**Plasma Physics**
**Prof. V. K. Tripathi**
**Department of Physics**
**Indian Institute of Technology, Delhi**

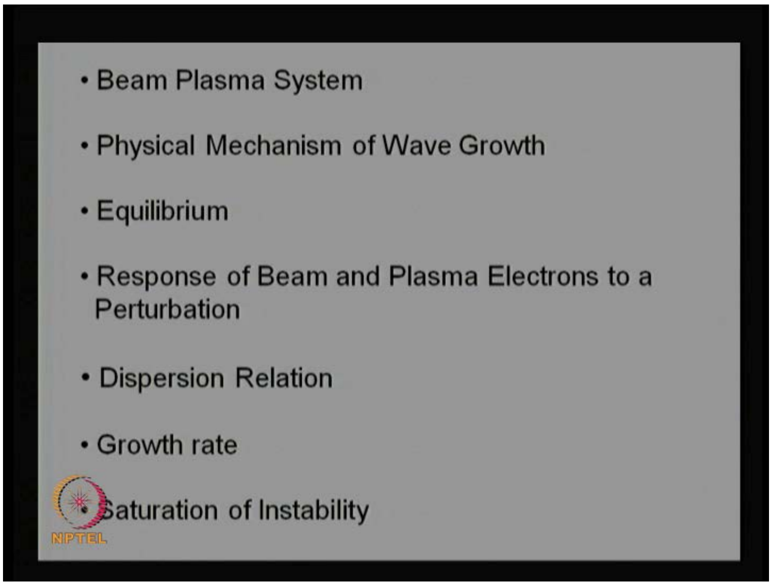
**Module No. # 01**

**Lecture No. # 12**

# Two Stream Instability

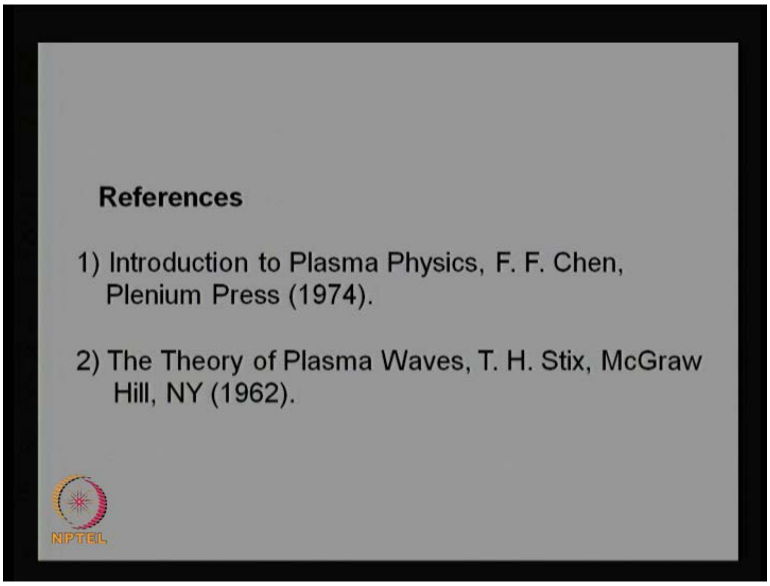
Today, I would like to discuss an unusual situation where the wave amplitude grows with time. Any wave electromagnetic or electrostatic when propagates through a medium and if its amplitude grows with time, then we call this wave as a unstable wave and this entire phenomenon is called instability. The easiest way to excite a wave is by using an electron beam and so in the plasma there will be two kinds of particles. The plasma electrons which are not having initially, a drift velocity and there is another stream of electrons which is launched from outside which is moving with initial finite velocity, finite drift velocity. So we call them as two steams of electrons, a stationary stream and a moving stream. And hence the sustainability is called two stream instability.

(Refer Slide Time: 01:34).



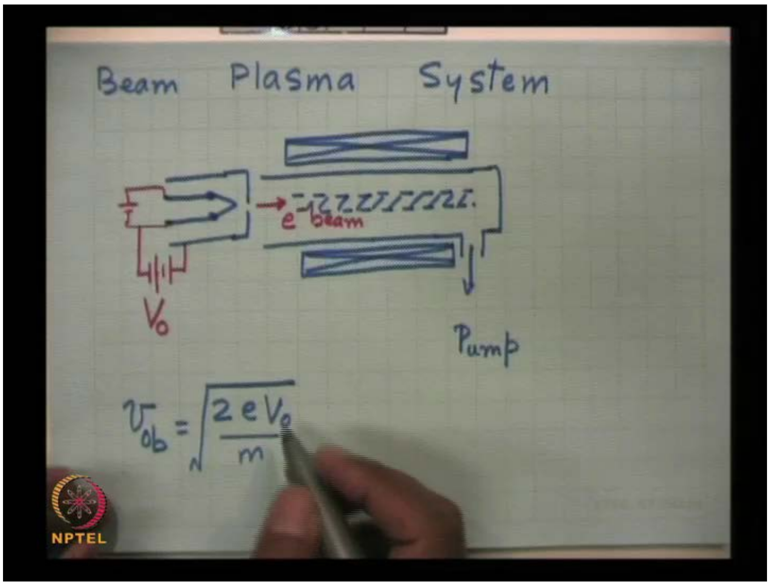

The system in which, you have a beam and a plasma is called a beam plasma system. I would like to begin today by describing a simple beam plasma system. Then, in such a system what is the physical mechanism of growth of a wave, we shall discuss that. Then when we begin the solution of Maxwell’s the equation of motion and equation of continuity, we have to start with a equilibrium, so we shall describe an equilibrium and then use the perturbation theory to obtain plasma response and beam response to an electrostatic wave. We will deduce a dispersion relation and obtain the growth rate and if time permits, I will discuss the saturation of instability.

(Refer Slide Time: 02:45)




Presentation is based on two books, introduction to plasma physics by F. F. Chen and the theory of plasma waves by T. H. Stix. But you can find this treatment in any book in plasma physics; this is a very common problem in plasma physics.

