Plasma Physics

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Module No. # 01

Lecture No. # 15

Free Electron Laser

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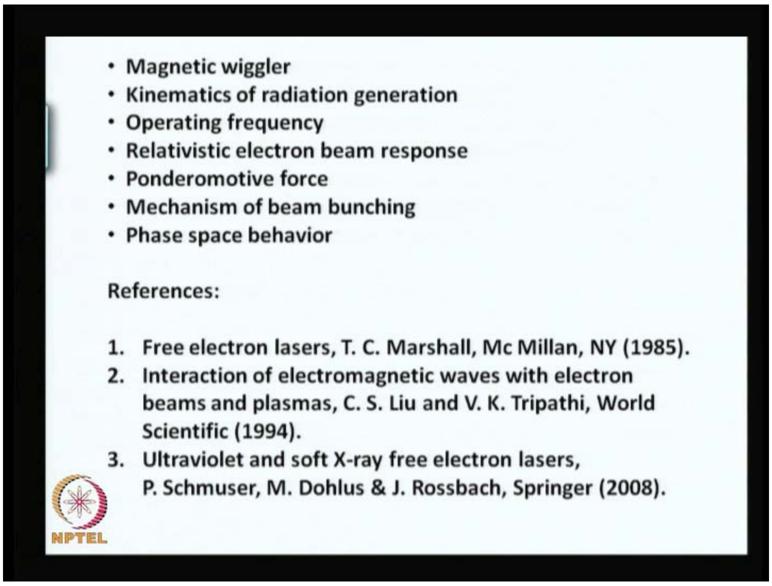

Well, today I am going to talk about free electron laser and in this I will discuss the following. Magnetic wiggler which, is a component very essential for phase matching in a free electron laser and also for non-linear interaction of electron beam with radiation. Then, we will discuss the kinematics of radiation generation deduce an expression for operating frequency of a free electron laser, then we will discuss the scheme of obtaining response of an electron beam, relativistic electron beam to radiation and magnetic wiggler and the process we will formulate a quantity called Ponderomotive force.

We will discuss the mechanism of beam bunching and then, discuss the phase space behavior of electrons beam electrons and the presence of ponderomotive force. Well, the references for presentation are a book by T C Marshall free electron lasers, then a book by professor C S liu and myself interaction of electromagnetic waves with electrons beams and plasmas. And, another book ultraviolet and soft x-ray free electron lasers by Schmuser, Dohlus and Rossbach.

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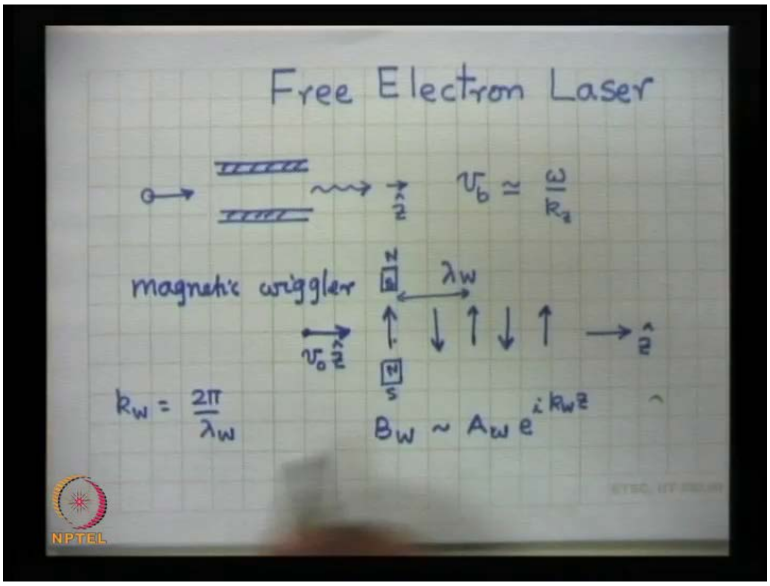

Well, we have already learnt a device called Cerenkov free electron laser in which we learnt, that an electron beam which was sent through a wave guide and in electron beam much launched from here and this dielectric loaded wave guide, which had dielectric lining in the interior this dielectric lining here. And, when the electron beams bus passing through this wave guide with appropriate velocity, it radiated electromagnetic waves. And, the condition for radiation was, that the beam velocity v b has to be close to or slightly more than the wave frequency upon z component of k, where z is the direction of along the axis of the wave guide.

So, whenever the wave phase velocity in the direction of beam propagation equals the beam velocity then there is a resonant interaction between the electron and the wave and it can produce Cherenkov radiation.

Now, obviously, in free space where the velocity of light is c, then this condition cannot be matched and hence a free electron or an electron beam travelling in free space by itself cannot radiate electromagnetic waves. The issue is that, is it possible without slowing down the wave to generate radiation, coherent radiation by an electron a beam?

Well, if we really examine the scheme of the fundamental issue of energy conservation, momentum conservation in a radiation generation process, then it turns out that Cherenkov condition essentially implies momentum conservation and energy conservation in a radiation generation process. And, that cannot be satisfied in free space by a non, by an electron which is not moving with a velocity c or higher than c.

Consequently, people discovered a scheme, that if there is some agency to account for momentum conservation, then one can have radiation generation. In a free electron laser, it is the magnetic wiggler that does this job.

Magnetic wiggler in principle is a kind of magnetic field whose polarity is in one direction here and then the reverse direction here, than in this direction here and then in this direction there. These are the lines of force I am drawing in certain region of space. If you can create a magnetic field of the sort, then we can call this magnetic field to have 0 frequency because it is static in time, but this has a variation in suppose z direction.

So, if you can produce a magnetic field of this sort by putting some magnets here. Suppose, this is a bar magnetic here, I put a bar magnet here, this may be my north pole here. This may be South Pole here and North Pole of this magnetic is there and South Pole is here.

So, if I put magnets of alternate polarities, then when we can generate a magnetic field of this sort which we can call as wiggler magnetic field, whose dependence on a space will be of this form, some amplitude exponential i K w z, where the distance between two these vertical upward lines, I will call as the wiggler period from here to here or wiggler wave length lambda w and K w i will define as 2 pi by lambda w. So, what happens is that if you launch an electron beam in this configuration in the system in this direction with a velocity say v 0.

