**Plasma Physics**
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**Lecture No. # 10**
# Electrostatic Waves in Plasmas

Well friends, so far we have discussed the propagation of electromagnetic waves in plasmas, well last time we were discussing a very special kind of wave phenomena, the W K B solution to wave propagation inhomogeneous plasmas, and we considered the propagation of wave in the direction of density gradient.

When we were considering the propagation of waves at oblique angle to density gradient, we came across a situation where the wave turns around, and there is a possibility of generation of plasma waves. So before I take over to the discussion of wave propagation at oblique angles to density gradient, I think it is necessary that we understand the character of electrostatic waves.

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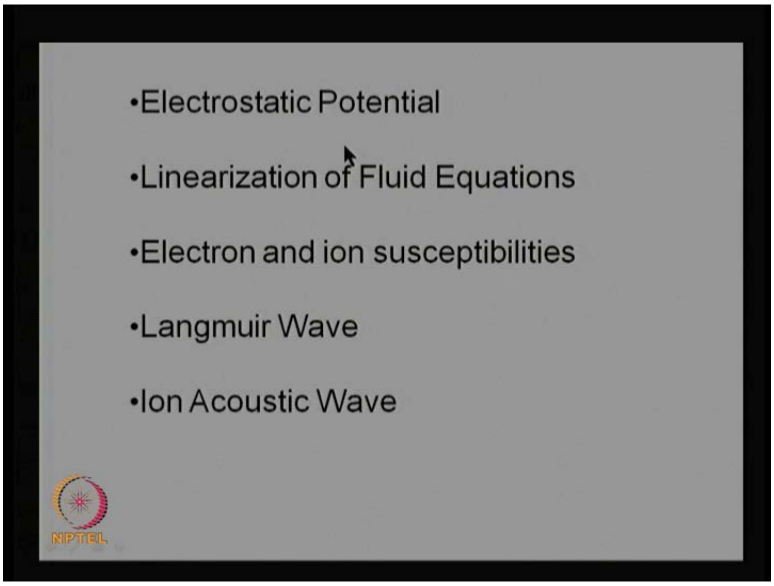


So, today I am going to start rather a new subject that is the propagation of electrostatic waves in plasmas, in this we will discuss how to express an electric field in terms of a potential.

Then, how to linearize the fluid equations? The equation of motion, and equation of continuity; and deduce expressions for electron and ion susceptibilities. And then to discuss specifications of Langmuir waves and ion acoustic waves, which exist in un magnetized plasmas as well as in magnetized plasmas also. So, today our presentation will focus on waves in un magnetized plasmas.

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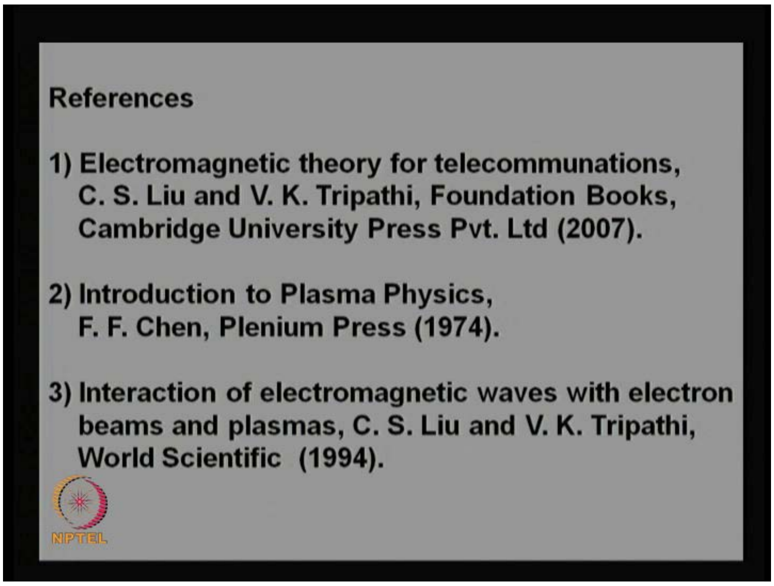


Well, I would like to refer to three books; one is electromagnetic theory for telecommunications written by professor C S Liu and myself published by foundation books Cambridge university press. Second is introduction to plasma physics by F F Chen published by Plenium press and third one is interaction of electromagnetic wave with electron beams in plasmas by professor C S Liu and myself published by world scientific.

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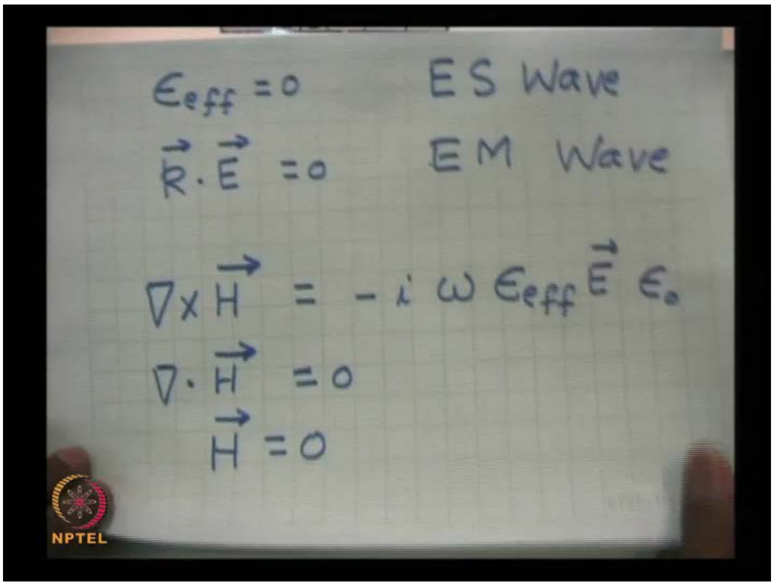


Well, as I mentioned that in order to satisfy the Maxwell’s equations for plane wave solutions it was necessary that either you satisfy epsilon effective equal to 0 condition or k should be perpendicular to E or k dot E equal to 0, the second is the case when electric field is perpendicular to k vector of the wave is called E M wave electromagnetic wave and we have discussed this case. Second possibility is that plane wave solution can satisfy Maxwell’s equations, when epsilon effective is 0 and in this case the waves are known as electrostatic waves.

For these waves please understand one thing that according to forth Maxwell equation curl of H is equal to minus i omega epsilon effective E into epsilon 0 of course, So, if right hand side is 0 then curl of H is also 0 and according to second Maxwell’s equation curl of B is also 0 a dot of divergence of H is also 0 .

When curl of a vector is 0 and divergence also 0 then the vector must vanish, well we had also see this explicitly last time that H is 0 for these waves. So these are purely electric waves and we call them as electrostatic waves.