(Refer Slide Time: 03:08)



Now, consider a beam plasma system, a simplest beam plasma system. (No Audio From: 03:08 to 03:22) Normally, you have a tube, plasma tube which is a vacuum chamber, long vacuum chamber may be more than a meter long and there may be a pump here to create a vacuum. Obviously, you create a vacuum of the order of 10 to minus 4 torr and 1 torr is 1 millimeter of mercury pressure. And, here you have to launch an electron beam, so this is a gas at low pressure and then you have a very special kind of gun which has two rods to which a tungsten filament is tied and then you have a outer cover a cylinder which has a hole here.

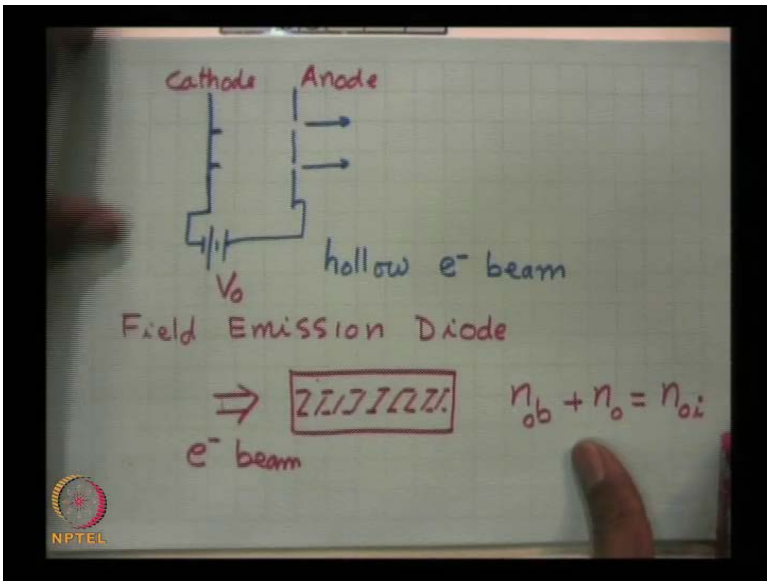
The potential difference between these two rods of the order of 9 volts is given, so I will put a battery of 9 volt here, between these two. And this hole, this outer cover is treated as anode is normally grounded, but a potential difference between these two, a huge potential difference between these two is given. So, what happens that, this small battery heats the electrons in the tungsten filament, the electrons come out from here. They are pulled by the electric field in this gap region which is applied a potential difference of V 0, then the electrons come out and they form a beam.

When the beam travels through a gas, it ionizes the gas and creates a plasma. So typically, it will produce a plasma in this region, the gas is ionized outside, but in the inner region this is, so this is plasma. So, you have created a beam plasma system, in this region there are electrons of the beam, there are electrons of the plasma and there are ions, this is goes to the pump. In order to have a better confinement of the beam so they does not diverge normally many systems have a magnetic field here by using a coil here, this is the schematic of a magnetic field coil.

So, by passing means this is a actually a solid order turn that is wrapped over this chamber and by passing a current in that you can produce a magnetic field in this direction. But magnetic field is not necessary for the growth of instabilities. In fact in the presence of magnetic field a new, a very new different kinds of plasma modes exist and a new kind of instabilities appear. But even if this magnetic field is not there, you will have the two steam instabilities, the basic instability, the growth of plasma wave by the electron beam.

This is the simplest schematic. The beam velocity is controlled by the beam voltage; the voltage of this gun, this entire thing is called electron gun or a diode. And the beam velocity you can easily say that half m V square, the beam velocity is equal to twice e V 0 upon m under the root. Here e is the magnitude of electron charge, m is the electron mass rest mass and this is the velocity that you will get there. Usually such beams have only milliampere current. If you want a large current and one can produce beams which can carry currents in the several amperes or even kilo amperes, then you require to design a different kind of beam.

(Refer Slide Time: 08:20).




One possible configuration is called field emission diode. A field emission diode is something like this; you do not have to heat the cathode. A field emission diode has a plate here and a hole here. On this cathode plate you have a knife edge a circular knife edge, very thin knife edge. I have drawn these two lines here, but essentially it is a ring. So, this is a circular ring of this diameter, being a knife edge and the distance between these two this is a cathode and this is anode. The potential differences between these two if you give of the order of a million volt and the separation you keep here of the range of the order of about a few millimeter, then the even if this is cold due to field emission electrons are emitted from here from these knife edges and they will get out here. They will give you, what we call as the hollow electron beam.

So the electrons will be coming out here, actually you can have a circular hole here, they will come out here, electron here and electron there and this is a circular symmetry. So, this is we call as the hallow electron beam. And this is what we call as the field emission diode. This is cathode, this is anode, this is field emission diode and it is used in free electron laser. Free electron laser is a very powerful device to produce coherent radiation; coherent electromagnetic waves over a wide range of frequencies and the frequency of radiation can be controlled by the beam energy which is controlled by the beam voltage, the voltage that we apply here. So, this becomes a tuning parameter, you can this brings in tune ability of the frequency of radiation.

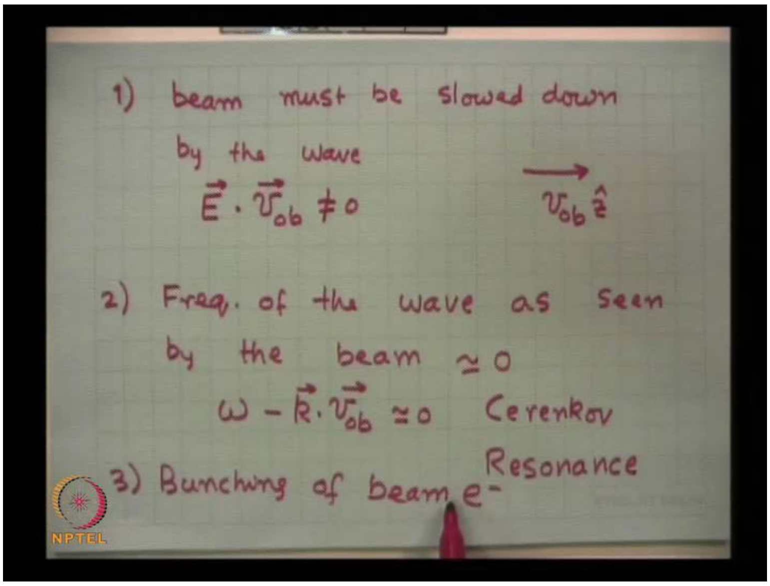
Anyway today, I like to discuss the physics of how an electron beam can generate a electrostatic wave? In one of the next lectures, I will then discuss how an electron beam can generate an electromagnetic wave? In a traveling wave tube or Cerenkov free electron laser normally, we have an electron beam that produces an electromagnetic radiation. So, these two issues I am going to address in these lectures, so let me begin with a simple problem of generation of a wave by an electron beam. But before I venture into that analysis instability analysis, I will I have to discuss the physics, but before that a word about equilibrium.

