Plasma Physics

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Module No.# 01

Lecture No. # 16

Free Electron Laser: Energy Gain

Today, I will discuss the energy gain in a free electron laser in my last lecture. I introduced free the electron laser as a very fascinating device that produces tunable coherent high power radiation at the expense of energy of an electron beam. And now, we would like to make some estimate of how much energy is transferred from electron beam to radiation. We shall discuss the following issues. The Ponderomotive force that is exerted by, the regular magnetic field and the radiation signal on the electrons. Andthat causes bunching of the electrons in the retarding phases of the Ponderomotive wave. And then we like to learn how the electron energy and the phase of the Ponderomotive wave. As seen by the electron, evolve in time or in a space as the beam travels. We shall also discuss electron trapping in these potential energy minima, and have some estimate of energy gain.

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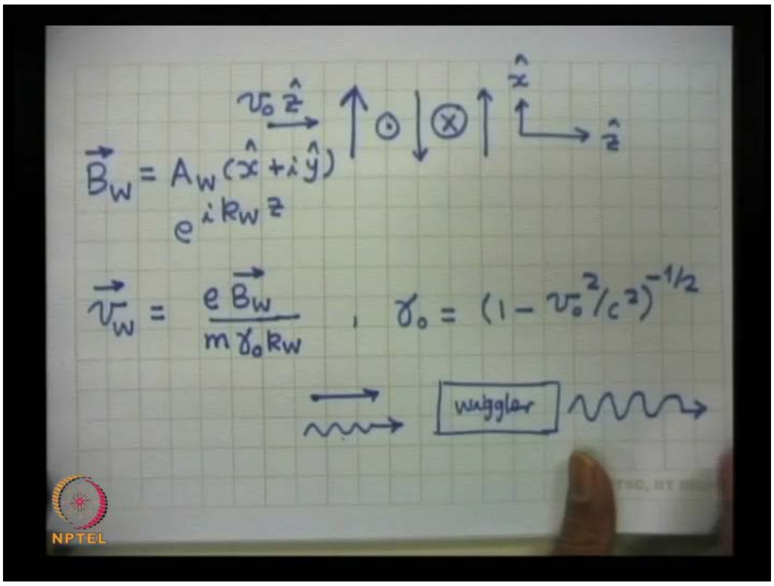

Well let me revisit a few things, we considered the propagation of an electron beam in a regular magnetic field. And a regular magnetic have lines of force like this, this was vertical magnetic line of force this is perpendicular to the plain of the paper this is downwards this is like this. This I call as the helical regular and magnetic regular has a magnetic field given by B w is equal to some amplitude A w x cap plusiy cap into exponential ofik w z. This is called a right circularly polarized regular. And we are launching an electron beam into the system like this. With the velocity v 0 and I call the direction of beam propagation as z axis like this is z axis. I can call the vertical direction as x axis.

So, when the electron beam travels through a regular magnetic field it acquires a velocity that we deduced last time. This turns out to be equal to v w is equal to charge of the electron. Magnitude of the charge of the electron into B w upon m mass of the free electron rest mass of the free electron gamma 0 into K w. Where gamma 0 is the relativistic gamma factor related to electron velocity v 0 by this relation, gamma 0 is equal to 1 minus v 0 square by c square to the power minus half. This is the important quantity when the electron velocity approaches c gamma 0 is quite large.And that is the case ofour interest.

So, the regular velocity these days, thatthe charged particles can acquirevelocity could be comparable to some fraction of c. Andin addition to the regular we launch an electromagnetic wave called seed signal or laser signal. So, what we have we have a laser signal that you want to amplify, we have an electron beam both are launched into the regular region this is my regular region. So, this is my regular.And I expect that there will be a large output here.The laser signal should get amplified the laser field that I wrote down last time was of this form.Let me write down on a next sheet.

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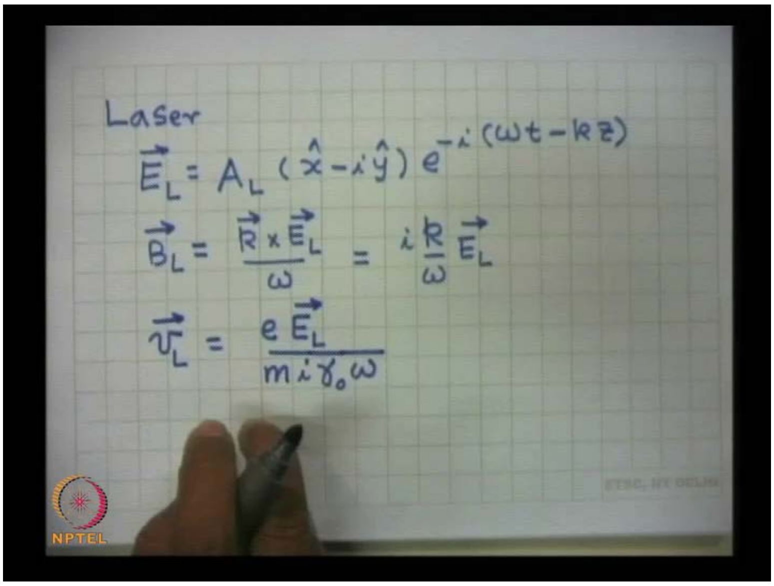

So, the laser fieldwas something like E L which wasleft circularly polarized,x minusiy exponential minusiomega t minus k z. This is a purely transverse electromagnetic wave with phase velocity omega by k almost nearly c.No electron can be is in phase synchronism with this wave because no electron can move with this velocity. It is a fast wave device in which omega by k is close to c. And that is inaccessible this velocity is inaccessible, by any charged particle. This wave also has a magnetic field B L which is equal to k crosse from Maxwells equation upon omega. And if you simplify this put the value of k as k z cap and simplify this expression it turns out to be equal toi, k upon omega into E L.

