Plasma Physics

Prof. V. K. Tripathi
Department of Physics
Indian Institute of Technology, Delhi

Lecture No. # 14

Cerenkov Free Electron Laser

Today, I will discuss the possibility of generation of electromagnetic waves by electron beams. The device that employs a slow wave is called Cerenkov free electron laser. This is a electromagnetic wave which is slower than the… which moves slower than the electromagnetic wave in free space and electron beam can generate these waves, the device is quite interesting.

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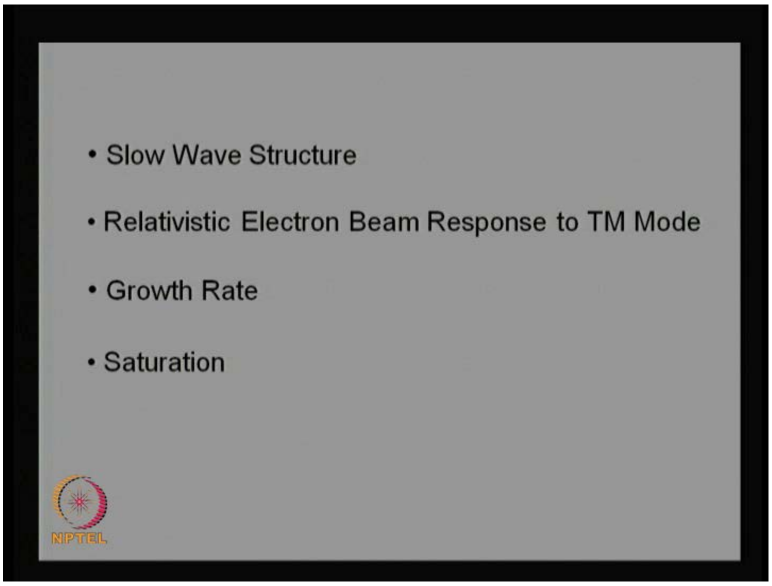

I will discuss a simplest structure that can slow down a wave. And then I will discuss the interaction of an electron beam to a transverse magnetic mode of this guiding a structure will deduce a dispersant relation, obtain the growth rate. And discuss the saturation of this instability. Well, this is the reference (Refer Slide Time: 01:34). Well, electromagnetic waves are little difficult to handle to be excited by electron beam. The reason is that, if beam is travelling suppose in this direction and it sees an electromagnetic wave.

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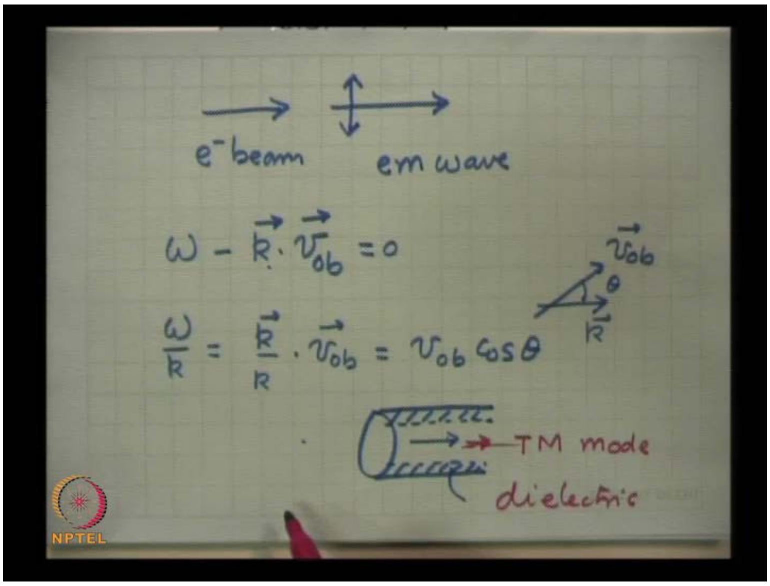

So, this is my electron beam and if my electromagnetic wave also travels in the same direction in free space. Then this electromagnetic wave will have a transverse electric field. So, first of all there is no longitudinal component of this electric field of this wave. And hence, there is no direct retardation of this electron beam and hence this scheme is not working cannot this, wave cannot be cannot extract energy from the electron beam. Second problem is that, Cerenkov resonance from momentum, for momentum in energy conservation we found that, if the wave is of frequency omega, and of wave number k then momentum in energy conservation if the electrons, is to emit a photon of frequency omega and wave number k then, this condition must be satisfied. This is known as the Cerenkov resonance condition, only when this is satisfied the momentum and energy are conserved between the electron and the photon. Because in free space k is omega by c so, if I divide this equation by k magnitude of k this becomes omega by k has to be equal to k vector upon k magnitude into v 0 b. So, if there is an angle theta between k and v 0 b then this, becomes v 0 b cos theta, because v 0 b is less than c, cos theta is also less than c. So, this is the angle suppose this is my k and this is my beam velocity so, if the angle between the two is theta then, this condition cannot be satisfied in free space, because omega by k is c.

So there are two problems, one you require a component of the electromagnetic wave in this direction, electric field of this wave in the direction of beam propagation. And secondly, we want to slowdown the wave, a wave can be slowdown if you pass this wave through a wave guide, suppose this is a wave guide. If it is having vacuum in the interior then, the phase velocity of the waves guided by a wave guide are always bigger than c so, this condition cannot be satisfied. However, if you line the wave guide by dielectric suppose, if a dielectric here in this region, near the walls dielectric, this is the dielectric here loaded dielectric here, then there is a possibility that the electromagnetic wave that travels through, this structure can have a velocity phase, velocity less than c.

And this condition can be satisfied, this is what is really happening in a device called Cerenkov free electron laser. You slowdown the wave by putting a dielectric lining, this is dielectric lining here, near the walls, through the effective velocity of the wave, in this region becomes less than c. And then this, such a structure guided structure supports, a special kind of wave called T M mode. And those modes have a finite electric field in this direction called e zee, if this the direction z axis. Then, transverse magnetic mode has a component of wave electric field, in the direction of axis or along the axis of the wave guide. And that can retard the beam electrons and convert, their energy into electromagnetic energy this is what happens in the Cerenkov free electron laser.

