is a function of z increasing rather v_p is a decreasing function of z γ_r becomes an increasing function of z .

And you can choose this. So, in a tapered wiggler FEL $d\gamma_r/dz$ is bigger than 0 we choose a finite value. So, what is happening? You want that the ponderomotive potential energy. Well; should be able to localize or confine the particles and then the entire potential energy minima are slow down, this is like trapping water in the bucket. In a bucket if you have something and slow down the bucket then the energy from the particle will go to the force that is retarding and that the beauty here.

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$$mc^2 \frac{d\gamma}{dt} = -e \vec{E}_L \cdot \vec{v}_w$$

$$\Gamma = \gamma - \gamma_r$$

$$\gamma = \gamma_r + \Gamma$$

$$\frac{d\Gamma}{dz} = \frac{F_{pz}}{mc^2} - \frac{d\gamma_r}{dz}$$

$$\frac{dP}{d\xi} = -A \cos\psi + \alpha_0, \quad \frac{d\psi}{d\xi} = P$$

So, you want to retard the particles and the equation of motion would be something like this, $d\gamma/dt$ this is the energy equation with mc^2 is equal to minus $e \vec{E}_L \cdot \vec{v}_w$. This is the energy equation that we have been writing for free electron laser. We define a quantity γ as, γ minus γ_r , but treat this γ_r as a function of z . So, what I am writing is γ , I write in this equation as γ_r plus capital γ then this equation for capital γ takes the following form, $d\gamma/dz$ turns out to be equal to ponderomotive force upon mc^2 . This is the z direction minus $d\gamma_r/dz$ this I have to write explicitly; here this is the additional term that you get here.

Subsequently, when you normalize your quantities and write this equation in terms of a quantity P by $d\xi$ this equation is minus $A \cos\psi$ plus there is an additional term because of this because $d\gamma_r/dz$ is positive. So, this term out something like minus α_0 . If the wiggler is not tapered then this is 0, but, because if this is tapered then this turns out to be like this. And the equation governing ψ is $d\psi/d\xi$ is equal to P .

So, basically, there is a change that occurs in this set of equations through this quantity α_0 . You can combine these two equations to obtain $dP/d\xi$ and integrate that equation and what you get is this. So, when you integrate that equation you obtain I made a mistake here, this γ_r really I want a decreasing function of z ,

because I want gamma the particle ponder motive force phase velocity to be decreasing so, I want d gamma by d r to be negative hence this becomes positive; I made a mistake in there.

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$$\frac{P^2}{2} + A \sin \psi - \alpha_0 \psi = H$$

$$H = \frac{P_{in}^2}{2} + A \sin \psi_{in} - \alpha_0 \psi_{in}$$

$$V(\psi) = A \sin \psi - \alpha_0 \psi$$

$$\frac{dV}{d\psi} = 0$$

$$A \geq \alpha_0$$

So, when you solve this set you get as the integral P square by 2 plus a sin psi minus alpha psi is equal to constant of integration, which I called H and this can be evaluated by using the initial conditions at some point. When at the entry point psi as psi i n P is P i n or P 0, then, you get H is equal to P initially square upon 2 plus A sin psi initial minus alpha psi initial. This equation again represents trapped particles, but the characters quite different now. First of all you can consider this to be some sort of a kinetic energy of a particle and this some sort of a potential energy of a particle, this some sort of a Hamiltonian.

So, this quantity is denoted by V a function of psi, which is equal to A sin psi minus alpha this alpha 0; actually, I made a mistake this alpha 0 into psi. If I plot this quantity as a function of psi plot V psi as a function of psi, this turns out to be having the sort of shape. These are maxima and minima in the potential energy as a function of psi. The particles can be trapped between these, potential energy minima around this potential energy minima and when this potential energy minima is slow down as the entire potential, this bucket is slow down. The energy goes from particles loose energy and

after all this potential energy is essentially, is a function of laser amplitude. So, the energy really goes into laser.

So, you are forcing the electrons to slow down and the condition for trapping turns out to be the particles should be trapped. That there should be minimum here, in potential energy curves only the particles will be trapped. And the condition turns out to be if we want $d\psi/dx = 0$ for minima to occur then you require that ω should be bigger than or comparable to α_0 . This is a necessary condition. So, if you can choose a bucket or laser field of finite amplitude then this condition can be satisfied. And the electrons that are trapped in these buckets, those electrons will be retarded forcibly by the retardation of the ponderomotive wave and then the energy that they gain is of this order.