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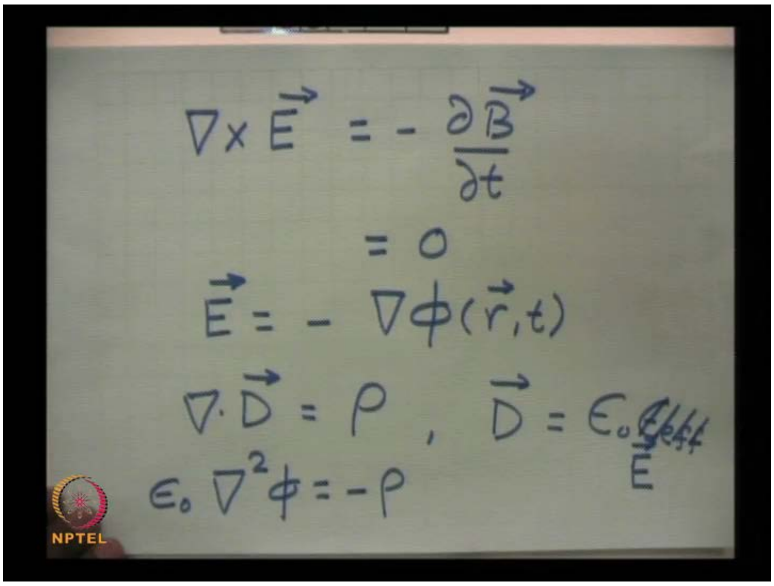


Now, before I move further I would like to examine the third Maxwell equation which is curl of E is equal to minus delta B by delta t.

If, there is no magnetic field associated with the wave then delta B by delta t is 0 and curl of E is 0. any vector whose curl is 0 is expressible in terms of a gradient of a scalar quantity I express this E then as minus gradient of a function phi which may depend on position and time in general, so an electrostatic wave electric field is expressible in terms of a potential phi which in general depends on position and time.

I would like to understand in terms of phi, what is the wave equation governing phi? And in terms of phi what is the response of electrons and ions to this potential. first of all a word about the Maxwell's equations there are four Maxwell's equations, second Maxwell equation I had already written divergence of H is 0, so when H is 0 this equation is not meaningful.

Third equation is this equation, so if H is 0 then curl of E is 0 so when I express E in terms of phi this equation is automatically satisfied. how about the first Maxwell equation when you use this E in the first Maxwell equation which is divergence of D is equal to rho and for plasmas we write D as epsilon 0 into effective sorry this is for this is not true I made a mistake let me delete this for plasmas D is equal to epsilon 0 into E Simple product of free space permittivity and electric field but polarization we ignore.

So, if I write E as grade phi this equation becomes epsilon 0 outside del square phi is equal to minus rho, and rho is expressible in terms of electron and ion densities thus this is the equation that governs, the variation of potential in terms of electron and ion densities and if we can express electron and ion densities in terms of phi then this becomes the wave equation. So as far as the electrostatic waves are concerned this is equivalent of a wave equation.

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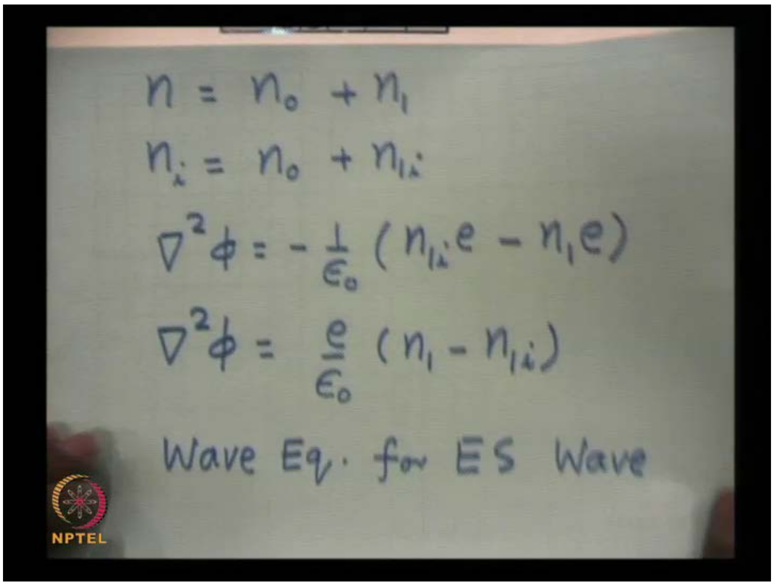


Let me write this, in terms of electron and ion densities initially the electron density in the plasma is n 0 and ion density also is n 0. however when a wave travels these densities are modified suppose this becomes n 1 and this becomes n 1 i in that case the Poisson equation or first Maxwell equation becomes del square phi is equal to minus 1 upon epsilon 0 into charged density electron density. If, I multiply by the electron charge which is minus E and the ion density by ion charge which is minus E then it becomes ion charge is n 1 i into e Minus n 1 into e.

Because, the n 0 and n 0 terms here will cancel each other, so take minus E out you can write this as e upon epsilon 0 into n 1 minus n 1 i this is the wave equation for electrostatic waves in a plasma, if we could write n 1 and n 1 i in terms of phi well this Poisson equation we will use little later.

Let us go over to the fluid equations which governed the response of electron to phi but, a word about the character of these waves probably will be in order at this stage.

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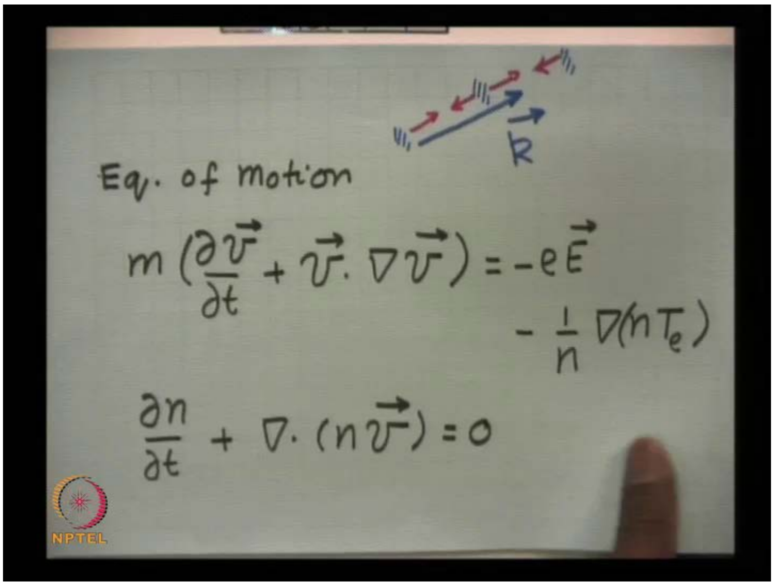


We are talking of waves going in some direction, which I call as the direction of propagation k. Because, the electric field of these waves in the direction of k at any instant if you look then the field of these waves will appear like somewhere in this direction somewhere in this direction somewhere in this direction somewhere in this direction at a given instant of time.

And then these arrows will move in the forward direction with time with the phase velocity of the wave, what is the consequence of these fields having opposite directions in different positions that for instance the electrons they will experience a force opposite to the electric field they will move in this direction whereas, the electrons which are here they will move in the in this direction.

So, in this region the electrons are moving away from each other and there will be a decrease in electron density. In this region there will be a buildup of electron density, so what you would expect is that in course of wave propagation charges accumulate in these regions they are accumulated there they are accumulating there they are accumulating there and they are there is a rarefaction there. So, with the propagation of electrostatic wave you are going to expect electron density oscillations and electron density oscillations are connected to velocity through the Poisson through the equation of continuity and hence let us write down the equation of motion.

Equation of momentum balance or equation of motion that is normally called is written as m delta v by delta t plus v dot del v is equal to the electric force, which I write as minus e E, I will ignore the presence of D C magnetic field and there is no magnetic field (( )) the wave so I will not write the magnetic force. But, then there is a collisional force for the sake of simplicity for the moment i may ignore the collisions also then there is a term called pressure gradient force which is minus 1 upon n gradient of n t.