But I am considering some sort of a plasma and suppose this is a region of plasma in which an electron beam is launched. What will happen? Initially, when the beam was not there the plasma has, suppose a free formed plasma, it has equal number of electrons and ions. So, there is no space charge anywhere, but when the beam goes in certain regions, suppose this is the region where the beam goes, so temporarily there is an excess electrons because additional electrons have arrived here, so there is a net space charge. That net space charge will do what? It will repel the electrons of this region, they really outward.

The beam electrons will also be repel, but they already have a large velocity in this direction, so there are not move much in the transverse direction, during their passage but the background electrons will move. So, a steady state will be realized pretty soon when the net space charge in this region is not there, additional space charge move to the edges outside. So, in the region of interaction, this is called the region of interaction I can still presume a space charge neutrality, means the density of beam electrons, initial density of beam electrons, initial density of plasma electrons is equal to initial density of ions in the system. This is the equilibrium that you will obtain.

So, my issue now is, I do not want to consider the effect of finite transverse size, I will consider the propagation of a large cross section beam through a large plasma ignoring any boundary effects. How the beam gives rise to the growth of amplitude of this, a plasma wave in the system? This is the issue.

(Refer Slide Time: 13:51)



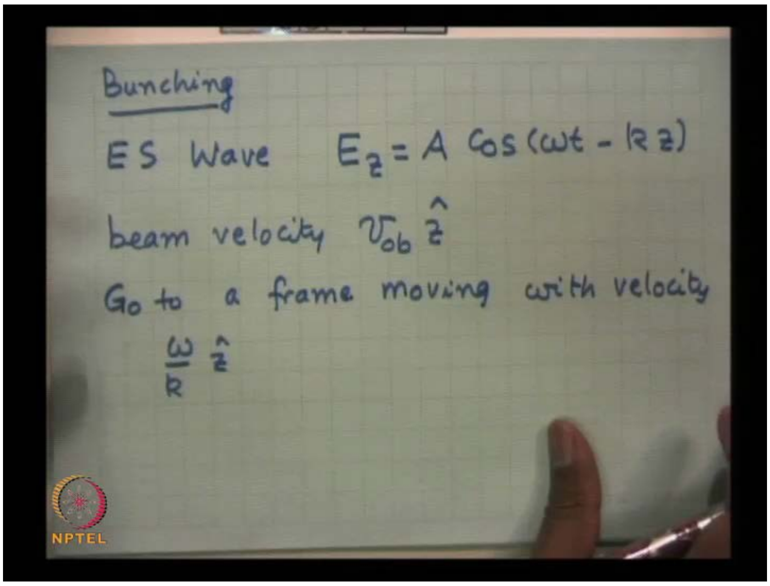

Now, physically what do you except? If a beam has to generate a wave it should have three properties. First property is that the beam should be slow down by the wave, beam must be slow down by the wave. How can it slow down the wave? If my beam is going in this direction for instance z direction, this is the beam velocity equilibrium velocity in the z direction, then the wave electric field must also be in the same direction. So, that this velocity beam is slowed down, means E must have a component in the direction of the beam velocity. This is one should not be 0, this is very important condition.

Second condition is that the frequency of the wave as seen by the beam should be small. We have seen that the response of electrons to any electrostatic field is inversely proportional to frequency. So, what you require is that, the frequency of the wave as seen by the beam should be nearly 0. If the wave is of frequency omega and k is the wave number, then the Doppler shifted frequency is omega minus k dot beam velocity. This should be nearly 0. Because we know that response of electrons, the drift velocity electrons acquired due to a wave or density potential, there is a acquire is inversely proportional to frequency, smaller the frequency larger the response. So, the effective frequency, this is the effective frequency of the wave as seen by moving electron, this should be nearly 0. This condition is also known as Cerenkov resonance condition, Cerenkov resonance (No Audio From: 16:24 to 16:31) this is a very important condition.

And third thing, any electrostatic wave has electric field in one direction in some region and the electric field opposite direction in some region. So, physically you would except that, if I launch a wave and the beam together then somewhere the beam will be retarded by the wave and somewhere this will be exceeded by the wave. So, net energy transfer is possible when the more electrons are retarded by the wave, so that they given (( )) wave. So, there should be net bunching of electrons in the retarding zones, bunching of the beam electrons.

These three conditions must be satisfied. Well, with an electrostatic wave this is possible, because if I launch an electrostatic wave in the direction of a beam, then the wave has a longitudinal component so this condition is satisfied. This is also possible because phase velocity of a plasma wave could be less than beam velocity or comparable to beam velocity so this condition can be satisfied. And, we have to see how bunching is possible. This physically I would like to explain before I venture into the mathematical analysis.

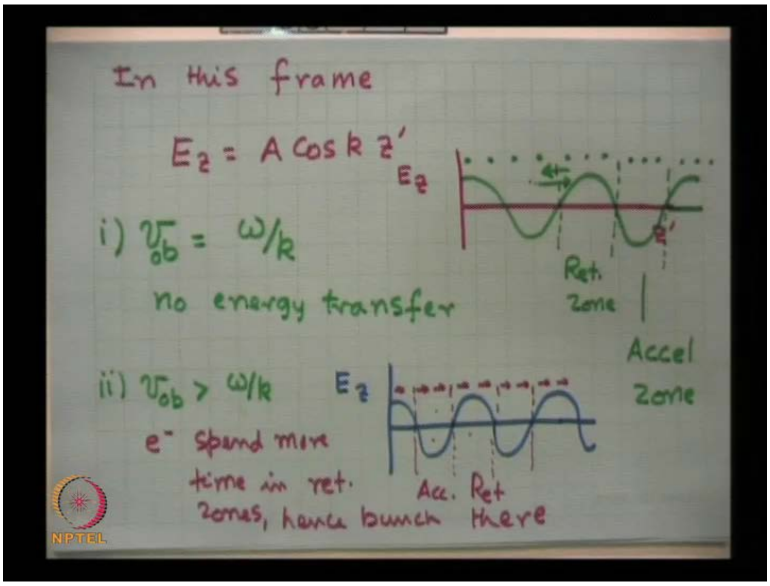
(Refer Slide Time: 18:13)




So, let me explain this mechanism of electron bunching. (No Audio From: 18:11 to 18:18) I think this is the crux of two stream instability. But I am considering a wave, electrostatic wave with electric field E z with some amplitude A say cos omega t minus k z, this wave has a frequency omega and it is travelling in the direction of z with phase velocity omega by k. I would like to understand the inter effect of this wave on a electron beam which is moving with velocity v 0 v in the z direction. In order to understand this interaction between electron beam and the wave go to a frame of reference moving with velocity of the wave, moving with velocity of value omega by k, magnitude in the z direction.

