

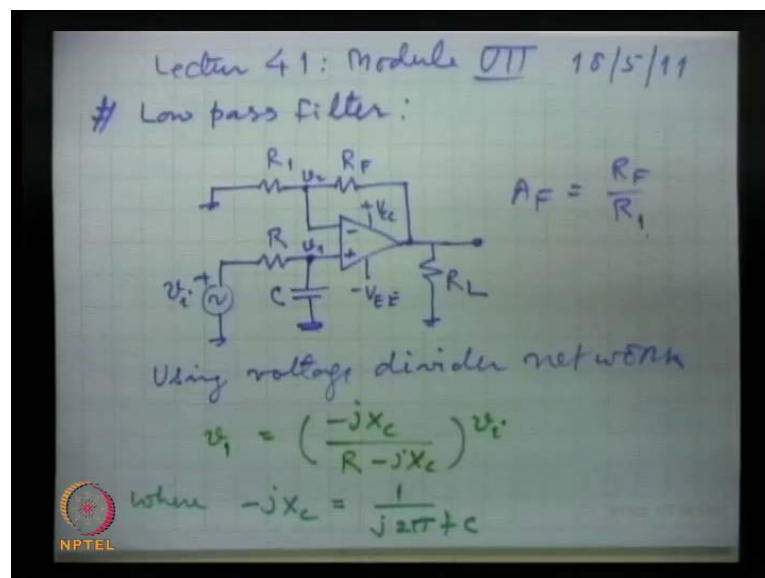
**Electronics**  
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**Module No. # 07**  
**Differential and Operational Amplifiers**  
**Lecture No. # 08**  
**Filters**

Selection of frequencies from a wide spectrum of the **receiving** received signal is an important activity, which is required in **in** signal processing. Now, these circuits are as we have talked filters and active filters are much more useful, they give a more accurate selection of frequencies and they are very efficient.

Now, four types of filters we talked, the low pass filter, high pass filter, band pass and band reject or band stop filter.

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So, having said that, now we take how to realize the circuit for these filters using operational amplifiers. Operational amplifiers are as it has been said several times are

very convenient to use, just few resistors and sometimes capacitors are externally connected appropriately that is it, and rest of the job is done by the operational amplifier.

So, first we take the low pass filter (No audio from 01:53 to 02:02) **low pass filter**. In low pass filter, frequencies will propagate, that means, they will go into the circuit from very low frequencies to a certain high frequency. After that, there will be a cut off and the circuit can be realized **in a** by using operational amplifiers like that.

The circuit is (No audio from 02:33 to 03:15) (Refer Slide time: 02:32), this is the circuit (No audio from 03:19 to 03:28). The inverting input is connected with these two resistors,  $R_1$  and  $R_F$  and the **signal** incoming signal  $v_i$ ,  $i$  for input signal, this is connected at the non inverting input through resistance  $R$  and a capacitor  $C$ .

Now, so this is the circuit for the low pass filter and we will see that frequencies, where we want to cut, they can be adjusted by appropriately choosing the values of  $R$  and  $C$ , while the overall gain of this **of this** circuit, of the amplifier will be governed by these two values and this is  $R_F$  by  $R_1$ .

Now, what are these voltages? It is the non inverting input  $v_1$  and the voltage at the inverting input  $v_2$ . Now, this is simply a voltage divider circuit, these are two impedances which are seen by this (Refer Slide Time: 04:31). So, what is the voltage which will be developed between non inverting input and ground that will be equal to  $v_1$ ?

Now, this such kind of things we have done several times that the current, which will be flowing in this circuit will be  $v_i$  divided by, because this source is seeing them in series,  $R$  and  $C$  are in series, so this is the **the** two impedances in series. So,  $v_i$  divided by the series combination that is the current into the impedance of this capacitance that is the voltage  $v_1$ . So, obviously, using voltage divider network, the  $v_1$  is equal to  $\frac{1}{R - jX_C} v_i$ ; this is where  $jX_C$ , this is  $X_C$  is the reactance of the capacitance and this is equal to  $\frac{1}{1 + j2\pi fC}$ .

We know that this capacitance impedance is  $\frac{1}{j\omega C}$ ,  $\omega$  is  $2\pi f$ , so here this is this, so now substituting for a simplifying this equation; if we simplify this equation, here the imaginary terms are appearing at two places (Refer Slide Time: 06:45).

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Simplifying the above equation

$$v_1 = \frac{v_i}{(1 + j \cdot 2\pi f R C)}$$
$$v_0 = A_F \cdot v_1$$
$$= \left(1 + \frac{R_F}{R_1}\right) v_1$$

Substituting for  $v_1$

$$v_0 = \left(1 + \frac{R_F}{R_1}\right) \frac{v_i}{1 + j \cdot 2\pi f R C}$$

So, simplifying **simplifying** the above equation, we get  $v_1$  is equal to  $v_i$  by  $1 + j \cdot 2\pi f R C$ , this is the value of  $v_1$ . And the output voltage  $v_0$  here, this is  $v_0$ , the  $v_0$  we can see;  $v_0$  is gain into  $v_1$ .