Then, what will happen? If this electron beam is traveling through this structure in the presence of an electromagnetic wave then the interaction of the electron beam is not nearly with the electromagnetic wave. But it is in the presence of the wiggler magnetic field. So, it is possible that the photon that is to be generated or added to the system, the energy it is gaining from the electron beam a momentum also it is gaining from the electron beam, but part of the momentum is transferring to the wiggler magnetic field.

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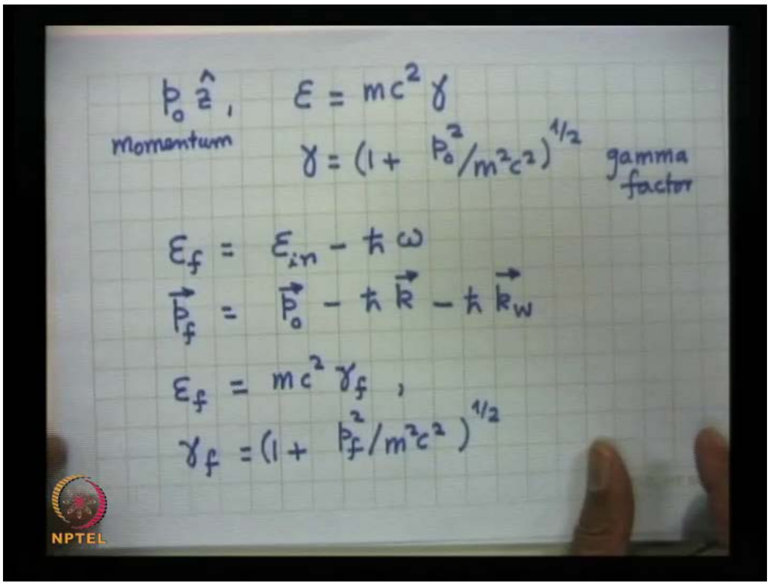

So, magnetic field plays a role of momentum balance. And, I can write down this expression in a slightly different way. Suppose I have an electron coming with momentum P 0 which is in the z direction. And, energy of this electron I can certainly write as E is equal to mass into c square, this is the energy of an electron beam. If m is the relativistic mass which is equal to rest mass multiplied by gamma relativistic gamma factor and gamma factor is equal to 1 plus P 0 square upon m square c square to the power half. This is the electron momentum and this is the energy, this is called the gamma factor. (No Audio From: 08:06 to 08:13)

Now, what we expect? That if an electron has to emit a photon, then energy of the electron after emission of photon should be equal to initial energy minus h cross omega, if omega is the frequency of radiation of photon that is the generating. And final momentum should be equal to initial momentum minus h cross k of the laser, the photon it is generated. But because electron looses more momentum then a photon can take and hence what we do, we say that part of this momentum electron initially had a momentum p 0. h cross k momentum it has given to a photon and plus some momentum h cross K w momentum, it has given to wiggler magnetic field.

So, the total momentum lost by the electron is sum of these two terms, this is the addition quantity I have added because the interaction is taking place in the presence of a magnetic wiggler. And, electron is under a constant Lorentz force due to the wiggler magnetic field. So, if this is possible, then if you work out that final energy of the electron should also be related to final momentum by the same relation which is m c square gamma f, final gamma. And, gamma f should be equal to 1 plus final momentums square upon m square c square to the power half.

If one puts it in there and rises that h cross k and h cross K w are much much smaller than p 0, maybe million times or at least tens or thousand times smaller than this quantity. So, one can use binomial expansion and once you put gamma f, a P f value in this expression and solve this equation, then one obtains is essentially using equation number 1 and 2.

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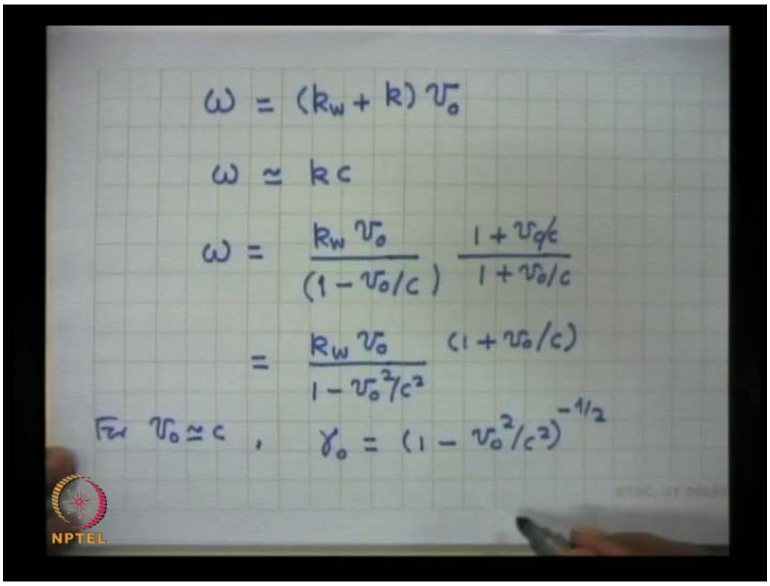

One obtains the condition for radiation generation is equal to omega equal to K w plus k into beam velocity, which we call as v 0. So, the electrons which are moving with momentum p 0, they will have a drift velocity v 0 and the energy and momentum conservations considerations give you this equation, which is a modified version of Cherenkov resonance condition. So, electron is not in Cherenkov resonance with the laser, but it is in resonance with a strange kind of force called a ponderomotive force, that is arising due to the laser and the wiggler magnetic field. And, we shall introduce this concept of ponderomotive force in a little while.

But this is a important condition and if we are having this interaction taking place in free space, obviously, in the presence of the wiggler. Then, I can assume that omega is nearly equal to k c, due to boundary effects this is slightly modified. But if boundary effects are not considered, then omega is exactly equal to k c the frequency of radiation and wave number of radiation, they are related by this relation. If I put this expression in this equation, but I get is omega turns out to be equal to, just put this equal to k is equal to omega by c and then this becomes equals to K w v 0 divided by 1 minus v 0 by c. And, if you can make v 0 close to c, then you can have this frequency as large as you wish.