This is the equation of motion for electrons of charge minus e, Mass m, density n and temperature T, I can write T e electron temperature T e.

So, equation of momentum balance involves density and the equations density is the equation of continuity delta n by delta t plus divergence of n v is equal to 0, these two equations are non-linear, why? Because, in this equations I can treat E as the source and v as the response and n also as the response. So whenever there is a electric field produced by a wave in the medium the electrons will respond to the field and they are called the response the velocity is the effect E is the cause.

Similarly, I can treat density also as variation in density also as the effect caused by this electric field, if electric field of the wave is small then response will also be v and product of two perturbed quantities or weak quantities I can ignore this process is called linearization. So, in order to solve a non-linear set of equations the first step is to linearize them linearization means neglect the terms which are of first order, even prior to that let us find out what is the equilibrium values of various quantities in these equations.

These equation govern the density and drift velocity average velocity of electrons. In equilibrium when there was no electrostatic wave in the system obviously I was not expecting in drift velocity, so in equilibrium v 0 and the density of electrons was n 0 and ion dense was also n 0 because the plasma was electrically neutral and how about temperature. If the electric field is of very small amplitude then in does not cause significant heating, so electron temperature also remains constant.

So, first whenever you have to solve a set of non-linear coupled differential equation or partial differential equations, you must always start with the equilibrium it is a general technique employed for electromagnetic waves as well as electrostatic waves.

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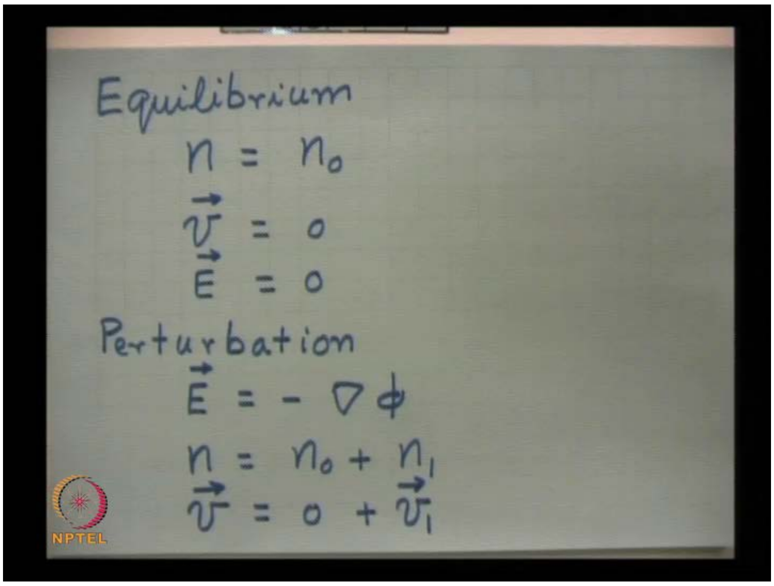


So, first of all let me write down the equilibrium an equilibrium my density of electrons is equal to n 0. And drift velocity of electrons is 0 and temperature just a constant, so I do not bother about it then I perturbed the system there is no electric field.

Now, I perturb this equilibrium so let me write the perturbation, I am writing my electric field E is equal minus grade phi and I am treating these E to be a perturbed quantity and phi, well I will explicitly write phi in terms of position and time little later but, then when the electric field of the wave is there in the system density I would expect to become n 0 plus some change called n 1. And how about the drift velocity? The drift velocity was 0 in the beginning there is a value given it acquires because of the wave called v 1 but, I will treat this phi n 1 and v 1 all small quantities and in the process of linearization of those equations of motion and continuity I will ignore the product of these quantities.

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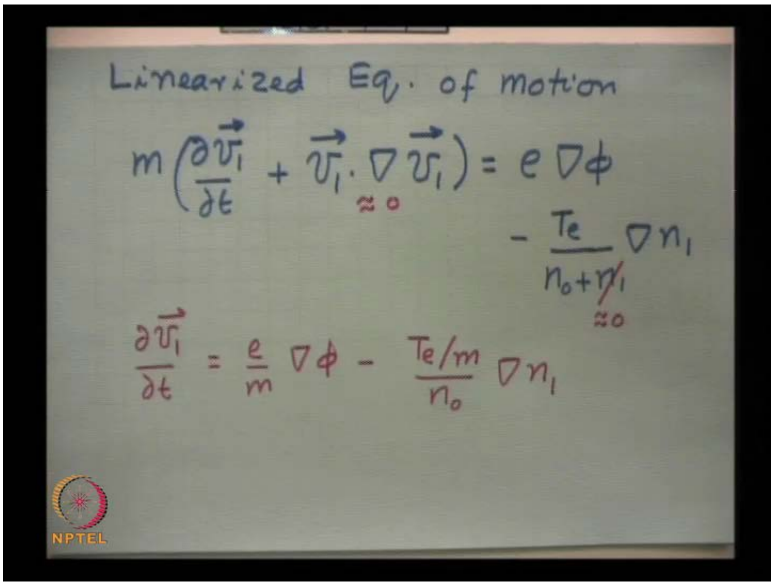


So, let me first write down the linearized equations of motion and continuity, so linearized equation of motion this equation is m delta v 1 by delta t plus v 1 dot del v 1 just substituting for v equal to v 1 in the equation of motion right hand side is e grade phi is the electric field electric force then there is a pressure term which is I take constant temperature so T comes out of this upon n 0 plus n 1 gradient of n 1, because gradient of n 0 is 0 this is the acquisition of motion.

Now by linearization, what do I mean? I am going to ignore the product of perturbed quantities v 1 is a perturbed velocity so the product of two perturbed quantities will be ignored so I take this term to be 0. Now, as far as this term is concerned n 1 is the perturbation and n 0 is the original value of density, so n 1 is very small and n 0 is large. If, I carry out a binomial expansion of this factor n 1 term will go up and product of n 1 with n 1 will be ignored.

So rather than going through that step I just ignore this here so this term is nearly 0, because this is already perturbed quantities there. So with their neglect of this term and this term this equation becomes a linear equation or linearized equation, which I can write down as delta v 1 by delta t is equal to divide by m, e upon m grade phi minus T e upon m upon n 0 into gradient of n 1; this is the equation of motion after linearization.

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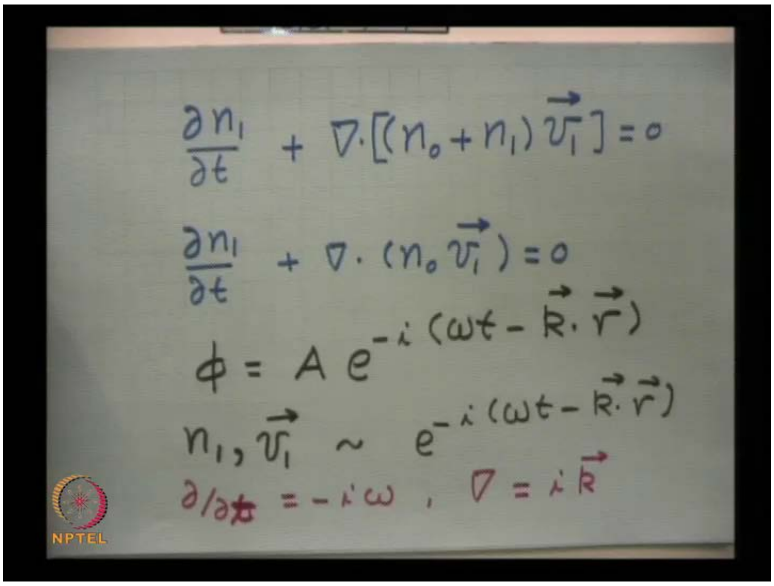


Similarly, the equation of continuity the equation of continuity is delta n 1 by delta t, because n 0 delta n 0 by delta t is 0 plus divergence of n 0 plus n 1 into v 1, the velocity n v is equal to 0 but, this n 1 is a perturbed quantity this v 1 is a perturbed, so I will ignore the product and hence this equation becomes delta n 1 by delta t plus divergence of n 0 v 1 is equal to 0, this is the linearized equation of continuity.