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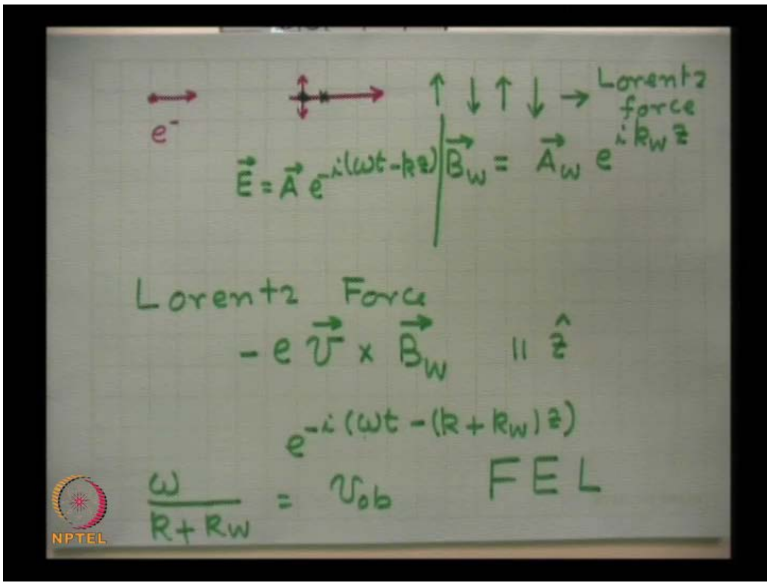

There is another interesting device, that does not slowdown the wave, but it says that, I want to generate an electromagnetic wave going in this direction with a transverse electric field. And a electron beam is coming from here like this, there is a possibility of energy transfer from here to here, if there is a magnetic field in the system, which can take momentum. So, if you can have a structure like this, with alternate polarity suppose, you put a magnetic here that produces a magnetic field in the vertically upward direction. And another set of magnets, which produces a magnetic field in the downward direction.

So, if you have this kind of esthetic magnetic field configuration, which is called a wiggler. And if wiggler is of this form some amplitude and exponential of I k W z. This is your electromagnetic wave, which will have an electric field of A exponential minus I omega t minus k z. So, this is electromagnetic wave that you want to excide, like the beam and this is in the presence of a magnetic field, called wiggler magnetic field what is the these two. If the electron sees this electromagnetic wave, the electron will oscillate in the direction of the electric field of the wave, well rather than I have not done this properly. This is these two are chosen perpendicular to each other so, rather than, I will choose that this electric field is really in this direction, perpendicular to the plane of the paper.

So, if this electromagnetic wave makes the electron oscillate in this direction, then those electrons will experience, a Lorentz force due to this magnetic field and that, will be in this direction so, this is the Lorentz Force. Due to the v cross B interaction of the electrons, which are oscillating due to one velocity and the magnetic field due to the wiggler. So, this force is in the same direction, as the direction of electron beam propagation. So, Lorentz Force which is a product of minus e the electron charge and the electron velocity and magnetic field. So, this force is parallel to z axis and if I multiply v w here and with this velocity due to this, then this force has a variation multiply, these two exponents of the form exponential minus i omega t minus k plus k w into z.

So, the force now does not have a phase velocity omega by k, the phase velocity the force is omega upon k plus k W. So, this force has to as a velocity k plus k W and whenever this is equal to be velocity. A resonant excitation of this wave can take place. So, you are not slowdown the wave, you have not introduced any longitudinal component in the electric field of the wave, but just by steer presence of a magnetic field, you can create a force on the electrons, due to the mutual interaction of the electromagnetic wave. And the wiggler magnetic field this force whose frequency is the same as the frequency of the wave, but wave number is a sum of the wave number of the wave, plus that of the wiggler and phase matching condition then becomes this much.

And such a device is called free electron laser or F E L free electron laser. We shall discuss, in substantial retail the mechanism of a free electron laser, but today I would like to focus my attention on the, previous device that I call as Cerenkov free electron laser.

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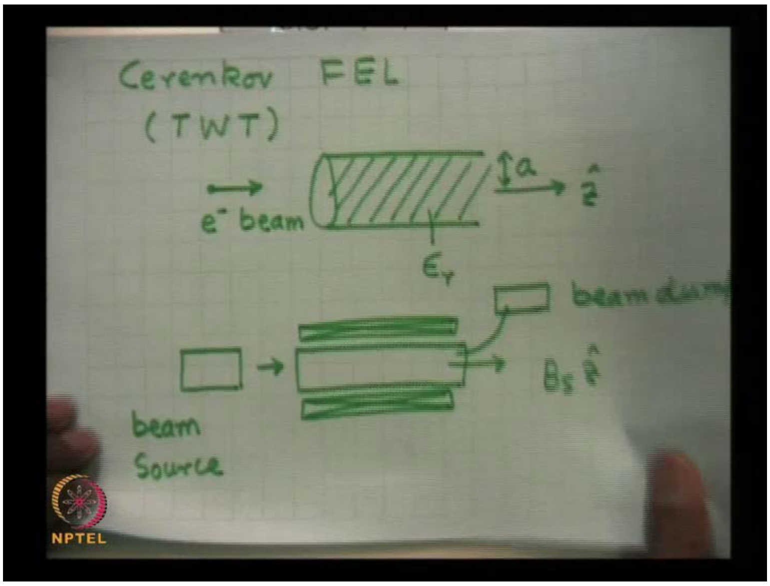

If the device employs a non-relativistic electron beam and a different kind of a structure, is also known as traveling wave tube, but I will be focusing my attention on Cerenkov wave F E L T W T will treatment is very similar. Now, in order to simplify the mathematics, I want to really highlight the physics of this process. So, I will rather than considering the dielectric to be confined, near the boundary as if the dielectric effective permittivity medium is full in the entire region.

So, I will presume that, this dielectric of effective permittivity epsilon r is filling in the wave guide, this as I will take a z axis and the radius of the wave guide, I will choose as a and I will be considering the excitation of a T M mode of the wave guide. T M mode is the mode, which has a component of electric field in the z direction also, electron beam will be launched into this device and we would like to see, how the wave amplitude grows in this a structure. A schematic of a Cerenkov wave F E L is like this, you have a gun that produces an electron beam then you have a interaction chamber this region, in which you have a dielectric lining of slow wave structure. And in order to guide the electron beam you put a usually, you put a field coil here that raps as a winding rapping round the plasma, this a chamber and that produces an excel magnetic field.

So, this is written like this, there is no plasm a here simply electron beam is traveling. And so, there is a guide magnetic field in this direction and after the beam emerges out of this a structure, the beam is taken to some where here called beam dump, this is called a spent beam. The initial beam is launched here, this is called gun or beam source and beam dump is there, I am focusing my interest, the attention on the interaction of an electromagnetic wave with this electron beam. And see how this wave is amplified is excited. Well the physics of energy transfer is very similar to two stream instability, that you need the electrons to be bunched in slow in retarding phases and they, must give net energy to the wave, well what is the equation of motion for this case, the same equation as I had written d p by d t is equal to the electric force.