So, just by bringing an electron beam of velocity very close to c, one can produce radiation, tunable radiation of desired frequency. If I multiply the denominator and numerator by a factor 1 plus v 0 by c 1 plus v 0 by c, then this can be written as K w v 0 upon 1 minus v 0 square by c square into 1 plus v 0 by c. This quantity for a beam whose velocity is very close to c of the order of c, v 0 by c can be taken like 1. This v 0 can be replaced by one; obviously, one has to be careful in this quantity. But by definition gamma 0, the initial Lorentz factor of the electron beam can also be written as 1 minus v 0 square by c square to the power minus half. And, hence this frequency of radiation of F E L can be written as operating.

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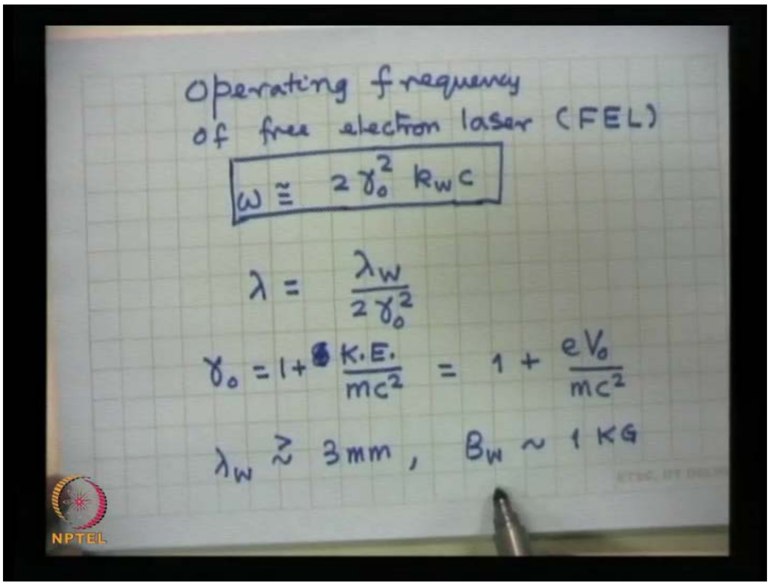

This is called operating frequency of F E L of free electron laser, which is usually called as F E L is omega is equal to or approximately equal to 2 gamma 0 square K w c, well this is all, this is the expression.(No Audio From: 14:55 to 15:03) There are two important issues in here. Number one, the frequency can be controlled by K w. In terms of wave length if I have to write, then the wave length of radiation turns out to be wiggler period divided by 2 gamma 0 square. If you choose shorter wiggler period, then the radiation frequency will be shorter and you can also reduce the wave length of generated radiation by increasing the energy of the electron beam.

People would prefer to write gamma 0 in terms of the beam voltage, because after all these electron beams are produced by some acceleration field. So, people normally write down in terms of, well energy, we write down in terms of million volts or something. So, I can write down gamma 0 in terms of energy, if I have to write this is equal to e 0. Well, this is 1 plus kinetic energy rather, kinetic energy upon m c square. So, if you are producing an electron beam by accelerating through a potential difference, suppose the potential difference is v 0 and e 0 is the, e is the magnitude of electron charge. Then, this is equal to m c square and m c square is about half million electron volt.

So, if you have accelerate the electrons to energy of the or kinetic energy of the order of 1 m e v, then gamma is 3. If you have an electron beam of 10 m e v kinetic energy, then this is 21, gamma 0 is 21. So, what I am saying is that, if you can increase the value of electron energy, then you can increase the radiation frequency immensely typically like a square of energy. So, usually the wigglers that you can construct have wiggler period lambda w of the order of may be 3 millimeter or bigger. Most wigglers have wiggler period around 1 or 2 centimeter. But one can also construct wigglers, with wiggler period around 3 millimeter or 2 millimeter. And, typically strength of the wiggler magnetic field is, modulus of this quantity is typically of the order of one can have a few kilogauss like 0.1 tesla.

So, this is the kind of magnetic field available by permanent magnets, nickel magnets of one use bar magnets, one can construct a wiggler magnetic field. And, one can produce coherent radiation in the infrared visible even at ultraviolet frequency depending on the energy of the electron beam. If one chooses gamma 0 around 100, which means if you choose a beam of about 50 million electron volt energy, then this quantity turns out to be let me put some numbers here for your, for some clarity.

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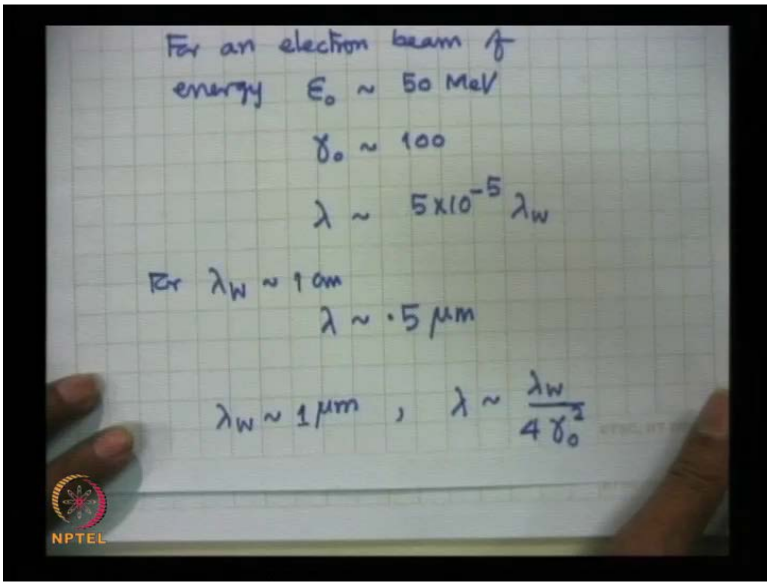

So, if I choose for an electron beam of energy E 0 of the order of say 50 M e V energy. gamma 0 is of the order of 100 and wave length of radiation is around 5 into 10 to minus 5 lambda w. So, if lambda w of the order of 1 centimeter, we are talking of lambda of the order of 0.5 micron, half micron radiation which is visible light. So, you require for production of visible radiation using a wiggler of period 1 centimeter, one would require an electron beam of 50 m e v energy. For ultraviolet radiation either you reduce the value of lambda w or increase the value of beam voltage or beam energy and certainly this is possible.