So, if you go to a frame of reference moving with this velocity in that frame, the frequency of the wave Doppler shift frequency of the wave will be 0. So, in this frame the electric field will be like this. So, let me write down the electric field of the wave in this frame.

(Refer Slide Time: 20:22)




So, in the moving frame you get E z is equal to A cos k z prime, z prime is the coordinate z coordinate the moving frame. Now, please plot this as a function of z prime. If I plot E z as a function of z prime, what do I get? This is A cos function, it will have the sort of variation. (No Audio From: 21:05 to 21:16)

Please remember this is the region where E z is negative, so this is the electric field in the backward direction. And, because the electron has a negative charge so this is called the accelerating zone. This is called the retarding zone, because the electric field is positive in the forward direction. So, here I can call this as retarding zone, because if the electrons are going in the forward direction the force on them will be in the backward direction, they will be retarded. And, this is the region called accelerating zone, this is accelerating zone, this is also accelerating zone here. So, there are alternate zones where electrons are retarded and accelerated.

I want to understand the interaction of an electron beam with such a wave. In the plasma electrons are all everywhere and the wave also exists everywhere, so suppose those electrons are there, these are the electrons I am locating and so on. These are the electrons that I have located indicated. There are three possibilities that these electrons are travelling with the same velocity as the wave. So, one case when beam velocity is equal omega by k v 0 b there, equilibrium beam velocity is equal to phase velocity of the wave, what happens in this case?

In the moving frame, the electrons will look like a stationary. So, in this region the electric field is positive and in this region the electric field is negative, so what happens? Electrons here would like to move in the backward direction and here the electrons will like to move in the forward direction. So, this is the boundary, actually what happens? The electrons here, they are experiencing a forward force. So, if you look at any boundary, the number of electrons crossing the boundary will be same. Let us see how?

The electrons which are here they will be going from here to here, because the force is opposite direction, the one which are here, they will goes in this direction. So, the net transfer of electrons from one area to another area will be 0. And hence, there is no net increase in the number of electrons in the retarding zones or accelerating zones. So, roughly half the electrons are retarded by the wave, half the electrons will be accelerated by the wave and there is no energy transfer.

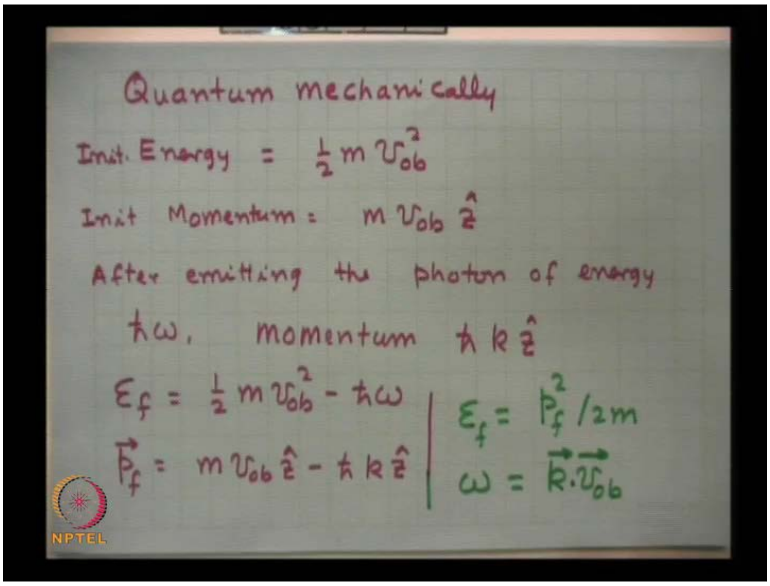
For the growth of the wave, you require more electrons should be retarded, so that their energy goes into the wave. So, no energy transfer, because same number of electrons go from retarding zone to accelerating zone, as go from accelerating zone to retarding zone, so no growth of the wave. Second possibility is that the beam velocity is slightly bigger than omega by k. In this case, what will happen? If you plot the same graph E z as a function of z prime, the variation is like this. So, these are our zones retarding zones, accelerating zones, alternately. So, this is my retarding zone and this is my accelerating zone, this is my accelerating zone, this my retarding zone and accelerating zone etcetera.

Now, here the electrons are moving slightly faster than the wave, so in the moving frame they will look like moving. When the wave is not there, they are start looking like moving in the new frame. Now, the electrons which are in the accelerating zone, they are accelerated further. So, they will quickly move from the accelerating zone to retarding zone. Whereas, the ones which are in the retarding zone they are retarded, so they will spend more time in the retarding zone and will take long time to get out from there. As a result, there is a reduction of electrons in the accelerating zone and a bunching of electrons or increase in their number in the retarding zones.

So, in this case, electrons spend more time in retarding zones, hence bunch over there. This is a important thing. So, when more electrons are getting retarded in the retarding zones, there is a net transfer of energy from the electrons with the beam and wave grows. So, this is the basic physics of energy transfer from electrons to the wave, that there velocity should be slightly more than the phase velocity of the wave; only then this phenomenon is possible.

So, physically we have seen that the condition for energy transfer from particles to wave is that the beam velocity should be more than the phase velocity of the wave. Now, let us carry out a mathematical analysis. I think before I take up a mathematical analysis, it will be very useful to look at the energy and momentum conservation in this process, because one can view the generation of wave as a quantum process.