In order to solve these equations I presume a solution to the Poisson equation or the potential as A exponential minus i omega t minus k dot r. a wave travelling in the k direction with frequency omega suppose it has a potential of this form, potential is a scalar quantity so a is also a scalar quantity.

Now, phi is the source and n 1 and v 1 are the responses. So, I expect n 1 and v 1 also to vary with time and position as exponential minus i omega t minus k dot r and this helps us a great deal, because wherever delta delta t occurs I can replace by minus i omega.

So, then after linearization replace delta delta t operating over perturbed quantity as minus i omega and del operator by i k this is time operator and this is space operator. So then these equations the equation of motion continuity with this substitution of these operators in terms of the algebraic quantities they become algebraic equations and let me write them.

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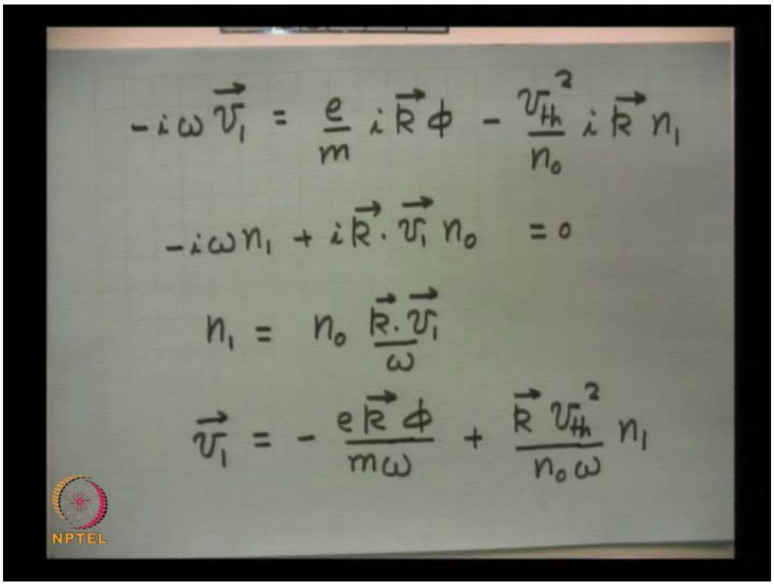


Quickly you will see, that the equation of motion becomes delta v by delta t is minus i omega into v 1 is equal to e upon m grade phi grade is i k vector phi minus there is a pressure term which is T e upon m which i call as v thermal square thermal velocity square upon n 0 gradient of n 1 grad i write as i k into density perturbation and the equation of continuity becomes minus i omega into n 1 plus divergence of n 0 v 1 divergence del operator is i k dot v 1 into n 0 is equal to 0 .

This equation is straight away gives n 1 is equal to n 0 k dot v 1 upon omega and this equation gives me v 1 is equal to two terms; one is minus e k upon m omega into phi, the other 1 is a plus term k vector v thermal square thermal velocity of electrons is square upon n 0 into omega into n 1.

So, from the above equation if I write down the value of n 1 in terms of v 1, I can or rather from the lower equation I use the value of v 1, I can write down n 1 in terms of phi, because in order to solve the Poisson equation or Maxwell’s equation I need the density perturbation, so n 1 is that is what I require and I will write down using these two equations n 1 in terms of phi and that turns out to be I just simplify these two equations and write the solution.

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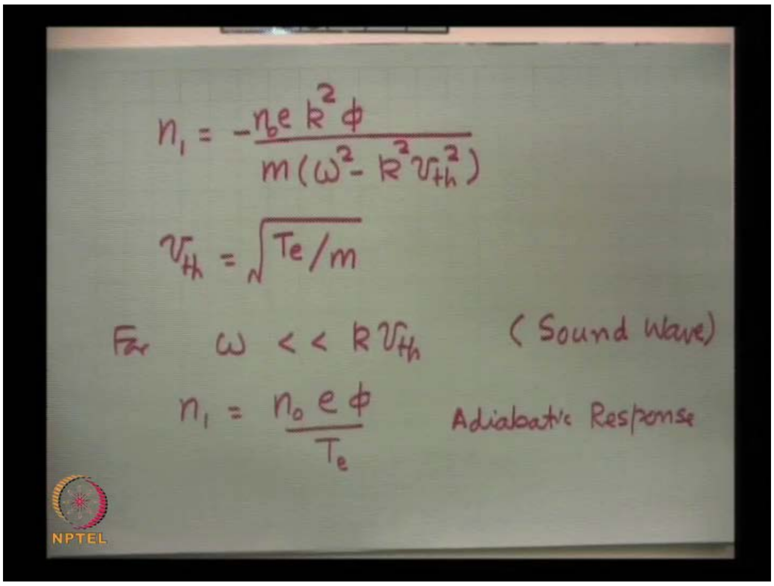


You obtain n 1 is equal to minus e k square phi there is n 0 also here, n 0 into E k square phi upon m omega square minus k square v thermal square thermal velocity of electrons is square, where I have written v thermal is equal to T e upon m under root.

Here, please remember minus e is the electron charge, n 0 is the equilibrium value of electron density, k is the propagation constant of the wave, m is the mass of the electron, v thermal is the thermal velocity of the electrons. This n 1 will be out of phase by phi with respect to phi if this term is smaller than omega square, on the other hand if k v thermal is bigger than omega square. I can ignore this term and response becomes very simple in the particular case or k v thermal much bigger than omega for low frequency response this is positive and this reduces to simply.

Let me write this, for low frequencies you get n 1 is equal to n 0 e phi k square will cancel out and upon T this is called adiabatic response. and you may recall this omega square terms comes from delta v by delta t in the equation of motion called inertia term and this comes because of the pressure term.

So, at low frequencies or when omega by k the phase velocity of the wave is less than thermal velocity of electrons, in that case the electrons respond almost instantaneously to the field of the electro static wave this is called Boltzmann distribution they all obey some sort of a Boltzmann distribution or their response is called adiabatic response.

Quickly, where the potential is large electrons will move there and so on this is the n 1 is bigger where phi is bigger same phase they oscillate.

But this is a case waves that satisfy this condition are called sound waves or ion acoustic wave but, today I will begin my discussion with a high frequency wave for which omega is bigger than k v thermal that wave is called (( )) wave and for that high frequency wave you may note that, if I write down a similar expression for ion density perturbation let me write down that expression I think then you will appreciate it better.