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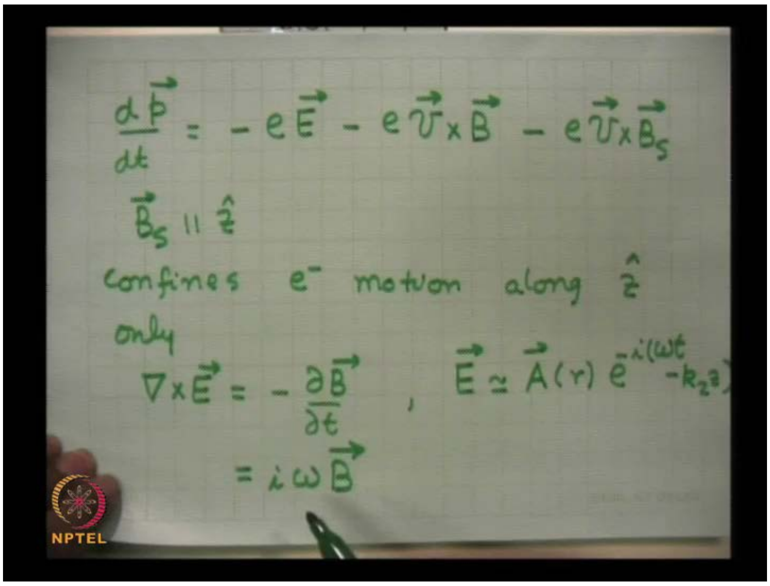

And the magnetic force, there are two magnetic fields in the system the magnetic field of the wave and there is a magnetic field that you have externally, applied d c magnetic field so, e v cross B s esthetic magnetic field that, I have applied in the z direction. To make mathematics simpler, I will presume that, this guide magnetic field that I have applied is so strong, that it does not allow, because B s I have chosen to be parallel to z axis and I choose this to be very large, it does not allow the electron to travel perpendicular to lines of force.

So, this magnetic field confines electron motion along z only. So, I do not have to worry about the x component of momentum or y component of momentum just consider, this z component of equation of motion and how about this b field here. B field of the wave, I can recover from the electric field of the electromagnetic wave, because from fourth third Maxwell equation curl of E is equal to minus delta B by delta t. If I t take my fields to be of this form, which is some function of r exponential minus i omega t minus k z Z.

So, I am presuming my wave to be travelling in the z direction with some phase variation like this, but the amplitude may not be uniform inside the wave guide it may depend on r, it may also depend on the azimuthal angle pi, but I take I consider only the azimuthally symmetric mode so, I ignore the pi dependence. If you have a electric field of this sort, then magnetic field will also have an same t and z dependence. And you can replace delta, delta t by minus i omega so, it becomes i omega B. So, if you know E B can be written my issue, is to obtain the response of beam electrons, to this electromagnetic wave with electric field E and magnetic field B first of all. Let me linearize this equation.

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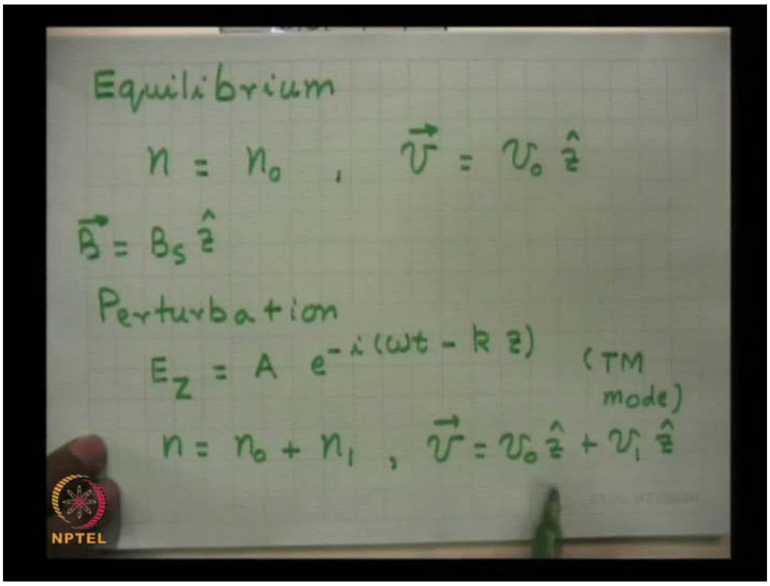

Let me say that equilibrium is like this, there has no plasma only beam electrons are there. So, the equilibrium has beam density is equal to n 0 and beam velocity is equal to v 0 in the z direction, this may beam perturbation and obviously, there is a esthetic magnetic field also in the equilibrium, there is no other field B s z this is the magnetic field in equilibrium. Now, I perturb the equilibrium so perturbation I will write electric field of the wave, certainly we will have many components e r E Z e pi, but they are all expressible, in terms of the longitudinal components. So, let me write E Z is equal to some amplitude A which depends on r exponential minus I omega t minus k z, this is my T M mode, means h z is 0, but other components of magnetic field are finite.

In presence of this, I will have density is equal to n 0 plus n 1, the velocity will be v 0 z plus v 1 z, because magnetic field does not permit. I am presuming that, any velocity of electrons perpendicular to z axis so this is the velocity. Let me linearize the equation of motion, treating electric field to be a perturbation n 1 to be a perturbation b 1 v 1 to be a perturbation and magnetic field of the wave, also to be a perturbation.

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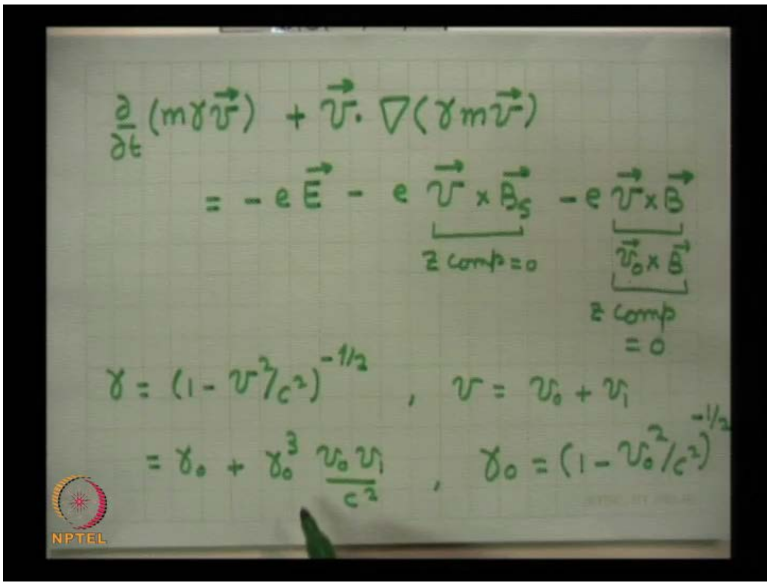

So, then my equation of motion would become; delta, delta t of p which is m gamma v this is the momentum plus v dot del gamma m v, it is again momentum here is equal to the electric force, which is minus e E minus e v cross B esthetic minus e v cross B of the wave. First of all examine this term, I am writing only z component, z component of this term is 0, because v s is in the z direction. So, this force will be a perpendicular to z axis. So, z component is 0. How about this to one, if I write v equal t v 0 plus v 1 and ignore the product of v 1 with b because this is a perturbed quantity, then v 0 is also in the z direction so, this force will be a perpendicular to z axis.