See your laser has electric field magnetic field.It imparts in oscillatory velocity to electrons also which turns out to be v L which is equal to charge magnitude charge of the electron into the laser field upon m rest mass of the electroni (( )) gamma 0 into omega. This is the electron velocity due to the laser signal.If the magnetic regular is not there, butbecause both the regular andlaser are present simultaneously. The electrons experience a force due to both of them called Lorentz force and that force is important.

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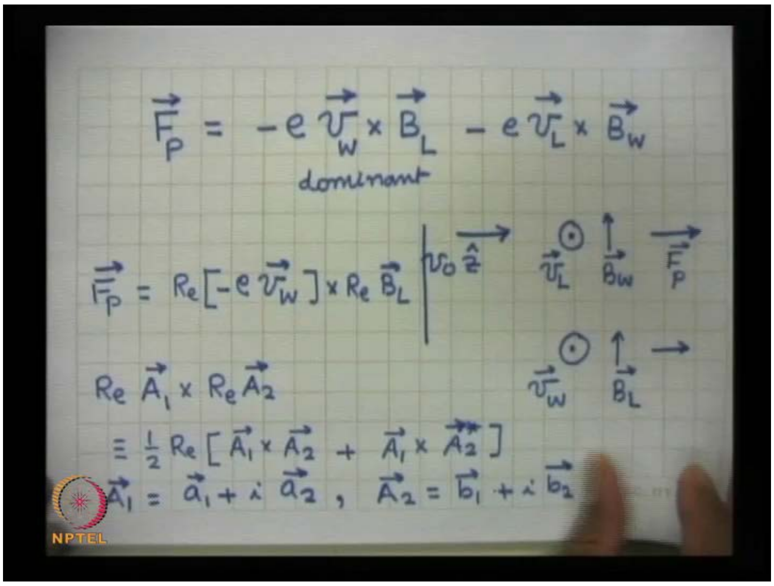

Let me write that force as (No audio from 07:17 to 07:25) F p given their name called Ponderomotive force. And that is caused by the interaction of well this is minus e into v cross b. This is called the Lorentz force, in which this is due to the regular and this is to the laser. And there is another term minus e v due to the laser and b due to the wiggler.Well,it is implied in here that isthat is understandthe origin of this, when the electrons are travelling in this direction.And they are seeing in electromagnetic wave then the electrons are acquiring a velocity. Suppose, perpendicular the plane of the board this I call as v L at some place.

And somewhere, some instant of time the magnetic field due to the wiggler is B w. They are perpendicular to each other and their cross product will have a direction this is the direction. So, this will exert a force along, the direction of propagation of the beam B L cross B w because both are transverse forces or transverse quantities.And hence their cross product will produce a z component of F p same thing happens because of this velocity sorry this isthis term similarly, this term also happens in the same way.The electron velocity due to the wiggler is v w and the laser magnetic field is like this at some location at some time. They also produce contribute to F p in the direction.

So, these twoforces are in the direction of the electron beam it is a great advantage. Number 1 that this force is in the direction of v 0 the original electron velocity and hence, this force can retard the electrons. And gain energy or this radiation which is usually the dominator here can gain energy from the field. Out of these two terms this term turns out to be stronger this turns out to be weaker. So, this is the dominant term and I can ignore the other one. For the sake of completeness or exactness one can retain the other term as well; however, for us this should be all right that I will be written this only this term.

Hereoneshould remember one thing that when I multiply velocity and magnetic field. Then I am really talking about real quantities it is a real because in our representation. We have expressed these quantities as complex numbers complex quantities. Butactual velocity the particle is not a complex quantity. It is the real part of the complex quantity its implied means, this F p primarily isreal part of minus e v w and real part of B L.This is important R e symbolizes real part of a complex quantity. And ifoneusesa complex number identity which says that if there are twoquantities a 1 and a 2.

And you want to multiply their real parts then this can be shown to be exactly equal to halfreal part of a 1 cross a 2 plus a 1 cross a 2 star, star means, complex conjugate of the quantityonecan verify this. By considering a 1 to be a 1 plus I times a 2where a 1 and a 2 are real quantities and similarly, a 2 you can write down asb 1vector plus I times b 2 where b 1 b 2 are also real quantities. Just substitute these in the in these equation and you will verify that this identity is satisfied.