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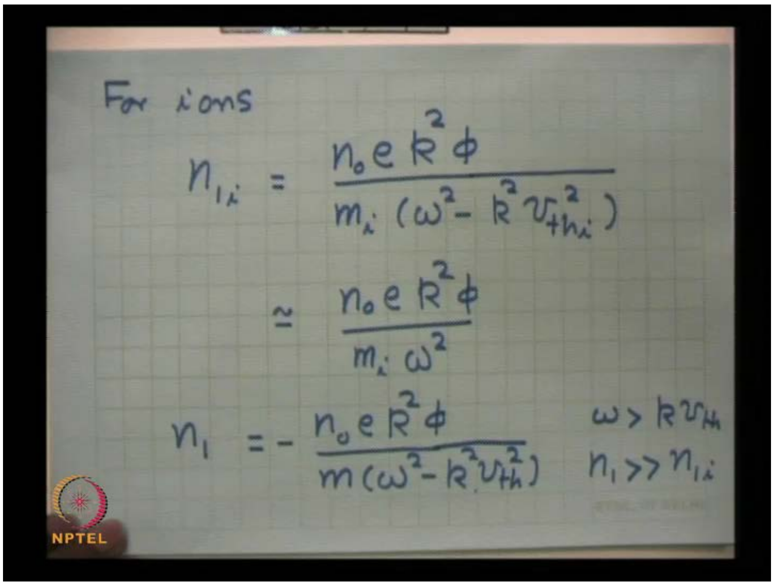


So far, ions the density perturbation for ions would be n 1 i would be equal to n 0 charge of the ion which I am taking to be singly ionized into k square phi upon mass of the ion into omega square minus k square thermal velocity of ions square. But thermal velocity of ions is very small as compared to electron thermal velocity so this term is usually very small and you can ignore this, so I can approximate this as n 0 e k square phi upon m i omega square. So far, a high frequency wave where n e and one of electrons is equal to n minus n 0 e k square phi upon m omega square minus k square v thermal square, because I am choosing this term to be less than omega square.

So, numerators of both these terms are equal in magnitude this denominator is small as compared to this denominator, because of mass and ion mass in very heavy. So this term is very large as compared to this term so for high frequency waves or omega bigger than k v thermal, we have n 1 much bigger than ion density perturbation and hence I can ignore the ion motion.

So, the high frequency waves satisfying this condition for those waves. I can completely ignore the ion density perturbation and electron density perturbation is expressed in terms of phi well people like to write down this n 1 in terms of a quantity called susceptibility. So rather before proceeding further, I can write this in a slightly different way density perturbation you know I can talk of a polarization.

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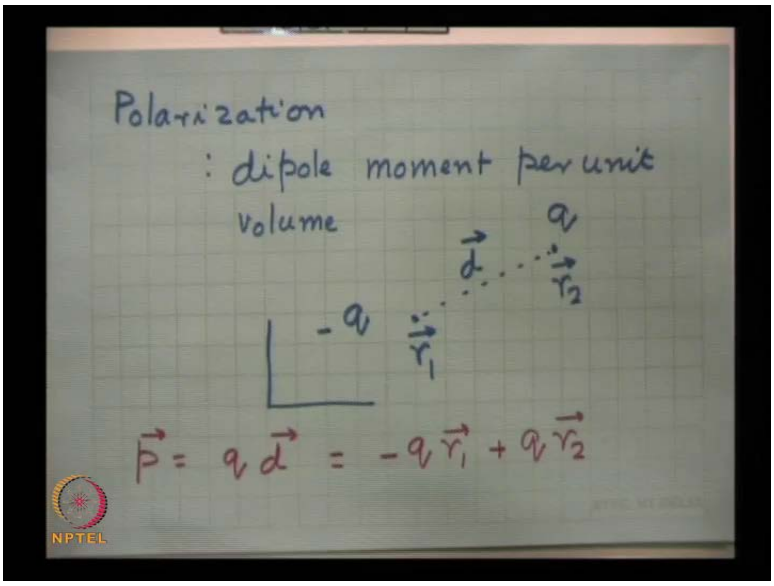


Polarization normally is a quantity that is employed in dielectrics, what does this mean? This is called dipole moment per unit volume per unit volume and what is a dipole moment, suppose I have a pair of charges one placed here the other placed there the charge is suppose minus q here and plus q there. The position coordinate of this in a coordinate system suppose this coordinate is r1 and the coordinate of this point is r 2. So, in a system if I have two charges minus q and q plus that a distance at positions r1 and r 2 then the distance between them is called d which is equal to r 2 minus r1. And you define the dipole moment of this (( )) of charges v is equal to magnitude of these either of the two charges q multiply by the distance d, starting from the negative charge to positive charge this is the definition of dipole moment. Which, I can also write down simply as product of charge and position coordinate this is minus q r 1 plus q r 2, if you write down this way this also the same thing, because r 2 minus r 1 is d.

So, this is the same way of writing of dipole moment of an atom. If there are many such particles you just multiply the position coordinate of each particle with the charge of the particle and add like this quantity is called dipole moment per unit volume. Now, in case of plasmas we are talking about charges per unit volume with density n or n I, so what I should have.

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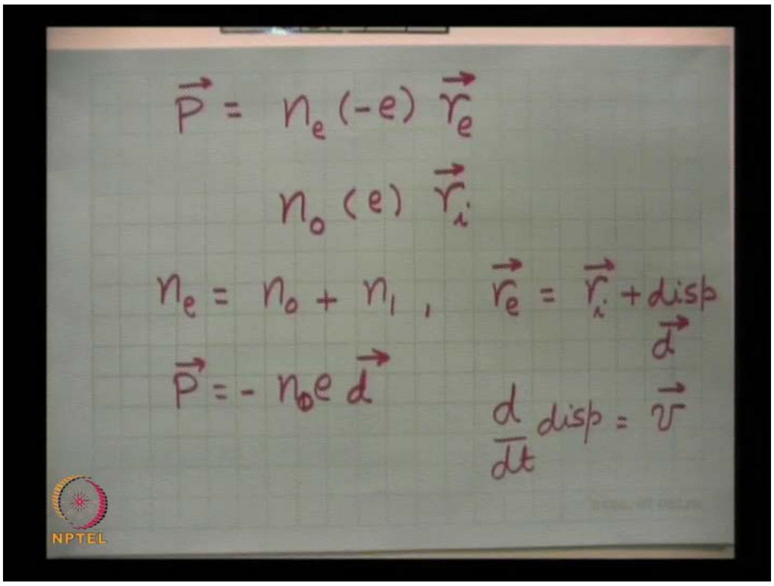


So polarization p in plasmas is defined as electron charge is n e for instance and minus e is the electron charge, so this is the density of electrons into charge of each electrons this the charged density per unit volume multiply by the position of the particle.

Now, position of the electrons they are distributed all over, so I should really consider because this is I have to let me call this r e of electron position and there should be ion position also, because ions are located with density n 0 charge plus e and ion position say r I, what should i do? Because n e is equal to n 0 plus n 1 this is the electron density, so n 0 term will cancel out because ions and electrons are the same place or r i, r e I can write down volume element position is equal to initial position where ions (( )) some displacement.

So, just put this displacement equal to d this becomes p is equal to minus n 1 e, well actually this is r n 0 e into d plus there will be a term which is a product of two small quantities I can ignore that.