What has been recognized is that this wiggler could be a static wiggler. It could also be in electromagnetic wave, after all electromagnetic wave also has a strong magnetic field and may be that can also play a role of momentum balance between electron and free electron laser radiation. So, in that case you can really use laser as a wiggler. So, lambda w can be of the order of even 1 micron neodymium glass laser if one use, then the wiggler period wiggler wavelength is of the order of 1 micron. And in that case, if you have taken into consideration the finite frequency of radiation, in this case lambda turns out to be around wiggler wave length divided by 4 gamma 0 square and one can produce x-rays and that is a very fascinating possibility.

So, people have been trying for quite some time, producing coherent x-rays by using laser as a wiggler and using electron beams of 100, around 100 m e v energy. I think that is the exiting possibility, that you can increase the frequency of radiation immensely. Some experiments were conducted little while ago on generation of ultraviolet or violet or visible radiation by using gyrotron as a pump, because gyrotron can produce radiation in the millimeter wave length regime. So, you do not really have to put a d c magnetic wiggler, one can use electromagnetic waves as wiggler also.

So, I think this is a interesting device. It has a tremendous advantage over the conventional lasers. Because the frequency of this device is frequency tunable, wave length can be changed by changing the energy of the electron beam. This tunability is a very important consideration, very fascinating consideration. And, more over the efficiency of the device if one uses tapered wigglers that we shall learn later, could be much higher than the conventional laser efficiency. So, it is I think a very important device that can produce radiation over a very wide frequency range. Free electron lasers are operating over a very wide frequency range, ranging from sub millimeter waves, sub millimeter wave lengths.

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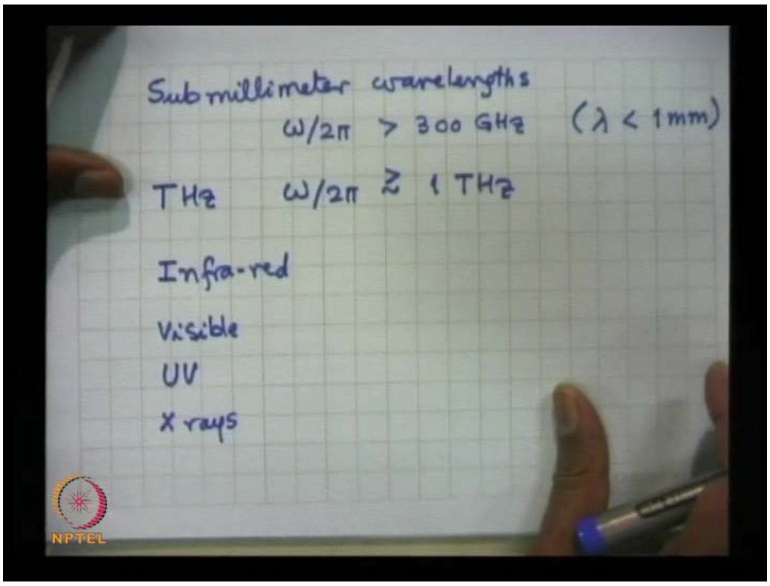

Please remember, that a frequency of 300 gigahertz is 1 millimeter wave length. So, when you are talking of frequency greater than 300 gigahertz, then you are talking about lambda less than 1 millimeter wave length. And, a special interest as come up at terahertz frequencies, when the wave length is or the frequency is omega upon 2 pi is around 1 terahertz means 1000 hertz. And, then infrared, then visible, then ultraviolet x-rays. So, these are all the frequency bands in which free electron lasers can produce high powers. Well, x-rays I would say that, that is still in the experimental stage. But you can have very high powers at millimeter, sub millimeter wave lengths and free electrons lasers are a good candidate for plasma heating. They are good candidate for terahertz generation, for various applications.

Terahertz radiation is really become very important from two perspectives recently. One is for explosive detection and also for medical imaging. Obviously, the powers that you require are not that much as are produced by free electron laser. So, people are thinking of other ways to produce terahertz radiation. But wherever you require high powers, then free electron laser is a device that can give you high powers. Well, let me give you a schematic of a free electron laser before I proceed further.

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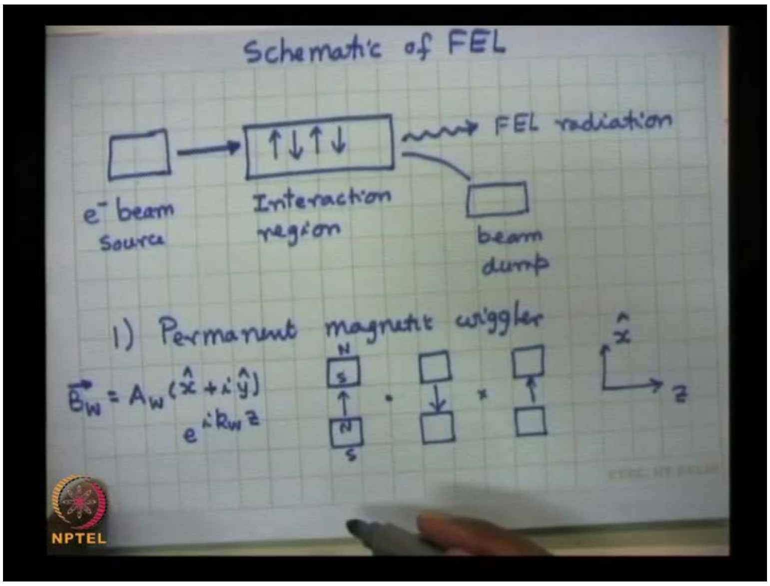

So, free electron laser will certainly require a interaction region. Let me call this as the schematic of a free electron laser, schematic of F E L. This will have essentially a interaction region which will have a wiggler magnetic field, symbolically denoted like this. This is called interaction region. (No Audio From: 25:56 to 26:05) Then you require it to be fed by means of an electron gun or beam source. So, this is the electron beam source, that produces an electron beam and the beam has to be relativistic. It could be a linear accelerator or some other device that can produce m e v energy electron beam, the current in the electron beam has to be about a ampere or at least 100 of milliampere.