Now, this was the gain for the inverting, but for the non inverting, **no** for the inverting here,  $R_F$  by  $R_1$  with the minus sign (Refer Slide time: 08:16), so this is for the inverting input. And here this is equal to  $A_F$  for the, when we apply the signal to the non inverting input, this is simply  $1 + R_F$  by  $R_1$  into  $v_1$ . We substitute the value of  $v_1$  from here this equation, so substituting **substituting** for  $v_1$ , we get  $v_0$  equal to  $1 + R_F$  by  $R_1$ ,  $v_i$  by  $1 + j \cdot 2\pi f R C$ .

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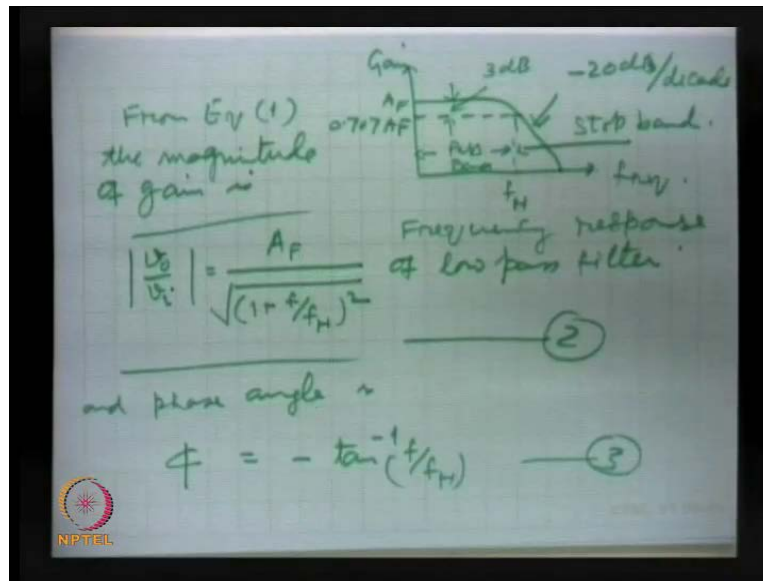
The image shows handwritten notes on a whiteboard. At the top, the transfer function is given as  $\frac{V_o}{V_i} = \frac{A_p}{(1 + jf/f_H)}$ . To the right is a graph of Gain versus frequency  $f$ . The gain is constant at  $A_p$  in the pass band and then rolls off at a rate of -20 dB/decade after the high-frequency cutoff  $f_H$ . Below the equation, it says  $\frac{V_o}{V_i} =$  Gain of the filter which is freq. dependent. Then,  $A_p = (1 + \frac{R_F}{R_1})$  is the gain in the pass band.  $f =$  frequency of the input signal.  $f_H = \frac{1}{2\pi RC}$  is the high frequency cutoff. An NPTEL logo is visible in the bottom left corner of the whiteboard image.

And this can be written **this can be written as**  $v_0$  by  $v_i$ , the gain **gain** of the amplifier that is  $A_F$  by  $1 + jf$  by  $f_H$ .

This is the fundamental relation for low pass filter where the ratio of output voltage to input voltage that is the gain. So,  $v_0$  by  $v_i$ , this is gain of the filter **gain of the filter** which is of course, which is frequency dependent, here the frequency, this is the frequency of the signal, I will write that which is frequency dependent (Refer Slide Time: 10:40).

And  $A_F$  is the gain of the pass band, this is  $1 + R_F$  by  $R_1$ , this we have talked that for the non inverting amplifier the gain is  $1 + R_F$  by  $R_1$  (Refer Slide time: 11:04). So, this is the gain and is the gain in the pass band **in the pass band** you remember that, of course we will draw the plot that here, so this is the pass band, this is the stop band, this is gain, and here is frequency is stop band, pass band (Refer Slide time: 11:28). So, this is the gain here in the pass band and  $f$  is the frequency of the input signal **frequency of the input signal** and  $f_H$  we have written for  $1$  by  $2\pi RC$  is the high frequency cutoff, is the here this is **h**  $f_H$  is the high frequency cutoff.

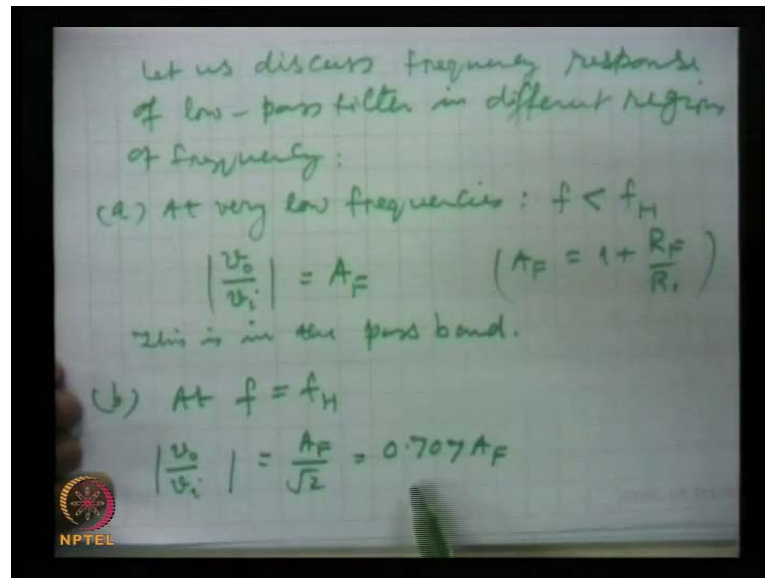
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And the figure, the exact figure comes out to be this (Refer Slide Time: 12:46), here it is  $A_F$ , this is gain and this is frequency and this is 3 dB fall **this is 3 dB fall** and here it is  $0.707 A_F$ , that is 70.7 percent of the gain, the maximum gain of the pass band and here this is  $f_H$ , this is the pass band from here to here, this is pass band and this is the stop band, from here onwards this is stop band. So, this is the frequency response (No audio from 13:58 to 14:04) **frequency response** of low pass filter and this fall in frequency, this is 20 dB fall. So, minus 20 dB per decade **per decade** of change in frequency, this is 20 dB.