Well this is the way one can proceed and what is the displacement? This is related to velocity d t of displacement is equal to velocity, if you work out from there you can find a connection between this dipole moment and density perturbation or velocity perturbation. Well, I just want to write this I do not want to give you a mathematical derivation, I just want to say that one can easily show that the density perturbation.

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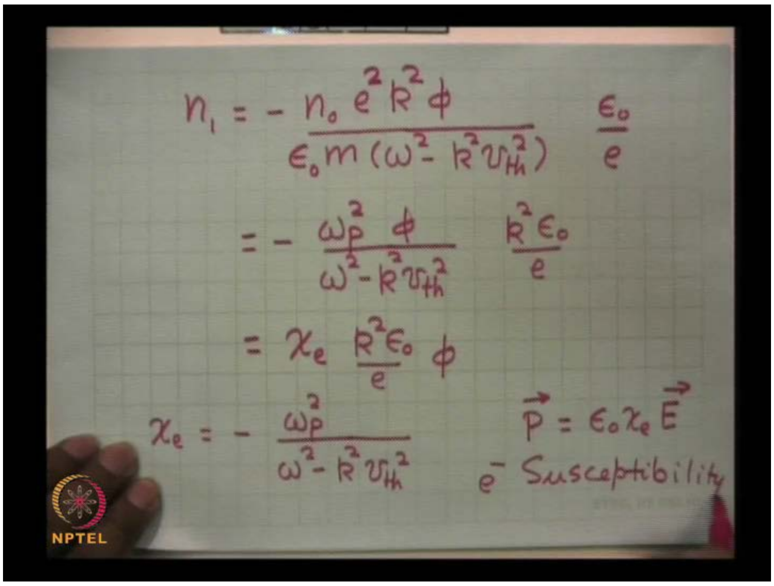


That I get n 1 which I obtained as minus n 0 e k square phi upon m into omega square minus k square v thermal square. if I multiply this equation by e up and e down and epsilon 0 up and down then this can be written as so you multiply e here, and divided by e here multiply by epsilon 0 here and multiply by epsilon 0 there then you know that n 0 e Square by m epsilon 0 is omega p square upon omega square minus k square v thermal square into phi the remaining term is k square epsilon 0 divided by e, this quantity which is coefficient of phi in this first term is called electron susceptibility.

As we will see that, this is dimensionless quantity and susceptibility is also dimensionless quantity. So we have taken care of this extra factor, if I write explicitly so people write this is equal to chi e into k square epsilon 0 by e into phi and this susceptibility I have written as minus omega p square upon omega square minus k square v thermal square, one can show that the polarization p that I was deriving in the last page is equal to epsilon 0 into chi e into E.

That is why? This constant independent of time and position etcetera, this depends only on electron density in the equilibrium, electron temperature in equilibrium and omega and k of the wave this quantity is a characteristic of a plasma depends on its density and temperature and on wave parameters omega and k.

Similarly, I can write down the density perturbation for ions if it is required, in the present case of high frequency wave when omega is much bigger then k v thermal you do not require there. So, if I know the density perturbation I can go to the wave equation this quantity is known as electron susceptibility.

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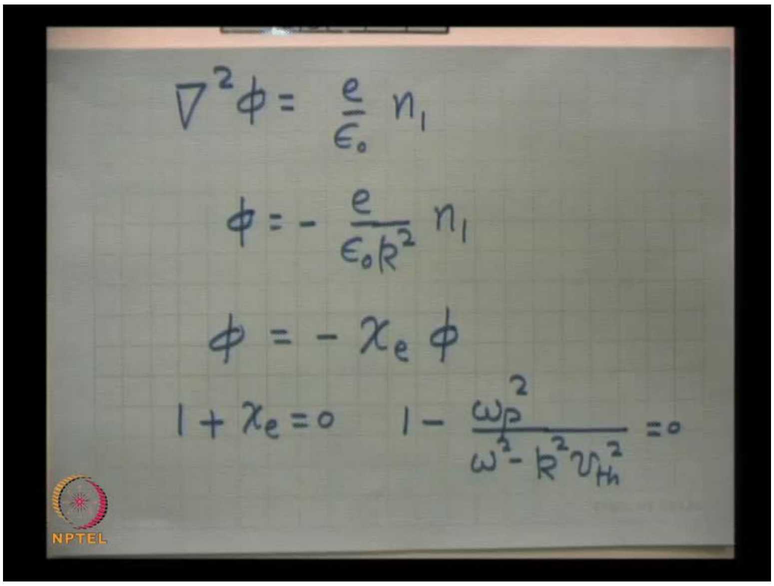


And my wave equation would be or the Poisson equation would be del square phi is equal to e upon epsilon 0 into n 1, this was my Poisson equation if I ignore the ion density perturbation del square. Again I will replace by minus k square, so this becomes phi is equal to minus e upon epsilon 0 into k square n 1 and put the value of n 1 in terms of chi this becomes minus chi e into phi. You may see, the reasoning here that this is a same factor with n1 that was in the expression for n1, in the reverse way so that this cancels out and if I want a non trivial solution then phi will cancel out on both sides and one is should be equal to this or the dispersion relation becomes 1 plus chi e is equal to 0

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If, you put the value of chi e this gives 1 minus omega p square upon omega square minus k square v thermal square is equal to 0, you can simplify this and you get the dispersion relation for the langhmuir wave or plasma wave.

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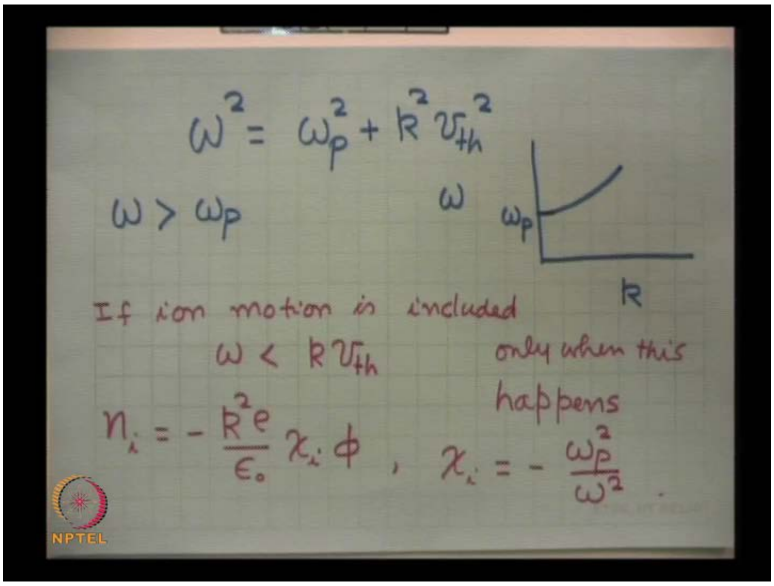


And that is omega square is equal to omega p square plus k square v thermal square. if I plot this plot k here, and omega there there is a frequency omega p the value of omega cannot be less than omega p, because this is a positive term and the disperse relation goes like this. So, omega increases with k, the minimum value omega can take it omega p so these high frequency waves can have frequencies only bigger than omega p. So, this is one branch of electrostatic waves where ion motion is ignored, if we have included the ion motion; if ion motion is included, what do I get? well as I mentioned ion motion will be important only when the electron response is such that omega is less than k v thermal.