(Refer Slide Time: 28:19)

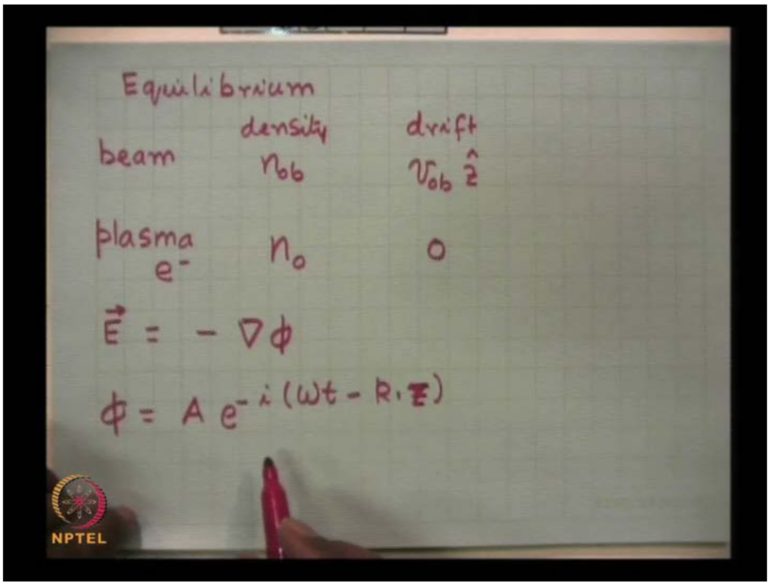



So, quantum mechanically, suppose the electron beam initial energy is equal to half m v 0 b square. Initial momentum you can easily write is equal to m v 0 b and suppose this is z direction. After this electron is emitted or generated a photon, so after emitting the photon or phonon plasma or whatever you call, of energy h cross omega and momentum h cross k. Electron energy is final energy is half m v 0 square minus h cross omega and final momentum is equal to initial momentum m v 0 b z minus h cross k. But kinetic energy and momentum are related to each other and the relation is energy is equal to momentum square upon 2 m where m is the mass of the electron.

Just substitute here and recognize that usually the value of h cross omega is less than an electron volte whereas, the value of this quantity is much much bigger. If you are launching an electron beam of 200 electron volt energy then this quantity 200 as compare to this is even bigger. So, you can ignore when you square this momentum square you can ignore the higher power of this and just substitute this there. And, it turns out to be; omega should be equal to k v 0 b. and, if there were a angle between the two between k vector and velocity of the electron, then there is a dot product here.

So, this is the Cerenkov resonance condition, the same condition which we say that the Doppler shifted frequency of the wave as seen by the electrons should be nearly 0, the same condition. So, this is a another way of looking at the generation process, that if the electron has to really generate a wave in a resonant fashion, the wave frequency that it want to generate and the beam velocity should have this relationship. So, this is the quantum mechanical interpretation also.

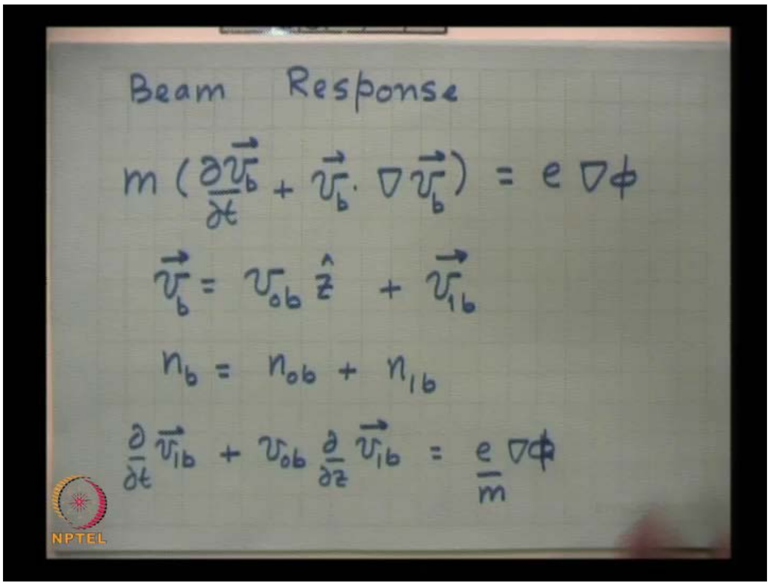
(Refer Slide Time: 32:23)




Now, let us carryout the instability analysis which is rather simple. What we have to do? We have to begin with an equilibrium and solve the equation of motion. What is our equilibrium? First of all, I would write down the beam equilibrium. The beam has a density, n 0 b in equilibrium. It is a drift velocity; I will call this as v 0 b and this in the z direction. The beam has no temperature, no collisions. Let one; let us understand the problem in a very simple way. And, plasma electrons what do they have? They have density, suppose is n 0 and their drift velocity is 0, they are not moving. I will ignore the temperature term also for the time being and later on we will generalize, that what is the consequence of electron temperature? So, simple problem.

And then, I want to say that suppose my system has an electrostatic wave, which is like I think I should express this in terms of the gradient of this potential. And, let the potential be of this form, some amplitude exponential minus i omega t minus k dot R. For the sake of simplicity, let me choose this is in the z direction, so k z. So, I want to find out that in the presence of this wave, what is the modification in the electron density and what is the modification in plasma electron density? What is the modification beam drift velocity and plasma drift velocity? So, first of all I will write down the equation for the beam.