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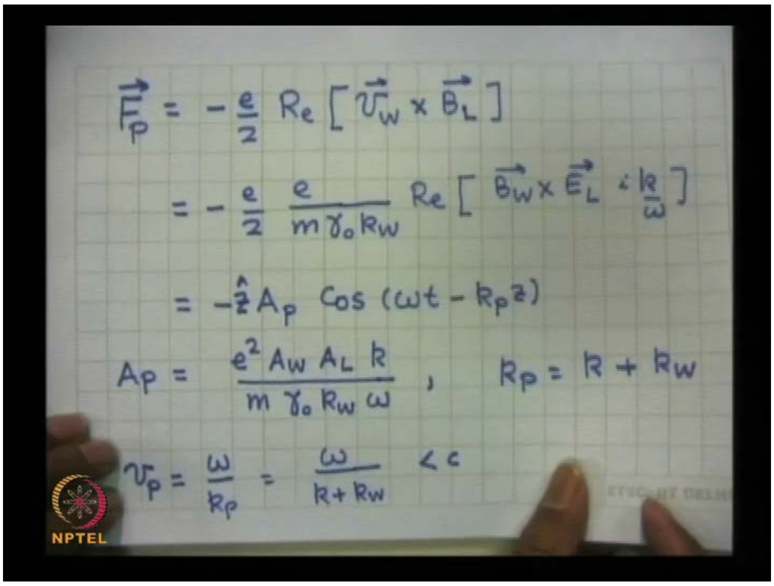

I would like to employ the same identity to evaluate this product of real parts oftwocomplex vectors.And once you do this you obtain the Ponderomotive force F p to be equal to minus e by 2 and real part of v w cross B L. (No audio from 12:56 to 13:03) The complex conjugate term in which B L star is there does not contribute because the real part turns out to be zerothis is the only term that survives. And when I put the values of this quantities the expression turns out to be put the value of v w and B L and it turns out to be minus e by 2e upon m gamma 0 K w this is the value of v w magnitude multiplied by real part of you will get here B w coming from B w cross E L intoik upon omega.

And simplify this expression it turns out to be equal to minus A p into cos omega t minus K P z where, A p is equal to the quantities from there you get is equal to e square A w A L into k divided by m gamma 0 K w into omega. So, this is a force that is x on this particle in the z direction this is in the z direction amplitude is A p. And it has a phase term here whose frequency is omega, butwavenumber is K P and K P is the sum of laser wave number plus regular wave number. You may recognize here that the phase velocity of this force which we call as v p is defined as omega upon K P which is equal to omega upon k plus K w.

Since omega by k is c this quantity is less than c this may be just smaller than c, butcertainly its less than c. And hence it is possible for the particles to respond to this force resonantly. And that is the beauty of free electron laser that is Ponderomotive force can resonantly interact with the particles. The amplitude of this quantity is proportional to the amplitude of the regular field. It is also proportional to the amplitudeof the laser field; obviously, in the device laser amplitude evolves as the laser travels.

So, 1 has to be careful in examining the energy gain by the laser. That this force will retired the particles and the force amplitude will also go up. These two things should be taken into consideration simultaneously, self consistently. However, a lot of information about energy gain can be received can be obtained by treating A L to be constant for a while. So, let us examine the response of electrons to this force. This force will give rise to will influence the z component of equation of motion.

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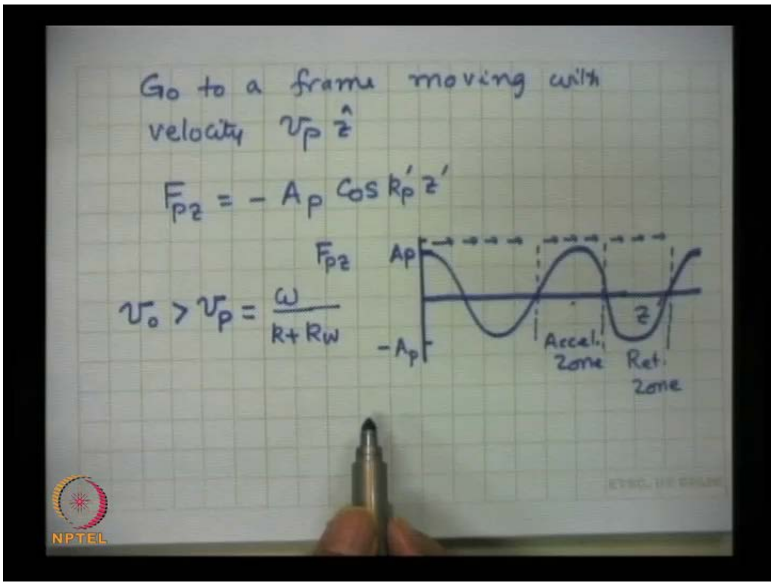

But I can doif I move to a frame of reference, go to a frame moving with the velocity of the Ponderomotive force moving with velocity v p in the z direction. Suppose I go to a frame in this direction then, the Ponderomotive force will appear likeF pz will appear as minus A p cos K P z prime where, Doppler shifted frequency of this force is 0. Because it is moving the frame is moving with the velocity of the phase velocity of the force. And this is the Lorentz transformed wave number of the force this is a sort of expression you will get. Now, in order to appreciate the particle motion in such a force let me draw a figure here.

I will plot F p as a function of z prime plot F p z here as a function of z prime. But you will get isthat suppose the maximum value of F p z is a p. So, plus A p minus A p I can put some lines here suppose, this is A p and this is minus A p here. But I can do if I plot this you will this have a periodic variation like this. (No audio from 18:50 to 18:59) You may note here that at some values of z prime F p z is positive and other values of x prime F p z is negative. The regions of values of x prime for which F p z is positive those regions are calledthe accelerating zones. So, F p this is positive here. So, I will call this as the accelerating zone accelerating zone. And this region I will call as retarding zone because F p z is in the reverse direction.

