Plasma Physics

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Lecture No. # 13

Relativistic Electron Beam – plasma Interaction

Today we will discuss, relativistic electron beam plasma interaction extending our work that we did earlier on non relativistic electron bean plasma interaction.

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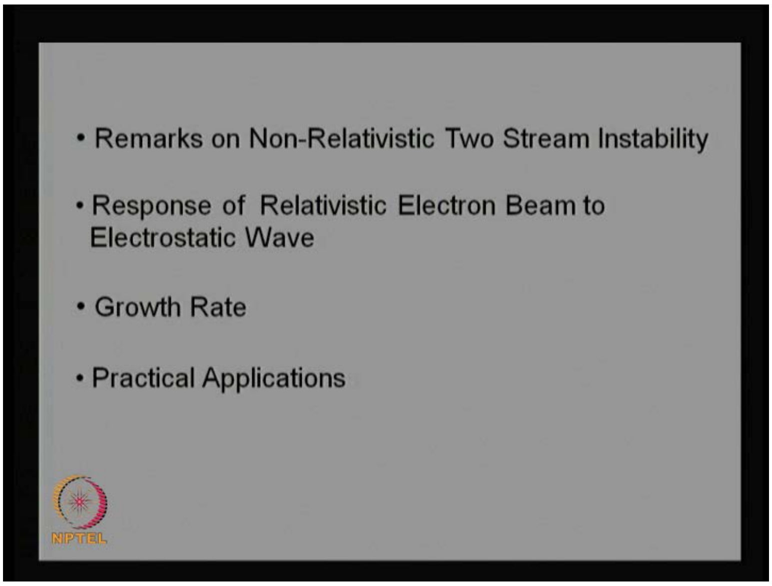

Well, delving into the problem of relativistic electron beam plasma interaction I will make a few remarks on non relativistic two stream instability. And then I will go over to discuss, the response of relativistic electron beam to an electrostatic wave or a plasma wave. We will deduce a dispersion relation and obtain the growth rate and then we will sight some practical applications of this study.

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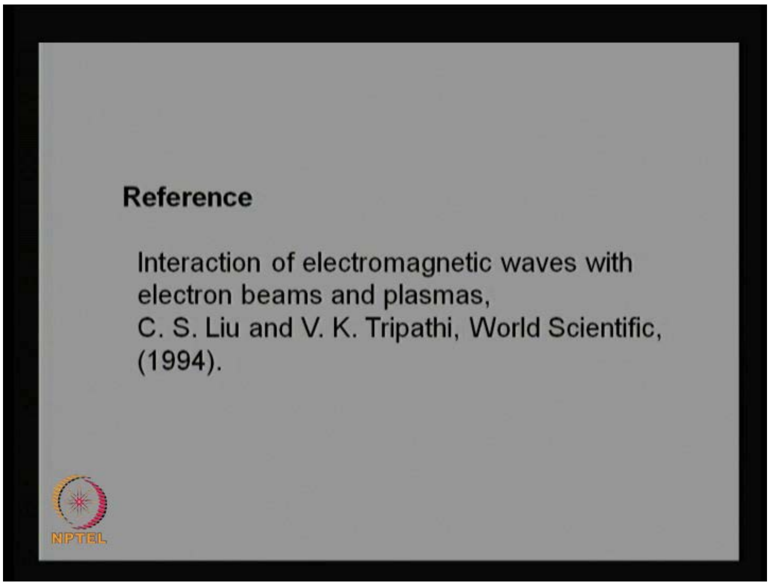

Well the difference for today's presentation is this book interaction of electromagnetic waves with electron beams in plasmas by professor C S Liu and myself.

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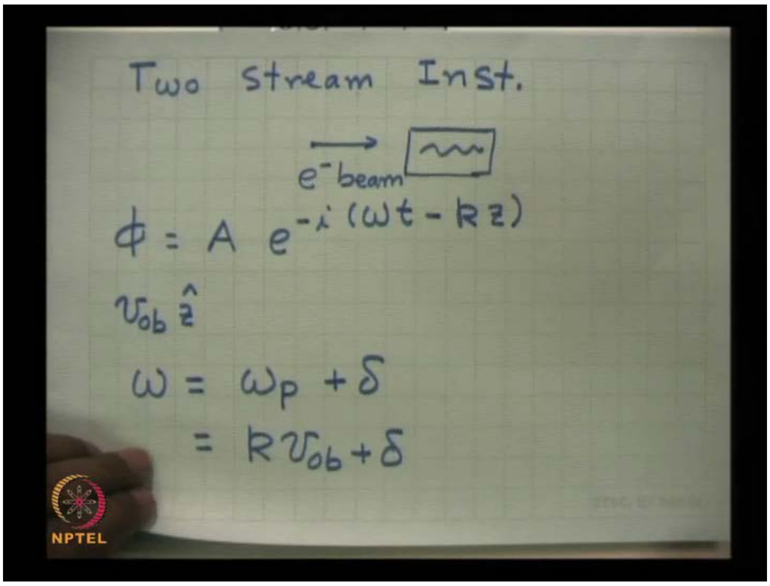

Let me begin with the remarks on two stream instability this we studied earlier for a non relativistic electron beam, we considered that, if there is a plasma through which an electron beam is launched then it gives rise to generation of waves in that plasma. The wave in the plasma we called it as a plasma wave with a potential phi we wrote down as some A exponential minus i omega t minus k z. And the beam had a initial velocity v 0 b along z direction as a consequence of the interaction of the beam with the plasma, we found that the frequency of this mode omega was expressible as omega p the plasma frequency of electrons of the plasma plus a quantity delta which was also equal to k v 0 b plus delta and delta v obtained?

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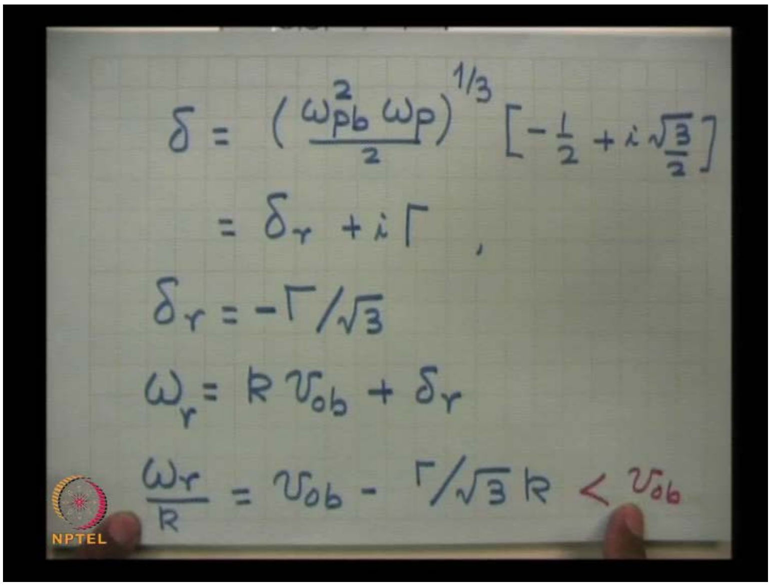

The value of delta that we obtained was is equal to omega p b square the beam plasma frequency square into plasma frequency of the plasma electrons divided by 2 raise to the power one third and for the unstable mode this was minus half plus i root 3 by 2. I can call this quantity as real part of delta plus imaginary part of delta that I called as gamma. But if I look at this expression delta r is simply equal to gamma upon root 3 this term is root 3 times bigger than this one. So, we find and this is negative (( )) what is the consequence of this? The consequence of this is that the omega that I had written is equal to k v 0 b plus delta.