So, the electron beam is launched into this device. And, then you require a seed signal F E L seed signal that has to be amplified or it has to be oscillator, it could be an oscillator. Then, you do not require because there is some noise signal that is amplified by the electron beam. And then, there were two things here radiation comes out here. So, you require a diagnostics or rather a system to channelize radiation and then you require a beam dump. The spent beam is dumped here, so, called beam dump and this is the F E L radiation, free electron laser radiation. Very special techniques have to be employed to produce good quality electron beam. The energy is spread tolerable in the electron beam is not more than a few percent and that is a very serious issue.

The transportation of the beam from the source to the interaction region is also a serious issue. The beam should not suffer excessive divergence on its way to interaction region to arrive at the interaction region. So, some care has to be excised here. And the wiggler magnetic fields, there are two kinds of wigglers. One is a permanent magnetic wiggler, (No Audio From: 28:07 to 28:19) in which you have essentially an arrangement of magnets as I showed you earlier like this. This magnet has a, suppose this has a North Pole here and South Pole there or North Pole. Suppose this has North Pole here, the South Pole here, the North Pole other North Pole is here, South Pole is here.

And then, you can have there in the say reverse polarity like this. So, alternatively you can have these kind of magnets. But what really people do? They do not use this is called a linear wiggler, if the lines of force are all vertically upward or vertically downward, then such a magnetic field is called linear wiggler. People normally have a circularly polarized wiggler. So, what they do? They put a magnet here, a magnet there. Then, they put suppose this is my z direction and this is my x direction. So, these two magnets are these pairs of magnets are placed on x axis, here parallel to x axis, but then in between if you have magnets placed on the y axis so that the line of force here is like this, here is like this. But here the line of force is vertically upward and here is the vertically downward.

So, by having magnets one placed on top here and one placed underneath on the y axis, then one can produce a magnetic field of this form v w is equal to some constant A w x cap plus i y cap exponential of i K w z. I have used a complex notation here, which certainly this expression implies that the real part of the right hand side is to be taken. It means that the wiggler magnetic field will have a x component because of these magnets and y component because the magnets placed along the y axis, with alternate polarities. And, the net magnetic field in this system is, take the real part.

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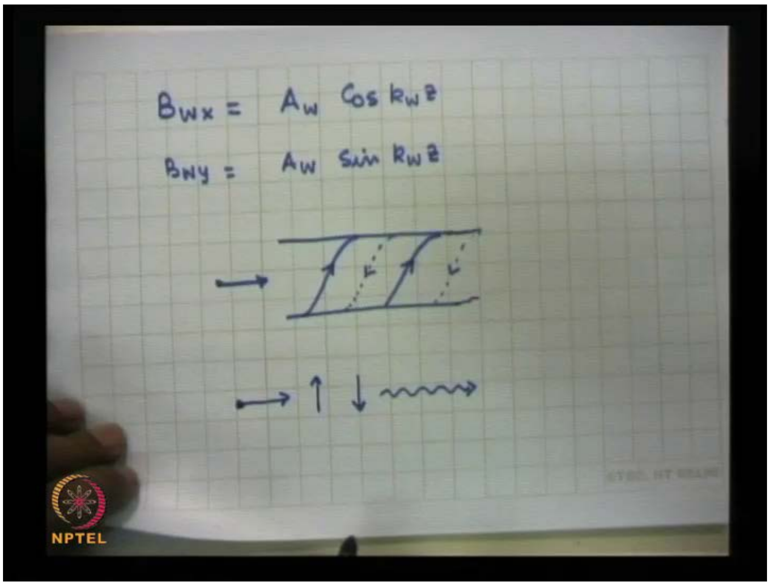

So, v wiggler magnetic field has x component, which is equal to amplitude A w cos K w z and v w y which is A w sin K w z. K w is the wiggler number, A w is the wiggler amplitude and these two components are out of phase by pi by 2, this is called circularly polarized wiggler. Wiggler magnetic field will also have a z component, but you can choose z component of magnetic field to be 0 on the axis of the system. And, away from that there is finite, but you are launching your beam through the center of the device where v w z is 0. So, it is a reasonable approximation for this field, this is how you produce a by permanent magnets you can produce a wiggler.

Well, you can also produce a wiggler by having winding on a tube. Take a tube and have helical windings on it, like this. So, helical windings are going over here, but then you have windings carrying current in opposite directions and in between, there is a reverse polarity magnetic field and just this kind of configuration also. So, by carrying currents like this, you can produce a wiggler magnetic field of form. And, that is really a quite common practice to employ permanent rather electromagnets rather current coils helical current coils, to produce this sort of magnetic field. And a important issue is to examine the kinetics of the process, how this wiggler is going to help the momentum conservation in the radiation generation process?

Well one thing that one can clearly see that if you are launch an electron beam here, if there were no wiggler magnetic field then the electron beam will travel through the system without any deviation. If you ignore the space charge effect of the beam electrons they are ripples on each other then this beam will travel without any modification and its velocity. However when this is a magnetic field perpendicular to the direction of propagation either v w x or v w y or a combination of both, in that case the particle trajectory will be modified it becomes curved. So, if electrons are going like this is and suppose it sees a magnetic field like this. Then this electron will try to gyrate about this line of force, it will be lifted upwards, but as it traverses further then the force is reversed inability downward.

So, the ion the electron will go up and down up and down up and down. So, the path will be wiggler wiggled. So, a straight electron which was moving with a constant velocity with a unidirectional velocity, it is going up and down ,away from this plane of the paper and then into the mobile zone. This is oscillating like this in the case of a wiggler, this will have a motion in x direction as well. So, that is a important issue that the electron which was moving with a d c velocity it has acquired some sort of a velocity which is a function of time or distance.

Now, this radiation any you know that any accelerated electron radiates. So, if you are launching millions of electrons, each of them will radiate; obviously, the radiation emitted by this is called undulator radiation. There is another term for wiggler called undulator and this radiation is called undulator radiation and the frequency of that will be essentially the frequency of this wig. But that is not of much interest. More interesting thing would be the coherent radiation they generated by the electron beam not the spontaneous radiation or not the radiation by different electrons and acceleration. So, let me examine the issue of coherent radiation by this electron beam.