Now, the expression which we have got, we can write for the magnitude of gain, we can call it some expressions say equation 1 (Refer Slide Time: 14:43). So, from equation 1, we can write the magnitude of gain, the magnitude of gain is  $V_o/V_i$ , this is equal to  $A_F$  and here  $1 + f/f_H$  square, this is another important equation, equation 2 which is the magnitude of the gain. And the phase angle can be written as phase angle is  $\phi$  equal to minus  $\tan^{-1}(f/f_H)$ , this is equation 3. From gain magnitude of the low pass filter, the different regions of you can see, may be discussed.

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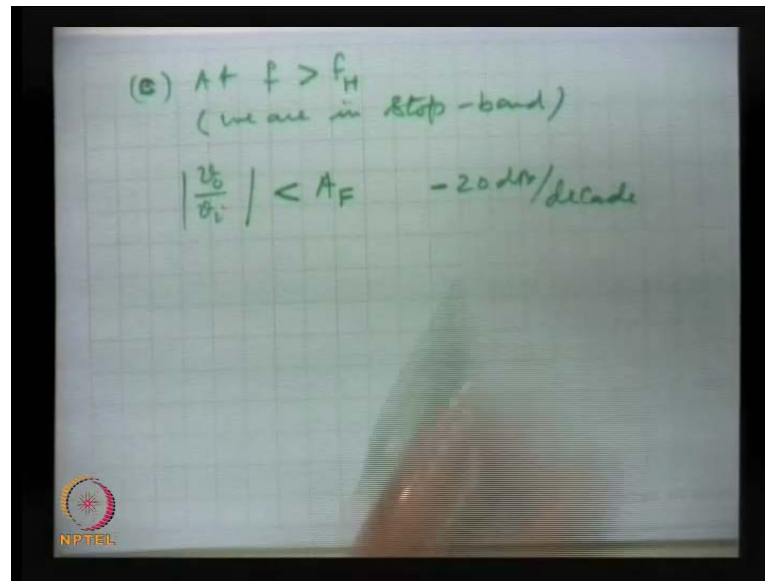


So, let us discuss **let us discuss** the frequency response of low pass filter in different regions of frequency, first is at very low frequencies **at very low frequencies**. By low, we mean that signal frequency is much less or less than **H f**, the cutoff frequency; in this frequency region (Refer Slide time: 17:28), from this expression we will see when  $f$  is a very low than this factor will be a very small, this square is only here. So, this will be very small, it can be dropped and in this region,  $v_0$  by  $v_i$  magnitude, this will be constant and equal to  $A_F$ , and  $A_F$  we have seen  $A_F$  is  $1 + R_F$  by  $R_i$  as we have chosen in the circuit.

So, this is the region here and frequency of signal is lower than **h f H**, so we are talking of pass band, so this is the constant gain in the pass band. So, this is in the pass band **this is in the pass band**, then let us see what happens, second case at frequency signal frequency equal to  $f_H$ , the cutoff frequency; then from this expression, this is equal to 1, so  $1 + 1$  is equal to 2. So,  $v_0$   $v_i$  magnitude is  $A_F$  by root 2 which is  $0.707 A_F$ ; in terms of d b s, this amounts to 3 d B fall.

So, exactly at frequency of signal frequency equal to the high frequency cutoff, this is the point which is 3 d B down as compared to  $A_F$  (Refer Slide Time: 19:38). So, this is the second case and this is the low pass filter, the gain falls at  $H f$  by **by** it from 100, it becomes 70.7 percent.

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Then we have third region C at signal frequency very high in comparison to  $f_H$ . So, obviously, we are talking of a stop band, a stop band starts from here that is  $f_H$  onwards, the frequency is higher than this, this is under discussion. So, this is about we are in a stop band and here, when we substitute that **that** factor, the gain falls. So,  $v_o/v_i$  is less than  $A_F$ , it falls, it continuously falls with raise in frequency here (Refer Slide Time: 20:10); from here onwards as we increase the frequency, it falls and the fall is of a 20 dB and because it is fall. So, we write minus 20 dB per decade fall in the, this is the slope of that fall.