So, if I write down the real part of (( )) should be equal to delta r. But because delta r is negative you may note here that, if I calculate the quantity omega r upon k it is equal to v 0 b minus gamma upon root 3 k. So, this is less than the beam velocity the ratio of real part of frequency to wave number is called phase velocity of the wave. So, phase velocity of the wave is less than v 0 b for the unstable plasma wave. The wave whose amplitude grows with time must necessarily, have a phase velocity less than v 0 b and this is the same condition that we deduced physically for charge bunching. So, this condition essentially ensures that the electrons are bunched in retarding phases of the wave.

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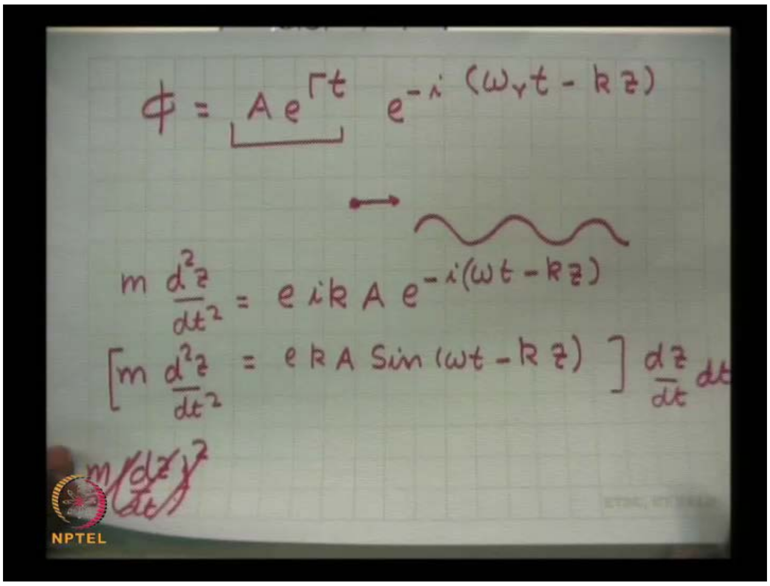

Now, with this the put amplitude of the wave will grow so what happens that phi now appears like A e to the power gamma t exponential minus i omega r t minus k z this is the growth in the wave amplitude with time. The question arises how long this amplitude will continue to grow, after all when a plasma generates a wave of growing amplitude is so much now then it becomes larger and larger. So, when the waves has acquired substantial amplitude then obviously, the electron beam may get trapped inside the wave these are the potential energy minima. So, what can happen, that when the electron can be trapped in this wave then it will back bounce back and forth and it will no longer give energy to the wave.

So, the instability will stop when the wave has acquired large amplitude and the electrons are trapped beam electrons are trapped in the potential energy minima of the potential of the wave. Now, in order to understand the trapping of particles we can do one thing, lets write down the equation of motion, the equation of motion is m d for a single particle I am writing so d 2 z by d t square this is acceleration is equal to the electrical force due to the wave on the beam electron. Which is equal to e grad phi and grad phi is simply i k into phi which is A exponential minus i omega t minus k z. I am examining the behavior of the electron in some sort of a steady state when the amplitude of the wave is no longer growing.

And I want to find out how the electrons behave in the potential energy minima of the wave. So, in order to understand this let me write down the actual force on the electron is the real part of this expression. So, it becomes e k A and I can write down this as sin omega t minus k z, if I divide multiply this equation now by d z by d t. So, this is m is already there d 2 z by d t square is there I am multiplying both side of this equation by d z by d t and in and d t and integrate on time. This first term gives you d d t of how much you get? You will get d z by d t whole square into m by 2. I think I made an error this integration stuff, because t is involved in there.

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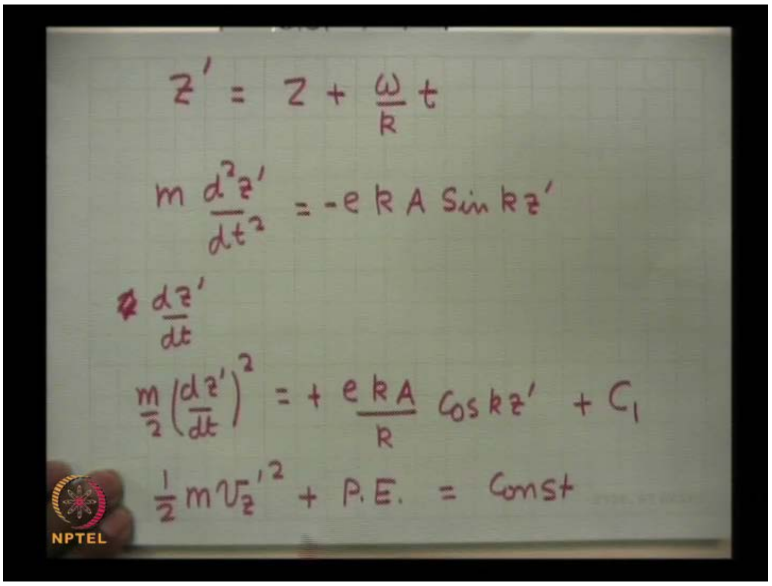

So, first of all I must get rid of this so I think I must do something else, before I multiply this, I will do the following. I introduce a new variable z prime, which is the coordinate of any point in moving frame moving with the velocity of the wave. So, let me call this quantity z prime is equal to z plus omega upon k into t, let me introduce a new variable. When I substitute this in that equation, my equation becomes m d 2 z prime d t square is equal to e k A sin k z prime and with a negative sin here this is the equation that I get. Now, this equation can be integrated, because there is no explicit time dependence on the right hand side.

So, just by making a transformation in from z to z prime given by this expression, my equation of motion for the electron of the beam in the presence of an electrostatic wave become so much. Now, multiply this equation by twice or rather d z prime by d t and integrate this two is not required you can delete this multiply this and integrate. This equation essentially becomes d z prime by d t whole square into m by 2 is equal to minus e k A. Now, when you integrate sine k z prime it gives k here and becomes cos function of k z prime and plus a constant here, constant of integration c1 this equation is a simple interpretation.

This is the velocity of electron in the moving frame. So, half m v square is the kinetic energy of the electron in the frame moving with velocity omega by k the phase velocity of the wave. This is the potential energy because a cos phi cos k z prime is the potential of the wave. So, k obviously will cancel out and this becomes the potential energy when you take on the left hand side. So, this can be written as kinetic energy means, half m v z prime square plus potential energy, which is negative of this quantity is equal to constant. Which means that, if I plot potential energy let me write down potential energy.