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Energy gain efficiency

$$\eta = \frac{\Delta\psi}{2\pi} \frac{\gamma_r(x_0) - \gamma_r(x_1)}{\gamma_r(x_0) - 1} + \dots$$

$\lambda_L = \frac{\lambda_W}{2\gamma_0^2}$, $\lambda_L = \frac{\lambda_0}{4\gamma_0^2}$

The image also contains a small diagram of a potential well with a wavy line representing an electron's path and a double-headed arrow indicating the width of the well, labeled $\Delta\psi$. An NPTEL logo is visible in the bottom left corner of the slide.

Let me give you final result about energy gain. The energy gain turns out to be typically, of this order energy gain efficiency actually, this is defined as the energy gain by radiation divide by the initial kinetic energy of the of the electron beam. And this turns out to be $\Delta\psi / 2\pi$ and γ_r at the entry point minus γ_r at the exit point where x_i is equal to unity divided by γ_r at 0 minus 1. If we multiply by $m c^2$ this is the initial kinetic energy of the electron beam. This is the if the electron beam I consider to be moving very close to the velocity v_p . And this is the energy is has lost. So, this is the efficiency $\Delta\psi$ is the spread the width of the bucket, as I showed

to you that potential energy curve goes like this. So, this is the width this is the width $\Delta\psi$ from here to here is $\Delta\psi$.

So, you get a significant number of this $\Delta\psi$ by 2π is a significant quantity like may be 0.2, 0.3 and this, could be significant and efficiency can be achieved to be to the tune of something like 30 percent. To this, you should also add the efficiency due to P 2 effect that I mentioned earlier in a plus the efficiency due to uniform or untapered wiggler plus some more. But, this is the prime term so by trapping a significant number of electrons in potential energy minima of the ponderomotive force, you can extract much higher energy by tapering the wiggler. So, I think this is one thing that I wanted to discuss with you. That tapered free electron laser is a very fascinating concept and it has improved the efficiency of radiation generation a great deal.

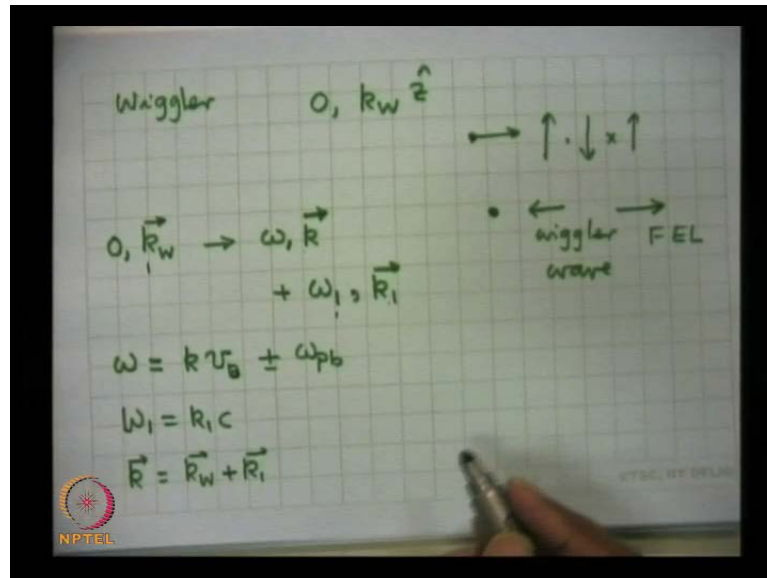
Well; there are many variations of this device, the conventional or the major threat of free electron laser search is employing magnetic wigglers. However, people can use different kind of ripples or waves as wigglers. For instance, people are thinking of having a x-ray free electron laser. Now, the problem is that for x-ray generation you require the x-ray, the laser wavelength that you generate is equal to wiggler wavelength divided by $2\gamma^2$. So, if you want to produce a short wavelength you should require either, a very short wiggler period or very large beam energy. γ is the Lorentz factor of the electron beam so, if you multiply mc^2 it is the energy.

Now, that is a very serious matter. You cannot have a conventional wiggler less than a few millimeter wiggler periods. However, if you can choose a laser for this purpose in that case it turns out that, because laser has a frequency and wave number both so from momentum conservation conditions, you can find an energy conservation you get λ of pump laser divided by $2\gamma^4$. And this could be a very short wavelength because $\gamma\lambda_0$ could be like 1 micron so this is a one interesting thing. Secondly, as for as generation of millimeter waves some millimeter waves is concerned, terahertz wave is concerned, one can use a gyrotron as a wiggler.

Gyrotron can produce microwaves of very large amplitude and use them as a wiggler. One can also use a density ripple. Some people have examined the generation of terahertz radiation in a plasma with density ripple then ripple can also act as a wiggler. The thing is that you need to compensate for the momentum and energy balance or energy

and momentum loss by the particle. And that can be done in a variety of ways, though the most effective way today is to employ a magnetic wiggler. But other schemes are also coming up. Now, before I close I would like to mention that one can view, the generation of radiation by free electron laser as a parametric instability also.