So that the electron term is strongly suppressed as compared to ion term only when this happens and in this case what happens? The ion response turns out to be minus k square e upon epsilon 0 ion susceptibility into phi and ion susceptibility is written as minus omega p i square upon omega square, you do not have to include the ion thermal effects, because the ion thermal velocity is very small because of the mass coming in this denominator. So, if I know the this n 1 i rather this n 1 i.

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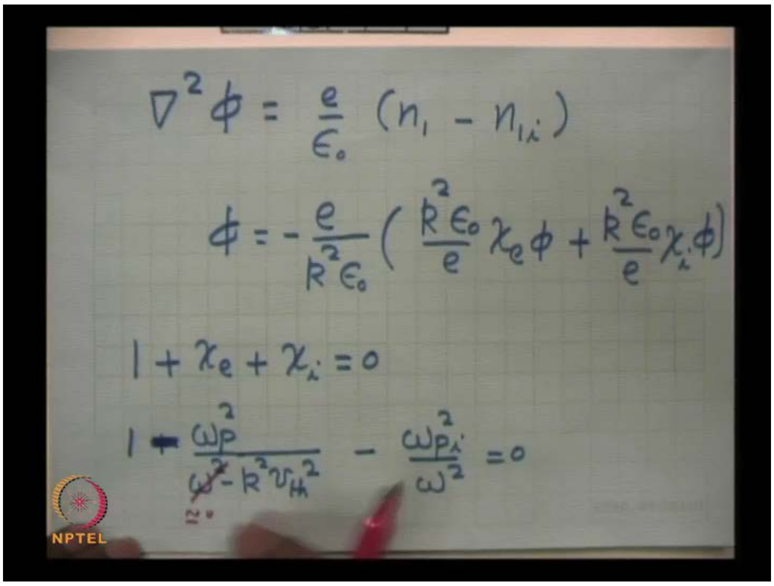


So, if I know the electron and ion density perturbations then the Poisson equation becomes del square phi is equal to e upon epsilon 0, n 1 minus n 1 of ions put these replaced by minus k square this becomes phi is equal to e upon k square epsilon 0, the negative sign and this becomes k square epsilon 0 by E into chi e into phi and this also becomes plus then because 1 minus is there 1 is in this becomes k square epsilon 0 by e chi i into phi.

All this will cancel out and the dispersion relation is 1 plus chi e plus chi i is equal 0, if I put these values this becomes 1 plus omega p square upon omega square minus k square v thermal square of electrons minus omega p i square, this is minus here I am sorry this is minus sign upon omega square is equal to 0.

But, as I mentioned this term is very huge as compared to this term unless this is negligible as compared to this, so I ignore this so suppress this term as compared to this one, when this is comparable to this term the wave is called langhmuir wave, that I have already discussed, so only when this is less than this.

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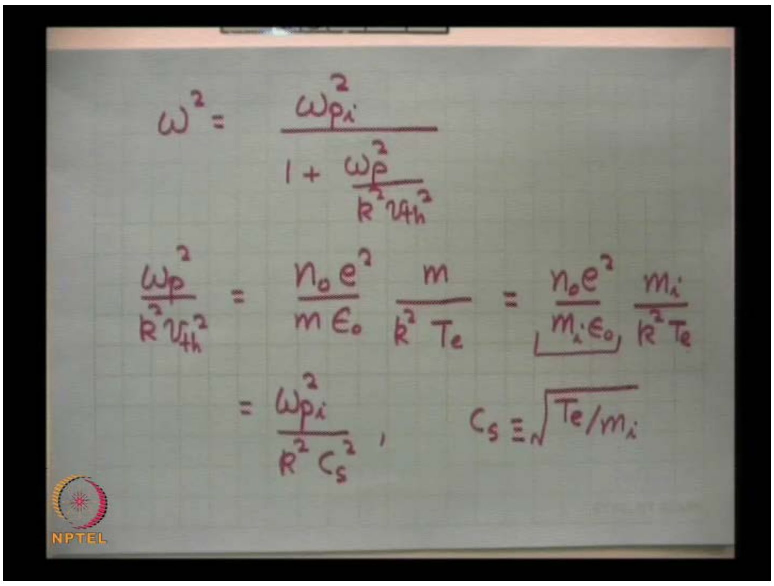


This ion term is important and in that case this equation gives you omega square is equal to omega p i square upon 1 plus omega p square upon k square v thermal square. but you know omega p square upon k square v thermal square is the same thing as n 0 e Square upon m epsilon 0, this is for omega p square k square is there and v thermal square is T e upon m, what you can do? m will cancel with this m you can write this as same thing e Square upon m i epsilon 0 into m i upon k square T e, because m i will cancel this m i so they are equal exactly same this quantity I call as ion plasma frequency square k square is there T e upon m i the ratio of electron temperature to ion mass.

This quantity is denoted as C s square, so C s is equal to T e upon m i this is known as the sound speed plus we shall learn about it why this is called sound speed I am just defining a constant at the moment T e upon m i under root as C s. So, if I use this omega p square by k square v thermal square as this quantity when omega square this dispersion relation for the ion acoustic wave.

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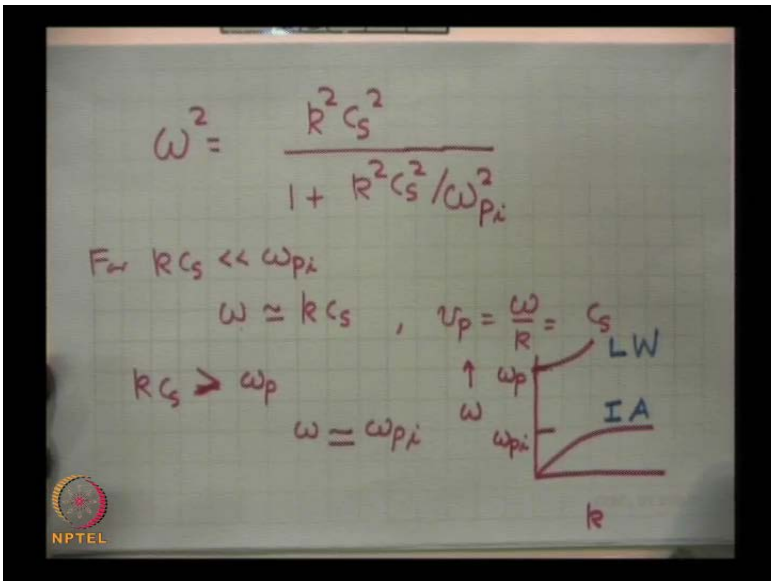


Can be written as you can rewrite this slightly in a different way omega is equal square is equal to k square C s square upon 1 plus k square C s square upon omega square, for k C s less than omega you can ignore this second term in the denominator and so for k C s much less than omega this omega is nearly equal to k C s. This is a linear relationship between omega and k and the phase velocity is omega by k equal to C s sound velocity.

So, this the wave whose phase velocity at low frequency is sorry this is sorry this is wrong I am sorry I made a mistake this is omega p i square hence omega p i. So, when k C s is much less than omega p I, the frequency of the wave is like k C s and phase velocity is like C s but, when k C s is large. So as k C s becomes bigger than omega p i omega approaches equal to omega p i ion plasma frequency.

So, let me plot a graph here, I am plotting k here and omega there and ion plasma frequency is here, please remember ion plasma frequency is very small as compared to electron plasma frequency. So these waves always have a frequency less than omega p i that is disperson relation goes like this and electron plasma wave was omega p was much higher here the dispersion relation was like this.