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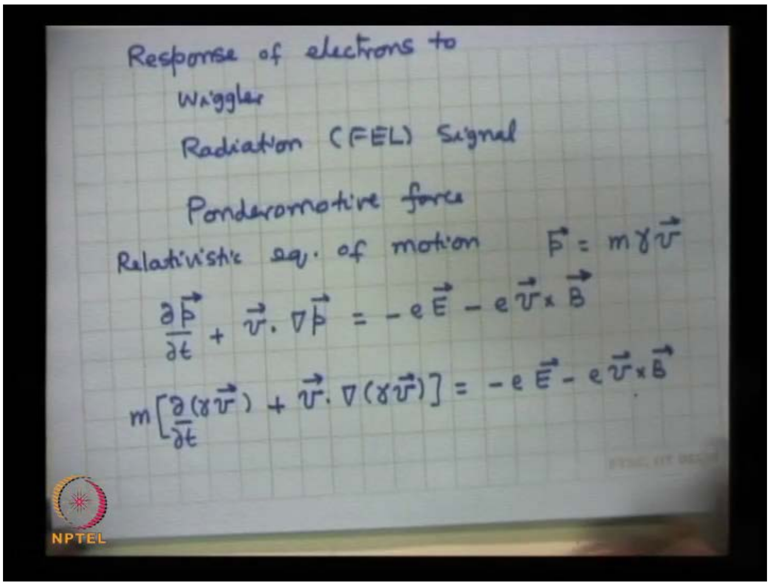

So, I would like to examine the kinetics of this process by examining the response of electrons to wiggler and then the radiation signal called F E L signal. And first of all I would like to study the response independently that if the electron sees only wiggler what kind of change in velocity will occur and if it is only the radiation signal what kind of change in its velocity will occur? But when both are present simultaneously what kind of effect will arise. So, that term I call as the ponderomotive force.

So, remaining part of my discussion will be focused on these three issues the response of electrons to a wiggler magnetic field. Response of electrons to a radiation field, as if there is no wiggler and then what is the modification caused when these two are present simultaneously. So, I would like to evaluate the non-linear force that arise due to the coupling between the wiggler and the radiation signal. As we are dealing with millions of electron energy volt energy of the beam. We are to considered the relativistic equation of motion. So, I will write down the relativistic equation of motion for beam electrons and I will ignore the effect of space charge, d c space charge of the beam.

Now, this equation is rate of change of momentum. So, momentum I will define p is equal to rest mass, then gamma factor, Lorentz factor into v. And the equation is for fluid electron beam as a fluid, the equation is delta p upon delta t plus v dot del p this 2 terms together as d p by d t, rate of change of momentum is equal to the force on the electrons due to electric field plus the force on the electron due to magnetic field which is minus e v cross b. This equation has two terms on the right hand side the first is called the force due to the electric or rather the electric force on electrons of charge minus e and this is called the magnetic force.

And v is the drift velocity of electrons; I have ignored the pressure term assuming that the electrons have no thermal energy spread. Now, if I put this p is equal to m gamma v in this equation is normally written like this m outside delta delta t of gamma v multiplied by plus v dot del of gamma v is equal to minus e E minus e v cross b. To solve such equation, what we do ?We use a process the procedure of linearization that I discussed in my earlier lectures that first write down the equilibrium and in equilibrium there was no d c magnetic field which was uniform and treat the wiggler and the laser fields to be as perturbed quantities.

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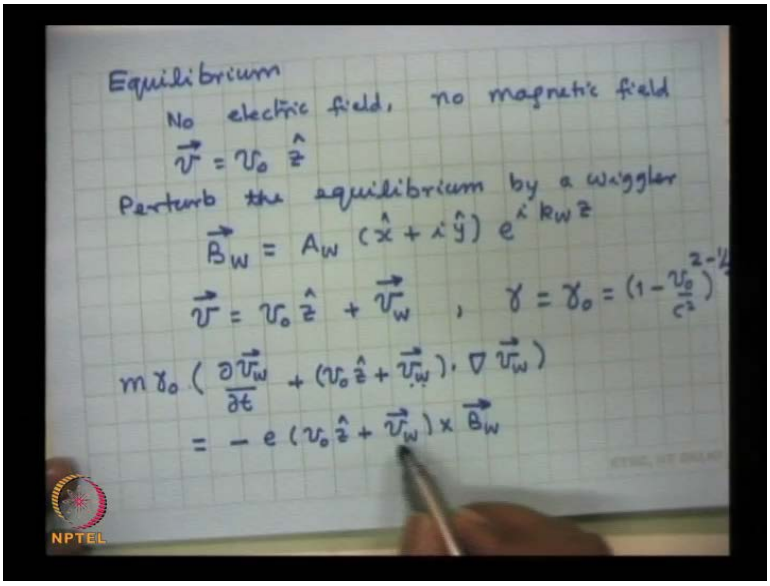

So, what we do? first we write down the equilibrium .Equilibrium is that the plasma has no field, no electric field, no magnetic field and as a consequence the beam velocity is just uniform velocity v 0, and that is in the z direction. So, electrons are travelling with velocity v 0 that is the 0 the order particle motion particle velocity Then we say that perturb this equilibrium by a wiggler, perturb the equilibrium by magnetic wiggler. And I am assuming my wiggler magnetic field to be of this form v w is equal to A w x cap plus I times y cap exponential of I K w z, this my wiggler.

So, I say that the electron response to this wiggler field would be v will be equal to initial velocity v 0 plus some perturbation caused by this. And I will call this perturbed quantities v w subscript simply implies that this is the response due to wiggler field. So, what I do? I presume that A w is a small quantity and hence the response of electrons to A w or v w will also be a small quantity and we ignore the products of perturbed quantities, this is called the process of linearization. So, if I substitute this in the equation of motion, I can get a linearised equation of motion, but, how about gamma?

You know that if the magnetic field is time independent then the force on the electron is always perpendicular to v cross b rather force on the electron is v cross b which is perpendicular to velocity and this does not give rise to any energy exchange energy change. So, gamma is a constant that becomes a great simplification. So, the velocity is changing, but, magnitude of velocity does not change. So, we say that gamma is a constant gamma equal to gamma 0, which is equal to 1 minus v 0 squares by C Square to the power half minus half. This is a quantity which I am presuming to be constant, well certainly it is a constant exactly constant in a magnetic field, d c magnetic field.