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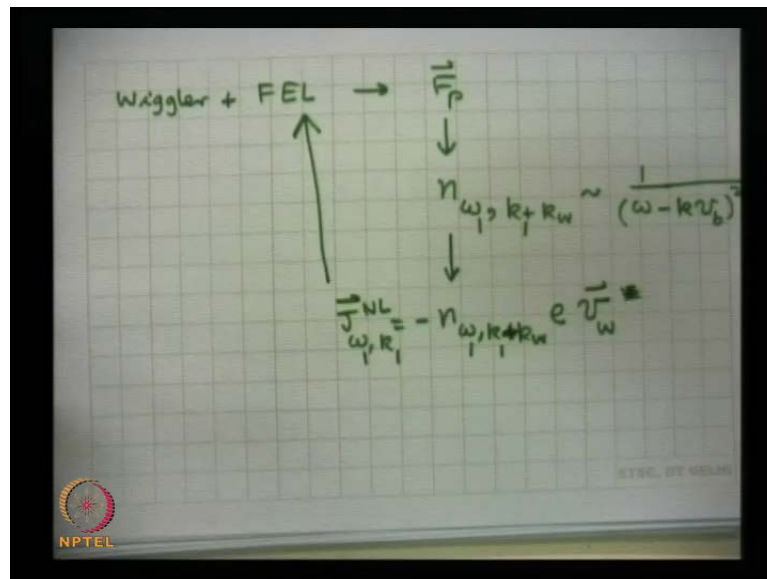
What you can do? You can treat the wiggler as a wave of frequency 0 and wave number k_w . But to a moving electron this will look an electromagnetic wave traveling backwards so suppose, there is a wiggler like this and an electron is moving in this direction. To this electron, this wiggler will appear like an electromagnetic wave traveling in the backward direction. And Compton is scattering stimulator Compton scattering of that wave can produce a radiation in the opposite direction. So, to this electron the wave is coming like this is the wiggler, which appears like a wave and electron beam is stationary and Compton scattering of these, wiggler wave photons in the backward direction will produce F E L.

Well, analysis of this processes is not difficult and let me just give an outline of how to handle this problem. Suppose, I consider my wiggler of frequency 0 and wave number k_w as a quasi photon of 0 energy and finite momentum, it is exciting a beam space charge mode of wave number ω frequency ω wave number k plus it produces, a free electron laser photon or radiation of frequency say ω_1 and wave number k_1 . This,

my radiation mode, this my space charge beam mode. And beam space charge mode is something like that this is equal to typically $k v_{\text{beam}} \pm \omega_p$.

This is the beam plasma frequency, this is the beam velocity, which I actually is v_0 I should call it v_0 . And this radiation mode has $\omega_1 = k_1 c$. The connection is that this k_1 is the or this k is the difference of this and this so actually, we have k is equal to $k_W \pm k_1$. So, what happens? The radiation wave and the pump wave; both these waves, this pump wave means wiggler they produce oscillatory velocities to electrons, they also exert a ponderomotive force at this frequency. So, the dynamics of the instabilities like this.

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That wiggler plus F E L radiation they produce a ponderomotive force and the ponderomotive force gives rise to density bunching compression rarefaction of electrons. So, this produces radiation bunching at frequency, ω and wave number $k \pm k_W$. And this density bunching has a very strong this produces a strong field and this gives rise to a non-linear current $n_{\omega, k \pm k_W} e^{jv_w}$ the electron velocity due to the wiggler. So, and this produces a current at frequency ω and wave number $k \pm k_W$, this is ω is the same as the ω_1 .

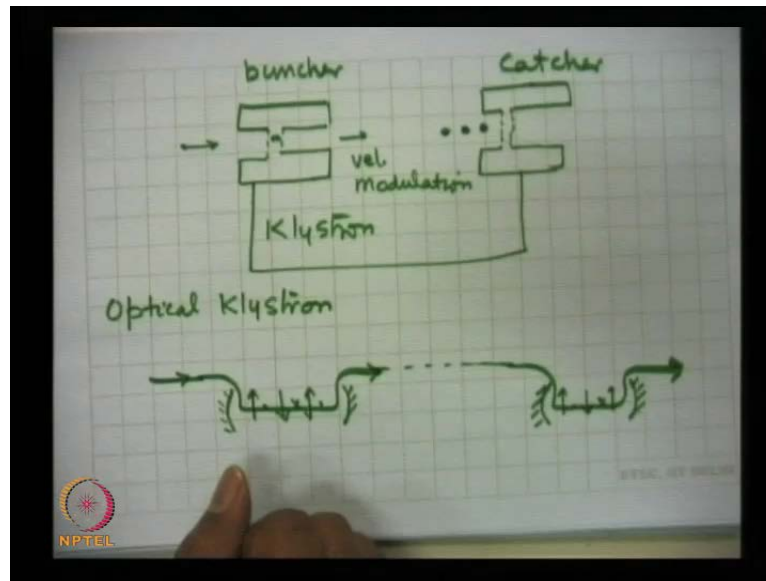
So, and this is k_1 let me just write down this is a k_1 and ω_1 here $\omega_1 = k_1 c$ there and this is minus k_W actually, and sorry this is $k_1 \pm k_W$ and $k_1 \pm k_W$ not the complex conjugate; this is same thing. And this produces a radiation a non-linear