So, this will give you v 0 cross B and this is the z direction. So, here has no z component so z component equal to 0 here so this does not give you a z component this also does not give you a z component. So, this is only place where z component will be there. So, when I write down here and divide this equation by mass, which is the rest mass gamma is the lorentz factor v is the electron velocity. So, I think in order to simplify these terms here, I need to find the products of gamma v. Let us see what is gamma, gamma has before is 1 minus velocity square by c square to the power minus half. So, when I write velocity is equal to v 0 plus v 1 all in z direction, then this becomes equal to, if you just substitute and carryout a binomial expansion it becomes equal to gamma 0 plus gamma 0 cube into v 0 v 1 by c square, where gamma 0 I have written as 1 minus v 0 square by c square to the power minus half.

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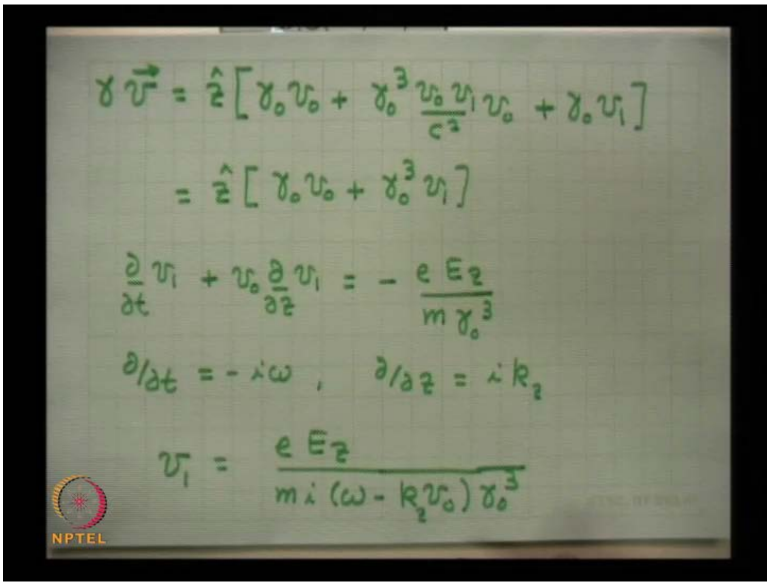

Just the way, we have done this for two stream instability; we can carry out this expansion of gamma. Then linearize this equation, what we get gamma v turns out to be having actually only z component I am interested. So, this becomes a gamma 0 v 0 plus there is a term called gamma 0 cube v 0 v 1 upon c square into v 0 from here, this is the one thing plus gamma 0 into v 1. And when you combine these terms, it becomes is equal to z gamma 0 v 0 plus gamma 0 cube v 1, use this in the equation of motion and linearize you will get delta, delta t of v 1 plus v 0 delta, delta z of v 1 is equal to minus e E z upon m gamma 0 cube, this is the linearized equation of motion including relativistic effect, it comes through gamma 0 cube.

Now, replace delta, delta t by minus i omega, because the response should have same t and z dependence as the source, E z is the source and v 1 is the response. And replace delta, delta z by i k and this equation, then gives you velocity which is equal to e E z over m i omega minus k k z rather k z v 0 into gamma 0 cube.

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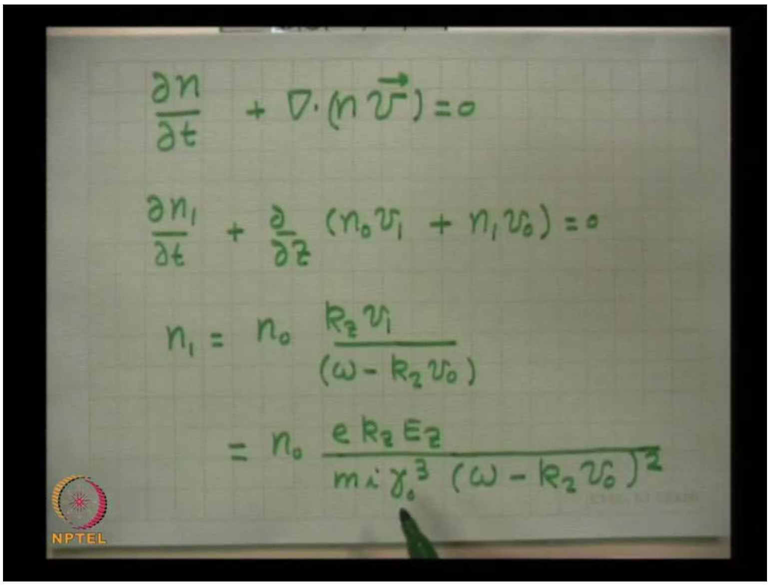

This v 1 is the z component of electron velocity, due to the electromagnetic wave. Go over to the equation of continuity, which is reads as delta n by delta t plus divergence of n v is equal to 0, because this v is in the z direction all the velocities are in z direction only delta, delta z will be will survive here then linearize this term. So, you will get delta n 1 by delta t plus delta, delta z of n 0 v 1 plus n 1 v 0. Linearization here means, I have ignored the produce of n 1 v 1, I substituted n equal to n 0 plus n 1 v equal to v 0 plus v 1. And ignore the products of n 1 v and n 0 v 0 terms does not contribute, because it is uniform. So, when you operate by this dial operator that will be a vanish.

So, this is the equation now replace this by minus i omega delta delta z by i k. And you will get n 1 is equal to n 0 k z v 1 upon omega minus k z v 0. If you use the value of v 1, that we had obtained earlier this becomes n 0 e k z E Z upon m i gamma 0 cube and omega minus k z v 0. This is square density perturbation becomes very large. In the vicinity when omega upon k z becomes equal to v 0, when the phase velocity of the wave in z direction becomes equal to beam velocity. This is a important point here and obviously, it decreases with increase in gamma 0. So, if you are having a beam of larger and larger energy, larger kinetic energy then gamma 0 will be larger and this effect will be suppressed, well for generating electromagnetic waves you require a current.