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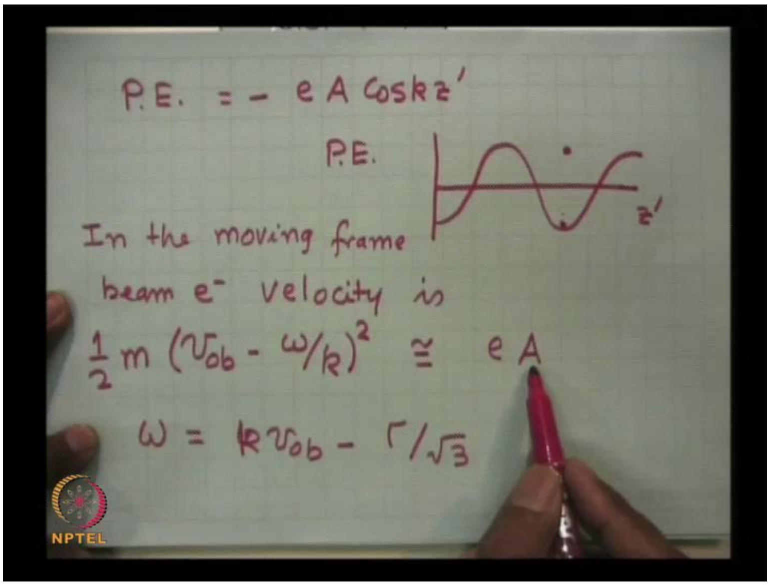

Potential energy is minus e A cos k z prime and if I plot this quantity as a function of z prime potential energy plotted here then this quantity will be like this. So, there are regions of minima and this is where the potential, which is minimum what will happen, as the wave amplitude grows with time initial then this amplitude well this potential energy well will increase. So, the height from here to here this will increase with time and it is possible that in the moving frame the electron velocity is how much. So, in the moving frame beam electron velocity is v 0 v minus omega by k is the velocity.

Whenever, so what is the kinetic energy this is half m and this quantity square. So, this is the kinetic energy of the electron in the moving frame. Whenever, the difference in potential energy from here to here becomes of this order the electron will be totally trapped. So, the condition for trapping is when this becomes of the order of e A of that order may be you can put e A from bottom to top typically. So, when this is of this order the electron will be trapped in the potential energy minima of the plasma wave and this gives the amplitude of the plasma wave.

So, if you want to calculate the amplitude of the plasma wave this is the condition and this is called saturation amplitude of the plasma wave and what you can do omega by k. We already obtained in terms of because we just wrote down omega is equal this is the phase velocity. So, it is actually omega r by k so omega r by k we just calculate was this quantity was omega v or k v 0 v minus gamma upon root 3, this is what we had seen.

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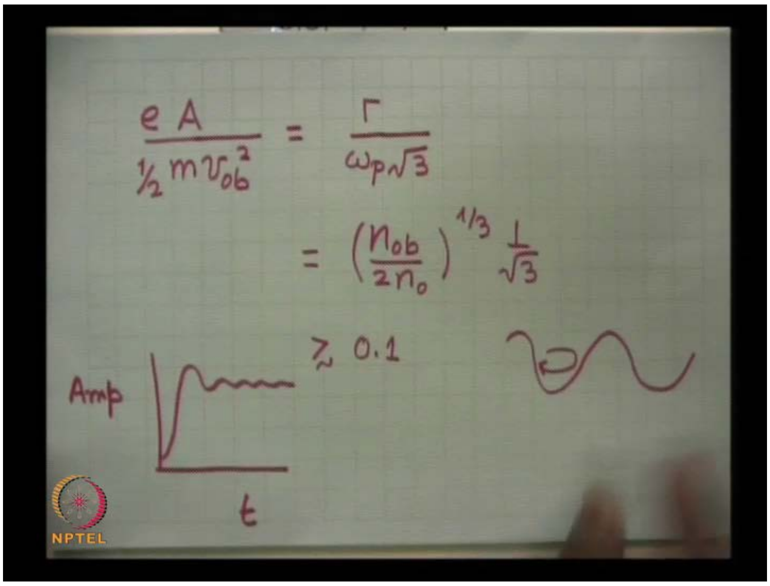

So, use this value of omega in this expression, then you can get the value of A the amplitude of the plasma wave and the result turns out to be let me write the result. It turns out to be e A upon half m v 0 b square, this is the potential energy of an electron maximum potential energy of an electron divided by the initial energy of the kinetic energy of the electron beam this becomes is equal to gamma upon omega p root 3. And, if I put the value of growth rate gamma it turns out to be beam density upon 2 times the electron density to the power one third into 1 upon root 3.

In most of the experiments beam density has a few percent of electron density and this quantity could be of the order of greater than around 0.1. So, if you are launching an electron beam of say kinetic energy of the order of 1 kilo electron volt, then the amplitude of the wave would be of the order of about 100 volt per 100 volt potential amplitude will be about 100 volt. So, you are really generating a very large amplitude plasma wave by this method. So, an electron beam can very easily generate a plasma wave in a plasma and the plasma wave can acquire large amplitude so, what will happen in the potential energy minima? The electrons will look like trapped like they are going back and forth.

So, they are no longer transferring the energy of the wave on an average they will stop giving energy to the wave and wave growth stops. What has been experimentally observed that, if you plot the amplitude of the plasma wave as a function of time then initially, the amplitude was very low and the wave amplitude actually, oscillates like this, then acquires a constant wave like this. This is a commonly observed feature of a two stream instability, here I have ignored the effects of plasma a magnetic field of the plasma.

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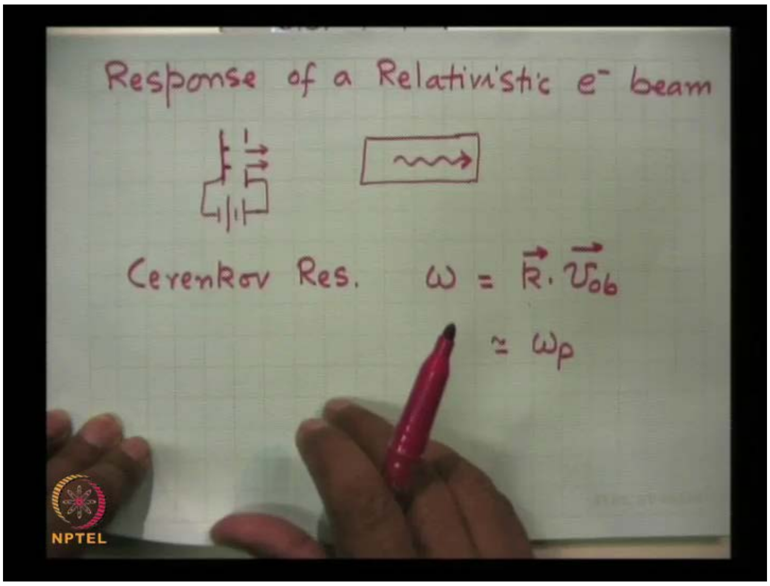

Now, I would like to see what will happen? If the beam velocity was relativistic, if the initial beam velocity v 0 v was of the order of c. So, I am studying the response of a relativistic electron beam to a plasma wave. Well, geometry is very similar, but the electron gun that you have to employ for this purpose, is essentially a gun which, can produce large velocity electrons and a very special kind of gun is usually made for this purpose. As I mentioned yesterday this is the gun which has a cold cathode with a knife edge, circular knife edge and that emits electrons.