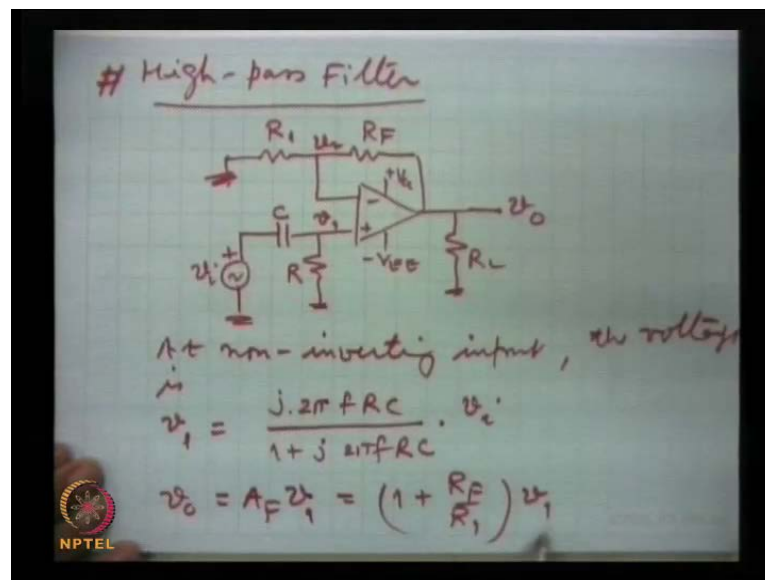
So, this about the low pass filter, what is the circuit that we have discussed, this is the circuit and how much gain we want (Refer Slide Time: 21:45), this will be decided by **by** the values of  $R_F$  and  $R_1$  and the cutoff frequency as we have seen that is given by the component  $R$  and  $C$ ,  $f_H$  is equal to  $1/2\pi RC$ . So, the components this  $r$  and  $c$  (refer Slide Time: 22:18), what values we choose or if we decide that what should be the cutoff, then one of the components we can take arbitrarily and using that expression for the cutoff, we can choose the other component.

So, normally capacitances are chosen, because they are not available for very large different values, but they vary for example, 1 micro farried, 1.5 micro, farried 2 micro farried and so on. But resistances are available in a larger range, so this is that.

Now, about the fall, the fall is 20 dB (Refer Slide Time: 23:05), this can be made 40 dB fall also. That means, it will be a sharper fall here, instead of this plot, this can be made sharper so that this fall is 40 dB. This fall may be 40 dB per decade; this can be achieved by what is called the second order low pass filter in which another R and C component added. So, there are two resistances here and two capacitances.

So, another network R C network that can be added and analysis is a is not a very different from his and that shows that the the fall in gain with frequency in the stop band can be achieved as 40 dB per decade, for the positive of time we cannot take all these second order and and so on. So, that is why just I have talked about it, so this is about the low pass filter.

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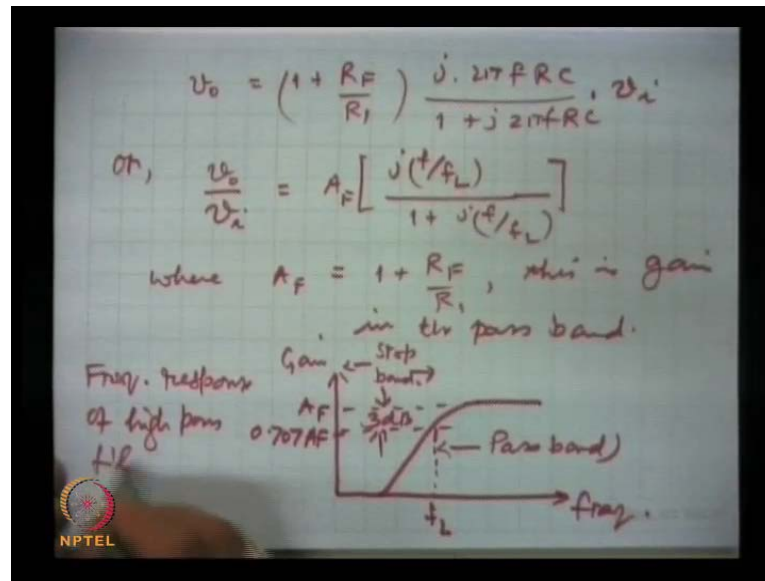


Then we talk we take high pass filter (No audio from 24:30 to 24:38), high pass filter, a high pass filter can be achieved by interchanging this capacitance and this resistance (Refer Slide Time: 24:47). So, the circuit is this (No audio from 24:54 to 25:08), we change the role, so the capacitance comes here and the resistance comes here, this is C, this is R, this is R F, this is R 1 and this is the load resistance R L and we take output here and input is connected here, this is v i and here this is v 1, this is v 2. So, this is our high pass filter, only difference is that we have interchanged this capacitance and resistance positions.



And then the analysis is same, similar type that this is the **volter** voltage divider network and at the non inverting input, the voltage at non inverting input, the voltage is  $v_1$  which again as we have done earlier in the previous case, this is  $j 2 \pi f R C$  by  $1 + j 2 \pi f R C$  into  $v_i$ . And  $v_0$  is equal to  $A_F$  by  $v_1$ , this is equal to gain is  $1 + R_F$  by  $R_1$  and this is  $v_1$ ,  $v_1$  we substitute from the above equation.

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Then the output becomes  $v_0$  is equal to  $1 + R_F R_1$  into  $j 2 \pi f R C$   $1 + j 2 \pi f R C$  into  $v_i$ . And which can be written in this form or  $v_0$  by  $v_i$  which is gain of the **of the** filter, this is equal to  $A_F$  and  $j f$  by  $f_L$   $1 + j f f_L$ , this is where  $A_F$  **A F** is  $1 + R_F$  by  $R_1$  and this is gain in the pass band, the we plot it and the graph is this (No audio from 29:02 to 29:13) (Refer Slide Time: 29:02).