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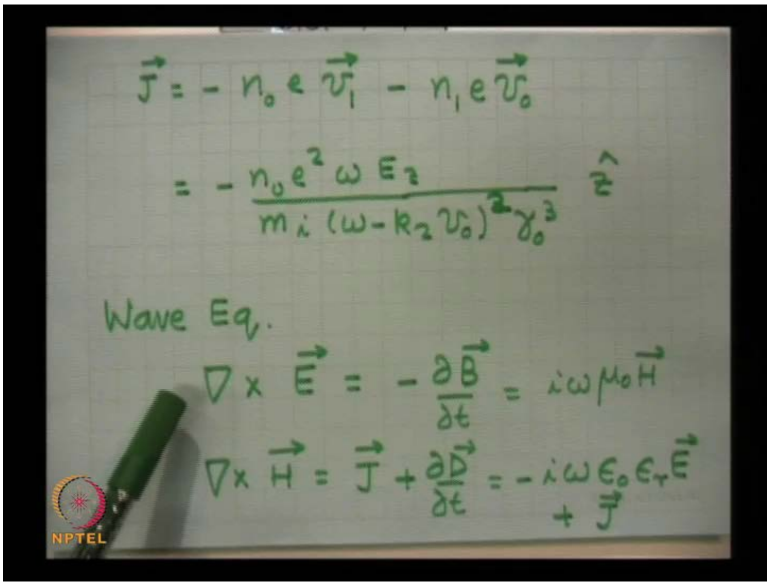

So, current is at frequency omega is equal to minus n the electron density in the equilibrium, electron charge which is minus e into velocity v 1, but there is a another term which is minus n 1 e v 0.Actually, it is a product of electron charge and electron density into electron velocity, electron density is n 0 plus n 1 electron velocity is v 0 plus v 1. So, multiply these n 0 v 0 term I have not written, because that is a d c term.

So, there will not produce any wave, it is only terms that have a time dependence, at the frequency of radiation that, we want to excite or amplify has to be retained. So, once you calculate this substitute the value of v 1 and v 0 sorry n 1 it turns out to be equal to minus n 0 e square omega E z upon m i omega minus k z v 0 cube sorry v 0 square into gamma 0 cube and that is the z direction.

So, a beam when travels in the presence of a low amplitude electromagnetic wave in a wave guide, it produce it acquires a oscillatory current density. And this oscillatory current density, will then give rise to the amplification of radiation. Now, let us see how does it happens. The wave equation for the electromagnetic wave in the wave guide, can be deduced from these two Maxwell equations, curl of E is equal to minus delta B by delta t which is equal to i omega mu 0 H and curl of H which is equal to J plus delta D by delta t. Now, dielectric is loaded in the wave guide so D I can write this term, I can write down this minus i omega for delta delta t and D is equal to epsilon 0 into epsilon r into E. This is the relative permittivity of the medium filled inside the wave guide or called dielectric constant plus J the current density which is given here.

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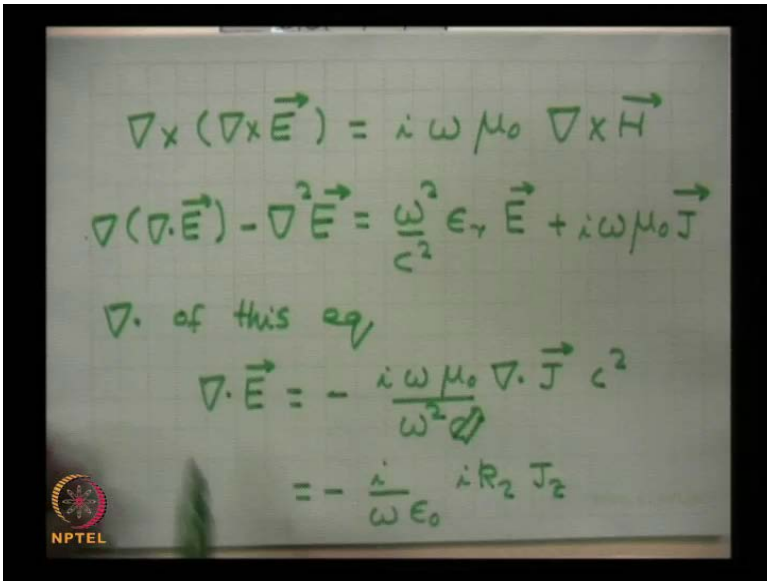

So, these two equations can be combined into one, if I take curl of the first equation. And the result is curl of curl of E is equal to i omega mu 0 curl of H use the value of curl of H from the second equation,it becomes omega square by c square epsilon r into E and plus i omega mu 0 into J. This can be expanded as gradient of divergence of E minus del square of E.

If I take divergence of E this is a inconvenient term. So, in order to obtain the value of divergence of E take del dot of this equation, then these two terms exactly cancel each other. And this gives me divergence of E turns out to be equal to here this is minus i omega mu 0 into divergence of J, because I am taking del dot so del dot of E and del dot of J omega mu 0 I have written, upon this quantity omega square by c square. If I put c square as I think this is c square is sorry c square will go up, it will not here I am sorry I made a mistake c square goes up. So, this can be simplified, because c square is one upon mu 0 epsilon 0. So, it becomes minus i upon omega epsilon 0 and J is in the z direction so only delta delta z which is i k z into J z will come there this is all.

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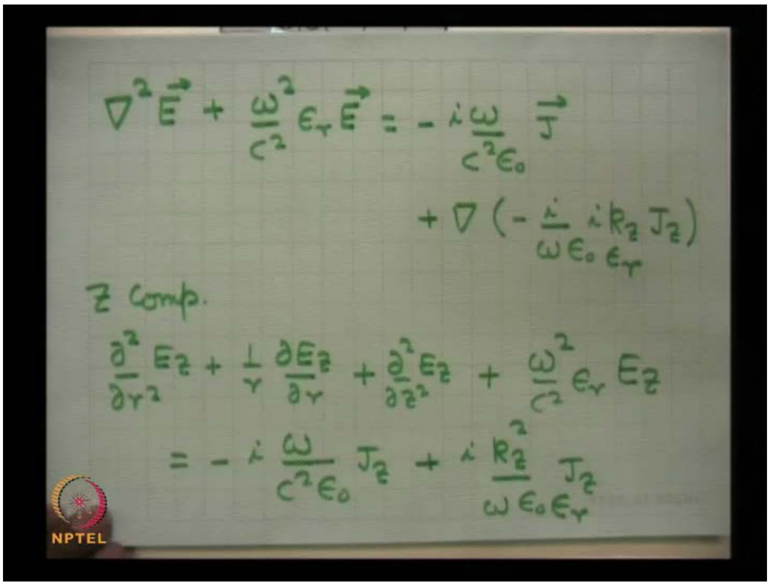

So, once I have obtain the value of divergence of E my equation becomes simple. And I can write this equation as del square of E plus omega square by c square epsilon r into E is equal to I can write that equation on the right as minus i omega by c square epsilon 0 into J. And then there is a term that, I have brought from left which is plus gradient of divergence of E divergence of E is minus i omega epsilon 0 i k z into J z, my goal is to write the equation for E z. So, z component of this equation I will write, this gives me and del square is I will explain a expand rate. So, d to d r square of E z plus 1 upon r delta E z by delta r plus d to d z square of E z this is the value of this quantity. If I presume that, there is no phi dependence.