The usually, the beam that you produce here is annular electron beam there are ways you can produce a solid electron beam also. But the effects of finite size of the beam or of the structure of the beam I will not consider at the moment, though we can easily include those effects they are called non local effects. So, today I will just consider a ideal situation so this is actually, a field emission diode this is cathode, this is anode and an electron beam is coming out. And we have a plasma somewhere, and its passing through the plasma and I want to examine the excitation of a wave in the plasma longitudinal electrostatic wave in the plasma in the presence of the beam.

One thing, you can easily see here, that the wave generation occurs due to Cerenkov resonance which means, that omega has to be equal to k dot beam velocity this condition turns out to be valid even in relativistic case. So, what happens, when v 0 b increases k must decrease to produce a given omega and as before we have seen as earlier you know we had seen that the frequency of the most considerable mode is of the order of omega p. So, if frequency is fixed by the plasma density. Then the k of the wave is fixed by the beam velocity.

So, the phase velocity of the wave has to be close to beam velocity and hence k will decrease with increasing beam velocity. In last few years lot of experiments have been done or with electron beams, which travel with velocities close to 0.9 c, 0.95 c, 0.99 c. And they found generation of very large amplitude plasma waves that can be used to accelerate the particles. And we shall discuss this as an application after I have discussed the effect of the relativistic mass dependence of electrons on two stream instability.

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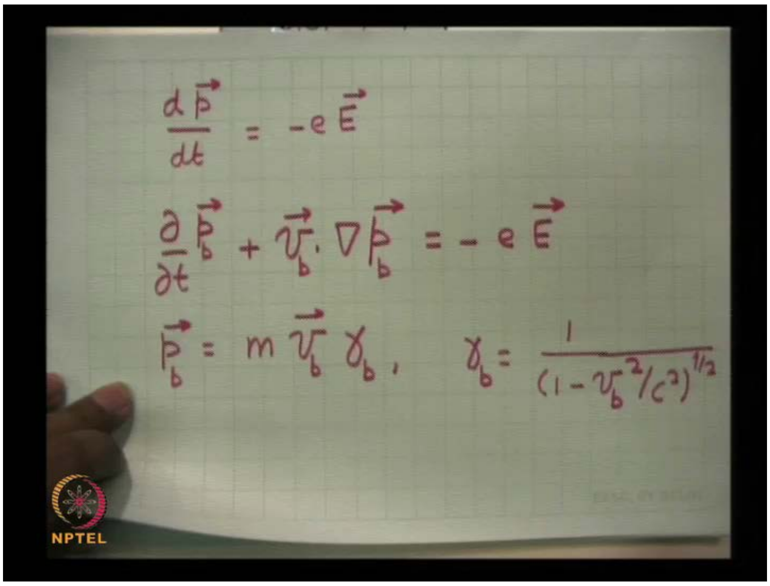

Well, the equation of motion in this case is modified, which says that d v by d t, if this is the momentum of an electron by d t is equal to the force, which is minus e into E. I forget the collisions, I forget the magnetic field, I forget the pressure term for the electron beam response. So, I want to stall this equation as I mentioned before total time derivative of momentum can be written as partial time derivative of momentum of the electron fluid. This is the average momentum of electron fluid plus v dot del v is equal minus e E. This equation we have to solve, where momentum I can write down as mass of the electron into velocity of the electron into law range factor gamma.

And gamma is related to v as gamma is equal to 1 upon under root of 1 minus v square by c square to the power half this is the law range factor. So, the fluid equation is now, modified by the introduction of a term called relativistic gamma factor and because Iam solving this equation for beam electron. So, I will put a subscript v on these quantity so particle velocity is v gamma v I will write down gamma v is beam velocity is quite like this. Here also this equation I have written for beam so I will put a subscript b here. Well, in order to solve this equation and the equation of continuity first of all, we have to identify the equilibrium and then the perturb the equilibrium by the plasma wave field.

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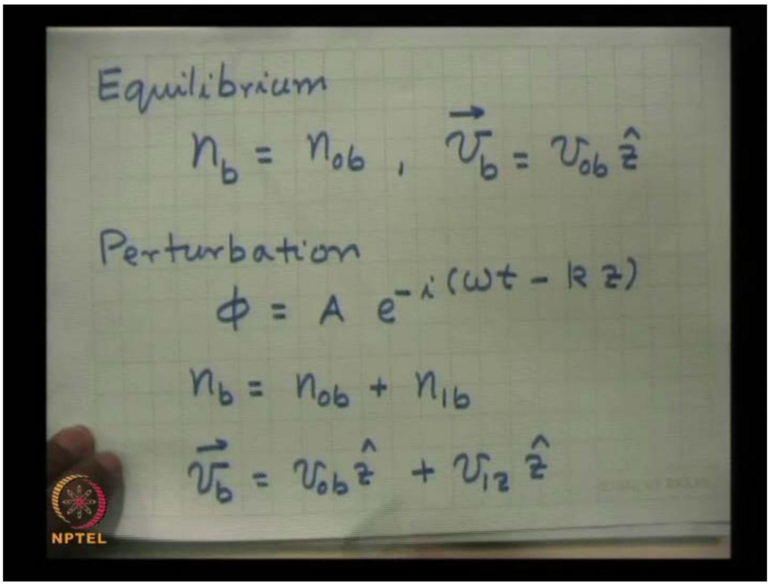

So, now what is our equilibrium is initially we have electron density of the beam is equal to n 0 b and beam velocity is v b is equal to v 0 v and that is in the z direction this is my equilibrium. Now, I perturb this by a plasma wave the potential of the plasma wave I will take as A e to the power minus i omega t minus k z as, before then we say that in the presence of this plasma wave my beam density has changed from n 0 b by n amount n 1 b and velocity has changed from v 0 b z cap to a quantity called v 1 z in z direction.

Because the potential I have chosen so like this, if you take the gradient of phi then the electric field will be only in z direction. So, we expect that the particle will gain velocity or lose velocity in the z direction, because particle motion is influenced only in the z direction you can verify this. What you have to do? You have to substitute these in the equation of motion continuity and linearize the equation of motion.