This is gain and this is  $A_F$  and 3 d B down, this is  $f_L$  and this is 3 d B and this is  $0.707 A_F$ , this is frequency and this is the stop band, this is the pass band, this is pass band and this is from here to here, this is a stop band **a stop band**, this is the frequency characteristics, frequency response **frequency response** of high pass filter and here we have explained the parameters.

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Handwritten notes on a whiteboard:

- $A_F = \text{Gain in Pass band}$
- $f_L (= \frac{1}{2\pi RC})$  is the low cutoff freq. of the filter.
- $f = \text{freq. of input signal}$ .
- Magnitude of voltage gain.
- $$\left| \frac{V_o}{V_i} \right| = \frac{A_F (f/f_L)}{\sqrt{1 + (f/f_L)^2}}$$

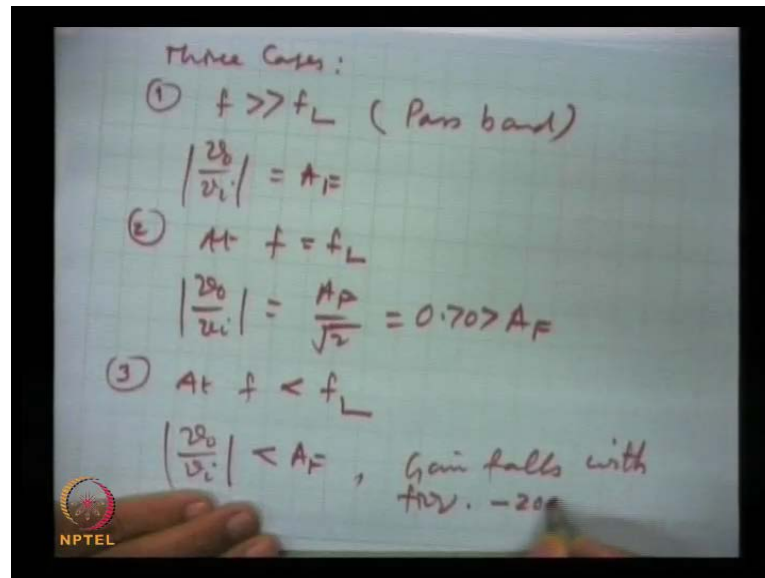
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Let me write them that here  $A_F$ , we have already talked,  $A_F$  is a gain in pass band **pass band** and  $f_L$  is equal to  $1 / 2\pi RC$  and is the low cutoff, low cutoff frequency of the filter and  $f$  is the frequency of input signal **input signal**.

That means, this frequency cutoff here (Refer Slide Time: 31:28), this we can choose what frequency we want depending on  $R$  and  $C$  in the circuit, this  $R$  and  $C$ ,  $R$  and  $C$  also decides the high cutoff in the low pass filter here also, but the interchanged values we have done. So, any way, so by choosing  $R$  and  $C$  in this network, we can choose the **the** cutoff and the magnitude can be written as magnitude of voltage gain, this is  $V_o / V_i$  is  $A_F f / f_L \sqrt{1 + (f/f_L)^2}$ , this is square root.

This is the gain and the frequency response we have already plotted and we can discuss in three different regions, this expression whether  $f$  is equal to  $f_L$  or lower than  $f_L$  or higher than  $f_L$  (Refer Slide time: 32:33), so three regions will be merged.

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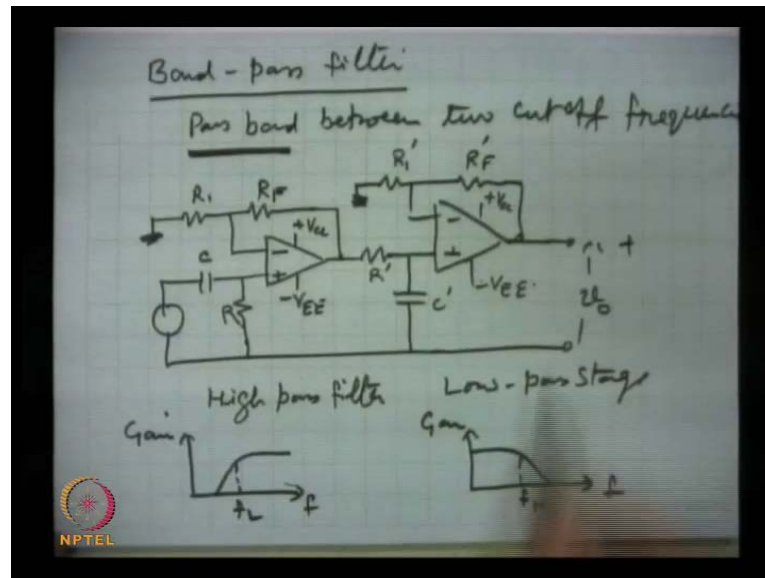
So, three cases, one is  $f$  very high as compared to  $f_L$ ; in this case, the magnitude of gain is equal to  $A_F$  as we have seen, this is this region pass band, so pass band.