This is the force experienced by the electrons due to the combination of regular and laser fields. Now, what will happen? If I choose the initial electron velocitythen I launch the electron into the regular region v 0 slightly bigger than the Ponderomotive velocity. So, if v 0 is slightly more than the phase velocity of the Ponderomotive force in the laboratory frames. In that case in this moving frame the electron velocity will be positive.So, the electrons will appear like moving here they are going in this direction the some finite velocities. So, an electrons are moving in this frame with some finite positive velocities.

What happens?The electrons in the accelerating zone will experience a forward force and they will quickly move out of from the accelerating zone and go over to the retarding zone.The electrons in the retarding zones will be slow down by this negative force Ponderomotive force, and they will spend more time over there. As a result more electrons cross from the accelerating zones to the retarding zones and less leave the retarding zone. So, there is a net bunching of electrons, in the retarding zones means there is a net retardation of charge particles of electrons.And there is a net transfer of energy from the electron to wave if this condition is met this is a mandatory condition, necessary condition.

So, any electron beam moving faster than the phase velocity of the Ponderomotive wave which is equal to omega upon k plus K w will move will bunch in the retarding phase. This is a very important thing that is built in this concept of in this Ponderomotive force. And this is how the energy is gained by the radiation field after all Ponderomotive force basically, is the radiation field. And the regular magnetic field that are implyexert in this force. So, now what we do we like to examine the energy exchange, between the electron beam and the leaser.

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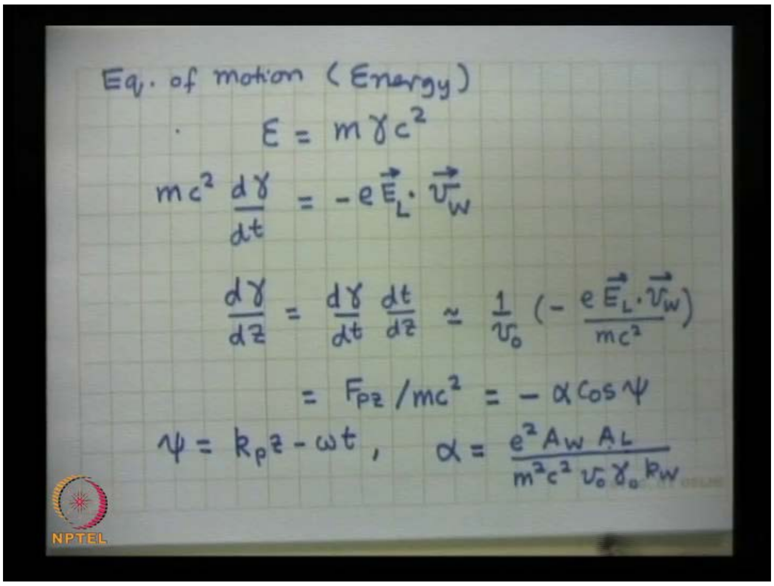

So, let us examine the equation of motion of the electron; equation of motion. I write down this equation for electron energy electron energy is actually not more if this is electron of equation of motion that there I will not write for momentum. I will write for energy. Energy of the electron is defined as rest mass into gamma into c square. So, m is the rest mass c is the velocity of light in free space gamma is the Lorentz factor. I can write this the equation of motion as or energy balance as m c square d gamma by d t. This is the rate of gain of energy by the electron per second is equal to the net electric force or that force on the dot velocity, the where the electron gains.

Now, there is onlyoneelectric field in the system that is the electric field of the laser. And this could be the velocity which is a sum of original velocity v 0 plus the velocity due to regular plus the velocity due to the laser itself. And if you take the dot product of E L with those quantities only theonethat survives is this quantity all other velocity is turned out to bezero. So, the electrons gain energy because of this term. And it is useful to write this equation in term for d gamma by d z the evaluation of the energy of the electron as the beam travels sowith distance.

And this can be written as d gamma by d t multiplied by d t by d z and d z by d t is the electron velocity and electron velocity initial velocity is v zero. And then it is slows down as that have to lose its energy to the radiation field, in the limit when the radiation theenergy lost to radiation is a small. I can replace it by simply 1 upon v 0. So, d t by d z is 1 upon v 0 the initial electron velocity and d gamma by d t from this equation is equal to minus eE L dot v w upon m c square. And if you substitute these, numbers these quantities E L in v w this turns out to be simply F p z component of Ponderomotive force divided by m c square.

So, Ponderomotive force is a very interesting thing here in terms of which you can get the evaluation of electron energy with the distance of propagation. Well, if you write down the value of F p z explicitly. Then this quantity turns out to be minus alpha cos psi where psi is the phase of the Ponderomotive force and I am defining phi as K P z minus omega t.And alpha is related to the amplitude of the Ponderomotive force and turns out to be equal to e square A w A L upon m square c square v 0 gamma 0 k w. So, the energy the electron loses, when alpha is and cos psi they are positive this product is positive; obviously, alpha is positive for sure. So, cos psi has to be positive only then gamma will decrease with z and that is what we want.