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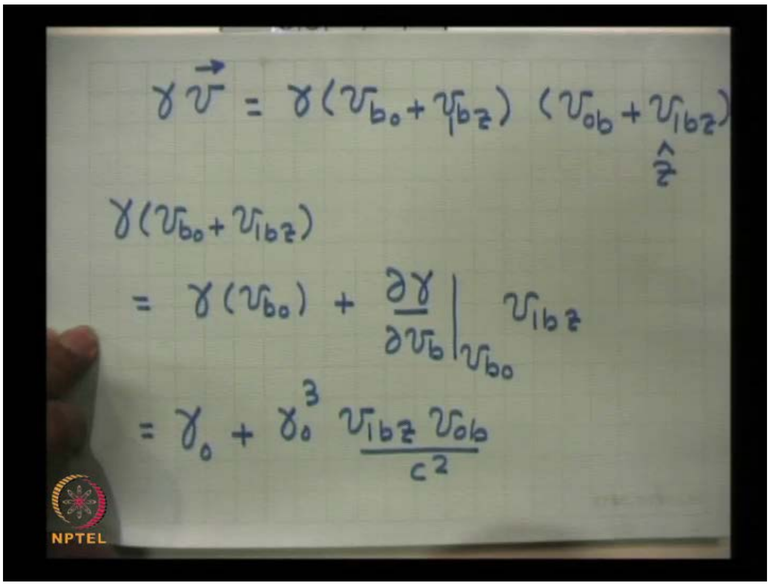

So, linearization is a very important step well the quantity that I get there is a quantity gamma v. So, let us see how this look, looks this is equal to the product of well gamma depends on velocity, which is v b 0 plus v b v 1 b z this is the gamma is a function of this quantity multiplied by v 0 b the initial velocity plus v 1 d z this is the velocity modified so this is the value of this quantity. What I can do gamma I can expand like this, so gamma at v b 0 plus v 1 b z, because gamma actually, is a function of so anyway, you can write down this, by Taylor expansion gamma at v b 0 plus delta gamma upon delta v b at v b 0 into v 1 b z simple.

And the expression for gamma that I had give to you, if you differentiate this turns out to be simply I will call this quantity as gamma 0. The initial value of Lawrence factor plus this turns out to be gamma 0 cube into v 1 b z into v 0 v upon c square. You can just verify this, just differentiate gamma with respect to beam velocity and replace this beam velocity by v 0 b and this is what you get.

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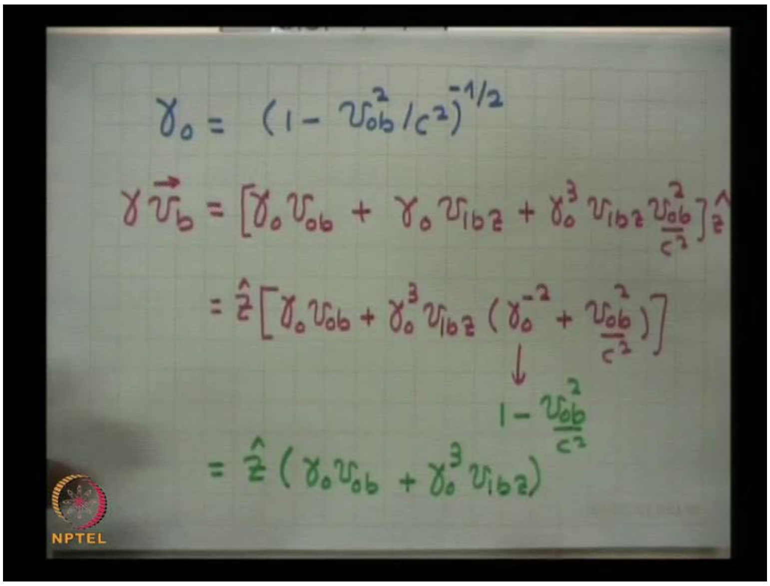

What is gamma 0, gamma 0 is equal to 1 minus v 0 b square by c square, the initial value of gamma factor and gamma 0 cube is the cube of this quantity, you can easily verify by differentiating. And when we substitute this in that expansion then what my gamma v becomes let us gamma into v for the beam I am writing it becomes gamma 0 v 0 z v 0 b plus you will get a term, which is gamma 0 into v 1 v z plus you will get a term gamma 0 cube v 1 b z v 0 b square by c square all these quantities are in the z direction.

You can combine these, two terms by taking gamma 0 cube common and this becomes z cap I will write here first term is gamma 0 v 0 b plus gamma 0 cube common take v 1 v also common then you are left with gamma 0 square minus square inverse plus v 0 b square by c square. This quantity you can write down as by this expression 1 minus v 0 b square by c square this will cancel with this so this becomes unity. And consequently and here I have ignored the product of v 1 b with v 1 b square terms I have ignored on linearization.

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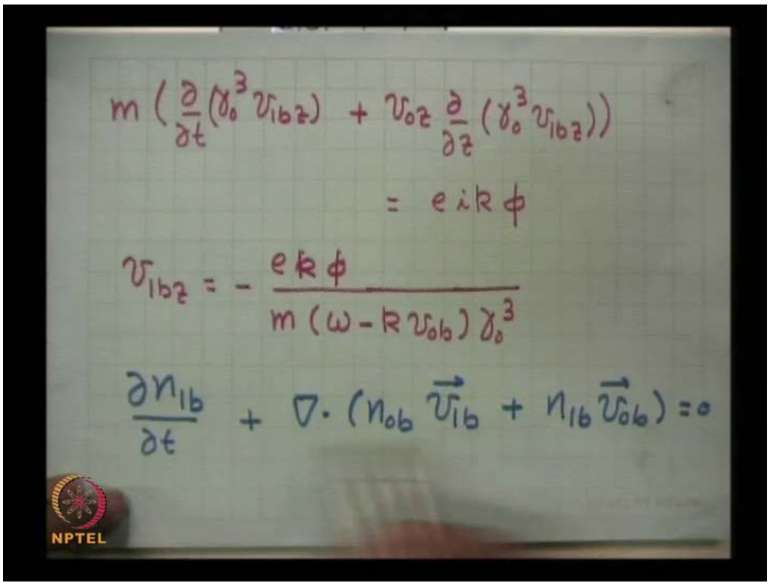

So, this becomes simply z gamma 0 v 0 b plus gamma 0 cube v 1 b z and when you substitute this in the equation of motion in linearized ,what you get? Let me write down this in the equation of motion, the equation of motion was m outside delta delta t of gamma v gamma v means, the first term that I had written gamma 0 v 0 b does not depend on time. So, I will not write it you will get gamma 0 cube v 1 b z. The second term is v 0 b delta delta z of this quantity gamma 0 cube v 1 b z, because gamma 0 v 0 term, when you differentiate by z is (( )) identically.

So, this is the only linearized term that you that survives and right hand side was the electrical force, which is e i k multiplied by phi. Once you have obtained the linearized equation of motion you can replace delta delta t by minus i omega and delta delta z as i k as we have been doing. And you will obtain from here v one b z is equal to e i rather e k phi is a negative sign into m omega minus k v 0 b this is v 0 z or v 0 b is the same thing multiplied by gamma 0 cube. An important note were the change brought in by the relativistic effect is the of this gamma 0 cube, if the beam was non relativistic then this term will not be there.