So there is no azimuthal dependence of E z so this is the z components term plus omega square by c square epsilon r into E z. The right hand side is minus i omega upon c square epsilon 0 J z how about this term dial operator z component is delta delta z which is i k z. So, if I put i k z here, then what do I get here i k z and minus i will become plus. So, it is becomes plus i k z square over omega epsilon 0 into J z. These two terms, can be combined. I think, did I make a I think I made an error. When I wrote down the expression for at divergence of E from here, I forgot to write epsilon r here put epsilon r there this epsilon r I missed. So, if I take correct for it then there will be epsilon r there and there will be epsilon r here. So, these terms can be combined and this d to d z square, I can write down as minus k z square.

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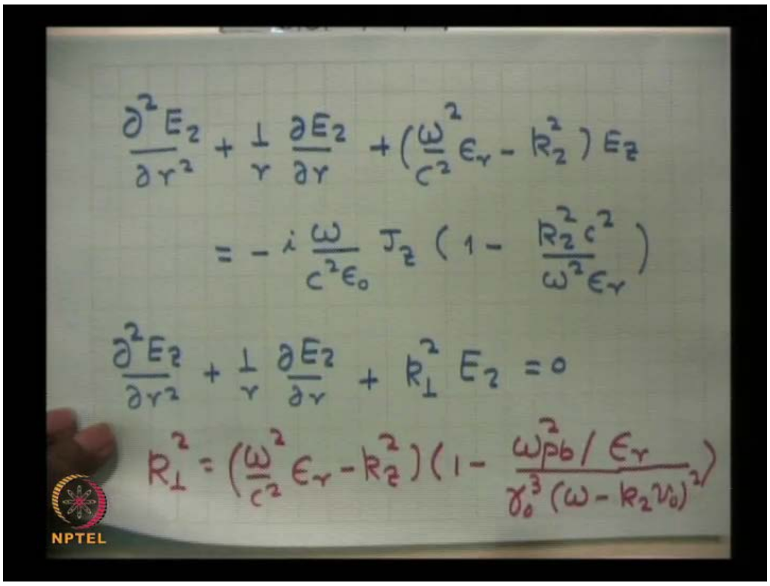

So, this equation becomes rather simple. And let me write down the value of this, rewrite this equation it becomes d 2 E z by delta r square plus 1 upon r delta E z by delta r plus omega square by c square epsilon r minus k z square into E z this is equal to, if I take minus i omega upon c square epsilon 0 common and J z also common, here you will get a term, which is 1 minus k z square c square upon omega square epsilon r. This is the similar factor as the factor over here, if I take this factor common, this is the same factor and J z already I have obtained, when you substitute for J z in terms of E z this equation take the following form.

D to E z by delta r square plus 1 upon r delta by E z by delta r plus some quantity a constant quantity, which I called as k perp square E z equal to 0. Where k perp square is a important quantity, that depends on frequency is a combination of this term and terms coming because of these and it is value turns out to be this much. Let me write down the value of this expression, it is a simple expression k perp square turns out to be a common factor like this, which I can write as omega square by c square epsilon r minus k z square multiplied by 1 minus omega p b square beam plasma frequency square upon epsilon r upon gamma 0 cube omega minus k z v 0 whole square.

This is something like, 1 minus omega p square by omega square kind of thing a beam velocity is not there, beam is not relativistic and epsilon r is unity. So, this looks like a plasma permittivity. And this is if plasma, if not there then this is simple k perp in a dielectric loaded wave guide. So, plasma contribution is contained in the second or the beam contribution is contained in the second term. And through this, we have to see how the beam transfers it is energy to the wave, how the wave is amplify, let us see.

So, I have to solve this equation with proper boundary condition, E z is the actual component of electric field at r equal to a the valve of the wave guide. It must vanish because if it is a metallic wave guide, then the tangential component of electric field must vanish. But before I do that what the solution of this equation, this equation is Bessel equation, whose solution is J 0 Bessel function of 0 th order.

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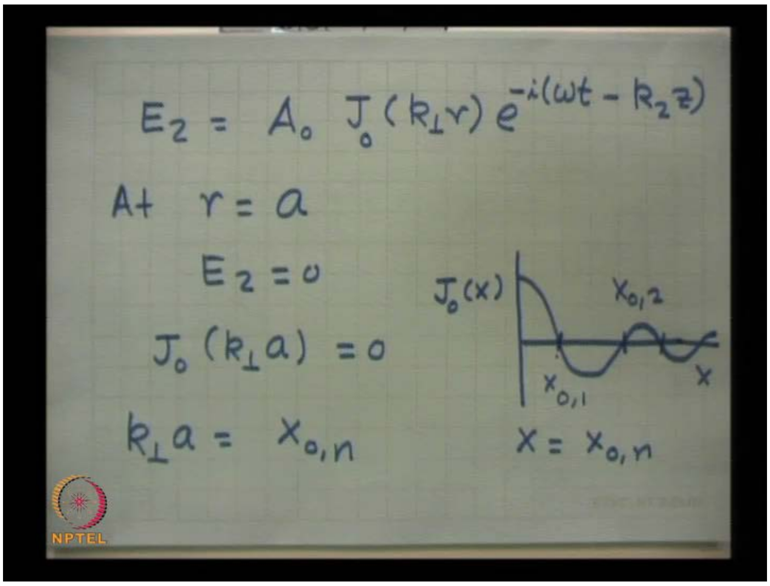

So, I can write down the solution of this equation as, E z is equal to some constant into Bessel function of 0 th order and argument k perp r and the dependence of this field on time is like this, omega t and this z is k Z z so this is my field. And this must be vanish, at plasma boundary or at the boundary the wave guide, at r is equal to a I want E Z to be 0 means J 0 of k perp a must be 0, but if you plot J 0 as a function of it is argument it goes like this. Suppose, I plot here J 0 if I plot of some argument X as a function of X, it goes like this, means the it is A 0 at many values of X this is called the first 0 of X the second 0 of X third 0 of X means J 0 is 0, when X is equal to some values they are called

X 0 n, this is called X 0 1, this is called X 0 2, this is called X 0 3 and so on. The value of X 0 1 is around three.

So, what you get from here, if this is to be 0, then k perp a cannot take arbitrary values, it can take values only equal to this much. So, k perp a has to be equal to the 0 of the Bessel function. Once, k perp is quantized by this condition then if I substitute the value of k perp in the definition of k perp, I get the dispersant relation for the electromagnetic mode. This is what I was we were looking for.