current at the frequency and wave number of the radiation and that drives this. So, that is a interesting thing in here. This is cyclic process; a feedback process in, which the wiggler and free electron laser radiation they exert a force on the electrons. That force drives a density perturbation, which is in resonance phase, resonance with the electron beam. And this is quite huge very small value of ponder motive force can produce very large density perturbation which goes as $1 \text{ upon } \omega - kv_{\text{beam}}$ whole square.

So, this could be very huge and this couples, with the electron velocity due to the wiggler to produce a non-linear current at the frequency and wave number of the laser and then they produce radiation. So, by this mechanism both the density fluctuation will grow as, well as the amplitude of the F E L wave will grow. This process is known as a Compton scattering and we have already learnt the two stream instability. In two stream instability also a beam drives a plasma wave via Cherenkov interaction. And this is the Cherenkov resonance that factor that appears, in the denominator for the density perturbation and that gives rise to very strong growth of radiation signal.

Well, lot of work on this process has gone. When the density of the electron beam is very large then self consistence or a space charge field of the this space charge mode should also be taken into consideration. And then the operation of free electron laser moves on to remain region. And I think there is lot of literature available on that topic. Before the end, I would like to mention that a device very similar to klystron has come up based on the concept of free electron laser.

(Refer Slide Time: 50:51)



In a klystron, what you have? You have a buncher cavity like this, and you have a catcher cavity. (No audio from 51:01 to 51:14) What happens you launch an electron beam through here? If there is a small signal here, then that signal will modulate the beam velocity. So, when the beam emerges out here so, if there is a r f signal in this gap that will accelerate some particles and after half the wave period will retard some particles. So, you are getting a velocity modulation here, in a klystron; this is a buncher cavity, this is a catcher cavity.

So, when you launch an electron beam in such a cavity and if there is a small noise signal here then that will modulate the electron beam velocity. Now, the slow moving electrons that have left this cavity earlier, they will be caught by the fast moving electron that come half wave period later and they will bunch. So, that will create density bunches here, that pass through the gap. So, density bunches are created like this, and they pass through this gap, they produce a huge electromagnetic wave at the frequency, same frequency at which the noise was here.

So, if you can couple this wave guide to this, then part of the signal will go from here and that will be more than the initial signal, and by this short feedback, you can have a net, transfer of d c energy of the electron beam into radiation energy into the catcher cavity and buncher as well. Now, this scheme has been utilized in the concept of optical klystron. The concept is as simple; this buncher cavity the klystron has a limitation that it

can operate only up to frequency like 10 gigahertz. If, for example, I want to produce radiation in the optical range, what should I do? I should reduce the buncher gap, but you cannot reduce it beyond a limit.

So, what people found was to replace this entire thing by some sort of an open resonator and bring a wiggler here. So, if I have a wiggler here and this catcher replaced by another wiggler, see these are the mirrors for open resonators, this is a wiggler here. And launch an electron beam. Suppose, I bring an electron beam here and by using magnets you make it move inside and let it go out here. And then when it goes here, again make it bend here. What is happening here? You are having an electron beam coming from here. The beam enters the first cavity where there is a wiggler and there is a small radiation signal. So, that will both of them exert a ponderomotive force on the electron beam and will change its velocity or modulate its velocity.

So, when the velocity modulated beam arrives in this space this is called drift space. Then these electrons which are fast moving they will catch up the slow moving electrons, which had left this cavity earlier. So, there is a charge bunching and those bunches of charge when they move through the second cavity they will produce much enhanced radiation here, and you can get optical amplification and this is a very fascinating device. So, here you have a split free electron laser into two segments a buncher cavity and a catcher cavity and I think this has a lot of promise.

So, I think I would like to conclude by saying that free electron laser has a lot of promise for terahertz generation, for infrared radiation generation as well as, for higher frequencies in the visible, in the ultraviolet and even x-rays, x-ray generation. And the main requirement is the electron beam, because the energy is coming from the electron beam. So, lots of efforts are being put into developing good quality high current, high voltage electron beams at reasonable cost. The two areas are really directly coupled with each other; the accelerators charge particle accelerators and coherent radiation generation. Thank you.