As a layman one would think that relativistic effect essentially modify the mass m is the rest mass of the electron. So, one would expect that this should be m into gamma 0, but in electrostatic response it is the gamma 0 cube that comes in. So, when you do a proper linearization of the equation of motion the effecting mass of the electron due to relativistic effect is not m gamma m is replaced by m gamma, but it (( )) m gamma cube. This is the important modification, please understand, what is the magnitude of gamma 0 for an electron having energy equal to rest mass energy rest mass of an electron is about half m e v.

So, if you have an electron with the energy kinetic energy of half m e v so total value of energy is rest mass energy plus kinetic energy is 1 m e v. So, this half m e v kinetic energy electron will have gamma 0 equal to 2. So, then gamma 0 is two, but the gamma 0 cube is 8 so effective mass as far as the response of electron to the wave is concerned is suppressed by gamma 0 cube a times, this is a very important reduction factor. And then we solve the equation of continuity, which is for the perturbed density turns out to be n delta n 1 b upon delta t plus divergence of n 0 v into v 1 b plus n 1 b into v 0 b this is the equation of continuity linearized equation of continuity replace this by i k this by minus i omega and solve this equation.

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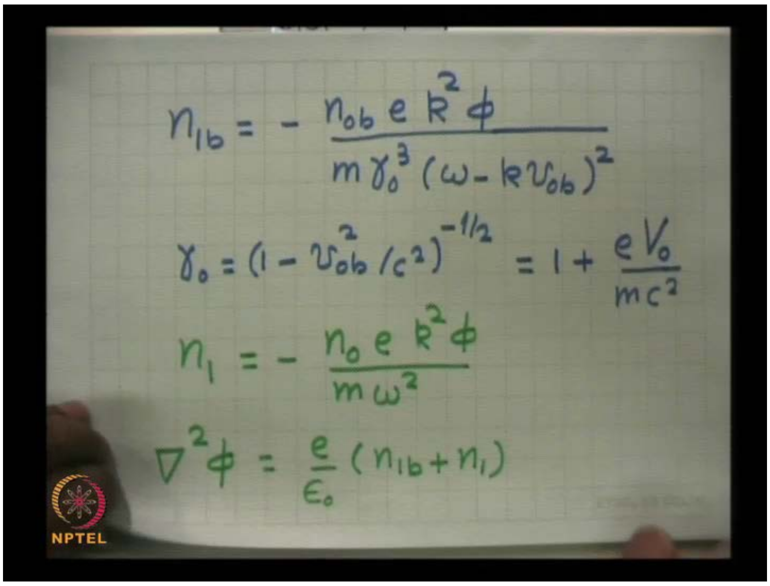

So, you will get an expression for perturbed beam density and that turns out to be very similar to before and the value is n 1 b is equal to minus n 0 b e k square phi upon m gamma 0 cube omega minus k v 0 b whole square. So, mathematically there is only one change in the density perturbation due to relativistic effect that the effective mass of the electron becomes rest mass multiplied by gamma 0 cube. And gamma 0 as I mentioned is equal to 1 minus v 0 b square by c square to the power minus half and, if you want to express this in terms of beam voltage it turns out to be about 1 plus magnitude of electron charge into accelerating voltage of the gun upon m c square.

So, this is the voltage that you apply across the diode or the electron gun. So, when you apply one million volt this quantity becomes 2 gamma becomes 3, when we apply 500 kilo volt then this ratio this quantity is unity gamma is equal to 2. So, depends on how much voltage you apply to the electron gun that is producing the electron beam gamma 0 can be controlled. So, by gamma 0 increasing gamma 0 v 0 b increases, but v 0 b cannot increase beyond c. So, the largest value v 0 b can take a c hence, the phase velocity of plasma wave cannot exceed c, if the wave is to generated by an electron beam.

Well, the plasma response is the same you can reduce from here the perturbation caused in the plasma response. Plasma electron response whose, electron density (( )) was n 0 then e k square phi upon m for the plasma electrons they are not moving in the equilibrium their velocity was 0 was gamma 0 is unity. And v 0 b is 0 so this becomes omega square, you have to use this in the poisson equation, which turns out to be equal to e upon epsilon 0 the free space permittivity into n 1 b plus n 1 replace this del square by minus k square.

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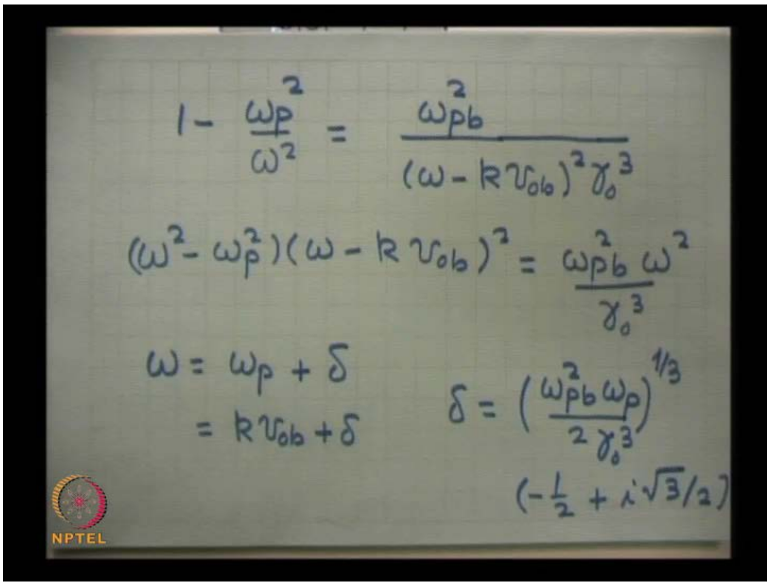

And you get that is poisson relation as before the dispersion relation I will write down, for the plasma wave is 1 minus omega p square by omega square is equal to beam plasma frequency square upon omega minus k v 0 b square multiplied by gamma 0 cube.

You follow the same procedure as we followed for the two stream instability due to a non relativistic electron beam to solve this equation. You get you write this equation as omega square minus omega p square multiplied by omega minus k v 0 b square is equal to omega p b square into omega square upon gamma 0 cube and the growth rate, that you obtain from here this factor is additional factor.

So, what you get omega we write as omega p plus delta and choose omega p equal to k v 0 b. In this case, this equation gives you delta is equal to omega p b square into omega p upon 2 to the power 1 third and for the unstable mode this into gamma 0 cube is there multiplied by minus half plus I root 3 by 2. So, the imaginary part of delta is the growth rate, which is down by the Lawrence factor. This is the main important effect of relativistic mass variation.