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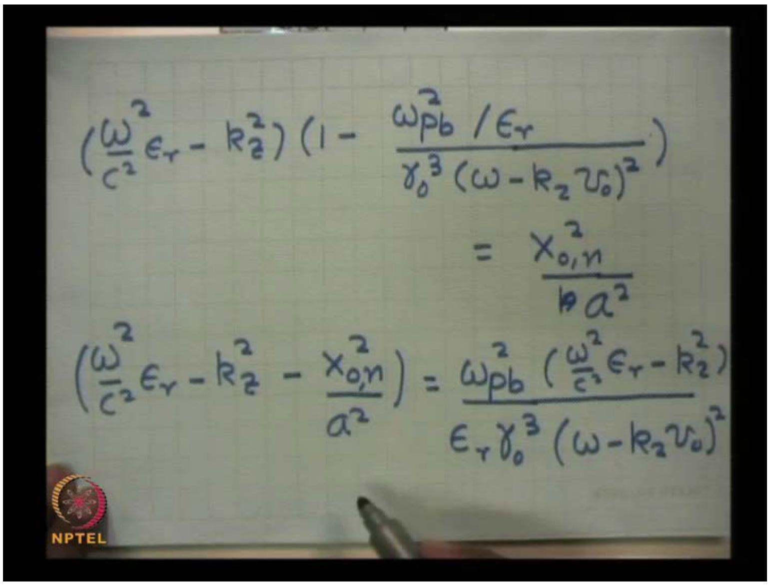

So, what you get is this, the put this value of k perp in that expression. And that becomes omega square upon c square epsilon r minus k z square I am putting the value of k perp square multiplied by 1 minus omega p b square divided by epsilon r divided by gamma 0 cube into omega minus k z v 0 whole square is equal to x 0 n square upon k perp upon a square. So, this is the dispersant relation this has, this is a fourth order equation in omega and this will give you four roots, in general and I am looking for the unstable root. Please understand, if the beam were not there this term will be 0.

So, what I want to do, if the beam density is very low I except that the modification caused in the dispersant relation by the beam to be small. So, I would like to take the beam from the right hand side. And non beam terms on the left hand side, then this equation becomes, so retain this into 1 and bring this on the left hand side, then this can be written in the following form, omega square by c square epsilon r minus k z square minus x 0 n square by a square. And then take this omega p b term on the right hand side, which becomes omega p b square upon epsilon 0 epsilon r into gamma 0 cube into this resonant denominator k z v 0 square multiplied by this factor, which is omega square by c square epsilon r minus k z square. This expression of this equation is very similar; to the one that we encountered in two stream instability of a plasma wave, what you can do, you may note that these streams are constants only omega square is in the first term.

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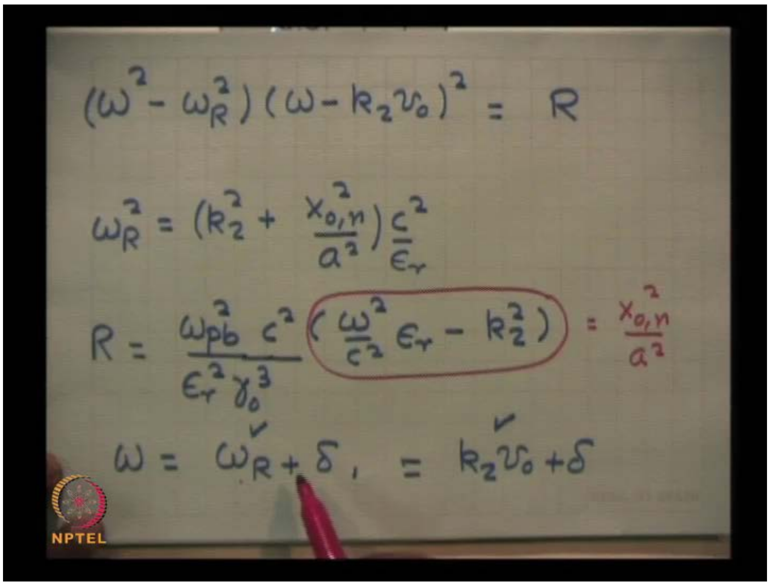

So, if I take epsilon r by c square common and divide that by this equation by that factor, then this equation can be caused in the similar form. And the form is like this, omega is square minus, I will call this omega R square, rather than omega p, this quantity I will write. And this can be put in this form minus k z v 0 square is equal to right hand side I will call this as R, what is omega R square, omega R square I am defining, as a term which is equal to k z square plus x 0 n square by a square multiplied by c square upon epsilon r, because this is factor I have taken common.

So, this thing and R turns out to be equal to omega p b square upon epsilon r square gamma 0 cube c square here multiplied by omega square by c square epsilon r minus k z square divided by well this is all. So, this is the kind of expression this equation we get, which is very similar to the two stream instability problem. And you except that, right hand side though very small, because the beam density is a small. So, this is small term, but it can significantly modify the electromagnetic wave, if these two factors are 0 simultaneously.

So, we solve this equation for a special value of k, k z when k z v 0 is exactly equal to omega R so we say that, because of this r or because of the beam. When beam term is retained the value of omega is modified, otherwise is not there omega equal to omega R or omega equal to k z v 0. But when this R is finite omega is not exactly equal to omega R we say that, omega is equal to omega R plus delta, which is also the same thing as k z v 0 plus delta, because I am choosing omega R is equal to k z v 0, I am solving this equation. So, that if this term is not there, they are simultaneously 0, because I am excepting the largest growth rate as in two stream instability, when this term equals to this omega R is equal to this factor. So, in this limit I will solve this equation.

Now, again assuming that this modification is a small as compare to omega R or k z v 0 as a result, when I substitute this here, in this first term delta square I ignore. And when I put this, in this term this becomes simply delta square. So, I get a delta cube term here, how about the right hand side, in the right hand side I will say that, well put the value of omega equal to simply omega R forget this delta term here. Then this term factor becomes quite simple, if you just put the value of omega R from here, it becomes simply this whole factor becomes is equal to. Let me just write the values of this at this factor.

This factor simplifies, to equal to x 0 n square by a square simple this factor, just substitute it. So, I will use this expression or replace this expression by this term and then r becomes quite simple. And just put this in there my equation terms out to be like this, I get delta cube is equal R divided by 2 omega R I multiply this quantity by exponential I twice l pi, because this is always unity l is an integer 0 1 2 3 they are the values.