A decrease in gamma implies a increase for transfer of energy from electron to the radiation field. This rate of decay of gamma with z depends, on laser amplitude regular amplitude and also depends on the regular wave number K w besides the energy of the electron beam. We would like to examine the evaluation of gamma and the phase of the wave. Because psi depends for the part this is a quantity that depends, on position of the particle and the instant at which you want to evaluate this. So, it is will be better to write an equation for d evaluation of psi with z d psi by d z and then you will get a coupled set gamma evaluation with z in terms of psi and psi evaluation in terms of may be gamma or something. So, let us write equation for d psi by d z.

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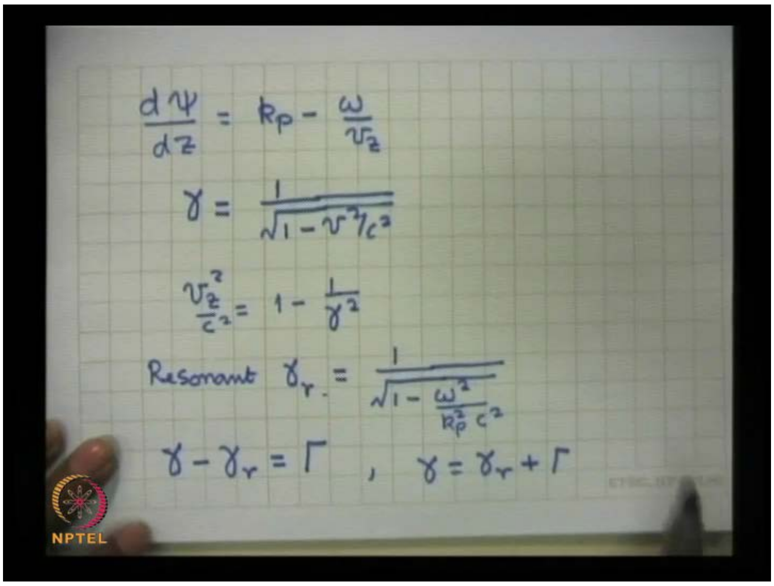

D psi by d z if you differentiate this partially with respect to z not partially fully because z is a function of time. So, this is equal to K P minus omega into d t by d z which is like v z. So, the phase of the wave as seen by the movingelectron evolves according to this equation. And one can write v z in terms of gamma by using this relation by the definition of gamma the Lorentz factor or realistic gamma factor. This is equal to 1 upon under root of 1 minus v square by c square.And if I take v square to be like v z square primarily other components of velocity are weak.Then from here I can get v z by c is equal to or a square of this quantity is equal to 1 minus 1 upon gamma square.What we are trying to do now?

That this set of equations involves gamma in a little complicated fashion. The equation for psi involves gamma through v z in a complicated fashion under root kind of thing will come there. And the equation governing gamma has cos psi on the right hand side. So, what we do that if the energy transfer from the electron to the wave is rather weak or a small. Then I can define twointeresting quantities one is the resonant gamma means, that electronLorentz factor of that electron that is moving with the phase velocity of the wave. So, I call gamma resonant is equal to 1 upon under root of1minus omega square by K P square this is the velocity square of the electron. If it is moving with velocity omega by K P and c square.

So, we define a resonant velocity equal to omega by K P the velocity of an electron moving with the phase velocity of the wave then the corresponding gamma factor for the electron would be gamma R. This is the resonant gamma factor. And we are expecting that your electron initially or later on will have energy close to gamma R it may be slightly more than gamma R in the beginning. But not too much large. So, we will introduce a quantity gamma minus gamma R is equal to some quantity delta. So, we are saying that I can write gamma as gamma R plus some small modification.And this is a quantity of interest do you would like to learn if this gamma has asome significant value in thebeginning as the electrons moves in the system then this gamma should decrease.

And how much is the reduction? That we would like, to estimate. So, what we are going to do I would expect I would rather expand gamma as gamma equal to gamma R plus delta in this equation for v z. And obtain the value of v z in terms of this gamma R and this new quantity. One may note here that this v z if I put it equal to v p then the right hand side is zerobecause omega by K P is the phase velocity of the wave. So, any departure if I have taken gamma equal to simply gamma, R then d psi by d z will be exactly zerobecause when the velocity of the electron is the same as v p. So, any departure in thisorany value of thisthe right hand side will have a finite value only because of this gamma. And if you just simplify and do it little bit Taylorto this binomial expansion you obtain this expression.

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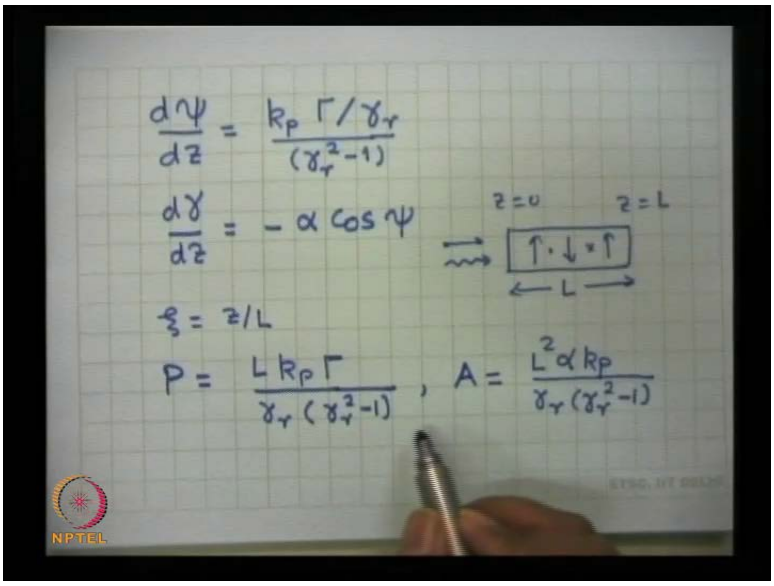