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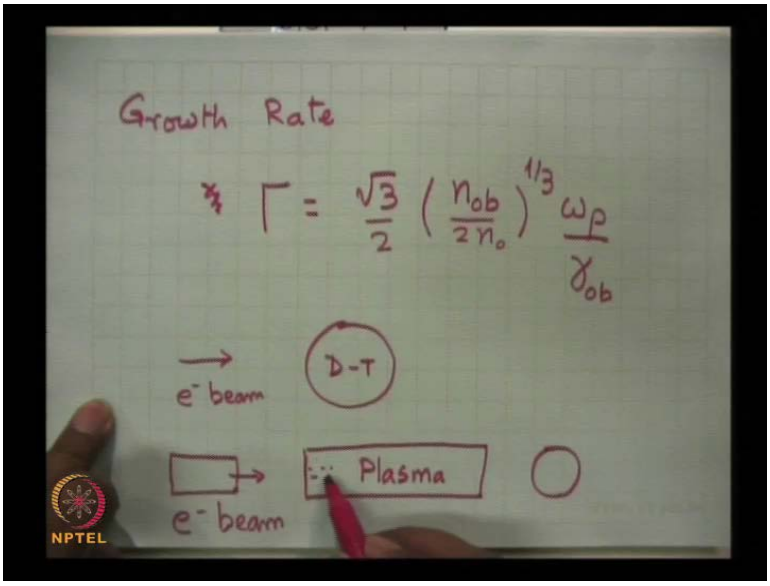

So, the growth rate is gamma does not this, this gamma is equal to root 3 by 2 and if I write in terms of beam density upon plasma density that is 1 third into omega p plasma frequency of the plasma electrons. And gamma 0 comes in there gamma 0 of the beam so this is the modification infact, modifying factor for the growth rate. Well, now I would like to elaborate some of the applications of this study, where are you really going to encounter relativistic electron beams (( )) use of those beams. Well, relativistic electron beams are employed in devices that produce coherent radiation like free electron laser is a very common device. Actually, at one time a relativistic electron beams were seen as a good candidate for thermal nuclear fusion also.

That you can have a pallet of deuterium tritium and you can bombard this by a electron beam and then this beam can heat this pallet to fusion temperature and one can achieve nuclear reactions and nuclear energy. However, later one people found that this scheme is not that efficient, because electrons are light in mass so rather than using electron beam people are employing iron beams. But there is something different, but electron beams are very commonly employed for generation of coherent radiation. Now, the issue is the you have a gun that produce the or the diode that produce the electron beam and this is the point where you have to deliver this beam. Suppose, this is the structure where the radiation is to be generated these two may not be same place there may be finite distance between the two.

So, this the electron gun electron beam is coming out from there and this is the delivery point the distance between these two. If you launch a high current beam then the beam will diverge, because of electrostatic repulsion electrons are negatively charged entity they will repel each other. So, what has been found that if you have to transport a beam from a source to a delivery point you must pass this through plasma, what the plasma will do? If the plasma density is large, larger than the density of beam electrons then as the beam enters here the electrons of the plasma move out to create a space charged neutralized region. So, what the plasma provides, what we call as the space charge neutralization.

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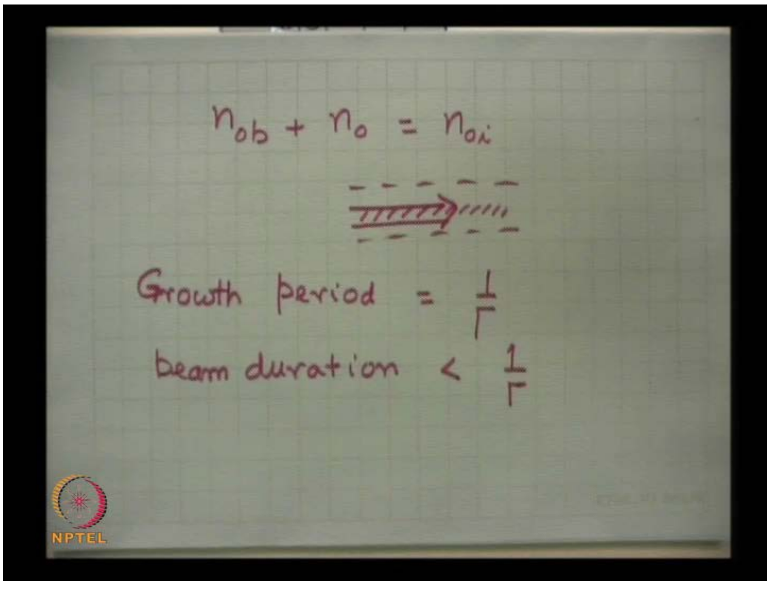

Because the plasma electrons move out so the electron density of the beam plus electron density of the electrons plasma electrons is equal to ion density. So, initially n 0 is equal to n 0 i, but once the beam enters the plasma this electron density of the plasma decreases such that this condition is satisfied. So, through the region where the beam travels in that region this is the electron beam going in the plasma like this. So, in the beam region there is a space charge neutralization and hence there is no repulsion of the electron beam. However, if the beam duration is long longer than the growth period of the two stream instability then this beam will give rise to generation of plasma waves and that puts a serious restriction. So, if you want to transport a beam through a plasma.

You must ensure that the beam duration is not too long otherwise, the beam will generate a plasma wave and that will trap the particles. The qualities of beam will be greatly deteriorate and lot of beam energy will go into the wave so that has to be avoided. So, you have to choose the beam pulse duration to be shorter than the growth period growth period of the plasma wave is inverse of growth time. So, what you require the beam duration should be less than one upon gamma, this is the condition. So, in all transports channels this is a important consideration. Another important area, where electron beam relativistic (( )) beam plasma interaction becomes important is the area of coherent of radius in generation as I just mentioned actually, the important area very similar to this is in the acceleration of electrons.

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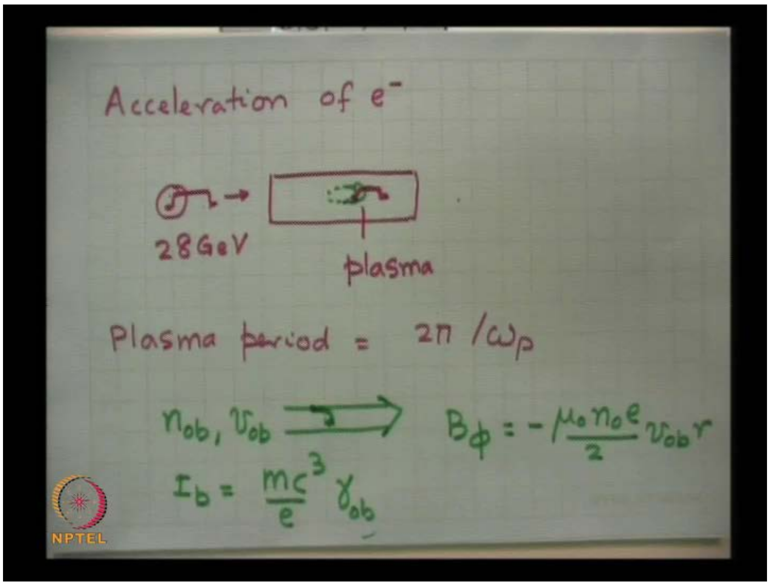

Let us see what do I what I mean by this I will consider give you an example of a recent experiment. People consider a plasma they launch an electron beam of typically so much length this the temporary length or special length of the beam the beam is going like this. The beam that was chosen for this study was about 28 GeV highly relativistic electron beam was launched into a plasma, this is plasma. What was seen this is the front of the beam this is the tail of the beam. The beam electrons lost little bit of energy, but the tail electrons gain energy and the tail electron gained energy typically from 28 GeV to about 33 GeV.