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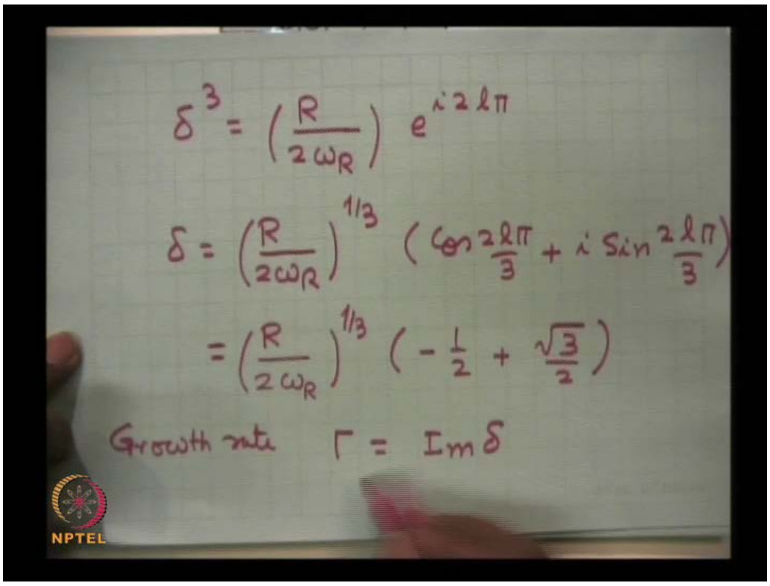

So, take the cube root of this equation and delta is the same similar expression as R upon twice omega R as 1 third power into cos twice l pi by 3 plus i sin twice l pi upon 3 for l equal to 1 this gives unstable root. And the value turns out to be equal to R upon twice omega subscript R to the power 1 third this is minus half plus root 3 upon 2. So, the growth rate is gamma, which is the imaginary part of delta. So, let me explicit the after substitute the value of omega R and R.

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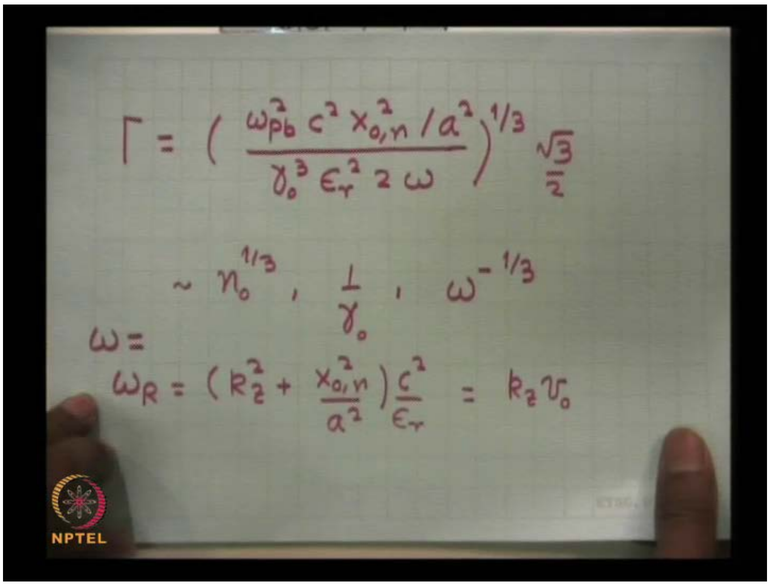

Let us see, how delta looks the delta turns out to be this expression let me just write this. The growth rate turns out to be omega p b square c square x 0 n square upon a square a is the wave guide radius upon gamma 0 cube epsilon r square into two omega omega R or omega same thing, this is the operating frequency of the Cerenkov wave F E L 1 third root 3 by 2. So, the growth rate scales as 1 third power of beam density, it is scales as 1 upon gamma 0. The beam energy it falls down, how was the frequency this is scales as omega to the power minus 1 third, higher the frequency smaller the growth rate, but who details the frequency, that is the important issue. The frequency of the wave is decided by this resonance condition, that omega R which was is equal to I had written the value of omega R. And let me just recall this value, which is k z square plus x 0 n square upon a square c square by epsilon r has to be equal to k z v 0.

So, from this equation you can obtain the value of k z. And put that value of k z multiply by v 0, you get the frequency of the operation. So, operation frequency of the Cerenkov device, Cerenkov wave F E L is this much. And as I mentioned in the case of two stream instability, the instability saturates by particle trapping, I did not prove it there, but one can easily show, that the same condition is equivalent, roughly equivalent not exactly equivalent to saying that, when the electron beam transfer it is energy.

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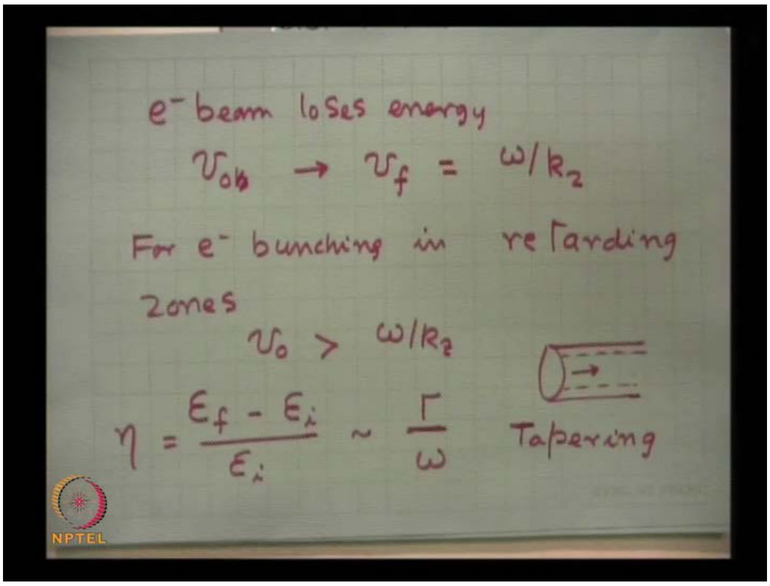

So, electron beam looses energy and it is velocity falls down from v 0 b, to say v from initial value it was v 0 and final value is v f, which becomes equal to omega by k z of the wave the growth where it stop. So, as the electron beam looses energy, it is velocity reduces from original velocity v 0 to say final velocity equal to omega by k z .And in that case, beyond this point it will not be able to generate, the electromagnetic radiation. Because the necessary condition for electron bunching and retarding phase is was for electron bunching, in retarding phases retarding zones, you require v 0 to be greater than omega by k z.

So, what is happening, initially the beam velocity is bigger than omega by k z as the beam on an average loose energy. And their velocity comes down to omega by k z, they stop giving energy, to the wave and, if you calculate so energy of the electrons, well it is little tricky dependence on v 0, because a relativistic case. So, final energy minus initial energy of the electron divided by the initial electron energy, this is the efficiency of the device. So, much fraction of electron energy will go to the wave and this quantity depends on the growth rate typically, I would say this of the order of growth rate upon omega, the frequency of radiation and this could be of the order of few percent.

So, Cerenkov free electron laser, can produce radiation with a reasonable efficiency, you can improve the efficiency by forcibly slow in how the wave. You know what we have done; in the wave guide you have a lining like this, there was a dielectric here, If you taper the wave guide. So, that when the wave travels it becomes slower and slower. So, what will happen, as the beam slows down if you slowdown the wave also, that wave velocity is always bigger than, the electromagnetic wave velocity, then the beam will continue to transfer it is energy to the wave and the wave will be amplified.

So, this by tapering you can increase the efficiency of the device, tapering is a very important scheme of increasing the efficiency of the device. When we discuss, free electron laser, we shall return to this issue of tapering and efficiency enhancement in some detail. Thank you very much.