So, then the linearism becomes very simple and you get m gamma 0 common and you will get delta v w upon delta t. The second term would be v 0 z cap plus v w dot Del v. But Del of v 0 is 0 because v 0 is a constant. So, Del of v w will be there and how about the right hand side, right hand side of the equation of motion will be there is no electric field. So, electric term force term is 0 the magnetic force is minus e. This is v 0 z cap plus v w cross B w. You may note here this small v w dot Del v w this is a product to perturbed quantity. So, we will ignore it and the product of v w and B w is also a product of perturbed quantity. So, we will ignore it. So, these two products we ignore this 1 and this 1 then this equation becomes.

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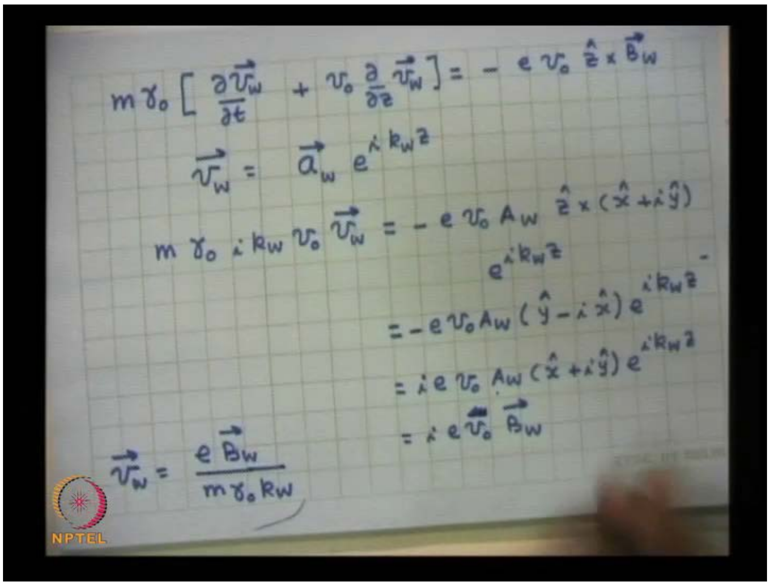

M gamma 0 delta v w by delta t plus v 0 delta delta z of v w is equal to minus e v 0 z cap cross B w. Please note v w is the response or the effect and B w is the cos, the right hand side is a force that is the cos that creates v w. So, in the quasi study state we believe that the response should have the same time and z dependence as the source. So, we say that v w can be written as say some quantity A w, the amplitude exponential of I K w z. Because the right hand side has a no time dependence and z dependence of this form. So, we presume a solution like this.

Then I substitute this here the first term goes to 0, because there is no time dependence. So, this is gone to 0 and delta delta z when I put here is replaced by I k w. So, this equation simply gives me m gamma 0 I K w v 0 v w is equal to minus e v 0 z cap v w v w is circularly polarize field. So, let me write down the value, this has A w outside and then this will be z cap cross x cap plus I y cap exponential of I K w z, this z cross x is minus y cap and z cross sorry z cross x is y cap and z cross y is minus x cap.

So, this can be written as minus e v 0 A w, just take this inside and this becomes z cross x is y cap and minus I x cap exponential of I K w z. If I can take minus I outside this becomes I times e v 0 A w x cap plus I y cap exponential of I K w z and this quantity A w into this quantity simply v w. So, this becomes rather simple. This become simply I times e v 0 and this is v w, sorry v 0 is a constant v w here. Then you can divide this coefficient and you will get v w is equal to simply I will cancel out, v 0 will cancel out.

You will get e v w v w is a vector here divided by m gamma 0 K w, this is a very simple neat expression.

If it were d c static electric field uniform electric field, magnetic field then e v upon m is known as cyclotron frequency. So, this is like effective cyclotron frequency due to the wiggler magnetic field divided by the wiggler wave number like omega by k divided gamma 0, the relativistic gamma factor of the electron. So, the wiggler has produced a drift velocity on electrons on the electron of the beam and this is circularly polarized also, if the wiggler is circularly polarized.

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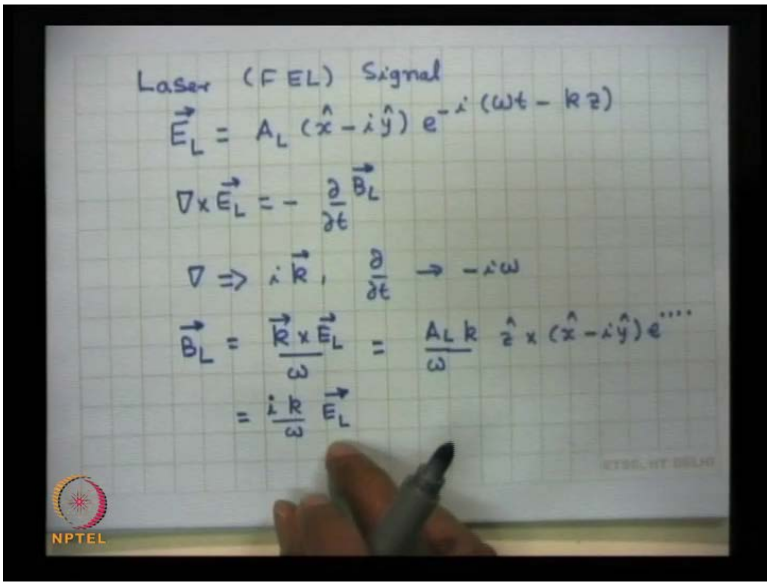

Similarly, I would like to obtain the response of beam electrons to free electron radiation. So, let me write down laser radiation these are all F E L signal, well 1 has to be little careful about radiation the wiggler we consider to be circularly polarized in the right handed sense . And if we choose the laser signal also to be circularly polarized the right handed sense, then the non-linear component turns out to be 0. So, but if you consider the laser to be left circularly polarized and we shall probably see the rational for this a little while. But right now I assume that the laser as an electromagnetic wave with E L is equal to suppose amplitude is a l x cap minus I times y cap and exponential minus I omega t minus k z.