Second case is at  $f$  equal to  $f_L$ , here  $v_o/v_i$  comes out to be  $A_F$  by root 2 which is 0.707  $A_F$ , this point where the gain has fallen from  $A_F$  to 0.707 of  $A_F$  and the  $f$  that is this (Refer Slide Time: 33:44).

And the third region is at  $f$  less than  $f_L$ , the gain, this continuously falls with frequency and gain falls with frequency at the rate of 20 dB per decade, this we have explained and again I repeat. That means, a frequency changes by 10 percent by 10 times, so if this frequency, for example is a 1 kilohertz, then at 100 hertz which is a 10 times smaller, the  $f$  gain will fall 1 tenth of  $A_F$  which comes out to be 20 dB per decade, so that is the meaning.

So, these are the basic two filters, low pass and high pass filter. By the combination of these, we can get the other two analysis remains the same, how to choose the cutoff frequencies in the low pass and high pass filters, those expressions are there in terms of  $R$  and  $C$  and let us see that, how we can get a band pass  $f$  filter (No audio from 35:29 to 35:37).

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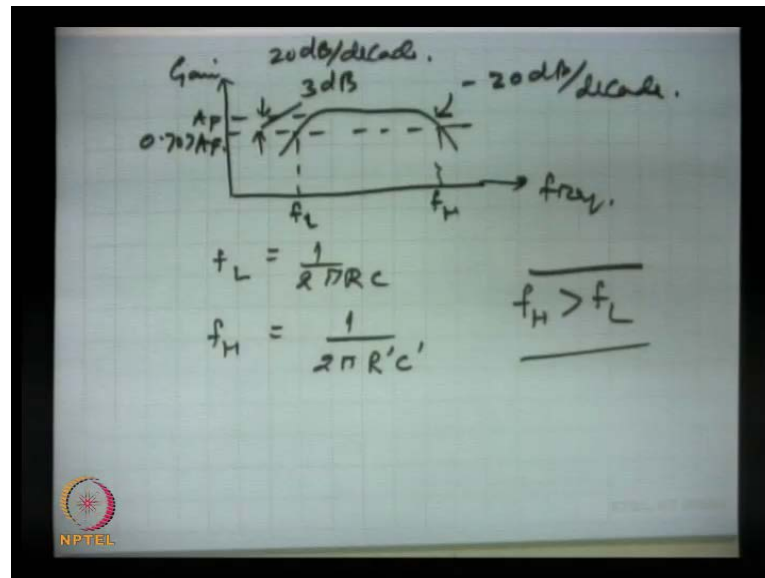
This band pass filter has a pass band between two cutoff frequencies **pass band between two cutoff frequencies between two cutoff frequencies**, any input frequency outside these two cutoff regions, this is highly attenuated while in the pass band, it goes without much attenuation and this can be achieved by cascading a high pass stage with another low pass stage.

So, the circuit is just what we have drawn, this two are cascaded, high pass first followed by a the next the low pass filter that is output is taken from low pass filter and input is given to the high pass filter.

So, the net circuit is this (No audio from 36:45 to 37:05) (Refer Slide time: 36:45), this is a C, this is R,  $R_1$ ,  $R_F$  plus V C C minus V E E and this is grounded and here (No audio from 37:26 to 37:32), this is R dash, C dash (No audio from 37:43 to 37:54), this is R 1 dash,  $R_F$  dash and output is taken here (No audio from 38:04 to 38:10), this is plus V C C minus V E E, this is the circuit we have just combined the two circuits.

And this is the high pass stage, high pass filter, this is the low pass, low pass circuit and you remember that for this high pass (Refer Slide time: 38:26), the response goes like this where this is  $f_L$  and this is the frequency, this is gain and for this, the response is like this where his is  $f_H$  frequency gain, this is for high pass filter, this is for low pass filter, we combine the two.

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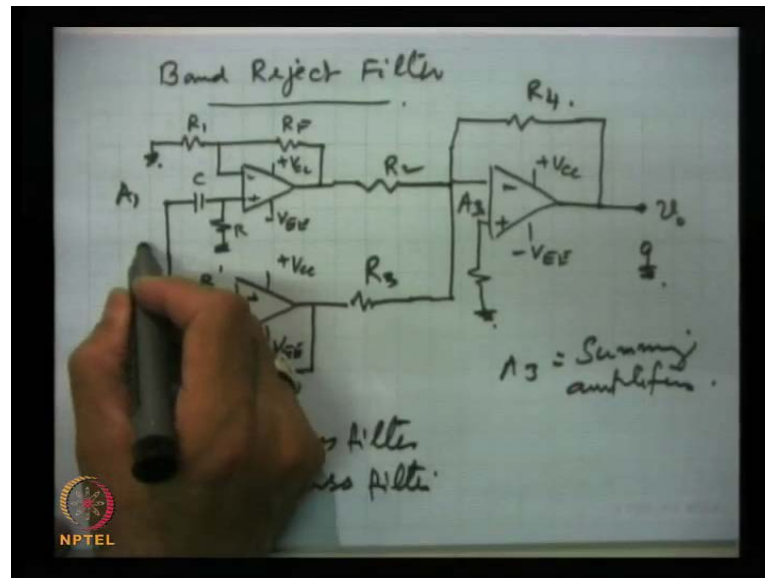


Then frequency response **frequency response** of the band pass filter becomes like this, this is the band pass filter, this gain is  $A_F$  gain and then 3 d B down, these are the cutoff frequencies  $f_L$  and  $f_H$ , this is frequency and this is 3 d B, this is  $0.707 A_F$  and these falls, this fall and this raise, rise with increase in frequency, fall within increase in frequency, these are 20 d B per decade. Similarly, here this rise will be 20 d B per decade, so this is the frequency response and here for the high pass filter, here we can choose the value of  $f_L$  given by the expression  $2\pi R C$  and this  $f_H$ ,  $f_H$  can be taken as  $2\pi R' C'$ .