Let me just write the value.It turns out to be d phi by d z is equal to K P into gamma, upon gamma R divided by gamma R square minus 1. This is the equation governing the evaluation of phase of the Ponderomotive force, as seen by moving electron and it is proportional to gamma. The difference in normalized energy of the electron from resonant energy gamma R. And let me. So, we have two couple equations this equation and the equation from gamma, d gamma by d z which was is equal to minus alpha cos psi. These two equations we need to solve. What we can do? We can defined normalized quantities normally in literature in F E L literature people define normalized quantities.

If the length of the chamber interaction chamber or regular region is L this is from here to here this is z equal to 0 and this is z equal to L. So, this is the interaction chamber interaction length where the regular exist this, sort of and you are launching electron beam here and you are launching radiation signal here. So, suppose L is the length of the interaction chamber then I can define a quantity which is z by L normalize to length normalized to the length of the system interaction chamber. And I can also define a normalized energy difference actually, if I put this gamma s gamma R plus delta this equation becomes d gamma by capital gamma by d z.

So, let me define a quantity p as normalized capital gamma, which is L into K P into this increment in gamma from gamma R value upon gamma R into gamma R square minus 1.And let me also; definenormalized amplitude of the Ponderomotive force or the energy term the driver term rather. It turns out to be this is equal to L square alpha K P divided by gamma R into gamma R square minus one. So, alpha contains the amplitudes of the wiggler and the laser. So, in terms of this A Pand psi thesetwo equations take a standard form.

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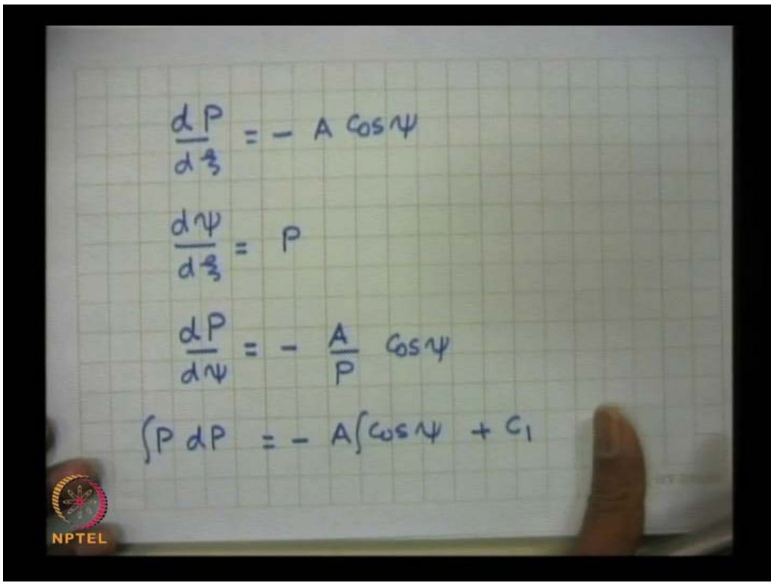

And let me write those equationsoneis d p upon d zi is equal to minus a cos psi this is the energy equation. And d psi by d zi is equal to p this is the phase evaluation equation.These are rather simple equations, butI can do from thesetwoequations I can divided the first equation by the second.And I can get this sort of equation d p by d psi is equal to minus a upon p cos psi I can take d psi on the right hand side, and p on this left hand side and integrate. So, what I get is p d p is equal to minus A cos psi and integrate on both sides add a constant of integration here c 1. Once you integrate this becomes p square by two.

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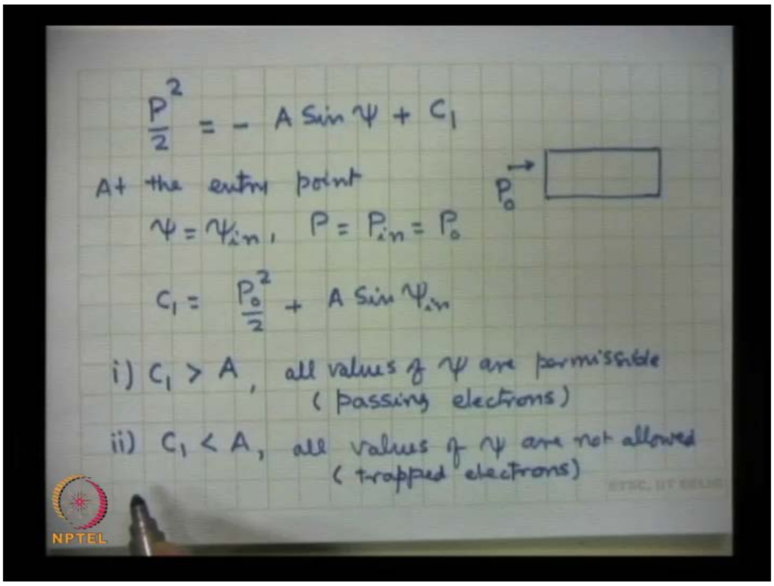

So, let me write this equation as p square by 2 is equal to minus A sin psi plus c 1 c 1 is A constant of integration and it can be determined in terms of the initial values of p. Suppose, I am launching in my interaction region electrons; obviously, different electrons are arriving at different times. So, the electrons, butI am considering a mono energetic electron beam suppose, all the electrons that are coming here to any instant of time. They have initial value of this normalized energy p equal to p 0.