So, this branch this lower branch is called ion acoustic wave, and this is called Langmuir wave there is a big forbidden gap between omega p i and omega p, the ion plasma frequency and electron plasma frequency there is no electrostatic wave possible. This is a very important consequence of ignoring thermal motions, ion acoustic wave will not exist if there is no thermal motion, because the electron response will always be more dominant than the ion response.

So, thermal motion is mandatory for the ion acoustic wave as far as the Langmuir wave is concerned. If thermal motion is not there then omega is always equal to omega p. There is no k dependence, so k dependence an omega of Langmuir wave arises because of temperature so temperature provides k dependence to Langmuir wave and it also makes the existence of ion acoustic wave possible. Now, let me mention a few words about the energy carried by these waves in my next lecture, I will deduce an expression for energy density but, today I will presume it.

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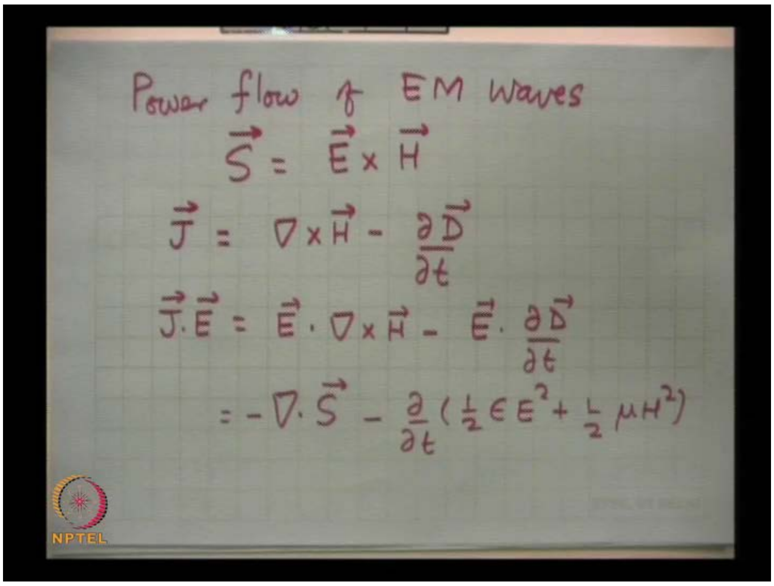


You know, for electromagnetic waves we know power flow of E M waves is given by pointing vector. S is equal to v cross H for electrostatic wave there is no magnetic field, H is not there, so this kind of pointing vector does not exist, does it mean that the waves do not carry any energy or momentum, this is not true. After all this S is equal to E cross H was deduced from Maxwell's equations, how to deduce this? You write down J is equal to from forth Maxwell's equation is equal to curl of H minus delta D by delta t.

And then you dot this equation with E, J dot E and you will get E dot curl of H minus E dot delta D by delta t and then you interpret this as the energy absorbed by electrons of charged particles per unit volume and then you write this curl of E dot curl of H in terms of a quantity called this becomes divergence of S, and then there is a term which gives you like delta delta t actually this is minus here and delta delta t of half epsilon E Square plus half mu H square, this is the way you interpret this saying that this is the pointing flux the amount of energy flowing per unit area per unit time, this is how you do it.

The entire (( )) is in the interpretation of J dot E we have see that in a collisionless plasma J equal to sigma E but, sigma is imaginary then this quantity does not represent a power dissipation, so it depends on the interpretation of this quantity. If, we ignore the interpretation of this quantity properly proper interpretation, if you ignore then you get a wrong result.

So, the waves which do not have a magnetic field by a proper interpretation of J dot E term you should be able to see, that the energy contained in in the electric field is not half epsilon E Square for electrostatic wave H is 0 so this term does not exist but, this is (( )) definition. rather in general in a dispersive medium where permittivity depends on frequency in that medium.

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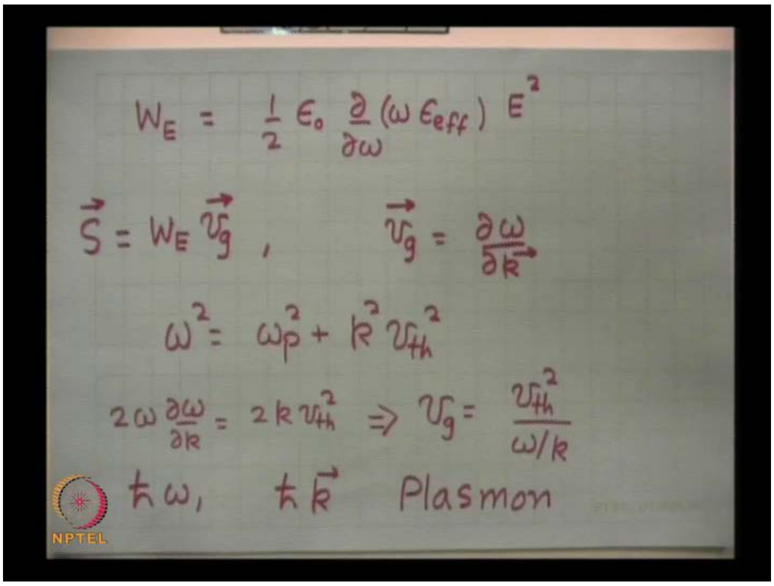


The energy contained in the electric field is denoted as W E is equal to half epsilon 0 into delta delta omega of omega epsilon effective into E Square rather than calling this half epsilon 0 into epsilon effective E Square, this is the right definition and when you use this we shall derive this expression next time then we will see that the energy contained with the wave is this quantity and energy propagating with the wave is W E into v g this is the equivalent pointing flux for a electro static wave.

And group velocity is delta omega by delta k magnitude wise, vector wise like this. Now, we have seen that for a plasma wave omega square is equal to omega p square plus k square v thermal square. So if I want to obtain the group velocity how much that would be you differentiate (( )) 2 k, so you will get twice omega delta omega by delta k is equal to twice k v thermal square.

So, this is the group velocity so you get group velocity is equal to v thermal square upon omega by k this is the phase velocity k, you can obtain from this relation and write this there so group velocity will vanish if v thermal is not there. So, it is the thermal motion which gives rise to finite energy propagation or finite improved velocity if this is not there pointing flux will be 0. So, thermal motions play very important role in energy transfer with plasma waves, plasma waves carry energy as well as momentum each quantum of plasma wave is called Plasmon with energy H cross omega and momentum H cross k called Plasmon.

The beauty of these waves is, in a thermalized plasma like this their dispersion relation is like this k can be very close to 0 or to be finite if it is very close to 0, the phase velocity will be very large they can have phase velocity close to c or close to a few times thermal velocity but, when they become very close to thermal velocity when the phase wave velocity becomes very close to thermal velocity a very important phenomenon occurs and that is called the phenomenon of landau damping that we can discuss on the basis of kinetic theory but, prior to discussing that I would like to discuss the phenomenon of two stream instability, because that will give you an insight into the landau damping but, that is a different matter.

For today, it will suffice to say that a plasma wave is strongly influenced by these thermal motions of electrons, they provide a group velocity these waves can have phase velocity close to c, and these waves can trap electrons and accelerate them to velocities close to c. And this is the basic mechanism of laser based electron acceleration. I think, I would like to stop here, for today, and then we will discuss other issues in next lecture. Thank you.