And obviously, we have to take  $f_H$  much higher than  $f_L$ , this we should remember and so, this is the band pass filter **band pass filter** where in the pass band, gain will be  $A_F$  and then it falls 20 d B per decade on either side.

So, this is how a band pass filter is defined that between two cutoff it passes and beyond that (Refer Slide Time: 41:28), it **it** stops, this is a stop band, this is also a stop band, this is the pass band, this is pass band between  $f_L$  and  $f_H$  lies pass band **pass band** of the band pass filter, so this is all we wanted to talk about that.

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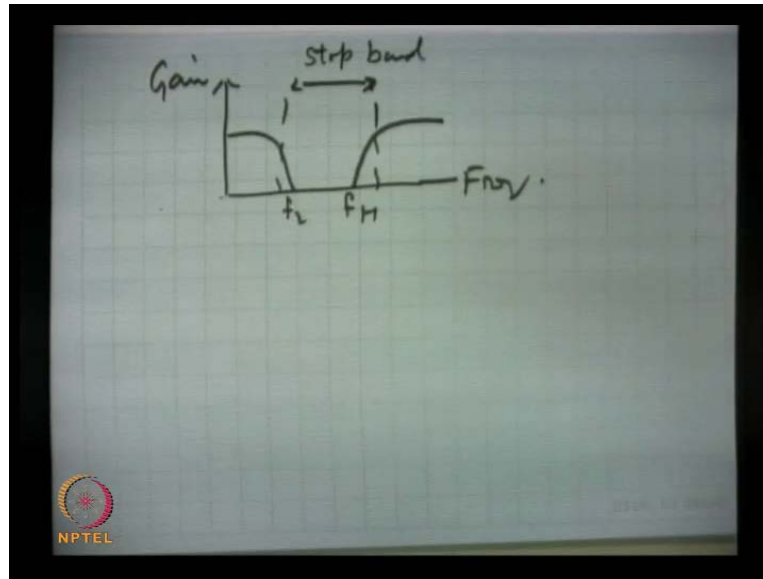


And next is band reject filter **band reject filter**, this is also called a band stop filter and it attenuates signals in the stop band **stop band**, we choose what is these stop band, it attenuates heavily at those frequencies and outside that band, the signal passes with little attenuation. So, that is band reject and band reject filter can be taken again by combining the effects of high pass, low pass and the output of these high pass and low pass is fed to a summing amplifier, so there are three op amps in this in the circuit is this (No audio from 43:02 to 43:11).

This is C, this is R and (No audio from 43:17 to 43:25), this is R 1, R F, this is grounded, here we attach, these are the power supplies V C C minus V E E and here is a another op amp and **and** this goes, this is R prime, C prime, this is plus and minus and from minus, we connect here, this is R 1 R F prime (Refer Slide time: 43:02). These two outputs are connected, R 2, R 3 (No audio from 44:43 to 45:29), there are 3 op amps which we are using the A 1, this is A 1 stage, this is A 2 and this is A 3 **A 3**, A 1 is high pass filter, this is A 2, this one, this is a low pass filter and this is A 3 is a summing amplifier.

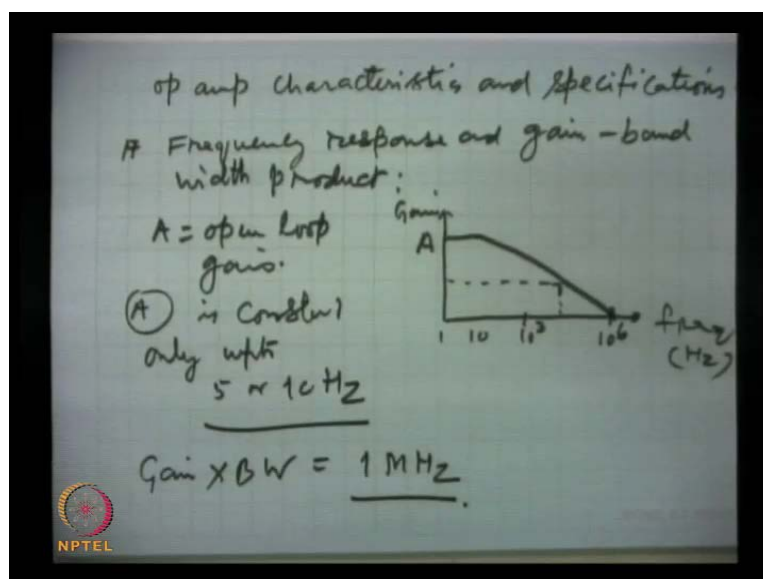
So, the low pass filter and the signal is connected here, this is v i and v out is taken from the summing amplifier. So, a low pass filter, high pass filter will give a output of this kind, so stop band or band reject filter.