(Refer Slide Time: 34:42)

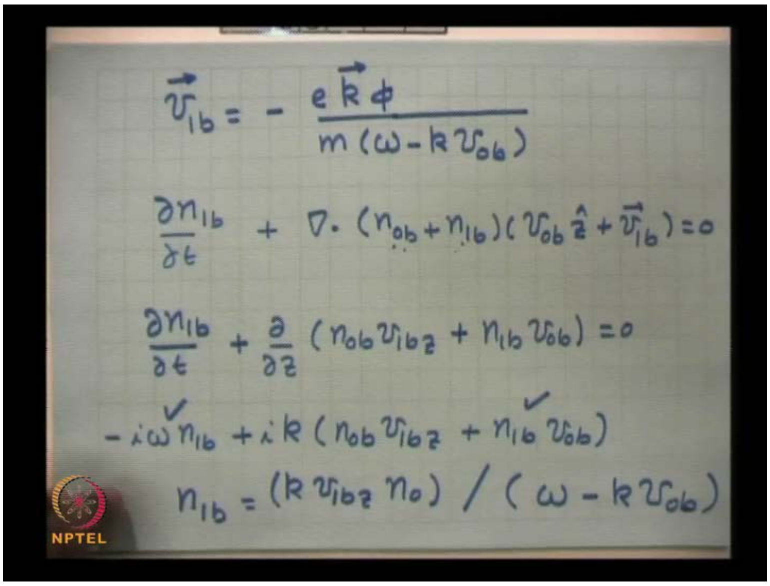



So, let me calculate the beam response. (No Audio From: 34:37 to 34:47) The equation of motion is m delta v by delta t plus dot del v is equal to minus e e which is e grad phi. There is no magnetic field; I am ignoring the pressure term and collision term. This is the beam response is governed by this equation. Please understand, phi is the potential of the electrostatic wave which is a small quantity and v is the response.

So, I can write down v as the original velocity of the beam. Actually, this equation is been written for beam, so I will use a subscript b to designate the beam quantity. So, beam velocity is equal to original beam velocity plus a change in beam velocity due to the wave. But this change is very small, because it is caused by a small perturbation phi. So, any product of v 1 v with v 1 v will be ignore and whenever we solve the equation of continuity, the density will be also expended like this, original density plus some perturbed beam density due to the wave.

Substitute this in this equation. So, when I differentiate delta v 0 b with time, this is 0 because this has no time dependence. So, this derivative goes away, only this term will survive. And, when I substitute these two terms in here, I get four terms. But please remember, v 0 b does not depend on position. So, when you simplify this four terms only one term survives, because product of v 1 v with v 1 v is ignored. So, this equation and I will divide this by m, take the following form delta delta t of v 1 v plus v 0 b, which in the z direction so only derivative with respect to z is to be retain and v 1 b is equal to e grad phi upon m. so, this is the equation of motion after linearization. In the quasi steady state response should have the same time and z dependence as the source.

(Refer Slide Time: 37:58)

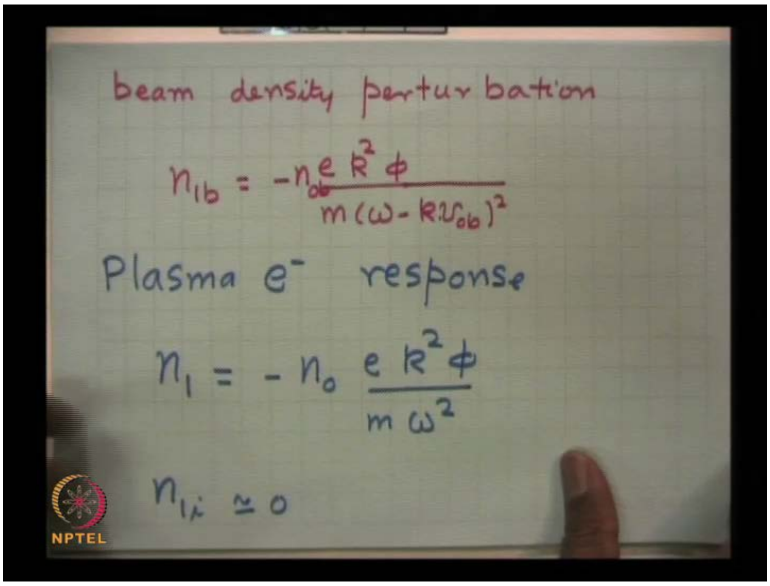



So I will replace delta delta t by minus i omega, delta delta z by i k and this equation gives v 1 b is equal to minus e k phi upon m, this is z component let me write this like this upon m. And, here you will get omega minus k v 0 b, you may appreciate the effectiveness of Cerenkov resonance condition, that whenever omega is close to k v 0 b, this becomes nearly 0 and this becomes huge. So, that is the advantage of having Cerenkov resonance condition that the response of electrons to a very small perturbation of the plasma wave will be huge, if this condition is satisfied.

Now, you can solve the equation of continuity which is delta n 1 for the beam delta delta t plus divergence of, there are many terms density into velocity products. So, this is n 0 b plus n 1 b into v 0 b plus v 1 b, this is the equation of continuity I have written. Please remember, n 0 b and n v 0 b product does not depend on position, so this will be 0. n 0 b into v 1 b certainly will be finite, n 1 b with v 0 b will be finite, but n 1 b and v 1 b product is too small, so we will ignore it. So, then this equation becomes delta n 1 b delta t plus this has only z variation, so just let me write down delta delta z of n 0 v into v 1 v z and this term plus n 1 v v 0 b is equal to 0.

Replace delta delta t by minus i omega and delta delta z by I k, you will get n 0 b v 1 b z plus n 1 b v 0 b. These two terms contain n 1 b, they combine together and you can simplify this, you will get n 1 b is equal to k v 1 b z component into n 0 divided by omega minus k v 0 b. Again the same denominator it was here comes in there and that gives rise to a resonant enhancement in density perturbation.

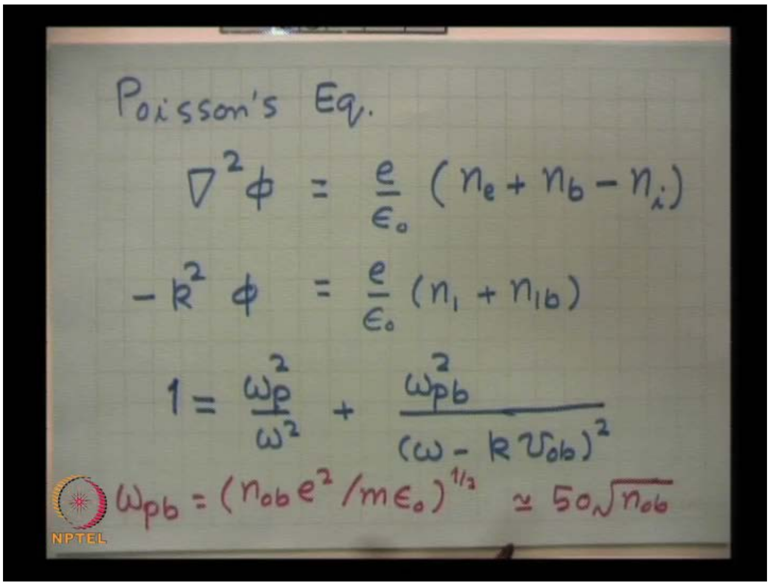
(Refer Slide Time: 41:38)




So, let me write down n 1 b expressively in terms of phi by using this equation. And, the result is that the beam density perturbation due to the wave (No Audio From: 41:40 to 41:49) is n 1 b is equal to minus e k square phi upon m omega minus k z or k v 0 b whole square and there is n 0 is also there, n 0 b. This is a important expression, which shows that the density perturbation scales as a square of this resonance denominator. It depends on mass also universally. For ions if I had written, this will be too small and you can ignore it. So, I have got the density perturbation due to the of beam electrons due to the wave.