So, if I consider p equal to p 0 as the initial value and psiwhen an electron arrives. So, suppose, at the entry point when an electron arrives at certain time. Suppose the phase of the wave is Ponderomotive wave is psiin. And initial momentum p is equal to initial momentuminitial energy which is equal to p 0. Suppose, then c 1 can be obtained from here and c 1 turns out to be equal to p 0 square by 2plus a sin psiin. This is the initial value of phase as seen by the of the wave as seen by the electron Ponderomotive wave.

Now, there isonething interesting in here, depending on the phasethe value of c 1 could be less than A bigger than A or equal to A. If c 1 is bigger than A and positive in that case this quantity will always remain less than this. So, there are three cases of interest hereonecase when c 1 is greater than Aplease noteonething, the left hand side of this equation is positive definite because p square cannot be less than zero. So, right hand side should also be positive, which is possible because sin psi can check values from minusoneto plus one.

So, when c 1 is bigger than A and positive in that case this require thisright hand side will also be positive. And hence all values of the phase of the wave are accessible. All values of psi R permissible and such electrons which satisfy this condition are called passing electrons or untapped electrons. In the second case if c 1 is less than A in that case all value psi are not permissible. Because left hand side is positive definite right hand side should also be positive. So, whenever c 1 is less than a all values of psi are not permitted all values of psi are not allowed means,the electrons cannot move in a way that they can see different phases of wave.

Such electrons are called trapped electrons. (No audio from 42:13 to 42:20) And third case is the 1 that is the boundary between trapped and untrapped electrons. And that is that decides the separatrix, c 1 is equal to a gives the separatrixit is instructive to plot the separatrix. And see what is the character ofthis curve p versus psi when c 1 is equal to a I take c 1 is equal to athen this equation takes the following form. P square is equal to twice A into 1 minus sin psi and let me plot this.

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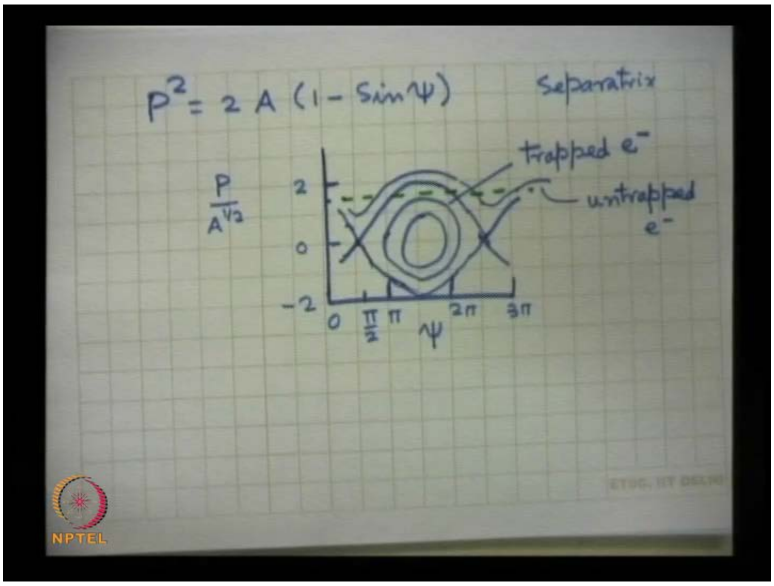

So, if I plot this quantity (No audio from 43:28 to 43:37) obviously,onemayrecognize that P square is positive, butP can take negative or positive values. And if I plot heresuppose, I plot psi here and p upon a, to the power half take the under root of this quantity write plus minus sign there and p by. So, the values that it can take are the maximum value of this quantity would be two. So, 2 into 2 isfour. So, this I put it 2here this is my 2here this is 0 and this is minus 2this minus 20 and 2and this is my phase is 0 here let me put pi here. Then I putletlittle elaborate this is pi here let me call this pi here. Then 2pi is there 3 pi is there this is pi by 2and pi is there.

So, let me have a plot of this separatrixcurve what you get you will obtain when sin psi is equal to pi by 2 then p is zero. So, this is somewhere here I will get and also, when this is 2 pi plus pi by 2 somewhere here. So, you will get2here and you will get a maximum here and if 0 here. So, this will be like this (No audio from 45:30 to 45:41)and this will be 0 here something like that. (No audio from 45:44 to 45:54)This is the curve of the separatrix. The electrons for which the initial conditions have such that c 1 is less than a they have trapped inside here and they go like this their curves are like this and like this.