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So, this is how this amplifier, this third A 3 this is a summing with that will take care that, when this is transmitting the first part, this will be available at the output, then the frequencies we have chosen that in between  $f_L$  and  $f_H$ , no signals will appear and then again the signal will appear from the high pass filter that is this one (Refer Slide Time: 47:00). So, this is how we get the band reject, this is pass band, this is also pass band and this band will be rejected. So, this is all about filters which we have taken and we will go for other things.

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The next thing is op amp characteristics and its specifications **op amp characteristics and its specifications its specification**, first we take frequency response and gain band width product **frequency response and gain band width product**, for a device the gain and normally we talk of voltage gain. So, voltage gain and the band width over which the gain **gain** is fixed is constant, this is limited, the product is a constant. And for a low for an open loop of op amp, because gain is very high, the frequency at which the gain remains constant is very low that is the response of an op amp is this, this is the open loop gain and then frequency, this falls and this is frequency on log scale (Refer Slide time: 49:25). So, 1, 10 this is hertz, but on the log scale and here this is 10 to power 3; that means, 1 kilo hertz and here is 1 mega hertz that is 10 to power 6 and this is gain.

Now,  $A$  is the open loop gain **open loop gain**, open loop gain is very high 10 to the power  $(\infty)$  and this the range over which it is constant,  $A$  is constant, gain is constant only up to 5 or 10 hertz **5 or 10 hertz** which is very a small band width, not of any practical use and this further tells you that why op amps are not used as an amplifier in open loop. We make feedback circuits and instead of 100000 gain, we get satisfied with gain of 50, 100, 200 that means some where here and then the **the** band width over which the gain is constant is very large.

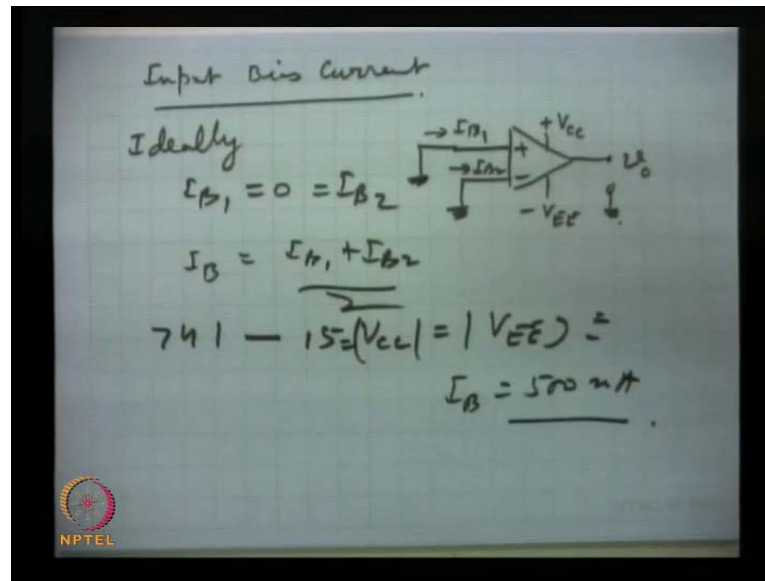
The gain band width product is defined as the **the** frequency at which gain falls to unity. I repeat, gain band width product is defined as the frequency at which the gain falls to unity. That means, here and for an op amp, this is around gain band width product, this is roughly 1 mega hertz.

So, in this region from say a few 100 hertz to a few several 100 or few 1000 coulombs, coulomb hertz frequencies can be used. So, here for example, if this is the gain, then we can use this much frequency band width which is very sizeable, this will be several tens or hundreds of kilo hertz (Refer Slide time: 51:52). So, this is the **the** frequency response and gain band width product **of the** of the amplifier.

And then we have some other input bias current, some other characteristics let us take that input, these are specifications provided by the manufacturer for the device.



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So, input bias current, here we take a op amp, we supply these voltages plus  $V_{CC}$  and minus  $V_{EE}$  and both the inputs, the non inverting and inverting they are grounded; that means, no external circuit is, no external voltage is connected, then this current  $I_{B1}$  and this is  $I_{B2}$  with end output  $v_0$  with respect to ground.

When we do not attach any external voltages to inverting and non inverting inputs, then we expect no waste currents flowing, because you remember that these are the currents which are going in the **two base base** two base of the two transistors which form the differential amplifier which is the first stage in the op amp.

So, ideally **ideally**  $I_{B1}$  is 0 and so is  $I_{B2}$ , but in practice this is not 0, in spite of the fact that we have not attached any **any** external voltage to these inputs, there is some finite current and this is given by the manufacturer. And why this arises? This arises from the miss match, **imperfect** imperfection in the in the total symmetry of the two transistors  $Q_1$  and  $Q_2$  of the differential, first stage differential amplifier and this is provided by the manufacturer as the average  $I_{B1}$  plus  $I_{B2}$  by 2, this is provided by the manufacturer and for 741 at 15  $V_{CC}$  magnitude and  $V_{EE}$ , this is equal to 500 nanoamperes,  $I_B$  is 500 nanoamperes.

And so, this is to be taken care of we will talk about that and few other specifications, we will talk. So, in this course, one more lecture remains that will be tomorrow will be the final lecture where we will finish this module seven completely.