For the plasma electrons the difference between the two is that beam density has to be replaced by electron density of the plasma and beam velocity is to be taken 0 for plasma response. So, plasma electrons response (No Audio From: 43:09 to 43:18) simply the perturbation plasma density would be minus n 0 of the plasma e k square phi upon m omega square. And, ion density perturbation I take to be 0, because ion mass is too heavy. So, if I know the density perturbations of all the three spaces, I can solve the Poisson equation.

(Refer Slide Time: 43:57)



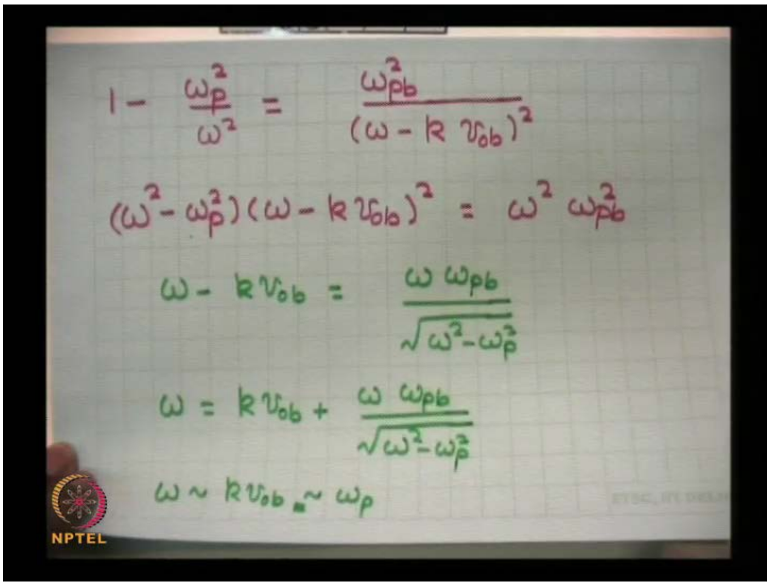

So, the Poisson equation is del square phi is equal to e upon epsilon 0 into electron density plus beam electron density minus ion density, in equilibrium all these sum to 0. But in case of perturbation this will be e upon epsilon 0, plasma electron density perturbation plus n 1 b del square I replaced by minus k square phi, use this n 1 and n 1 b in terms of phi. Then this equation takes the simple form 1 minus omega p square, 1 is equal to omega p square by omega square plus omega p b square upon omega minus k v 0 b whole square.

Where I have used two symbols omega p and omega p b, which I am going define in a little while. This is called beam plasma frequency, it depends on beam density. omega p is the plasma frequency of electrons of the plasma called electron plasma frequency, this has a term which is called resonant denominator. Even though the value of omega p b is tiny, I think it will be better I should write this omega p b here. Beam plasma frequency is defined as beam density in equilibrium e square upon mass of the electron into epsilon 0 to the power half which is of the order of 50 under root of beam density.

In beam plasma systems, beam density is much smaller than the electron density of the plasma. But because of this resonance this term could be comparable to this term that is the beauty. So, Cerenkov resonance is a important thing in bringing the response of beam electrons to comparable to the contribution made by the plasma electrons and not only that, this has in it built in this relation. A possibility of omega being complex, please see this is a fourth order equation in omega and it may have a complex root with positive imaginary part and that is called the instability. So, I would like to simply solve this equation. There are two cases in which this equation has been solved and studied in great depth.

One when omega p b is small and other case, when omega p b is the same as omega p. That is a very special case, people have studied that. But the most common case is the one in which beam density is small as compared to plasma density and hence this term is small. Numerator here is small than the numerator here, let us see.

(Refer Slide Time: 47:44)



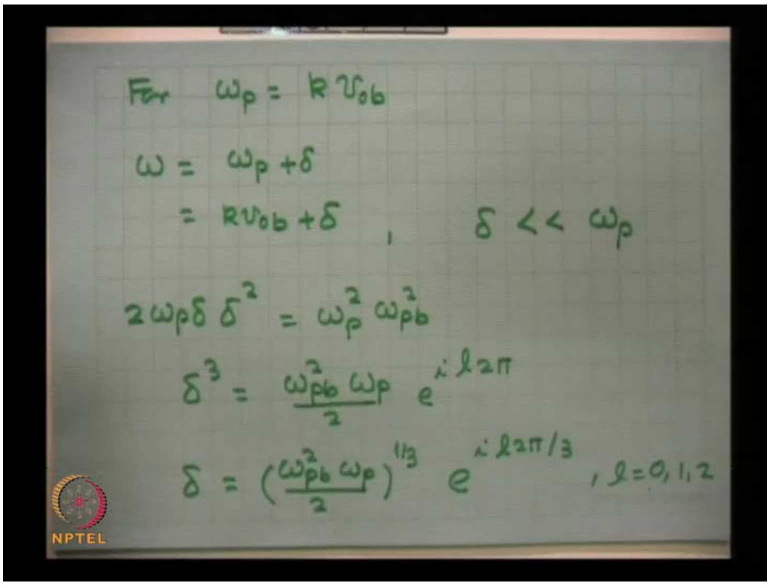

So, I will consider today, the case when omega p b is a small and I would like to solve this equation. I can rewrite this dispersion relation as 1 minus omega p square upon omega square means, I have taken the plasma term, plasma electron term to the left and beam term I will keep on the right. Omega minus k v 0 b whole square, I can take l c m here and rewrite this equation as omega a square minus omega p square into omega minus k v 0 b square is equal to omega square into omega p b square. Well, this equation is fourth order in omega and is very difficult to solve for arbitrary values of k. For a special value of k solution is simple and first of all I would like to find out, that the case for which this equation is solvable analytically. That is also the case of largest growth rate. Let us see how.