These are called trapped particles no audio from 46:20 to 46:26) and the electrons whose which are outside this they arelike the electrons the trajectory here are like this.They are called passing particles or untrapped. What really happens? Initially when you launch an electron beam suppose the initially you are launching electron beam is some finite energy. So, p has some values suppose, like this let me draw with a different color. So, the electron may be somewhere here. So, if the electrons are evenly distributed means, they are entering the wave at regular intervals some electrons when they enter the proper phase. They will be inside the separatrix and these will be trapped electrons, some ones will be outside the separatrix and they are called untrapped electrons.

So, the wave which is a the ponder motive wave will essentially trap some of the electrons. And these some of the electrons when they move from larger value of energy p to a smaller value of energy p on this inside this separatrix, then they lose energy and they giveit to radiation amplifying the radiation. So, this is how in (( )) space the character appears likeenergy electrons acquiringloosing energy and giving into radiation. Let me make some estimate of energy gain how much energy gain this is the physics. That the electrons can be trapped within the potential well of the Ponderomotive force and they can lose energy when they move on this curve from top towards the bottom.

Now, let me examine the two in order to examine the energy gain. Let me solve the equation of motion little more carefully this is the (( )) space plot p versus psi. I am interested in knowing the value evaluation of p with psi the distance of propagation normalized distance of propagation. So, what we have to do? For that I need to solve the equation of phase evaluation because psi if I can express in terms ofz. The distance of propagation then I can find out p as a function of z and I do that iteratively.

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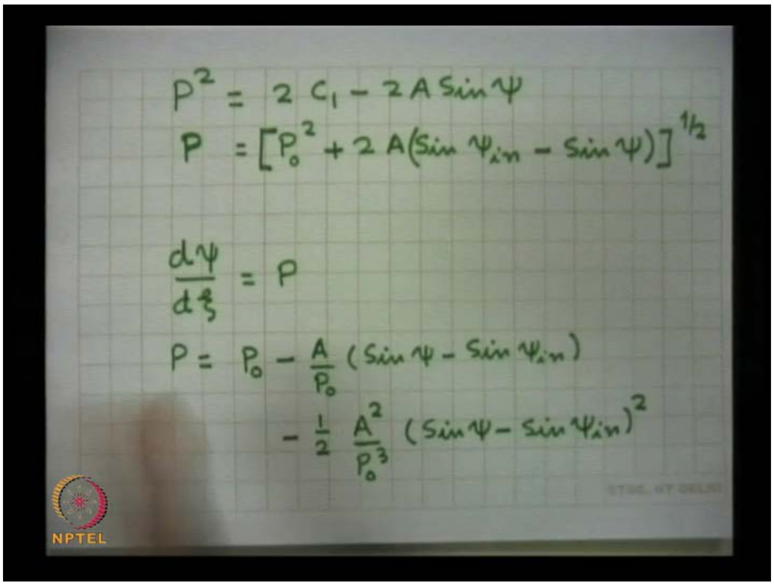

But I can always employ the expression that I just wrote down that p square is equal to for a trapped electron p square is equal to 2 c 1 minus 2 a sin psi. This is the equation that I had written a general equation. For c 1 is equal to A this became the equation for a separatrix otherwise a general equation for particle energy in terms of psi. But this equation I have to solve in conjunction with the equation for phase evaluation which is d psi by d zi is equal to p. And it will be better if I write this c 1 in terms of initial quantities initial momentenergy of the waveof the particle and initial phase. Then this becomes is equal to p 0 square by 2. So, this 2 c 1 is like p 0 square plus 2A sin psi initial.

And from this is the value of c 1 from this if I subtract this quantity this become simply this minus sin psi. So, this equation if I take the square root of the right hand side will give me the momentum. The energy of the particle at any positionzi at which the phase is psi when psi is equal to psi I n and these two terms cancel and p equal to p 0. So, this has to be solved in conjunction with this. So, p is a function of psi according to here, and I should put this in here andintegrate it.

But the problem is that because of the sin psi coming in such a complicated fashion the integration is not possible. So, what people do? They solve this equation iterativelyto different powers on a treating thatp 0 is square is much bigger than 2 a then you can binomial expand this and when you do this. First of all you can write this p by you carryingbinomial expansion asthis is approximately equal to p 0 minus A upon p 0 sin psi minus sin psi I n. And then you go to second term, which turns out to be equal to minus half a square upon p 0 cube sin psi minus sin psi initial phase whole square. This sort of expansion you get for p to different, powers in amplitude of the Ponderomotive force.

What we will do probably we will have to wait till the next lecture. I will solve this set oftwoequationsoneis a differential equation another is aAlgebraic expression or trigonometric expression iteratively. First I will consider to the zeroth order if there was no laser if Ais 0 then p equal to p 0 and I will find out psi is how much then I will consider that value of psi in here. And use that value of p again to obtain new value of psi. So, iteratively I will evaluate p 2 higher orders in a square and finally, I will like to find out what is the energy that the electron retains at the exit point at zi zi equal to 1 zi equal to 1 means z equal to n.

At the end of the interaction region and this will be a function of initial phase. So, we have to average out over all the phases, of all the electrons that are entering the interaction chamber. And that averaging, will give you the net energy gain or energy loss by the electron. And we will certainly want maximum energy loss by the electron. So, that the maximum energy is gained by the laser. And we shall discuss these issues in our next lecture thank you.