Suppose my electromagnetic wave that I want to amplify at the expanse of beam energy has the electric field like this .This is a transverse electromagnetic wave and obviously, if you work out the third Maxwell equation which says that curl of E L is equal to minus delta delta t of B L, this is the faraday's law of electromagnetic induction. And if I replaced del by I k and delta delta t by minus I omega. Because my field quantities are varying like this in exponential way like this in time and z. Then I can certainly do this then B L I get is equal to k cross E L upon omega and because k is in the z direction. So, if I take z cross E L then what you get is, it turns out to be equal to a l upon omega and k is there then z cap cross x cap minus I y cap into exponential term, the same factor as written there.

What you get here when you take this in the interior, it turns out to be z cross x is y cap and z cross y is minus x cap. So, you get here I common I k upon omega and the thing that is left out is simply equal to E L. So, this is the interesting thing here that the laser magnetic field is expressible in terms of E l, but, the coefficient outside is I k upon omega I comes because of this because z cross y is x cap minus x cap. So, I come because of that.

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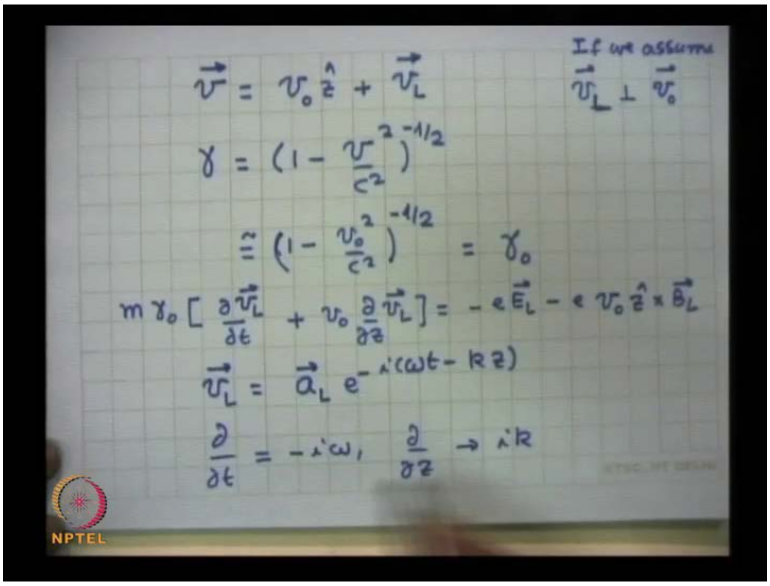

So, once I know my electric and magnetic fields and if I forget the wiggler then the response of beam electrons can be written as v. I can write down if I forget the wiggler response the original beam velocity v 0 z cap plus some modification due to the laser called this v l. Now, this is an interesting quantity, this is a term that I have added here I would except that this should have the same t z variation as the electric field of the laser signal. So, now, let me go over to our equation of motion, but, that involves gamma also. I should be little careful about gamma, gamma is equal to 1 minus v square by c square to the power half. But now, v is v 0 plus v l because this laser has an electric field. So, there is a possibility that the energy also changes

So, if this is a case here, but if I consider this v l to be very small as compare to v z v zero. So, that if we can ignore the square of this quantity, then you know because the we are expecting v l to be in the direction of electric field which is transverse. So, if this has no z component then this is simply equal to 1 minus v 0 squares by c square, there will be term v l square term, but, that is a product of perturbed quantity. So, we ignore it and if I consider that, then this remains minus half please minus remains the same as gamma 0. So, if v l is presumed to be perpendicular to v l is assume to be perpendicular to be 0. If we assume and we will justify later, we shall see that this indeed valid. So, in case v l is perpendicular to z axis then gamma turns out to be unmodified within the limit of perturbation analysis.

And by equation of motion then becomes simple, it becomes m gamma 0 outside, this will be delta v l by delta t plus v 0 delta delta z of v l is equal to the force on the electron due to the laser v l minus e linear zed force on would be v 0 z cap cross v of the laser. And, if you substitute the value of v l here and assume this v l to be of this form a l some constant amplitude, exponential minus I omega t minus k z, because the source E L or B L they have this kind of dependence in t and z. So, in the quasi study state v l which is the response to these fields must also have same t and z dependence and in that case life become simple.

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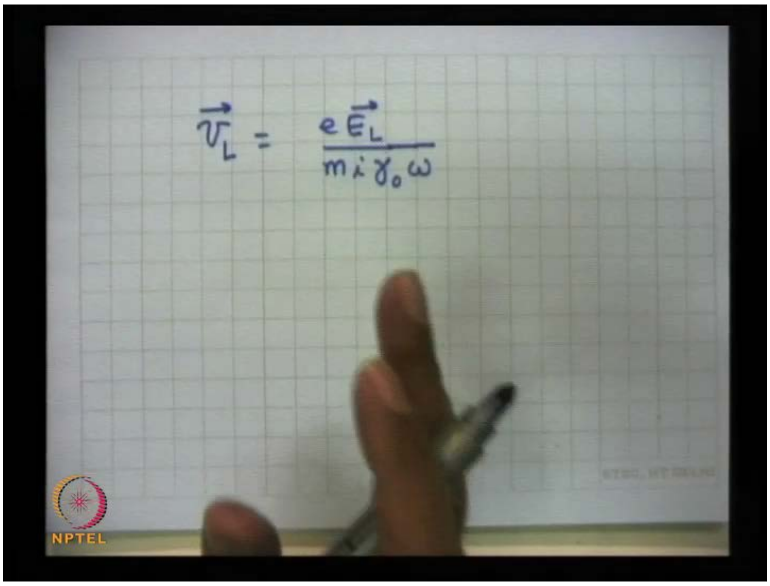

Replace delta delta t by minus I omega and delta delta z by I k and then we can write down the solution and I will write this solution as v l is equal to e E L upon m I gamma 0 omega this is really simple neat expression. So, I have obtained linear responses of electrons to wiggler magnetic field as well as to laser signal independently. Now, when the two are present simultaneously what will happen? We shall look into that issue in our next lecture, I think, I close at this point. Thank you.