Please note one thing in here. I can rewrite this equation by dividing both sides by this factor, so what do I get? omega minus k v 0 b, then I get is equal to omega omega p b into under root of omega square minus omega p square, means what I have done? I have divide this equation by this factor and take the square root. So, I get the sort of expression. This says that omega is equal to k v 0 b plus additional term. So, omega becomes is equal to k v 0 b plus omega omega p b upon under root of omega square minus omega p square. So, when omega p b is a small, this term will small and omega is nearly equal to k 0 k v 0 b. However, near omega equal to omega p, we expect that this contribution will be huge. And, if omega is close to omega p, but slightly less than omega p, this will be imaginary.

So, you expect a very large growth rate, when omega is close to omega p, but because I am presuming this term to be small so omega is also close to k v 0 b. Means, when omega is close to k v 0 b and it is also close to omega p, you expect a large growth rate. So, what I am going to do? I am going to solve this equation, in the special case when omega p is equal to k v 0 b. Because in that case, whenever this factor is small this is also very small and hence the right hand side though very small, will make a significant contribution or modification in these roots.

So, driven by this logic, I am going to solve this equation in a special case when k v 0 b is equal to omega p. It is a general technique of solving a complicated dispersion relation in plasma physics, that please look at the roots the zeros like here one 0 is that omega equal to omega p, second 0 is that omega is equal to k v 0 b. So, we are looking for the simultaneous possibility that this is nearly 0, this is also nearly 0. So, that a tiny term on the right hand side makes a significant contribution, because this is the only source that can bring instability. If this term is not there, if this term is not there, the root of this equation is simply real. So, the instability is expected to come from the beam, though the density of the beam is a small we are considering. But still, it could have a significant influence over the wave, if these two are 0 simultaneously.

(Refer Slide Time: 52:15)

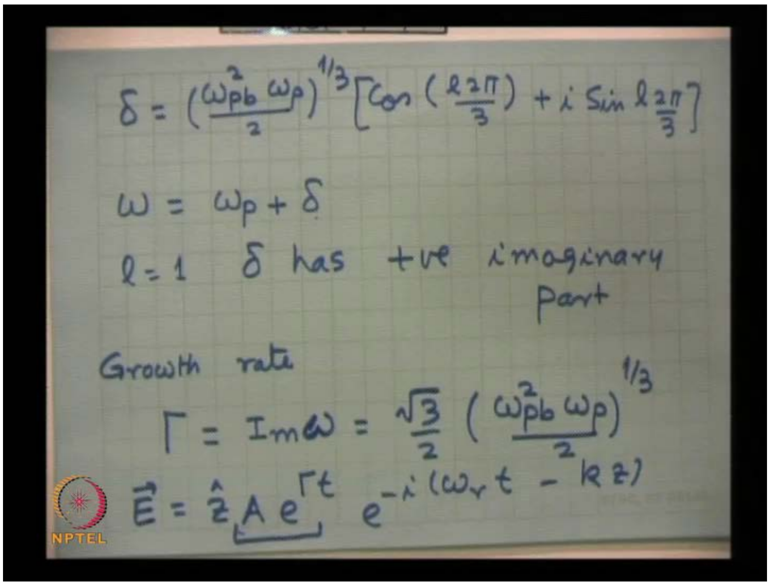



So, we solve this equation for omega p is equal to k v 0 b, then we say that omega is equal to say omega p plus some delta which is also equal to k v 0 b plus delta. Use this in that dispersion relation and assume that delta is smallest compare to omega p. So that, higher powers of delta can be ignored. When you use this in that expression, the first term omega square minus omega p square gives twice omega p delta. The second term is omega minus k v 0 b whole square which gives to delta square again, the right hand side is omega square omega p v square.

So, here you get delta q is equal to omega p b square and this omega I will put nearly equal to omega p, so, omega p upon two. Now, this is a cubic equation in delta you should have three roots to find three roots of this I multiply by this quantity by a quantity called i l into 2 pi, because for integers the values quantity is unity. So, you can always multiply this quantity by this say sort effective. So, delta turns out to be equal to omega p b square into omega p upon 2 to the power one-third into exponential of i l 2 pi by 3, take one-third of this cube root of this quantity.

This I can write as cos of l 2 pi by 3 plus i sin l 2 pi by 3 where l is 1, 2, 3, 0 1 2 3 4 5 etcetera. But you will note that for l equal to 3 this will have the same value as for l equal to 0. So, the three independent roots are l equal to 0, 1 and 2 others are then repeated.

(Refer Slide Time: 54:55)




Now, let me simplify this expression. Means write down this expression in terms of cos and sin. So, delta is equal to omega p b square omega p by 2 to the power one-third cos l into 2 pi by 3 plus i sin l 2 pi by 3. For l equal to 0 this has no imaginary part, but for l equal to 1 this has the imaginary part. Please remember, your frequency is omega p plus delta. I am looking for a solution where omega has a positive imaginary part, so that the wave amplitude grows with time, so I am looking for delta to be positive imaginary.

For l is equal to 1, delta has positive imaginary part. And hence, the frequency will be complex with a positive imaginary part. And, imaginary part of omega is the same thing as imaginary part of delta is called the growth rate. Growth rate is called gamma, which is the imaginary part of omega, which is the imaginary part of delta. And, if you calculate from here put l is equal to 1 in these expressions, imaginary part will give you root 3 by 2 and becomes root 3 by 2 into omega p b square into omega p by 2 to the power one-third.

So, growth rate and how is the amplitude vary? Electric field of the wave, which is z A e to the power gamma t into e to the power minus i real part of omega into t minus k z, this is the field. So, this is the amplitude of the wave, now it is growing with time and growth rate scales with beam density and electron density, it increases with both of them. And, this is a very important result. So, we have seen that if the beam velocity is very close to omega p, then the phase velocity of the plasma wave. In that case the beam can give energy to the wave. Further implications of this, we will discuss in our next lecture. Thank you very much.