So, this was the energy gain that how these tail electrons here gained energy when they come out here. The total duration of this pulse was about, one plasma period; one plasma period is inverse of omega p whether 2 pi upon omega p where omega p is the plasma frequency of the plasma. So, when you choose a beam passing through a plasma of typically one plasma period people found that a plasma wave was generated in the plasma. You can visualize the situation when the beam enter comes over here, when the beam comes over here in the wave or behind it is tail the beam produces a plasma wave something like this.

In this region the plasma wave generated and this extends up to here actually. So, this plasma wave then traps the electrons and accelerates them. And this result was a land mark, because this initiated lot of particle and cell simulations and lot of studies of electron acceleration by plasma waves driven by a highly relativistic electron beam. Well, this was to demonstrate that a plasma wave can accelerate electrons at a very fast rate total length of the plasma may be of the order of a centimeter or a few centimeters. And similarly, this scheme of electron acceleration got lot of attention that you may generate a plasma wave not by essentially launching a electron beam, but by launching lasers.

So, people are employing similar scheme to generate a large amplitude plasma wave that can trap the electrons and accelerate them, please note that an electron beam of such a high energy will produce a plasma wave of velocity very close to c. So, what you require for electron acceleration is a plasma wave of phase velocity close to c then, such plasma waves we shall learn in subsequent lectures can trap electrons and accelerate them to very high energies. So, this is a very major application of electron beams relativistic electron beams to produce large amplitude plasma waves. And then those waves are employed for various applications.

Well, in this context I would like to mention one thing, whenever, a beam has to travel through a plasma the beam does two things. Suppose, I am launching an electron beam in a plasma this is my electron beam, if the beam travels with under space charge neutralization then there is no electric fields seen by the electrons so their divergence is stopped. But when, the beam travels it carries a current, because beam essentially is an entity that carries current (( )) particles move. So, if I consider a beam of density n 0 b and velocity v 0 b going in this direction.

This will produce a magnetic field you can easily calculate the value of magnetic field and that will be in the phi direction azimuthal direction, phi direction. And it is value will be typically equal to let me just write the expression, it turns out to be minus mu 0 n 0 e upon 2 v 0 b into r. What is the consequence of this magnetic field the electrons ,which are at the periphery these electrons under the Lawrence force tendency to come down and they may even get you know they go like this. So, when the electron from the periphery goes to the actual region its actual velocity slows down and it may even come back.

So, the beams cannot carry too much current the maximum current an electron beam can carry is of this order otherwise it will turn around. Because it is self magnetic field is equal to m c cube upon e multiplied by gamma e is the magnitude of electron charge m is the rest mass of the electrons c is the velocity of light and free space and gamma is the Lawrence factor of the beam which I am calling gamma 0 b. So, if you want larger current to be flown you must choose beam with larger and larger Lawrence factor larger energy, this is a important thing I wanted to mention to you. And before I close a classic application of beam plasma interaction is in the understanding of a phenomenon called landau damping. Let me say few words about landau damping.

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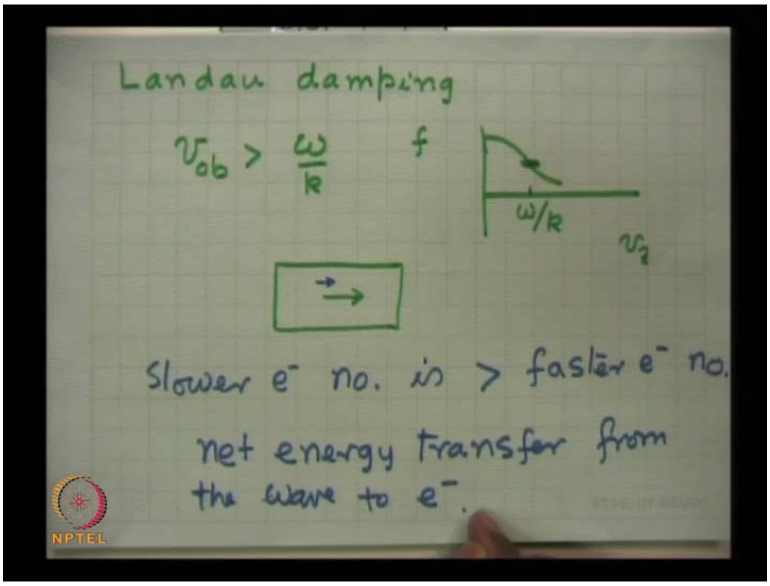

We have learnt that a beam can give energy to the wave whenever, the beam velocity is greater than the phase velocity of the wave or omega by k slightly greater than this. Now, suppose, I have a plasma which is maxwellian velocity distribution function suppose, a plasma is in thermal equilibrium there is no beam what you can do suppose, there is a plasma field somewhere, and if I plot the distribution function as a function of v z the distribution function for a maxwellian plasma is of this. So, there is a plasma here which has electrons of various different thermal velocities they are moving all around some of the electrons will be moving in the direction of the wave.

So, I am considering the propagation of wave in the z direction, then some of the electrons will be moving with the wave. And the ones which are moving with the wave with a velocity close to omega v by k will be resonantly interacted. So, suppose, the wave velocity is omega by k I write omega by k here, then this is the electrons in this neighborhood of omega by k. That are really resonantly interacting with the wave they can be (( )) resonance with the wave and can efficiently give energy to the wave or take energy from the wave.

Now, what you see from the slope of the distribution function that the electrons of lower velocity are more and higher velocity are less, because the slope of f with velocity is negative. Means, if there are many electrons which are moving all around the ones which are moving with the wave slightly with the larger phase velocity, larger velocity than the wave phase velocity their number is less. And the ones, which are moving slightly slower than the wave their number is more than these electrons, which are moving with larger velocity will give energy to the wave. The electron, which are moving slowly they will take energy from the wave. So, here what you are getting is slower electrons are more in number is more than the faster electron number.

What is going to happen because of this more electrons are there, to take energy from the wave fewer electrons are there, to give energy to the wave and there is a net energy transfer from the wave to the particles. Net energy transfer from the wave to the electrons this phenomenon is known as landau damping. And this is one of the major phenomenon observed in plasmas. It is a very important process that gives rise to heating efficient heating of plasmas. And I think we have to develop a kinetic theory to discuss this phenomenon at depth and that we shall take some time. And with this we close our discussion today. Thank you.